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20 Years (2024–2005) Solved Exam Papers

12th
CBSE
2025



CBSE 2025

PHYSICS

Including Case Based Questions

CLASS 12

Chapter-wise Question Bank
Based on Previous 20 Years 63 Papers

NODIA AND COMPANY

CBSE Physics Question Bank Class 12
Edition July 2024
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ISBN : 978-9384843243
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MRP Rs 699.00

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Published by :
NODIA AND COMPANY
125, Sector 6, Vidyadhar Nagar, Jaipur 302039
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Exam 2024 Solved Paper

Class XII 2023-24

Physics

Time: 3 Hours

Max. Marks: 80

General Instructions:

1. This question paper consists of 33 questions in 5 sections.
2. All questions are compulsory. However, an internal choice is provided in some questions. A student is expected to attempt only one of these questions.
3. This question paper is divided into five sections Sections A, B, C, D and E.
4. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
5. In Section B Questions no. 17 to 21 are Very Short Answer type questions. Each question carries 2 marks.
6. In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
7. In Section D Questions no. 29 and 30 are case study-based questions. Each question carries 4 marks.
8. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.

SECTION-A

1. An ammeter and a voltmeter are connected in series to a battery. Their readings are noted as A and V respectively. If a resistor is connected in parallel with the voltmeter, then
 - (a) A will increase, V will decrease.
 - (b) A will decrease, V will increase.
 - (c) Both A and V will decrease.
 - (d) Both A and V will increase.

Ans :

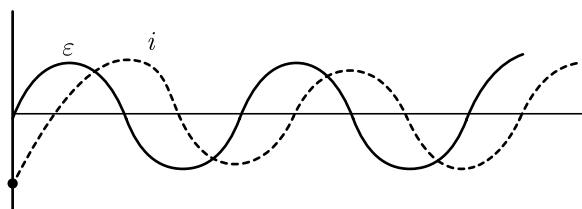
When the resistor is connected in parallel with the voltmeter then the equivalent resistance of the circuit decreases hence the current flowing through circuit increases. As a result of increase in the current through circuit the ammeter reading increases while the reading of voltmeter decreases as potential difference across it decreases.

Thus option (a) is correct

2. An ac voltage is applied across an ideal inductor. The current in it
 - (a) Leads the voltage by $\left(\frac{1}{4}\right)$ cycle.
 - (b) Lags the voltage by $\left(\frac{1}{4}\right)$ cycle.
 - (c) Leads the voltage by $\left(\frac{1}{4}\right)$ cycle.
 - (d) Lags the voltage by $\left(\frac{1}{4}\right)$ cycle.

Ans :

Let the A.C voltage applied across inductor be $\varepsilon = \varepsilon_0 \sin(\omega t)$



Current through inductor, $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

i.e., current lags by a phase difference of $\frac{\pi}{2}$

hence it lags the voltage by $\frac{1}{4}$ cycle

Thus option (b) is correct

3. An iron needle is kept near a strong bar magnet. It will experience
 - (a) A force of attraction and no torque.
 - (b) A force of attraction and a torque.
 - (c) A torque and no force.
 - (d) Neither a force nor a torque.

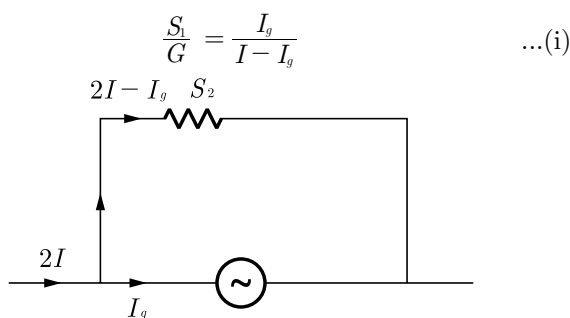
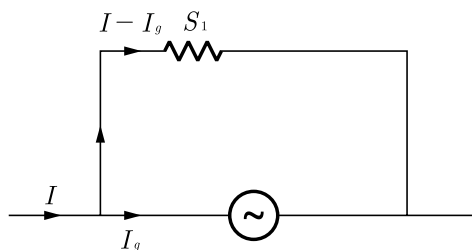
Ans :

The iron needle experiences a non-uniform field due to a bar magnet. There is induced magnetic moment in the nail hence it experiences an attractive force. Due to presence of magnetic moment, it will experience a net torque also.

Thus option (b) is correct

4. A galvanometer shows full scale deflection for a current I_g . If a shunt of resistance S_1 is connected to the galvanometer, it gets converted into an ammeter of range $(0 - I)$. When resistance of the shunt is made, S_2 its range becomes $(0 - 2I)$. Then $\left(\frac{S_1}{S_2}\right)$ is

- (a) $\frac{I+I_g}{I-I_g}$ (b) $\frac{I-I_g}{I+I_g}$
 (c) $\frac{2I-I_g}{I-I_g}$ (d) $\frac{I-I_g}{2I-I_g}$

Ans :

$$\frac{S_1}{G} = \frac{I_g}{I-I_g} \quad \dots(i)$$

$$\frac{S_2}{G} = \frac{I_g}{2I-I_g}$$

From equations (1) and (2)

$$\frac{S_1}{S_2} = \frac{2I-I_g}{I-I_g}$$

Thus option (c) is correct

5. A coil of area of cross-section 0.5 m^2 is placed in a magnetic field acting normally to its plane. The field varies as $B = 0.5t^2 + 2t$, where B is in tesla and tin seconds. The emf induced in the coil at $t = 1 \text{ s}$ is
 (a) 0.5 V (b) 1.0 V
 (c) 1.5 V (d) 3.0 V

Ans :magnetic flux $\phi = BA \cos 0^\circ$

$$\phi = B \times A = (0.5t^2 + 2t)(0.5)$$

$$\phi = 0.25t^2 + t,$$

$$\text{Now, } \varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(0.25t^2 + t)$$

$$\varepsilon = -[0.5t + 1]$$

$$\varepsilon_{at t=1} = -(0.5 \times 1 + 1) = 1.5 \text{ V}$$

This option (c) is correct

6. A pure Si crystal having $5 \times 10^{28} \text{ atoms m}^{-3}$ is doped with 1 ppm concentration of antimony. If the concentration of holes in the doped crystal is found to be $4.5 \times 10^9 \text{ m}^{-3}$, the concentration (in m^{-3}) of intrinsic charge carriers in Si crystal is about

- (a) 1.2×10^{15} (b) 1.5×10^{16}
 (c) 3.0×10^{15} (d) 2.0×10^{18}

Ans :

Number of atoms in pure Si crystal

$$= 5 \times 10^{28} \text{ atoms m}^{-3}$$

$$\text{Number of electrons, } n_e = \frac{5 \times 10^{28}}{10^8}$$

$$n_e = 5 \times 10^{22} \text{ m}^{-3}$$

$$\text{Hole concentration} = 4.5 \times 10^9 \text{ m}^{-3}$$

We know that,

$$n_i^2 = n_e \times n_h$$

$$= 5 \times 10^{22} \times 4.5 \times 10^9$$

$$n_i^2 = 22.5 \times 10^{31} = 225 \times 10^{30}$$

$$n = 15 \times 10^{15} = 1.5 \times 10^{16}$$

This option (b) is correct

7. The potential energy between two nucleons inside a nucleus is minimum at a distance of about
 (a) 0.8 fm (b) 1.6 fm
 (c) 2.0 fm (d) 2.8 fm

Ans :The potential energy is minimum at a distance r_0 of about 0.8 fm.

Thus option (a) is correct

8. In a Young's double-slit experiment in air, the fringe width is found to be 0.44 mm. If the entire setup is immersed in water ($n = \frac{4}{3}$), the fringe width will be
 (a) 0.88 mm (b) 0.59 mm
 (c) 0.33 mm (d) 0.44 mm

Ans :

$$\beta = \frac{\lambda D}{d} \quad \dots(i)$$

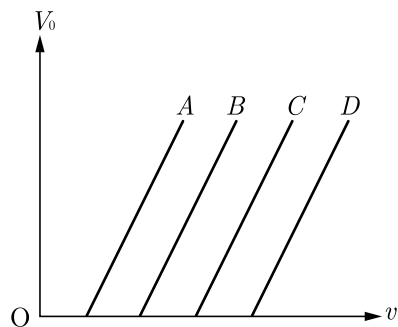
When the whole setup is immersed in water then

$$\lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{\frac{4}{3}} = \frac{3\lambda}{4}$$

$$\beta' = \frac{\lambda' D}{d} = \frac{3\lambda D}{4d} = \frac{3}{4} \times \beta = \frac{3}{4} \times 0.44 = 0.33 \text{ mm}$$

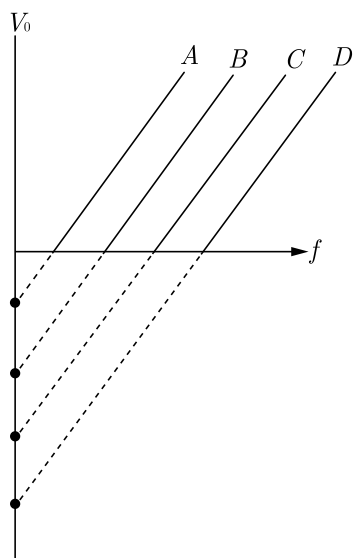
Thus option (c) is correct

9. The variation of the stopping potential (V_0) with the frequency (ν) of the incident radiation for four metals A, B, C and D is shown in the figure. For the same frequency of incident radiation producing photo-electrons in all metals, the kinetic energy of photo-electrons will be maximum for metal



- (a) A (b) B
(c) C (d) D

Ans :



As per the variation shown above, for the same frequency of incident radiation the work function for metal A is minimum hence the kinetic energy of photo electrons will be maximum for metal A .
This option (a) is correct

10. The energy of an electron in the ground state of hydrogen atom is -13.6 eV . The kinetic and potential energy of the electron in the first excited state will be
- (a) $-13.6 \text{ eV}, 27.2 \text{ eV}$
(b) $-6.8 \text{ eV}, 13.6 \text{ eV}$
(c) $3.4 \text{ eV}, -6.8 \text{ eV}$
(d) $6.8 \text{ eV}, -3.4 \text{ eV}$

Ans :

$$(T.E)_{\text{ground state}} = 13.6 \text{ eV}$$

$$(T.E)_{\text{first excited state}} = \frac{-13.6}{4} = -3.4 \text{ eV}$$

$$(P.E)_{\text{first excited state}} = 3.4 \times 2 = -6.8 \text{ eV}$$

$$(K.E)_{\text{first excited state}} = -(-3.4 \text{ eV}) = 3.4 \text{ eV}$$

Thus option (c) is correct

11. The electromagnetic waves used to purify water are
- (a) Infrared rays (b) Ultraviolet rays
(c) X-rays (d) Gamma rays

Ans :

Ultraviolet rays are used to kill germs in water purifiers.

Thus option (b) is correct

12. The focal lengths of the objective and the eyepiece of a compound microscope are 1 cm and 2 cm respectively. If the tube length of the microscope is 10 cm , the magnification obtained by the microscope for most suitable viewing by relaxed eye is :
- (a) 250 (b) 200
(c) 150 (d) 125

Ans :

When the image is formed at infinity,

$$m = \frac{L}{f_o} \left[\frac{D}{f_e} \right]$$

$$m = \frac{10}{1} \times \frac{25}{2} = 125$$

Thus option (d) is correct

For Questions 13 to 16, two statements are given - one labelled Assertion (A) and other labelled Reason (R). Select the correct answer to these questions from the options as given below.

- (a) If both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(b) If both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) If Assertion (A) is true but Reason (R) is false.
(d) If both Assertion (A) and Reason (R) are false.
13. **Assertion (A) :** An alpha particle is moving towards a gold nucleus. The impact parameter is maximum for the scattering angle of 180° .

Reason (R) : The impact parameter in an alpha particle scattering experiment does not depend upon the atomic number of the target nucleus.

Ans :

When $\theta = 180^\circ$, then the impact parameter would be zero.

$$b = \frac{ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi\epsilon_0 E}$$

Impact parameter is directly proportional to atomic number of target nucleus.

Hence both assertion and reason are incorrect.

Thus option (d) is correct

14. **Assertion (A) :** In a Young's double-slit experiment, interference pattern is not observed when two coherent sources are infinitely close to each other.

Reason (R) : The fringe width is proportional to the separation between the two sources.

Ans :

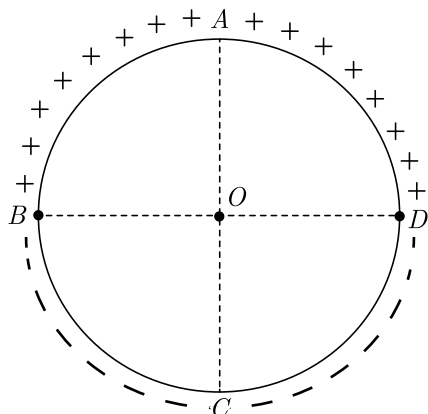
$\beta = \frac{\lambda D}{d}$ as $d \rightarrow 0, \beta \rightarrow \infty \Rightarrow$ since the fringe width is tending to infinity hence when two sources are placed infinitely close then interference pattern would not be observed. Hence assertion is correct.

$$\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \frac{1}{d} \Rightarrow \text{Reason incorrect.}$$

Thus option (c) is correct

15. **Assertion (A) :** Equal amount of positive and negative charges are distributed uniformly on two halves of a thin circular ring as shown in figure. The resultant electric field at the centre O of the ring is along OC.

Reason (R) : It is so because the net potential at O is not zero.



Ans :

The net field at point O is along OC due to vector sum of all the fields. Hence assertion is correct.

The potential at centre of ring will be zero.

Hence reason is incorrect.

Thus option (c) is correct

16. **Assertion (A) :** The energy of a charged particle moving in a magnetic field does not change.

Reason (R) : It is because the work done by the magnetic force on the charge moving in a magnetic field is zero.

Ans :

Yes, the energy of the moving charged particle in a magnetic field does not change as no work is done by the magnetic force on the charged particle. Hence both assertion and reason are correct and reason is correct explanation.

Thus option (a) is correct

SECTION-B

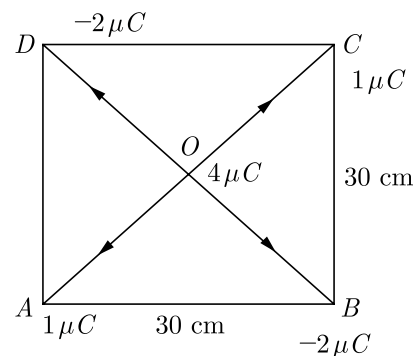
17. (a) Four-point charges of $1\alpha C, -2C, \alpha C, 1\alpha C$ and $-2\mu C$ are placed at the corners A, B, C and D respectively, of a square of side 30 cm. Find the net force acting on a charge of $4\mu C$ placed at the centre of the square.

or

- (b) Three-point charges, 1 pC each, are kept at the vertices of an equilateral triangle of side 10 cm. Find the net electric field at the centroid of triangle.

Ans :

(a)



$$\vec{F}_{Oa} = \frac{R \times 1 \times 4 \times 10^{-12}}{\left(\frac{0.3\sqrt{2}}{2}\right)}$$

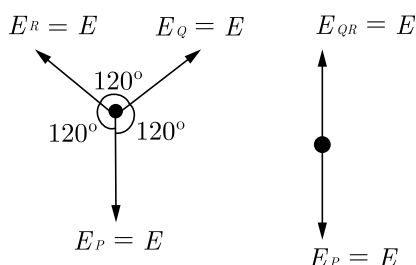
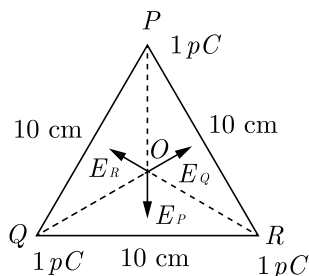
$$\vec{F}_{OC} = -\vec{F}_{OA}$$

$$\vec{F}_{OB} = \frac{R \times 4 \times 2 \times 10^{-12}}{\left(\frac{0.3\sqrt{2}}{2}\right)}$$

$$\vec{F}_{OB} = -\vec{F}_{OB}$$

Net force on $4 \mu\text{C}$ due to charges at corners is zero.

(b)



Net electric field at the centroid of equilateral triangle will be zero.

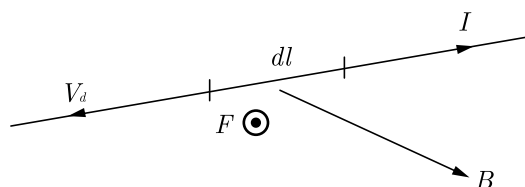
$$\vec{E}_{\text{net}} = 0$$

Thus option (a) is correct

18. Derive an expression for magnetic force $\vec{F}$ acting on a straight conductor of length L carrying current I in an external magnetic field $\vec{B}$. Is it valid when the conductor is in zig-zag form? Justify.

Ans :

Consider a conducting wire, carrying a current I is placed in a magnetic field B . Consider a small element dl of the wire. The free electrons drift with a speed V_d opposite to the direction of the current. The relation between the current I and the drift speed V_d is



$$I = jA = nev_d A \quad \dots(i)$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average magnetic force

$$\vec{f}_m = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered is $nAdl$. Thus, the magnetic force on the wire of length dl is

$$d\vec{F} = (nAdl)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $d\ell$, the above equation becomes

$$d\vec{F} = nAev_d d\ell \times \vec{B}$$

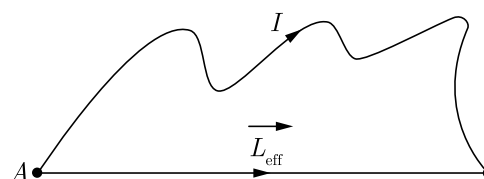
Using (i)

$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$

The quantity $I d\vec{\ell}$ is called a current element.

If a straight wire of length ℓ carrying a current I is placed in a uniform magnetic field $\vec{B}$, the force on it is

$$\vec{F} = I\vec{\ell} \times \vec{B}$$



Yes, it is valid for zig-zag wire as,

$$\vec{F}_m = I(\vec{L}_{\text{eff}} \times \vec{B})$$

For zig-zag wire effective length of wire is considered between initial and final point (L_{eff} shortest distance between initial and final point).

19. The radius of curvature of a convex mirror is 30 cm. It forms an image of an object which is half the size of the object. Find the separation between the object and the image.

Ans :

$$R = 2f$$

Here

$$R = 30 \text{ cm}$$

$$30 = 2f$$

$$f = 15 \text{ cm}$$

from mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{v}{15} = 1 + \frac{v}{u} \quad \dots(i)$$

Given

$$\frac{h_i}{h_o} = \frac{1}{2}$$

$$\frac{v}{u} = \frac{-1}{2}$$

from eq. (i)

$$\frac{v}{15} = 1 - \frac{1}{2}$$

$$\frac{v}{15} = \frac{1}{2}$$

$$2v = 15$$

$$v = \frac{15}{2} \text{ cm}$$

Now

$$u = -v \times 2$$

$$= -\frac{15}{2} \times 2$$

$$u = -15 \text{ cm}$$

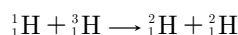
So, separation

$$d = 15 + \frac{15}{2}$$

$$= \frac{30 + 15}{2}$$

$$= \frac{45}{2} \text{ cm}$$

20. Calculate the energy released/absorbed (in MeV) in the nuclear reaction:

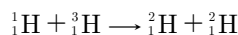


Given: $m({}^1_1\text{H}) = 1.007825 \text{ u}$

$$m({}^2_1\text{H}) = 2.014102 \text{ u}$$

$$m({}^3_1\text{H}) = 3.016049 \text{ u}$$

Ans :



$$\text{Mass of products} = 2(2.014102) \text{ u}$$

$$= 4.028204 \text{ u}$$

$$\text{Mass of reactants} = (3.016049 + 1.007825) \text{ u}$$

$$= 4.023874 \text{ u}$$

$$\text{Mass of products} > \text{Mass of reactants}$$

So, energy is absorbed.

$$\Delta m = 0.00433$$

$$E_{\text{abs}} = 0.00433 \times 931 \text{ MeV}$$

$$= 4.03123 \text{ MeV}$$

21. A proton of energy 1.6 MeV approaches a gold nucleus ($Z = 79$). Find the distance of its closest approach.

Ans :

Distance of closest approach

$$r = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{K.E}$$

Here,

$$K.E = 1.6 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 2.56 \times 10^{-13} \text{ J}$$

$$r = \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{2.56 \times 10^{-13}}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{2.56 \times 10^{-13}}$$

$$= \frac{9 \times 10^9 \times 79 \times 1.6 \times 1.6 \times 10^{-38}}{2.56 \times 10^{-13}}$$

$$= 9 \times 79 \times 10^{-16}$$

$$= 711 \times 10^{-16} \text{ m}$$

$$R \approx 71.1 \text{ fm}$$

SECTION-C

22. A photosensitive surface of work function 2.1 eV is irradiated by radiation of wavelength 150 nm. Calculate (i) the threshold wavelength, (ii) energy (in eV) of an incident photon, and (iii) maximum kinetic energy of emitted photoelectron.

Ans :

Given,

- (i) Threshold wavelength :

$$\phi_0 = 2.1 \text{ eV}$$

$$E = \frac{12400}{\lambda}$$

$$\lambda = \frac{12400}{2.1} = 5904.7 \text{ Å}$$

- (ii) Energy of an incident photon

$$\lambda = 150 \text{ nm} = 1500 \text{ Å}$$

Energy of photon

$$E = \frac{12400}{\lambda(\text{Å})} \text{ eV}$$

$$E = \frac{12400}{1500} \text{ eV} = 8.26 \text{ eV}$$

- (iii) Einstein's photoelectric equation:

$$E = \phi_0 + KE_{\text{max}}$$

$$KE_{\text{max}} = E - \phi_0$$

$$= 8.26 - 2.1$$

$$KE_{\text{max}} = 6.16 \text{ eV}$$

23. (a) (i) State Lenz's Law. In a closed circuit, the induced current opposes the change in magnetic flux that produced it as per the law of conservation of energy. Justify.

- (ii) A metal rod of length 2 m is rotated with a frequency 60 rev/s about an axis passing through its centre and perpendicular to its length. A uniform magnetic field of 2T perpendicular to its plane of rotation is switched-on in the region. Calculate the e.m.f. induced between the centre and the end of the rod.

or

- (b) (i) State and explain Ampere's circuital law.
 (ii) Two long straight parallel wires separated by 20 cm, carry 5 A and 10 A current respectively, in the same direction. Find the magnitude and direction of the net magnetic field at a point midway between them.

Ans :

- (a) (i) Lenz's law states that the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes change in the initial magnetic field.

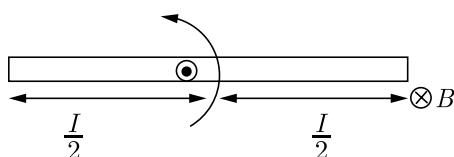
In a closed circuit, there will be extra effort to work against opposing force. This extra effort is transformed into electrical energy which satisfy the conservation of energy.

- (ii) Given $l = 2\text{m}$,

$$\omega = 60 \times 2\pi \text{ rad/s} = 120\pi \text{ rad/s}$$

$$B = 2\text{ T}$$

Emf between the centre and the end of the rod is given by



$$\varepsilon = \frac{B\omega r^2}{2}$$

$$\varepsilon = \frac{2 \times 120\pi \times (1)^2}{2} = 120\pi \text{ volt}$$

or

- (b) (i) Ampere's circuital law-Ampere's law states that the path integral or line integral over a closed loop in any magnetic field B produced by a current distribution is given by

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where I refers to the current enclosed by the loop.

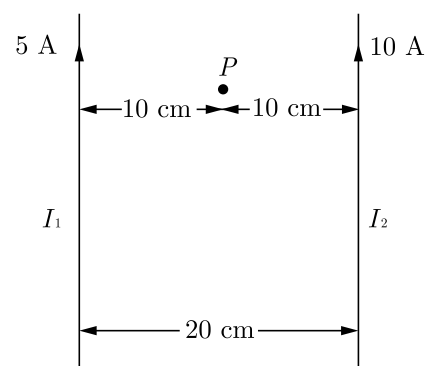
If B is directed along the tangent to every point on the perimeter L of a closed curve and is constant in magnitude along perimeter then

$$BL = \mu_0 I_e$$

where I_e is the net current enclosed by the closed circuit.

- (ii) Net magnetic field at point P

$$B_{\text{net}} = B_{\text{due to } I_1} + B_{\text{due to } I_2}$$



$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} = \frac{\mu_0}{4\pi} \frac{2 \times 5}{10^{-1}}$$

(perpendicularly downward)

$$B_2 = \frac{\mu_0}{4\pi} \frac{2I_2}{r} = \frac{\mu_0}{4\pi} \frac{2 \times 10}{10^{-1}}$$

(perpendicularly upward)

Hence,

$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{2}{10^{-1}} (10 - 5)$$

(perpendicularly upward)

$$= \frac{\mu_0}{4\pi} \times \frac{2 \times 5}{10^{-1}}$$

$$= 10^{-7} \times \frac{10}{10^{-1}} = 10^{-5} \text{ T}$$

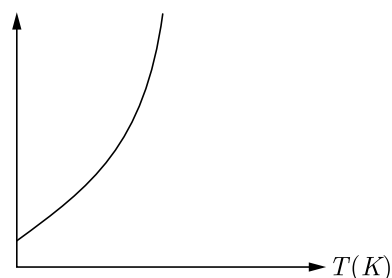
24. (i) Define 'temperature coefficient of resistance' of a metal.
 (ii) Show the variation of resistivity of copper with rise in temperature.
 (iii) The resistance of a wire is $10\ \Omega$ at 27°C . Find its resistance at -73°C . The temperature coefficient of resistance of the material of the wire is $1.70 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$.

Ans :

- (i) Temperature coefficient of resistance is defined as resistance change factor per degree celsius of temperature change.

$$\alpha = \frac{\Delta R}{R_0 \Delta T}$$

- (ii) Variation of resistivity of copper with temperature



- (iii) Given,

$$R_1 = 10\Omega \text{ at } T_1 = 27^\circ\text{C}$$

$$\alpha = 1.7 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$$

$$R_2 = ? \text{ at } T_2 = -73^\circ\text{C}$$

$$R_2 = R_1(1 + \alpha\Delta T)$$

$$R_2 = 10[1 + 1.7 \times 10^{-4}(-73 - 27)]$$

$$R_2 = 10[1 - 1.7 \times 10^{-4} \times 100]$$

$$R_2 = 10[1 - .017]$$

$$= 10 \times 0.983 = 9.83 \Omega$$

25. Name the part of the electromagnetic spectrum which are

- stopped by face mask worn by welders.
 - used in detectors in Earth satellites.
 - used in 'short-wave band' in communication.
- Also write the order of wavelengths, in each case.

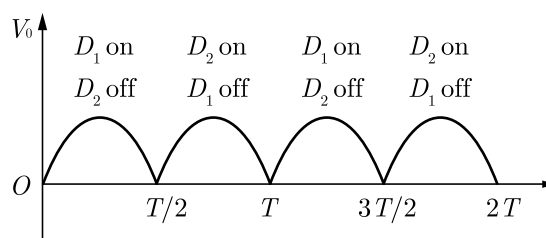
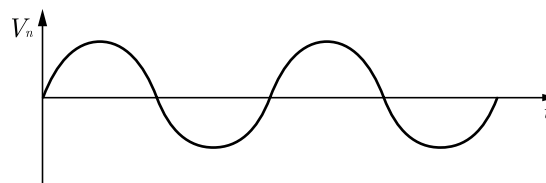
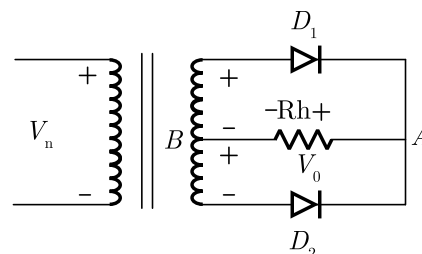
Ans :

- UV rays produced by welding arcs are stopped by face mask worn by welders. $1\text{nm} < \lambda_{\text{uv}} < 400\text{ nm}$
- Infrared detectors are used in Earth satellites. $700\text{ nm} < \lambda_{\text{IR}} < 1\text{ mm}$
- Radio waves are used in short wave band communication. $\lambda_R > 0.1\text{ m}$

26. (a) Explain the characteristics of a p-n junction diode that makes it suitable for its use as a rectifier.
(b) With the help of a circuit diagram, explain the working of a full wave rectifier.

Ans :

- An ideal $p-n$ junction diode is conducting or act as short circuit. When forward bias is applied and it becomes open at reverse bias. It acts as switch. So this allow current to pass in one direction and block in opposite direction
- Full wave rectifier : It convert A.C. current to D.C. current



During positive cycle diode D_1 is on and D_2 is off. The current will flow through load resistance (R_L) from A to B . For negative cycle diode D_2 is on and D_1 is off but direction of current through load resistance (R_L) is still from A to B . The output is always in same direction irrespective direction of input.

27. Explain the following giving reasons:

- A doped semiconductor is electrically neutral.
- In a p-n junction under equilibrium, there is no net current.
- In a diode, the reverse current is practically not dependent on the applied voltage.

Ans :

- In a doped semiconductor the total positive charge is equal to total negative charge so it is electrically neutral.
- Under equilibrium condition the diffusion current is equal to the drift current in magnitude and they are in opposite direction. So net current is zero.
- Under reverse bias, drift current is very small because it is due to the flow of minority charge carrier. Increasing voltage does increase the minority carrier density so current remains constant with respect to applied voltage.

28. An electron moving with a velocity $\vec{v} = (1.0 \times 10^7 \text{ m/s}) \hat{i} + (0.5 \times 10^7 \text{ m/s}) \hat{j}$ enters a region of uniform magnetic field $\vec{B} = (0.5 \text{ mT}) \hat{j}$. Find the radius of the circular path described by it. While rotating does the electron trace a linear path too? If so, calculate the linear distance covered by it during the period of one revolution.

Ans :

Given,

$$\vec{v} = (1.0 \times 10^7 \hat{i}) + (0.5 \times 10^7 \hat{j})$$

$$\vec{B} = (0.5 \times 10^{-3}) \text{ tesla}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$q_e = 1.6 \times 10^{-19} \text{ C}$$

Radius of path,

$$R = \frac{mv}{qB}$$

Where $v_{\perp}$ is component of velocity perpendicular to magnetic field.

$$R = \frac{9.1 \times 10^{-31} \times 1 \times 10^7}{1.6 \times 10^{-19} \times 0.5 \times 10^{-3}}$$

$$= \frac{9.1 \times 10^{-31+7+22}}{0.8}$$

$$R = 11.375 \times 10^{-2} \text{ m} = 11.375 \text{ cm}$$

Since the electron has perpendicular and parallel components of velocity, it will move on helical path, hence it will trace a linear path too virtually. The linear distance covered by it during one period of revolution is equal to pitch (P)

$$P = v_{\parallel} \times T \quad \left\{ T = \frac{2\pi m}{qB} \right\}$$

$$P = \frac{0.5 \times 10^7 \times 2\pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.5 \times 10^{-3}}$$

$$= \frac{18.2\pi \times 10^{-24}}{1.6 \times 10^{-22}}$$

$$= 11.375\pi \times 10^{-2} \text{ m or } 11.375\pi \text{ cm}$$

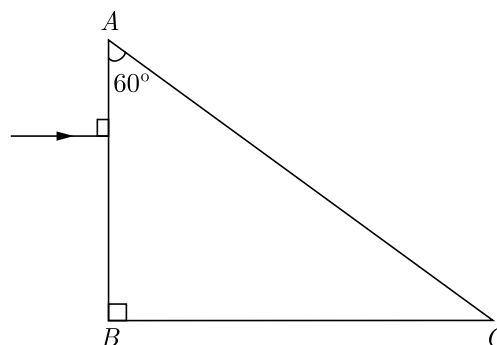
Section C

29. A prism is an optical medium bounded by three refracting plane surfaces. A ray of light suffers successive refractions on passing through its two surfaces and deviates by a certain angle from its original path. The refractive index of the material of the prism is given by $\mu = \frac{\sin(\frac{A+\delta_m}{2})}{\sin \frac{A}{2}}$. If the angle of incidence on the second surface is greater than an angle called critical angle, the ray will not be refracted from the second surface and is totally internally reflected.

- (i) The critical angle for glass is θ_1 and that for water is θ_2 . The critical angle for glass-water surface would be (given $n_g = 1.5, n_w = 1.33$)
- (a) less than θ_2 (b) between θ_1 and θ_2
 (c) greater than θ_2 (d) less than θ_1
- (ii) When a ray of light of wavelength λ and frequency ν is refracted into a denser medium
- (a) λ and ν both increase.
 (b) λ increases but ν is unchanged.
 (c) λ decreases but ν is unchanged.
 (d) λ and ν both decrease.
- (iii) (a) The critical angle for a ray of light passing from glass to water is minimum for
- (a) red colour (b) blue colour
 (c) yellow colour (d) violet colour

or

- (b) Three beams of red, yellow and violet colours are passed through a prism, one by one under the same condition. When the prism is in the position of minimum deviation, the angles of refraction from the second surface are r_R, r_Y and r_V respectively. Then
- (a) $r_V < r_Y < r_R$ (b) $r_Y < r_R < r_V$
 (c) $r_R < r_Y < r_V$ (d) $r_R < r_Y < r_V$
- (iv) A ray of light is incident normally on a prism ABC of refractive index $\sqrt{2}$ as shown in figure. After it strikes face AC, it will



- (a) go straight undeviated
 (b) refract and go out of the prism
 (c) just graze along the face AC
 (d) undergo total internal reflection

Ans :

(i) $\sin \theta_c = \frac{1}{\mu_g} = \frac{2}{3}$

$$\theta_c = \sin^{-1}\left(\frac{2}{3}\right)$$

$$\sin \theta_2 = \frac{1}{\mu_w} = \frac{3}{4}$$

$$\theta_2 = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\sin \theta_3 = \frac{\alpha_w}{\alpha_g} = \frac{4 \times 2}{3 \times 3} = \frac{8}{9}$$

$$\theta_3 = \sin^{-1}\left(\frac{8}{9}\right)$$

Thus option (c) is correct

- (ii) v = frequency remain unchanged for light ray in different medium.

$$\lambda' = \frac{\lambda}{\mu}$$

Thus option (c) is correct

- (iii) (a)

$$\sin \theta_c = \frac{\alpha_w}{\alpha_g} = \frac{1}{\alpha}$$

Refractive index of glass water is maximum for violet colour. So, critical angle is minimum for violet colour.

Thus option (d) is correct

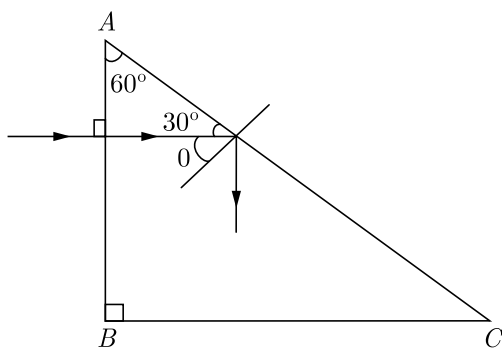
or

- (b) For second surface, angle of emergence is angle of refraction.

So $r_v > r_y > r_r$

Thus option (c) is correct

- (iv)



$$\theta = 60^\circ$$

$$\sin \theta_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$$

$$\theta_c = 45^\circ$$

$\theta > \theta_c$, so the ray will undergo total internal reflection at AC.

Thus option (d) is correct

30. Dielectrics play an important role in design of capacitors. The molecules of a dielectric may be polar or non-polar. When a dielectric slab is placed

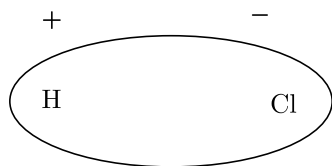
in an external electric field, opposite charges appear on the two surfaces of the slab perpendicular to electric field. Due to this an electric field is established inside the dielectric.

The capacitance of a capacitor is determined by the dielectric constant of the material that fills the space between the plates. Consequently, the energy storage capacity of a capacitor is also affected. Like resistors, capacitors can also be arranged in series and/or parallel.

- (i) Which of the following is a polar molecule?
 (a) O_2 (b) H_2
 (c) N_2 (d) HCl
- (ii) Which of the following statements about dielectrics is correct?
 (a) A polar dielectric has a net dipole moment in absence of an external electric field which gets modified due to the induced dipoles.
 (b) The net dipole moments of induced dipoles is along the direction of the applied electric field.
 (c) Dielectrics contain free charges.
 (d) The electric field produced due to induced surface charges inside a dielectric is along the external electric field.
- (iii) When a dielectric slab is inserted between the plates of an isolated charged capacitor, the energy stored in it:
 (a) Increases and the electric field inside it also increases.
 (b) Decreases and the electric field also decreases.
 (c) Decreases and the electric field increases.
 (d) Increases and the electric field decreases.
- (iv) (a) An air-filled capacitor with plate area A and plate separation d has capacitance C_0 . A slab of dielectric constant K , area A and thickness $\left(\frac{d}{5}\right)$ is inserted between the plates. The capacitance of the capacitor will become
 (a) $\left[\frac{4K}{5K+1}\right]C_0$ (b) $\left[\frac{K+5}{4}\right]C_0$
 (c) $\left[\frac{5K}{4K+1}\right]C_0$ (d) $\left[\frac{K+4}{5K}\right]C_0$
- (b) Two capacitors of capacitances $2C_0$ and $6C_0$ are first connected in series and then in parallel across the same battery. The ratio of energies stored in series combination to that in parallel is
 (a) $\frac{1}{4}$ (b) $\frac{1}{6}$
 (c) $\frac{2}{15}$ (d) $\frac{3}{16}$

Ans :

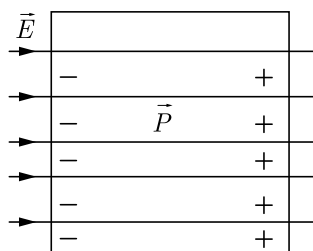
- (i) HCl is a polar molecule



because its one end is positive, other end is negative.

Thus option (d) is correct

- (ii) The net dipole moment of induced dipoles is along the direction of the applied electric field because dipole moment is along negative to positive charge



- (iii) For isolated charge capacitor Q is constant

$$C' = KC$$

$$V' = \frac{Q}{C'} = \frac{Q}{KC} = \frac{V}{K}$$

$$E' = \frac{E}{K}$$

The electric field decreases

$$U' = \frac{1}{2} \frac{Q^2}{C'} = \frac{1}{2} \frac{Q^2}{KC} = \frac{U}{K}$$

The energy also decreases

Thus option (b) is correct

- (iv) (a)

$$C' = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} \text{ and } C_0 = \frac{\epsilon_0 A}{d}$$

$$= \frac{\epsilon_0 A}{d - \frac{d}{5} + \frac{d}{5K}}$$

$$= \frac{\epsilon_0 A}{\frac{4d}{5} + \frac{d}{5K}} = \frac{\epsilon_0 A}{\frac{d}{5} \left(4 + \frac{1}{K} \right)}$$

$$\frac{\epsilon_0 A 5K}{d(4K+1)} = \frac{(5K) C_0}{4K+1}$$

Thus option (c) is correct

- (iv) (b) When the capacitors are connected in parallel combination then equivalent capacitors is given by

$$C_2 = 2C_0 + 6C_0 = 8C_0$$

When capacitors are connected in the series combination the equivalent resistance is given by

$$C_1 = \frac{2C_0(6C_0)}{2C_0 + 6C_0} = \frac{12C_0^2}{8C_0} = \frac{3}{2} C_0$$

energy stored in capacitor

$$U = \frac{1}{2} CV^2$$

$$\frac{U_1}{U_2} = \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2} C_0\right) V^2}{\left(\frac{1}{2}\right)(8C_0) V^2}$$

$$\frac{U_1}{U_2} = \frac{3}{16}$$

Thus option (d) is correct

SECTION-E

31. (a) (i) A plane light wave propagating from a rarer into a denser medium, is incident at an angle i on the surface separating two media. Using Huygen's principle, draw the refracted wave and hence verify Snell's law of refraction.
- (ii) In a Young's double slit experiment, the slits are separated by 0.30 mm and the screen is kept 1.5 m away. The wavelength of light used is 600 nm. Calculate the distance between the central bright fringe and the 4th dark fringe.

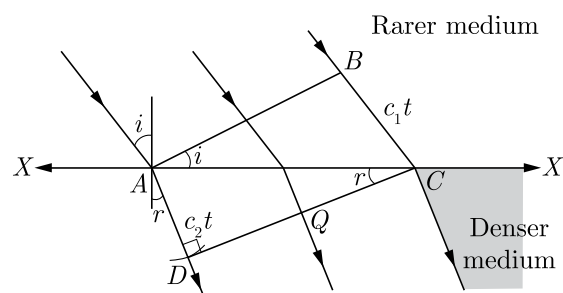
or

- (b) (i) Discuss briefly diffraction of light from a single slit and draw the shape of the diffraction pattern.
- (ii) An object is placed between the pole and the focus of a concave mirror. Using mirror formula, prove mathematically that it produces a virtual and an enlarged image.

Ans :

- (a)

- (i) **Laws of Refracting using Huygen's Principle**



where, XX' = two media separating surface

AB = incident wavefront

CD = refracted wavefront

$\angle i$ = incident angle

$\angle r$ = refracted angle

c_1 = speed of light in rarer medium

c_2 = speed of light in denser medium

The point P on incident wavefront is reached at Q on refracted wavefront in time t as taken by point B to reach at C and point A to reach at D .

$$\text{i.e., } t = \frac{PO}{c_1} + \frac{OQ}{c_2}$$

$$\text{In } \triangle APO, \sin i = \frac{PO}{AO}$$

$$PO = AO \sin i$$

$$\text{In } \triangle CQO, \sin r = \frac{OQ}{OC}$$

$$OQ = OC \sin r$$

$$\begin{aligned} \text{Hence, } t &= \frac{AO \sin i}{c_1} + \frac{OC \sin r}{c_2} \\ &= \frac{AO \sin i}{c_1} + \frac{(AC - AO) \sin r}{c_2} \\ &= AO \left(\frac{\sin i}{c_1} - \frac{\sin r}{c_2} \right) + \frac{AC \sin r}{c_2} \end{aligned}$$

AO and AC independent on time, hence we get

$$\frac{\sin i}{c_1} - \frac{\sin r}{c_2} = 0$$

$$\frac{\sin i}{c_1} = \frac{\sin r}{c_2}$$

$$\frac{c_1}{c_2} = \frac{\sin i}{\sin r}$$

The ratio of speed of light in rarer medium to denser medium is called refractive index of optical medium.

$$\text{i.e., } \mu = \frac{c_1}{c_2}$$

$$\text{Hence, } \mu = \frac{\sin i}{\sin r}$$

This is Snell's law.

$$(ii) \text{ Position of dark fringe } X_D = \frac{(2n-1)\lambda D}{2d}$$

For fourth dark $n = 4$

$$X_D = \frac{(2 \times 4 - 1)\lambda D}{2d} = \left(\frac{7\lambda D}{2d} \right)$$

$$D = 1.5 \text{ m}, d = 0.30 \text{ mm}, \lambda = 600 \text{ nm}$$

On putting these value in above equation

$$X_D = \frac{7 \times 6 \times 10^{-7} \times 1.5}{2 \times 3 \times 10^{-4}}$$

$$= 10.5 \times 10^{-3} \text{ m}$$

$$= 10.5 \text{ nm}$$

or

(b)

(i) The bending of light from the corner of small obstacles or apertures is called diffraction of light.

Diffraction due to a Single Slit :

When a parallel beam of light is incident normally on a single slit, the beam is diffracted from the slit and the diffraction pattern consists of a very intense central maximum and secondary maxima and minima on either side alternately.

If a is width of slit and θ the angle of diffraction, then for maxima

$$a \sin \theta = \left(n + \frac{1}{2} \right) \lambda$$

$$n = 1, 2, 3$$

The position of n^{th} minima is given by

$$a \sin \theta = n\lambda,$$

Where $n = \pm 1, \pm 2, \pm 3, \dots$ for various minima on either side of principal maxima.

Width of Central Maximum:

The width of central maximum is the separation between the first minima on either side.

The condition of minima is

$$a \sin \theta = \pm n\lambda \quad (n = 1, 2, 3, \dots)$$

The angular position of the first minimum ($n = 1$) on either side of central maximum is given by

$$a \sin \theta = \pm \lambda$$

$$\theta = \pm \sin^{-1} \left(\frac{\lambda}{a} \right)$$

$$\text{Half-width of central maximum, } \theta = \sin^{-1} \left(\frac{\lambda}{a} \right)$$

Total width of central maximum,

$$\beta = 2\theta = 2 \sin^{-1} \left(\frac{\lambda}{a} \right)$$

Linear width : If D is the distance of the screen from slit and y is the distance of n^{th} minima from the centre of the principal maxima, then

$$\sin \theta \simeq \tan \theta \simeq \theta = \frac{y}{D}$$

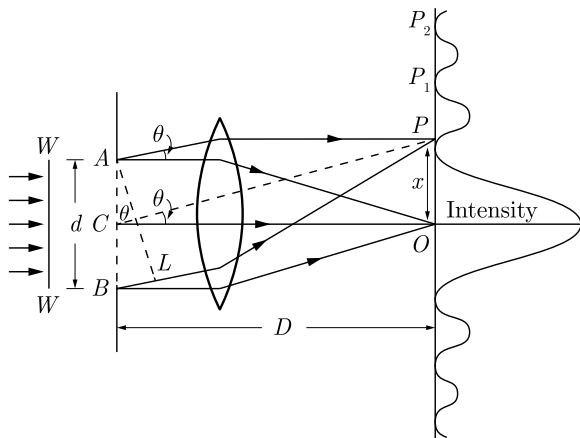
Now,

$$\lambda = a \sin \theta \simeq a\theta$$

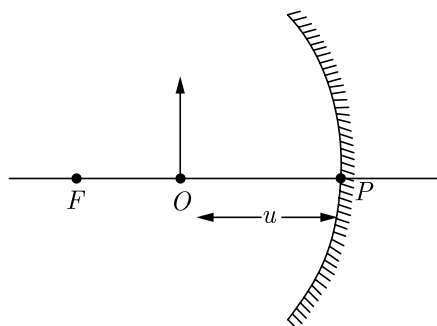
$$\theta = \frac{\lambda n}{a} = \frac{y_n}{D}$$

Linear half-width of central maximum, $y = \frac{\lambda D}{a}$

Total linear width of central maximum,
 $\beta = 2y = \frac{2\lambda D}{a}$



(ii)



Using mirror equation

$$\frac{1}{-f} = \frac{1}{v} + \frac{1}{-u}$$

$$\frac{1}{-f} + \frac{1}{u} = \frac{1}{v}$$

$$v = \frac{uf}{f-u}$$

$$|f| > |u|$$

v will always be positive i.e., virtual and erect

Also, magnification, $M = \frac{f}{u-f}$

$u < f$ Numerator will always be greater than denominator

Hence, $M > 1$

Enlarged image will be formed.

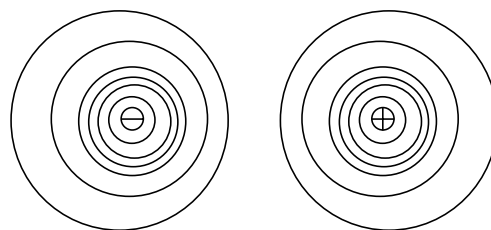
32. (a) (i) Draw equipotential surfaces for an electric dipole.
 (ii) Two-point charges q_1 and q_2 are located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field $\vec{E}$. Obtain an expression for the potential energy of the system.
 (iii) The dipole moment of a molecule is 10^{-30} cm. It is placed in an electric field $\vec{E}$ of 10^5 V/m such that its axis is along the electric field. The direction of $\vec{t}$ is suddenly changed by 60° at an instant. Find the change in the potential energy of the dipole, at that instant.

or

- (b) (i) A thin spherical shell of radius R has a uniform surface charge density σ . Using Gauss's law, deduce an expression for electric field (i) outside and (ii) inside the shell.
 (ii) Two long straight thin wires AB and CD have linear charge densities $10 \mu\text{C/m}$, and $-20 \mu\text{C/m}$ respectively. They are kept parallel to each other at a distance 1 m. Find magnitude and direction of the net electric field at a point midway between them.

Ans :

- (a)
 (i)



- (ii) Potential energy of a charge q at r in an external field $= qV(r)$
 Work done on q_1 to bringing it from infinity to r_1 against the electric field (E) $= q_1 V(r_1)$
 Work done on q_2 to bringing it from infinity to r_2 against the electric field (E) $= q_2 V(r_2)$
 Work done on q_2 against the electric field of q_1 is $\frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$

(where r_{12} is distance between the charges)

Potential energy of the system = the total work done in assembling the configuration

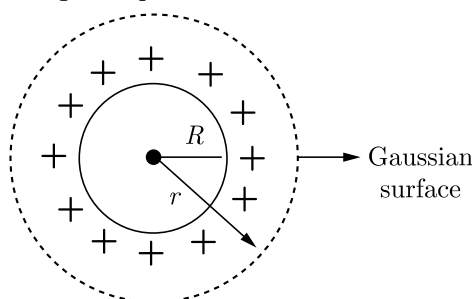
$$= q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

- (iii) The potential energy of a dipole in electric field is given by

$$\begin{aligned}
 U &= -\vec{P} \cdot \vec{E} \\
 U &= -PE \cos \theta \\
 U_i &= -PE \cos \theta \\
 &= -PE \\
 U_f &= -PE \cos 60^\circ \\
 &= -\frac{PE}{2} \\
 \Delta U &= U_f - U_i \\
 &= -\frac{PE}{2} + PE \\
 &= \frac{PE}{2} \\
 &= \frac{10^{-30} \times 10^5}{2} \\
 &= 5 \times 10^{-26} \text{ J}
 \end{aligned}$$

or

- (b)
(i) Consider a thin spherical shell of radius R
[Surface charge density
Charge on sphere



Here $q = \sigma A$
 $q = \sigma 4\pi R^2$

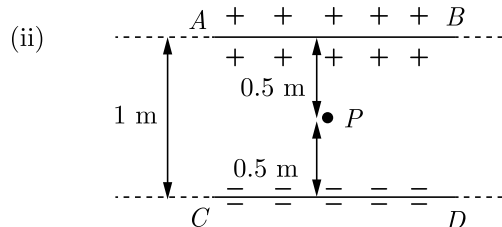
For : $r > R$

Using Gauss's law

$$\begin{aligned}
 \oint \vec{E} \cdot d\vec{s} &= \frac{q_{in}}{\epsilon_0} \\
 E \oint ds \cos 0^\circ &= \sigma A \\
 E(4\pi r^2) &= \sigma(4\pi R^2) \\
 E &= \frac{\sigma R^2}{r^2}
 \end{aligned}$$

For: $r < R$

$$\begin{aligned}
 \oint \vec{E} \cdot d\vec{s} &= \frac{q_{in}}{\epsilon_0} \\
 \text{But } q_{in} &= 0 [\text{shell}] \\
 E &= 0
 \end{aligned}$$



Net electric field at $P, E = E_{CD} + E_{AB}$

$$\begin{aligned}
 &= \frac{K\lambda_1}{2r} + \frac{K\lambda_2}{2r} \\
 E &= \frac{9 \times 10^9}{2\left(\frac{1}{2}\right)} [10 + 20] \times 10^{-6} \\
 E &= 9 \times 30 \times 10^3 \\
 E &= 270 \times 10^3 \\
 E &= 2.7 \times 10^5 \text{ N/C}
 \end{aligned}$$

33. (a) (i) You are given three circuit elements X , Y and Z . They are connected one by one across a given ac source. It is found that V and I are in phase for element X . V leads I by $(\frac{\pi}{4})$ for element Y while I leads V by $(\frac{\pi}{4})$ for element Z . Identify elements X , Y and Z .
- (ii) Establish the expression for impedance of circuit when elements, X , Y and Z are connected in series to an ac source. Show the variation of current in the circuit with the frequency of the applied ac source.
- (iii) In a series LCR circuit, obtain the conditions under which (1) impedance is minimum and (2) wattless current flows in the circuit.

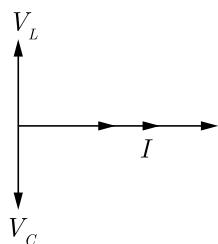
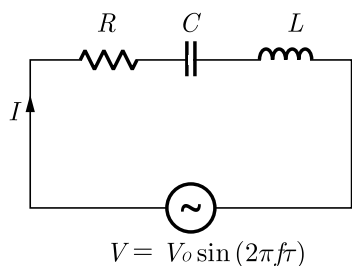
or

- (b) (i) Describe the construction and working of a transformer and hence obtain the relation for $(\frac{V_s}{V_p})$ terms of number of turns of primary and secondary.
- (ii) Discuss four main causes of energy loss in a real transformer.

Ans :

- (a)
(i) If an ac source applied across resistor, then current will be in phase of voltage.
- If an ac source applied across the capacitor, then current will lead with voltage.
 - If an ac source applied across the inductor, then current will lag with voltage.
- So X is resistor, Y is inductor and Z is capacitor.

- (ii) Consider a resistor of resistance R , capacitor of capacitance C and an inductor of inductance L is connected with a ac source $V = V_0 \sin(2\pi ft)$ in series given below



From Phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$$

$$V = IZ$$

$$IZ = \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = 2\pi fL$$

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

- (iii) 1. We know that impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For impedance

$$Z = Z_{\min}$$

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

Hence, for minimum impedance

$$X_L = X_C \text{ or } Z = R$$

2. Wattless current $I_1 = I \sin \phi$

$$\text{And } \cos \phi = \frac{R}{Z}$$

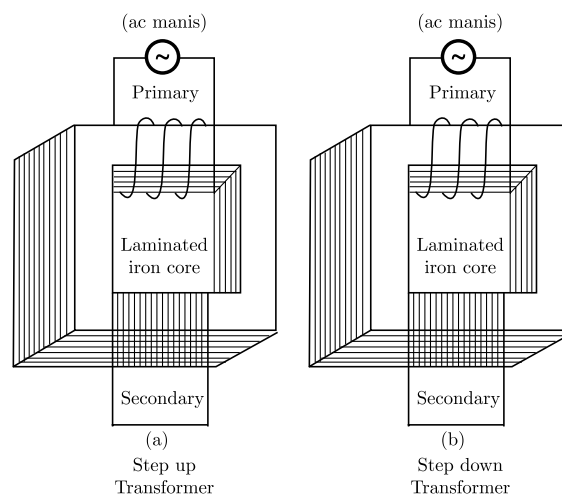
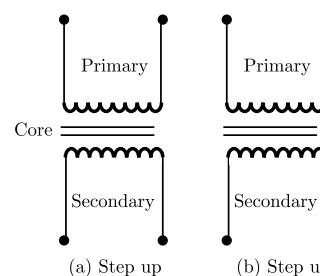
For Wattless current

$$\cos \phi = 0, \text{ it means } R = 0.$$

or

(b)

- (i) **Construction :** It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately, wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called primary coil and other is called the secondary coil. The terminals of primary coils are connected to ac mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types:



1. **Step up Transformer:** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil (*i.e.*, $N_s > N_p$).
2. **Step down Transformer:** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (*i.e.*, $N_s < N_p$).

Working: When alternating current source is connected to the ends of primary coil,

the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_p be the number of turns in primary coil, N_s the number of turns in secondary coil and ϕ the magnetic flux linked with each turn. We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same. According to Faraday's laws the emf induced in the primary coil.

$$\varepsilon_p = -N_p \frac{\Delta\phi}{\Delta t} \quad \dots(i)$$

and emf induced in the secondary coil.

$$\varepsilon_s = -N_s \frac{\Delta\phi}{\Delta t} \quad \dots(ii)$$

From (i) and (ii)

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \quad \dots(iii)$$

For step up transformer $\varepsilon_s > \varepsilon_p$ as $N_s > N_p$

For step down transformer $\varepsilon_s < \varepsilon_p$ as $N_s < N_p$

(ii) Energy Losses and Efficiency of a Transformer

1. **Copper Losses:** When current flows in primary and secondary coils, heat is produced. The power loss due to Joule's heating in coils will be $I^2 R$, where R is resistance and i is the current.
2. **Iron Losses (Eddy currents):** The varying magnetic flux produces eddy currents in iron-core, which leads to dissipation of energy in the core of transformer. This is minimised by using a laminated iron core or by cutting slots in the plate.
3. **Flux Leakage:** In actual transformer, the coupling of primary and secondary coils is never perfect, *i.e.*, the whole of magnetic flux generated in primary coil is never linked up with the secondary coil. This causes loss of energy.
4. **Hysteresis Loss:** The alternating current flowing through the coils magnetises and demagnetises the iron core repeatedly. The complete cycle of magnetisation and demagnetisation is termed as hysteresis. During each cycle some energy is dissipated. However, this loss of energy is minimised by choosing silicon-iron core having a thin hysteresis loop.

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CHAPTER 1

Electric Charges and Fields

SUMMARY

1. ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electric and magnetic effect. Benjamin Franklin introduced two types of charges namely positive charge and negative charge based on frictional electricity produced by rubbing two unlike objects like amber and wood.

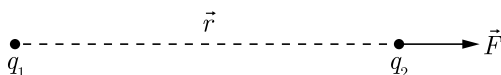
Properties of Electric Charge

Electric charge have the following properties-

1. **Additivity of Charge :** Charges are scalars and they add up like real numbers. It means of a system consists of n charges $q_1, q_2, q_3, \dots, q_n$ then total charge of the system will be $q_1 + q_2 + q_3 + \dots + q_n$.
2. **Quantisation of Charge :** The total charge on a body is integral multiple of fundamental charge e , i.e., $q = \pm ne$ where, n is an integer ($n = 1, 2, 3, \dots$).
3. **Conservation of Charge :** The total charge of an isolated system is always conserved, i.e., charge cannot be created or destroyed. It transfer from one body to another body.
4. Like charge repel each other while unlike charge attract each other.

2. COULOMB'S LAW

It states that the electrostatic force of attraction or repulsion acting between two stationary point charges is given by,



$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

where, q_1 and q_2 are the stationary point charges and r is the separation between them in air or vacuum.

Also,
$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

where, ϵ_0 = permittivity of free space
$$= 8.85419 \times 10^{12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

The force between two charges q_1 and q_2 located at a distance in a medium other than free space may be expressed as

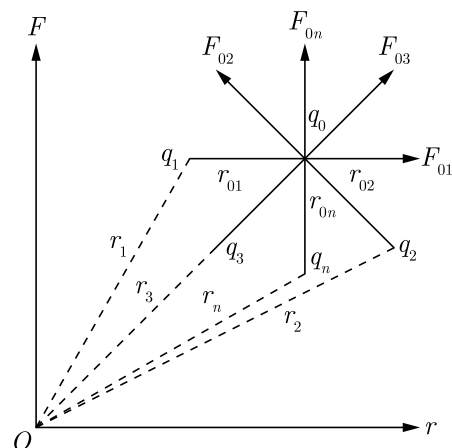
$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

In vector form,
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

Electrostatic forces are conservatives forces.

3. PRINCIPLE OF SUPERPOSITION OF ELECTROSTATIC FORCES

This principle states that the net electric force experienced by a given charge particle q_0 due to a system of charged particles is equal to the vector sum of all the forces exerted on it due to all the other charged particles of the system.



i.e.,
$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \dots + \vec{F}_{0n}$$

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_0}{|\vec{r}_{01}|^2} \vec{r}_{01} + \frac{q_2 q_0}{|\vec{r}_{02}|^2} \vec{r}_{02} + \dots + \frac{q_n q_0}{|\vec{r}_{0n}|^2} \vec{r}_{0n} \right]$$

where, $\vec{r}_{01} = \vec{r}_0 - \vec{r}_{01}$

$$\vec{F}_{01} = \text{Force on } q_0 \text{ due to } q_1.$$

$$\text{Similarly, } \vec{r}_{0n} = \vec{r}_0 - \vec{r}_n$$

$$\vec{F}_{0n} = \text{Force on } q_0 \text{ due to } q_n$$

$$\text{Hence, } \vec{F}_0 = \frac{q_0}{4\pi\epsilon_0} \left[\sum_{i=1}^n \frac{q_i}{|\vec{r}_0 - \vec{r}_i|^3} (\vec{r}_0 - \vec{r}_i) \right]$$

Electric field

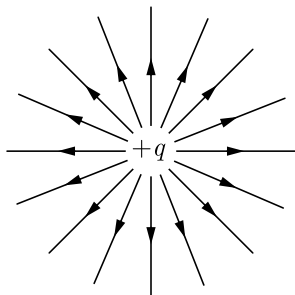
If $\vec{F}$ is the force acting on infinitesimal positive test charge q_0 then electric field strength $\vec{E} = \frac{\vec{F}}{q_0}$, electric field can be given as

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

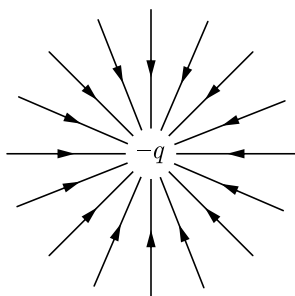
4. ELECTRIC FIELD LINES

Electric field lines are a way of pictorially mapping the electric field around a configuration of charge. These lines start from positive charge and end on negative charge. The tangent on these lines at any point gives the direction of field at that point.

Electric field lines due to positive and negative charge and their combinations are shown as below:

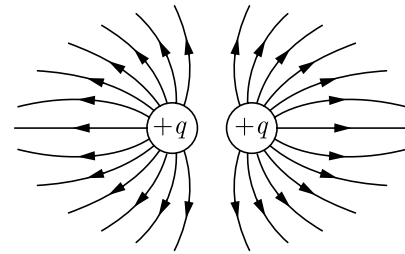


(a)

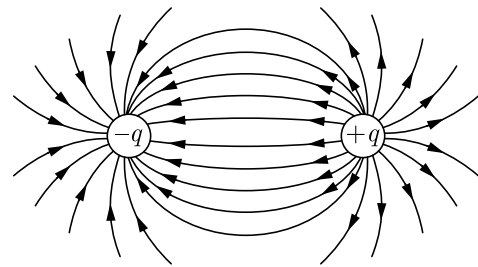


(b)

Electric Field Lines Due to Positive and Negative Charge



(c)



(d)

Electric Field Lines Due to Combination of Charge

5. ELECTRIC FLUX

The total number of electric field lines crossing (or diverging) a surface normally is called electric flux.

Electric flux surface element $\Delta \vec{S}$ is $\Delta \phi = \vec{E} \cdot \Delta \vec{S} = E \Delta S \cos \theta$

where, $\vec{E}$ is electric field strength.

Electric flux through entire closed surface is

$$\phi = \int_S \vec{E} \cdot d\vec{S}$$

6. ELECTRIC DIPOLE

Two point charges of same magnitude and opposite nature separated by a small distance altogether form an electric dipole.

$$\vec{p} = q \cdot 2\vec{l}$$

It is a vector quantity, directed from $-q$ and $+q$.

7. ELECTRIC FIELD DUE TO A SHORT DIPOLE

A point P on axis,

$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

At a point P' on equatorial plane,

$$E' = -\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

Torque on an electric dipole placed in a uniform electric field (E) is given by,

$$\tau = p \times E$$

$$|\tau| = pE \sin \theta$$

8. GAUSS'S THEOREM

It states that the total electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface

$$\text{i.e.,} \quad \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum q$$

Formulae for Electric Field Strength Calculated from

Electric field due to infinitely long straight wire of charge per unit length λ at a distance r from the wire is

$$E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$$

Electric field strength due to an infinite plane sheet of charge per unit are σ is

$$E = \frac{\sigma}{2\epsilon_0},$$

independent of distance of a point from the sheet.

Electric field strength due to a uniformly charged thin spherical shell or conducting sphere of radius R having total charge q , at a distance r from centre is

1. At external point $E_{ext} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} (r > R)$
2. At surface point $E_s = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$
3. At internal point $E_{int} = 0$.

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OBJECTIVE QUESTIONS

1. The magnitude of the electric field due to a point charge object at a distance of 4.0 m is 9 N/C. From the same charged object the electric field of magnitude 16 N/C will be at a distance of

- (a) 1 m (b) 2 m
(c) 3 m (d) 6 m

Ans :

OD 2023

Here, distance, $r = 4$ m

$$E = \frac{kq}{r^2} = 9 \text{ N/C}$$

$$\frac{kq}{(4)^2} = 9 \quad \dots(i)$$

and $\frac{kq}{r^2} = 16 \quad \dots(ii)$

where, r is the distance at which the given charge is kept

Divide Eq. (i) by (ii), we get

$$\frac{r^2}{4^2} = \frac{9}{16}$$

$$r = 3 \text{ m}$$

Thus (c) is correct option.

2. A point P lies at a distance x from the mid point of an electric dipole on its axis. The electric potential at Point P is proportional to

- (a) $\frac{1}{x^2}$ (b) $\frac{1}{x^3}$
(c) $\frac{1}{x^4}$ (d) $\frac{1}{x^{1/2}}$

Ans :

Foreign 2021, SQP 2012

The electric potential due to electric dipole, $\vec{p} = q\vec{d}$ at a distance x from the mid-point of the dipole is given as :

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{p \cos \theta}{x^2}$$

where, θ is the angle vector $\vec{r}$ forms with vector $\vec{d}$.

Hence, $V \propto \frac{1}{x^2}$

Thus (a) is correct option.

3. Ampere-hour is unit of
- (a) Power (b) Charge
(c) Energy (d) Potential difference

Ans :

OD 2020

An ampere-hour is the amount of energy charge in battery that will allow one ampere of current to flow for one hour.

Thus (b) is correct option.

4. When a body is connected to the earth, then electrons from the earth, flow into the body. It means that the body is

(a) unchanged (b) an insulator
(c) positively charged (d) negatively charged

Ans :

Delhi 2021

When a positively charged body is connected to the earth, then electrons from the earth flow into the body.

Thus (c) is correct option.

5. When a body is charged, its mass-

(a) increase
(b) decrease
(c) remains same
(d) may increase or decrease

Ans :

Foreign 2018, Delhi 2010

Charging of a body depends upon transfer of electrons so, if a body becomes negatively charged it will gain electrons that is its mass will increase and when body becomes positively charged it loose the electron that is its mass will decrease.

Thus (d) is correct option.

6. 1 stat-coulomb = ... coulomb.

(a) 3×10^9 (b) 3×10^{-9}
(c) $\frac{1}{3} \times 10^9$ (d) $\frac{1}{3} \times 10^{-9}$

Ans :

Delhi 2005

One stat-coulomb of charge is that charge which when placed at a distance of 1 cm in air from an equal and similar charge repels it and it is also repelled by it with a force of one dyne.

$$1 \text{ stat-coulomb} = 3 \times 10^9 \text{ coulomb}$$

Thus (a) is correct option.

7. The electrostatic field is

(a) conservative (b) non-conservative
(c) both (a) and (b) (d) none of these

Ans :

SQP 2000

A force is conservative if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end points. Hence, electrostatic force is conservative force.

Thus (a) is correct option.

8. 1 coulomb charge = ... esu.

(a) 3×10^9 (b) 9×10^9
(c) 8.85×10^{-12} (d) none of these

Ans :

Delhi 2017

One e.s.u of charge or one stat-coulomb is that charge which repels an identical charge in vacuum at a distance of one centimetre from it with a force of 1 dyne.

$$1 \text{ coulomb} = 3 \times 10^9 \text{ e.s.u}$$

Thus (a) is correct option.

9. The specific charge of electron is

(a) $1.8 \times 10^{-11} \text{ C/kg}$ (b) $1.8 \times 10^{-19} \text{ C/kg}$
(c) $1.9 \times 10^{-19} \text{ C/kg}$ (d) none of these

Ans :

OD 2017, SQP 2011

$$\begin{aligned} \text{Specific charge of electron} &= \frac{\text{Charge an electron}}{\text{Mass of the electron}} \\ &= \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \\ &= 0.1758 \times 10^{-12} \text{ C/kg} \\ &= 1.758 \times 10^{11} \text{ C/kg} \\ &\approx 1.8 \times 10^{11} \text{ C/kg} \end{aligned}$$

Thus (a) is correct option.

10. Two charged spheres separated by a distance d exert some force F on each other. If they are immersed in a liquid of dielectric constant 4, then what is the force exerted, if all other conditions are same?

(a) $2F$ (b) $4F$
(c) $\frac{F}{2}$ (d) $\frac{F}{4}$

Ans :

Foreign 2011, Comp 2006

Distance between two charged sphere = d

and dielectric constant of liquid, $K = 4$

Force between two charged spheres,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \dots(1)$$

and force between two charged spheres when they are immersed in a dielectric medium,

$$F' = \frac{1}{K} \times \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

From Eq. (1), we get

$$F' = \frac{F}{K} = \frac{F}{4}$$

Thus (d) is correct option.

11. Three point charges are placed at the corners of an equilateral triangle. Assuming only electrostatic forces are acting, the system

(a) Can never be in equilibrium
(b) Will be in equilibrium if charges rotate about the centre of the triangle
(c) Will be in equilibrium if charges have same magnitude but different signs

- (d) Will be in equilibrium if charges have different magnitudes and different signs

Ans :

OD 2017, Comp 2011

When three point charges are placed at the corners of an equilateral triangle, then some electrostatic forces will act on the system. As a result of this, the system can never be in equilibrium.

Thus (a) is correct option.

- 12.** Three charges, each equal to $+q$, are placed at the corners of an equilateral triangle. If the force between any two charges is F , then net force on either charge will be

- (a) $2F$ (b) $3F$
(c) $\sqrt{2}F$ (d) $\sqrt{3}F$

Ans :

SQP 2004

First charge, $q_1 = q$

Second charge, $q_2 = q$

Third charge, $q_3 = q$

Force between any two charge $F_1 = F_2 = F$

Force acting on a charge due to other two charges, placed at the corners of an equilateral triangle is F and it is acting at an angle 60° .

Therefore net force on either charge

$$\begin{aligned} &= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \\ &= \sqrt{F^2 + F^2 + 2F \cdot F \cos 60^\circ} \\ &= \sqrt{F^2 + F^2 + 2F^2 \times 0.5} \\ &= \sqrt{3F^2} \\ &= \sqrt{3}F \end{aligned}$$

Thus (d) is correct option.

- 13.** A charge q is placed at the centre of the line joining two equal point charges each equal to $+Q$. The system of three charges will be in equilibrium, if q is equal to

- (a) $-\frac{Q}{4}$ (b) $+\frac{Q}{4}$
(c) $-\frac{Q}{2}$ (d) $+\frac{Q}{2}$

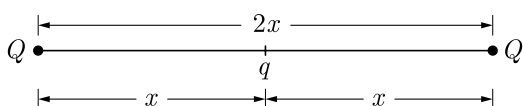
Ans :

Foreign 2009

Charge placed at centre $= q$

Two equal point charges $= +Q$

Electrostatic force at charge Q due to the other charge at Q .



$$F_1 = k \cdot \frac{Q \cdot Q}{(2x)^2} = k \cdot \frac{Q^2}{4x^2}$$

where $2x$ is distance between the point charges.

Similarly, force at the same charge Q due to charge q at the centre,

$$F_2 = k \cdot \frac{qQ}{x^2}$$

For equilibrium, sum of these two forces should be equal to zero. Therefore,

$$0 = k \cdot \frac{Q^2}{4x^2} + k \cdot \frac{qQ}{x^2}$$

or
$$q = -\frac{Q}{4}$$

Thus (a) is correct option.

- 14.** Permittivity of free space is

- (a) $9 \times 10^9 \text{ mF}^{-1}$ (b) $1.6 \times 10^{-19} \text{ C}$
(c) $8.85 \times 10^{-12} \text{ Fm}^{-1}$ (d) $8.85 \times 10^{-9} \text{ Fm}^{-1}$

Ans :

OD 2006

The permittivity of free space (vacuum) is a physical constant equal to approximately 8.85×10^{-12} farad per meter. It is symbolized ϵ_0 . It is constant of proportionality that exists between electric displacement and electric field intensity in free space. Thus (c) is correct option.

- 15.** Columbian force is

- (a) central force (b) electric force
(c) both a and b (d) none of these

Ans :

Delhi 2016

Columbian force also called electrostatic or electric force because this force are working between two charges. This force is central in nature because the force between the charges acts along the line joining the centers of two charges.

Thus (c) is correct option.

- 16.** The electric field at a distance 2 cm from the centre of a hollow spherical conducting shell of radius 4 cm having a charge of $2 \times 10^{-3} \text{ C}$ on its surface, is

- (a) zero (b) $1.1 \times 10^{10} \text{ V-m}^{-1}$
(c) $4.5 \times 10^{-10} \text{ V-m}^{-1}$ (d) $4.5 \times 10^{+10} \text{ V-m}^{-1}$

Ans :

SQP 2014

Distance of point from centre, $x = 2 \text{ cm}$

Radius of shell, $r = 4 \text{ cm}$

Electric field inside a hollow spherical conducting shell is zero as potential inside the spherical shell is same at all points.

Thus (a) is correct option.

17. Two point charges each equal to $2\mu\text{C}$ are 0.5 m apart. If both of them exist inside vacuum, then electrostatic force between them is

- (a) 0.144 N (b) 0.288 N
(c) 1.44 N (d) 2.88 N

Ans :

SQP 2006

First charge, $q_1 = 2\mu\text{C} = 2 \times 10^{-6}\text{ C}$

Second charge, $q_2 = 2\mu\text{C} = 2 \times 10^{-6}\text{ C}$

and distance between the charge, $r = 0.5\text{ m}$

Electrostatic force between two point charges in vacuum is given by,

$$F = k \cdot \frac{q_1 q_2}{r^2}$$

Here, $K = \text{Electrostatic force constant}$
 $= 9 \times 10^9\text{ N} - \text{m}^2 - \text{C}^{-2}$

Hence, $F = (9 \times 10^9) \times \frac{(2 \times 10^{-6}) \times (2 \times 10^{-6})}{(0.5)^2}$
 $= 0.144\text{ N}$

Thus (a) is correct option.

18. Two point charge Q and $-2Q$ are placed at some distance apart. If the electric field at the location of Q is E , then the electric field at the location of $-2Q$ will be

- (a) $-\frac{E}{2}$ (b) $-\frac{3E}{2}$
(c) $-E$ (d) $-2E$

Ans :

OD 2008, Delhi 2004

First charge, $q_1 = Q$

Second charge, $q_2 = -2Q$

and electric field at the location of first charge due to second charge only,

$$E_1 = E$$

Electric field at the location of Q due to second charge,

$$E_1 = k \cdot \frac{q_2}{r^2} \propto q_2$$

Therefore, $\frac{E_1}{E_2} = \frac{q_2}{q_1} = \frac{-2Q}{Q} = -2$

$$E_2 = -\frac{E_1}{2} = \frac{E}{2}$$

(where, $E_2 = \text{Electric field at the location of } -2Q$)

Thus (a) is correct option.

19. The electric field at a point near an infinite thin sheet of charged conductor is

- (a) $\varepsilon_0 \sigma$ (b) $\frac{\sigma}{\varepsilon_0}$

- (c) $\frac{\sigma}{2\varepsilon_0}$ (d) $\frac{1}{2}\sigma\varepsilon_0$

Ans :

Delhi 2004

The electric field at a point near an infinite thin sheet of charged conductor is given by,

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}$$

where, $\sigma = \text{Surface Charge Density}$

$\varepsilon_0 = \text{Permittivity of free space}$

Thus (c) is correct option.

20. The electric field required to keep a water drop of mass m just to remain suspended, when charged with one electron, is

- (a) $m g$ (b) $e m g$
(c) $\frac{m g}{e}$ (d) $\frac{e m}{g}$

Ans :

Foreign 2015

Mass of water drop $= m$

Weight of water drop, $w = mg$

Charge on drop, $q = e$

where, $e = \text{Charge on one electron}$

Electrostatic force on the water drop due to electric field,

$$= qE = eE$$

Since, the water drop is to remain suspended due to electrostatic force, therefore weight of the water drop should be equal to the electrostatic force.

Thus, $mg = eE$

or $E = \frac{mg}{e}$

(where, $E = \text{Electric field required}$)

Thus (c) is correct option.

21. The S.I. unit of electric field $\vec{E}$ is

- (a) Cm^{-2} (b) NC^{-1}
(c) Am (d) Vm^{-1}

Ans :

OD 2009

The electric field intensity at any point due to source charge is defined as the force experienced per unit positive charge placed at that point without disturbing the source charge i.e.,

$$E = \frac{F}{q} = \frac{N}{C} = \text{NC}^{-1}$$

Since, it is equivalent to volt per meter (Vm^{-1}).

Thus (b,d) is correct option.

22. Electric intensity due to an electric dipole varies with distance r as $E \propto r^n$, where n is equal to

- (a) 0 (b) -1
(c) -2 (d) -3

Ans :

SQP 2011

Electric intensity due to an electric dipole,

$$E \propto r^n \quad \dots(1)$$

Electric intensity due to an electric dipole varies inversely as cube of the distance of the point,

i.e., $E \propto \frac{1}{r^3} \quad \dots(2)$

Comparing the Eq. (1) with (2), we get

$$n = -3$$

Thus (d) is correct option.

23. An electric dipole consists of a positive and negative charge of $4 \mu\text{C}$ each placed at a distance of 5 mm. The dipole moment is

- (a) $2 \times 10^{-8} \text{ C-m}$ (b) $4 \times 10^{-8} \text{ C-m}$
(c) $6 \times 10^{-8} \text{ C-m}$ (d) $8 \times 10^{-8} \text{ C-m}$

Ans :

OD 2012, Delhi 2008

Magnitude of each charge on dipole,

$$q = 4 \mu\text{C} = 4 \times 10^{-6} \text{ C}$$

and distance between the charges,

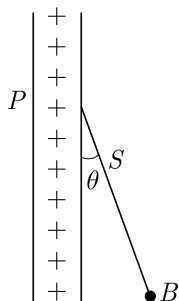
$$2a = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

Dipole moment, $p = q \times 2a$

$$\begin{aligned} &= (4 \times 10^{-6}) \times (5 \times 10^{-3}) \\ &= 2 \times 10^{-8} \text{ C-m} \end{aligned}$$

Thus (a) is correct option.

24. A charged ball B hangs from a silk thread S , which makes an angle θ with a large charged conducting sheet P , as shown in the figure. The surface charge density σ of the sheet is proportional to



- (a) $\sin \theta$ (b) $\cos \theta$
(c) $\tan \theta$ (d) $\cot \theta$

Ans :

Foreign 2017

Angle between thread and sheet $= \theta$ and surface charge density $= \sigma$.

Electric field due to conducting sheet,

$$E = \frac{\sigma}{2\epsilon_0}$$

Therefore, force on the ball due to conducting sheet,

$$F = qE = q \times \frac{\sigma}{2\epsilon_0} = \frac{q\sigma}{2\epsilon_0}$$

In equilibrium, horizontal component of tension in thread is equal to the force on ball due to conducting sheet i.e.,

$$T \sin \theta = \frac{q\sigma}{2\epsilon_0} \quad \dots(1)$$

Similarly, in equilibrium vertical component of the tension in thread is equal to the weight of the ball i.e.,

$$T \cos \theta = mg \quad \dots(2)$$

Dividing Eq. (1) by (2),

$$\begin{aligned} \tan \theta &= \frac{q\sigma}{2\epsilon_0 mg} \\ \sigma &= \frac{2\epsilon_0 mg \tan \theta}{q} \\ \sigma &\propto \tan \theta \end{aligned}$$

Thus (c) is correct option.

ASSERTION AND REASON

25. **Assertion (A)** : Work done in moving a charge around a closed path in an electric field is always zero.

Reason (R) : Electrostatic force is a conservative force.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is NOT the correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is also false.

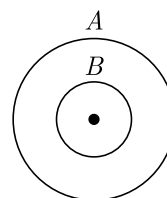
Ans :

OD 2023

We know that the work done by the conservative force around any closed path is zero is $\oint F \cdot dl = 0$ and electrostatic force is a conservative force.

Thus (a) is correct option.

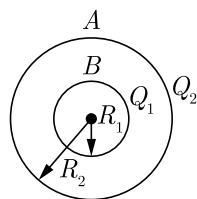
26. **Assertion (A)** : Two concentric charged shells are given. The potential difference between the shells depends on charge of inner shell.



Reason (R) : Potential due to charge of outer shell remains same at every point inside the sphere.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true and Reason (R) is NOT the correct explanation of Assertion (A).
 (c) Assertion (A) is true and Reason (R) is false.
 (d) Assertion (A) is false and Reason (R) is also false.

Ans :



$$V_A = \frac{1}{4\pi\epsilon_0} \frac{Q_1 + Q_2}{R_2}$$

$$V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2} \right)$$

$$V_B - V_A = \frac{1}{4\pi\epsilon_0} Q_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Thus (a) is correct option.

- 27. Assertion (A) :** For a non-uniformly charged thin circular ring with net charge is zero, the electric field at any point on axis of the ring is zero.

Reason (R) : For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true and Reason (R) is NOT the correct explanation of Assertion (A).
 (c) Assertion (A) is true and Reason (R) is false.
 (d) Assertion (A) is false and Reason (R) is also false.

Ans :

For a non-uniformly charged thin circular ring with net zero charge, electric potential at each point on its axis must be perpendicular to the axis.

Thus (d) is correct option.

- 28. Assertion (A) :** The electrostatic force between the plates of a charged isolated capacitor decreases when dielectric fills whole space between plates.

Reason (R) : The electric field between the plates of a charged isolated capacitance increases when dielectric fills whole space between plates.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (b) Both Assertion (A) and Reason (R) are true and Reason (R) is NOT the correct explanation of Assertion (A).
 (c) Assertion (A) is true and Reason (R) is false.
 (d) Assertion (A) is false and Reason (R) is also false.

Ans :

For isolated capacitor,

$$Q = \text{constant},$$

$$F = \text{constant},$$

But, $E = \frac{\sigma}{K\epsilon_0}$, hence E decreases

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

- 29.** Why must electrostatic field at the surface of a charged conductor be perpendicular to every point on it?

Ans :

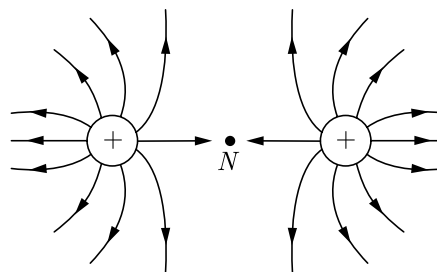
Delhi 2021

As, electric field inside a conductor is always zero. The electric line of force exert. Internal pressure on each other leads to repulsion between like charges. Thus, in order to stable spacing, the electric field lines are normal on the surface.

- 30.** Draw lines of force of electric field due to a system of two equal point charges.

Ans :

Foreign 2019



- 31.** Why must electrostatic field at the surface of a charged conductor be normal to the surface at every point? Give reason.

Ans :

SQP 2019

The work done in moving a charge from one point to another on an equipotential surface is zero. If electric field is not normal, it will have a non-zero component along the surface which would cause work to be done in moving a charge on an equipotential surface.

- 32.** Why should electrostatic field be zero inside a conductor?

Ans :

OD 2015

Charge on conductor resides on its surface. So if we consider a Gaussian surface inside the conductor to find the electrostatic field.

$$\phi = \frac{q}{\epsilon_0}$$

where, q = charge enclosed in Gaussian surface.

Hence $q = 0$,

inside the conductor, hence the electrostatic field inside the conductor is zero.

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\oint E \cdot ds \cos \theta = \frac{q}{\epsilon_0}$$

- 33.** A proton is placed in a uniform electric field directed along the position X-axis. In which direction will it bend to move?

Ans :

Comp 2021, OD 2004

Proton will tend to move along the X-axis in the direction of a uniform electric field.

- 34.** A charge q is placed at the centre of a cube of side l . What is the electric flux passing through two opposite faces of the cube?

Ans :

Delhi 2016

By symmetry, the flux through each of the six faces of the cube will be same when charge q is placed at its centre.

$$\phi_E = \frac{1}{6} \cdot \frac{q}{\epsilon_0}$$

- 35.** State and explain the superposition principle for electric fields?

Ans :

Delhi 2007

The electric intensity at a point due to several charges is the vector sum of electric intensities produced by each charge individually in the absence of other charge.

Let the electric field intensities due to individual charges are $\vec{E}_1, \vec{E}_2, \vec{E}_3, \dots, \vec{E}_n$ respectively, then resultant electric field intensity is given by,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$

- 36.** A comb after passing through dry hair attracts small pieces of paper. What happens if the hair is wet or if it is a rainy day?

Ans :

SQP 2016

A dry comb when passed through hair gets charged due to friction and because of this it induces an

opposite charge on the paper. The electrostatic force of attraction between the comb and the paper is much greater than the weight of paper therefore it is lifted.

On a wet day, the hair is wet, the friction is quite less and hence, comb does not get charged.

- 37.** What is electrostatic shielding? Give one of its practical applications.

Ans :

OD 2014

Electrostatic Shielding

The phenomena of protecting certain region of space from electric field is called electrostatic shielding.

It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

Application of Electrostatic Shielding

In a thunderstorm accompanied by lighting, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lighting.

- 38.** Two metallic spheres of exactly equal masses are taken. One is given a positive charge q coulomb and the other an equal negative charge by friction. Art their masses after charging equal?

Ans :

Foreign 2014

No. Because when two bodies are rubbed together, there is transfer of electrons from one body to another. Since electrons are material particles, so their transfer from one body to another causes a change in mass. The body from which electrons are removed becomes positively charged and body to which electrons get transferred becomes negatively charged. Hence the mass of negatively charged body will be more than positively charged body.

- 39.** Define electric dipole moment. Is it a scalar or a vector quantity? What are its SI unit?

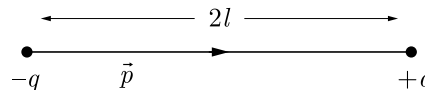
Ans :

Comp 2020

The electric dipole moment is defined as the product of either charge and the distance between the two charges. Its direction is from negative to positive charge.

i.e.,

$$|p| = q(2l)$$



- 40.** What is the electric flux through a cube of side 1 cm which encloses an electric dipole?

Ans :

Comp 2011

Net electric flux is zero.

Reason:

1. Independent to the shape and size.
2. Net charge of the electric dipole is zero.

41. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l , its axis coinciding with the length of the wires. Find the expression for the electric flux through the surface of the cylinder.

Ans :

SQP 2011

Charge enclosed by the cylindrical surface $= \lambda l$

By Gauss Theorem,

$$\begin{aligned}\text{Electric flux} &= \frac{1}{\epsilon_0} \times \text{charge enclosed} \\ &= \frac{1}{\epsilon_0} (\lambda l)\end{aligned}$$

42. Atmosphere is not electrically neutral. Explain why?

Ans :

OD 2020, SQP 2009

The atmosphere is continuously being charged by thunderstorms and lightning bolts all over the globe. This maintains an equilibrium with the discharge of the atmosphere in ordinary weather conditions.

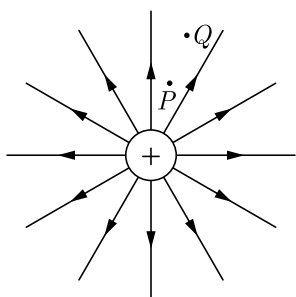
43. How rubbing of the two bodies produces electricity?

Ans :

Delhi 2016

When we rub two bodies, due to friction, some electrons are transferred from one body to another. The body which gains electrons become negatively charged and which loses electrons become positively charged by equal amount.

44. Figure shows the field lines on a positive charge is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason.



Ans :

Comp 2021

The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

45. What are the limitations of Coulomb's law ?

Ans :

Comp 2014

The limitation of coulomb's law are following:

1. Coulomb's law has been verified over distances ranging from nuclear dimension ($\approx 10^{-15}$ m) to macroscopic distance ($\approx 10^{18}$ m).

2. This law is applicable only to point charge.

46. What do you mean by additive nature of electric charges?

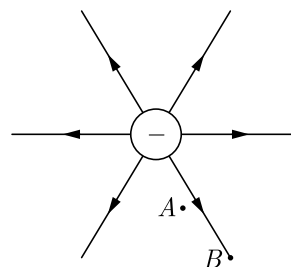
Ans :

SQP 2011

The additivity of electric charge means that the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system. If a system contains charges $q_1, q_2, \dots, q_n$; then its total charge is

$$q = q_1 + q_2 + \dots + q_n$$

47. Figure shows the field lines due to a negative point charge. Give the sign of the potential energy difference of a small negative charge between the point A and B .



Ans :

Foreign 2015

$$u = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$$

Here, $U_A > U_B$

Therefore, $U_A - U_B$ is positive.

48. Why do the electrostatic field lines not form closed loops?

Ans :

Delhi 2017

Electric field lines start from positive charge and terminate at negative charge. If there is a single positive charge, the field lines start from the charge and terminate at infinity. So, the electric field lines do not form closed loops.

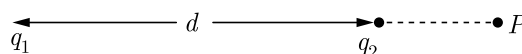
49. Why do the electric field lines never cross each other?

Ans :

Comp 2009

At the intersection point if electric field lines cross each other, then there would be two directions of electric field which is not possible so lines of forces never cross each other.

50. Two point charges q_1 and q_2 are placed at a distance d apart as shown in the figure. The electric field intensity is zero at the point P on the line joining them as shown. Write two conclusions that you can draw from this.



Ans :

Delhi 2005

- The two point charges (q_1 and q_2) should be of opposite nature.
- Magnitude of charge q_1 must be greater than magnitude of charge q_2 .

51. What is electric charge? Is it a scalar or vector quantity? Name its SI unit.

Ans :

Foreign 2018

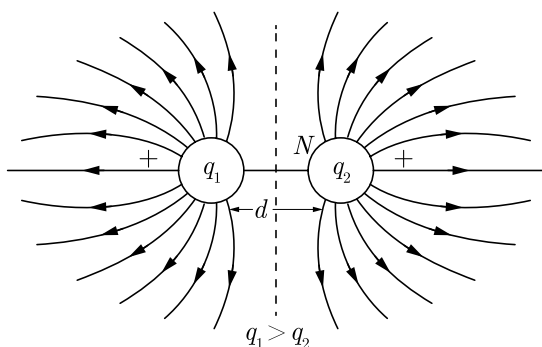
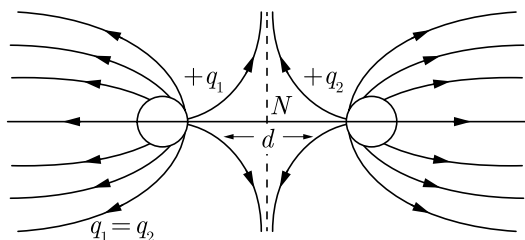
It is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects. It is a scalar quantity and its SI unit is coulomb(C).

52. Sketch the electric field lines for two point charges q_1 and q_2 for $q_1 = q_2$ and $q_1 > q_2$ separated by a distance d .

Ans :

OD 2013

When the charges are equal, the neutral point N lies at the centre of the joining the charges. However, when the charges are unequal, the point N is closer to the smaller charge.

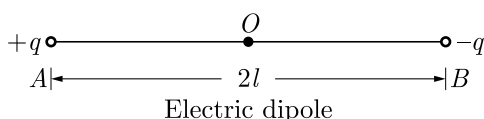


53. Define electric dipole moment and write its SI unit.

Ans :

OD 2012

The strength of an electric dipole is measure by a vector quantity known as electric dipole moment p which is the product of the charge q and separation between the charges $2l$.



i.e.,

$$p = q \times 2l$$

or

$$|p| = q(2l)$$

It is a vector quantity and its direction is always from negative charge to positive charge. The SI unit of dipole moment is coulomb-metre (C-m).

54. What do you mean by permittivity of a medium?

Ans :

OD 2008

Permittivity is a property of the medium which determines the electric force between two charges situated in that medium. For example, the force between two charges located some distance apart in water is about 1/80th of the force between them when they are separated by same distance in air. This is because the permittivity of water is about 80 times greater than the permittivity of air.

55. Compare the electric fields due to a monopole (single charge) and dipole.

Ans :

Delhi 2005, Comp 2010

	Electric Monopole	Electric Dipole
1.	It is due to charge where value is not zero.	Net charge is zero.
2.	$E \propto \frac{1}{r^2}$	$E \propto \frac{1}{r^3}$
3.	Electric field is spherically symmetric.	Electric field is not spherically symmetric.

56. Give the physical significance of electric dipoles.

Ans :

OD 2013

Electric dipoles have a common occurrence in nature. A molecule consisting of positive and negative ions is an electric dipole. Moreover, a complicated array of charges can be described and analysed in terms of electric dipoles. The concept of electric dipole is used

- In the study of the effect of electric field on an insulator, and
- In the study of radiation of energy from an antenna.

57. Write two basic properties of electric charge.

Ans :

Delhi 2012

Two properties of electric charge is following:

- Quantisation of Charge :** The total charge on a body is integral multiple of fundamental charge of electron e .

$$q = \pm ne$$

where, n is an integer ($n = 1, 2, 3, \dots$)

- Like charges repel while unlike charges attract each other.

SHORT ANSWER QUESTIONS

- 58.** Two charged conducting spheres of radii a and b are connected to each other by a wire. Find the ratio of the electric fields at their surfaces.

Ans :

2023

Let q_1 and q_2 be the charges C_1 and C_2 be the capacitance of two spheres. The charge flows from the sphere at higher potential to the other at lower potential, till their potentials become equal.

After sharing, the charges on two spheres, will be

$$\frac{q_1}{q_2} = \frac{C_1 V}{C_2 V} \quad \dots(i)$$

Also $\frac{C_1}{C_2} = \frac{a}{b} \quad \dots(ii)$

Then, ratio of surface charge on the two spheres

$$\begin{aligned} \frac{\sigma_1}{\sigma_2} &= \frac{q_1}{4\pi a^2} \times \frac{4\pi b^2}{q_2} = \frac{q_1}{q_2} \cdot \frac{b^2}{a^2} \\ &= \frac{a}{b} \cdot \frac{b^2}{a^2} \quad [\text{From Eqs. (i) and (ii)}] \\ &= \frac{a}{b} \end{aligned}$$

The, ratio of electric fields at the surfaces of two spheres $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{a}{b}$.

- 59.** Two equal balls having equal positive charge q coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

Ans :

OD 2020

Form Coulomb's law, electric force between the two charged bodies, in a medium,

$$F = \frac{1}{4\pi\epsilon_0 K} \frac{|q_1 q_2|}{r^2}$$

where, K = dielectric constant of the medium.

For vacuum, $K = 1$

For plastic, $K > 1$

Therefore, after insertion of plastic sheet, the force between the two balls will reduce.

- 60.** Give the physical significance of electric field.

Ans :

Comp 2021, 15

By knowing electric field at any point, we can determine the force on a charge placed at that point. The Coulomb force on a charge q_0 due to a source charge q may be treated as two stage process:

1. The source charge q produces a definite field $\vec{E}(r)$ at every point $\vec{r}$.
2. The value of $\vec{E}(r)$ at any point $\vec{r}$ determines the

force on charge q_0 at that point. This force is,

$$\vec{F} = q_0 \vec{E}(r)$$

i.e., Electrostatic force = Charge $\times$ Electric field.

Thus an electric field plays an intermediary role in the forces between two charges:

Charge $\longleftrightarrow$ Electric field $\longleftrightarrow$ Charge

- 61.** Distinguish between conductors and insulators. Give examples.

Ans :

SQP 2017, Comp 2011

Conductors

The substance through which electric charge can flow easily are called conductors. They contain a large number of free electrons. Metals, human and animal bodies, graphite, acids, alkalise, etc. are conductors.

Insulators

The substances through which electric charges cannot flow easily are called insulators. They contain very few free electrons. Most of the non-metals like glass, diamond, porcelain, plastic, nylon, wood, mica, etc. are insulators.

When some charge is transferred to a conductor, it readily gets distributed over its entire surface. On the other hand, if some charge is put on an insulator, it stays at the same place.

- 62.** State the law of conservation of charge. Give two examples to illustrate it.

Ans :

Foreign 2014

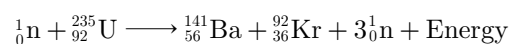
Law of Conservation of Charge

This law can be stated in a number of ways:

1. The total charge of an isolated system remains constant.
2. The electric charges can neither be created nor destroyed, they can only be transferred from one body to another.

Examples

1. Charge is conserved during the fission of a nucleus by a neutron.



Total charge before fission $(0 + 92) =$ Total charge after fission $(56 + 36 + 3 \times 0)$

2. Electric charge is conserved during the phenomenon of pair production in which a γ -ray photon materialises into an electron-positron pair.



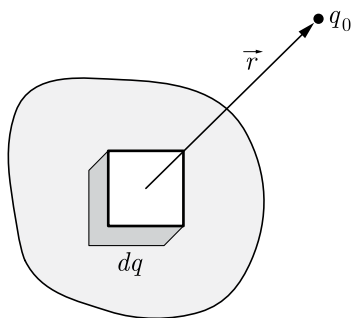
- 63.** What is a continuous charge distribution? How can we calculate the force on a point charge q due to a continuous charge distribution?

Ans :

Delhi 2019

When the charge involved is much greater than the charge on an electron, we can ignore its quantum nature and assume that the charge is distributed in a continuous manner. This is known as a continuous charge distribution.

Force on a point charge due to a continuous charge distribution. As shown in Fig. 1.8, consider a point charge q_0 lying near a region of continuous charge distribution. This continuous charge distribution can be imagined to consist of a large number of small charges dq . According to Coulomb's law, the force on point charge q_0 due to small charge dq is



Force on a Point Charge q_0 due to a continuous charge distribution,

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r^2} \cdot \hat{r}$$

where, $\hat{r} = \vec{r}/r$, is a unit vector pointing from the small charge dq towards the point charge q_0 . By the principle of superposition, the total force on charge q_0 will be the vector sum of the forces exerted by all such small charges and is given by,

$$\begin{aligned}\vec{F} &= \int d\vec{F} = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0 dq}{r^2} \cdot \hat{r} \\ &= \frac{q_0}{4\pi\epsilon_0} \int \frac{dq}{r^2} \cdot \hat{r}\end{aligned}$$

64. Compare the properties of electric charge and mass which are not similar.

Ans :

Foreign 2018

	Electric Charge	Mass
1.	Electric charge can be +ve, -ve or zero.	Strictly positive.
2.	Electric charge is always quantised i.e. the charge in a given body is always an integral multiple of a minimum value e , i.e. $q = \pm ne$.	Quantisation of mass is not so clear.

	Electric Charge	Mass
3.	Electric charge is invariant with respect to speed.	Mass increases with speed.
4.	Strictly conserved.	The phenomenon is more apparent at high speeds.
5.	A charge always possesses non-zero mass.	Not conserved by itself in relativistic process. A mass may have non-zero charge.

65. What is the nature of electrostatic force between two point electric charges q_1 and q_2 if

- $q_1 + q_2 > 0$?
- $q_1 + q_2 < 0$?

Ans :

SQP 2010, Delhi 2001

1. If both q_1 and q_2 are positive, the electrostatic force between these will be repulsive.

However, if one of these charges is positive and is greater than the other negative charge, the electrostatic force between them will be attractive.

Thus, the nature of force between them can be repulsive or attractive.

2. If both q_1 and q_2 are -ve, the force between these will be repulsive.

However, if one of them is -ve and it is greater than the second +ve charge, the force between them will be attractive.

Thus, the nature of force between them can be repulsive or attractive.

66. Define dielectric strength and relative permittivity.

Ans :

Foreign 2016

- Dielectric Strength :** The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.
- Relative Permittivity :** The dielectric constant or relative permittivity of medium may be defined as the ratio of the force between two charged placed some distance apart in free space to the force between the same two charges when they are placed the same distance apart in the given medium. It is given by,

$$\text{Relative permittivity, } \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

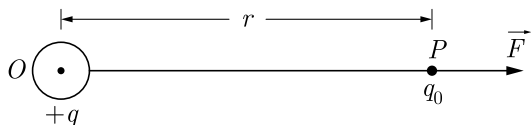
67. Derive the expression for electric field intensity at a point due to a point charge.

Ans :

OD 2014, SQP 2011

Suppose we have to calculate electric field intensity at any point P due to a point charge q at O , where

$OP = r$. Imagine a positive test charge q_0 at P . According to Coulomb's law, force at P is



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r}$$

(where, $\hat{r}$ is unit vector directed from q towards q_0)

As,
$$\vec{E} = \frac{\vec{F}}{q_0}$$

Hence,
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$\hat{r}$ is the unit vector in the direction of force.

If q is +ve, the field intensity is directed radially outwards along OP . If q is negative, the field intensity is directed towards q , along PO .

68. Show that charge always resides on the outer surface of a conductor.

or

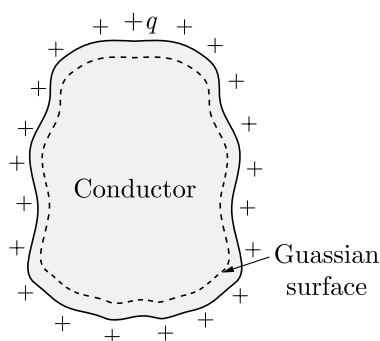
Show that net charge in the interior of a conductor is zero.

Ans :

Comp 2007

Consider a conductor of any shape or size, which is given a +ve charge $+q$. Consider a Gaussian closed surface just inside the outer surface of the conductor as shown in the figure by dotted lines.

From Gauss's theorem,



$$\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

But,
$$\vec{E} = 0 \text{ inside the conductor}$$

Hence,
$$q = 0$$

i.e., charge inside the conductor is zero, Hence we find that charge always resides on the outer surface of the conductor.

69. Define uniform and non-uniform electric fields. How are they represented geometrically?

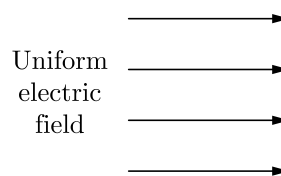
Ans :

OD 2009, Foreign 2014

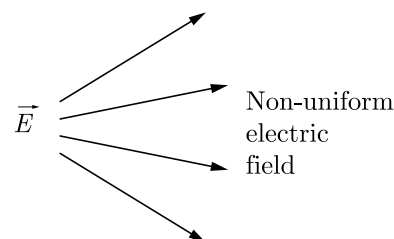
A uniform electric field has same intensity (both in magnitude and direction) at all points in it. Therefore, the electric field vectors at all the points must be of the same length and point in the same direction. Hence a uniform field is represented by a set of parallel lines pointing in the same direction.

A non-uniform field is that in which electric intensity is different at different parts, either in magnitude or direction or both.

A uniform and a non-uniform electric field are represented by the lines of force as shown in Figures (a) and (b) respectively.



(a)

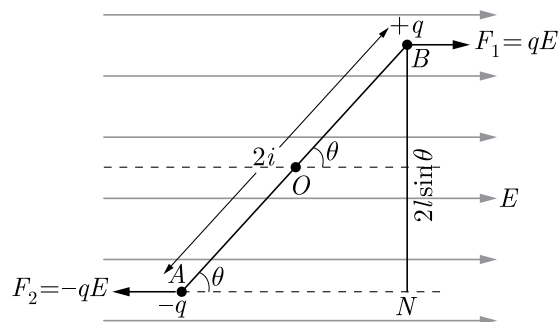


(b)

70. Derive an expression for the torque experienced by electric dipole in external electric field.

Ans :

Delhi 2010



The forces $\vec{F}_1$ and $\vec{F}_2$ from a couple (or torque) which tends to rotate and align the dipole along the direction of electric field. This couple is called the torque and is denoted by τ .

Hence, Torque

$$\begin{aligned}\tau &= \text{magnitude of one force} \times \text{perpendicular} \\ &\quad \text{distance between lines of action of forces} \\ &= qE(BN) = qE(2l \sin \theta) \\ &= (q \cdot 2l) E \sin \theta \\ &= pE \sin \theta \text{ [using (1)]} \quad \dots(2)\end{aligned}$$

Clearly, the magnitude of torque depends on orientation (θ) of the electric dipole relative to electric field. Torque (τ) is a vector quantity whose direction is perpendicular to the plane containing given by right hand screw rule $\vec{p}$ and $\vec{E}$.

In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$

71. An electric dipole is placed in a uniform electric field E with its dipole moment p parallel to the field. Find:

- The work done in turning the dipole till its dipole moment points in the direction opposite to E .
- The orientation of the dipole for which the torque acting on it becomes maximum.

Ans :

Foreign 2017

- We know that,

$$\begin{aligned}W &= \int_{\theta_1}^{\theta_2} \tau d\theta \\ &= \int_0^\pi pE \sin \theta d\theta \\ &= pE[-\cos \theta]_0^\pi \\ &= 2pE\end{aligned}$$

- We know that,

$$\tau = pE \sin \theta$$

If, $\theta = \frac{\pi}{2}$, then τ is maximum.

$$\text{i.e.,} \quad r = pE \sin \frac{\pi}{2}$$

$$\tau = pE \text{ (Maximum)}$$

- 72.** (i) Write down unit and dimension of permittivity of free space.
(ii) A Gaussian surface contains three charges ($-q$), $(+2q)$ and $(-q)$. Evaluate net electric flux through the surface.

Ans :

OD 2018, Comp 2015

- Unit and dimension of permittivity of free space.

According to the Coulomb's Law,

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

Hence, Permittivity

$$\epsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

$$= \frac{(\text{Coulomb})^2}{\text{Newton} \cdot (\text{meter})^2}$$

Here,

$$\begin{aligned}q &= [AT] \\ F &= [M^1 L^1 T^{-2}] \\ r &= [L]\end{aligned}$$

Hence,

$$\begin{aligned}\epsilon_0 &= \frac{[AT][AT]}{[M^1 L^1 T^{-2}][L]^2} \\ &= [M^{-1} L^{-3} T^4 A^2]\end{aligned}$$

- Net electric flux through the surface.

Total charge in Gaussian surface

$$q = q_1 + q_2 + q_3$$

Here,

$$\begin{aligned}q_1 &= -q \\ q_2 &= +2q \\ q_3 &= -q \\ &= -q + 2q - q = 0\end{aligned}$$

Net flux,

$$\phi = \frac{q}{\epsilon_0} = \frac{0}{\epsilon_0} = 0$$

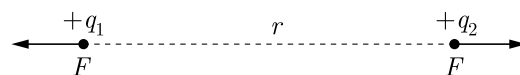
- 73.** State Coulomb's law. Define Coulomb's dielectric constant.

Ans :

Foreign 2008

The law states that two stationary electric point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

Let q_1 , q_2 two point like charges separated by a distance r , then the force between the two charges is



$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

Hence,

$$F \propto \frac{q_1 q_2}{r^2}$$

or

$$F = k \frac{q_1 q_2}{r^2} \quad \dots(1)$$

where k is the constant of proportionality and its value depends upon (1) system of unit adopted and (2) the nature of medium separating q_1 and q_2 .

In SI,

$$\begin{aligned}k &= \frac{1}{4\pi\epsilon} \\ &= \frac{1}{4\pi\epsilon_0\epsilon_r} \quad [\text{since, } \epsilon = \epsilon_0\epsilon_r]\end{aligned}$$

where ϵ is called absolute permittivity, and ϵ_0 is called permittivity of free space (vacuum) and ϵ_r is called relative permittivity (or dielectric constant K)

of the medium.

The value of $\epsilon_0 = 8.854 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2}$

So, Eq. (1) becomes,

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$$

In vacuum (or in air) $\epsilon_r = 1$.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\begin{aligned} \text{In SI,} \quad \frac{1}{4\pi\epsilon_0} &= \frac{1}{4 \times 3.142 \times 8.854 \times 10^{-12}} \\ &= 9 \times 10^9 \text{Nm}^2 \text{C}^{-2} \end{aligned}$$

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

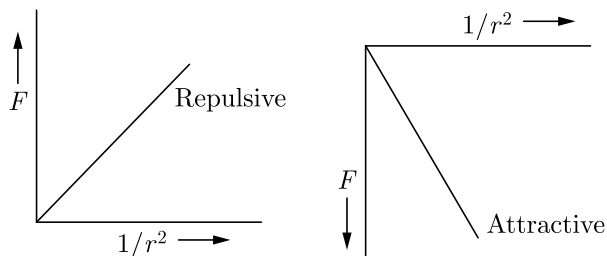
74. Plot a graph showing the variation of Coulomb force F , versus $(1/r^2)$, where r is the distance between the two charges of each pair of charges ($1\text{ }\mu\text{C}, 2\text{ }\mu\text{C}$) and ($2\text{ }\mu\text{C}, -3\text{ }\mu\text{C}$). Interpret the graphs obtained.

Ans :

Delhi 2014, OD 2004

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

The graph between F and $1/r^2$ is a straight line of slope $\frac{1}{4\pi\epsilon_0} q_1 q_2$ passing through origin in both the cases.



Since, magnitude of the slope is more for attraction, therefore, attractive force is greater than repulsive force.

75. Define electric field intensity in current carrying conductor and equipotential surface.

Ans :

SQP 2013

Electric Field Intensity

The electric field intensity at any point due to source charge is defined as the force experienced per unit positive charge placed at that point without disturbing the source charge.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

where, $\vec{E}$ = Electric field intensity

and $\vec{F}$ = Force experienced by the test charge q_0

Equipotential Surface

Any surface which has same electrostatic potential at all point, on it is called an equipotential surface.

76. Two point charges q_1 and q_2 are located at points $(a, 0, 0)$ and $(0, b, 0)$ respectively. Find the electric field due to both these charges at the point $(0, 0, c)$?

Ans :

OD 2012

$$\begin{aligned} \text{We have,} \quad \vec{E}_{\text{net}} &= \vec{E}_1 + \vec{E}_2 \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^3} \vec{r}_1 + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^3} \vec{r}_2 \end{aligned}$$

$$\text{where} \quad \vec{r}_1 = -a\hat{i} + c\hat{k}$$

$$\text{and} \quad \vec{r}_2 = -b\hat{j} + c\hat{k}$$

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1(-a\hat{i} + c\hat{k})}{(a^2 + c^2)^{3/2}} + \frac{q_2(b\hat{j} + c\hat{k})}{(b^2 + c^2)^{3/2}} \right]$$

77. A metallic spherical shell has an inner radius R_1 and outer radius R_2 . A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on

1. The inner surface
2. The outer surface?

Ans :

Comp 2006

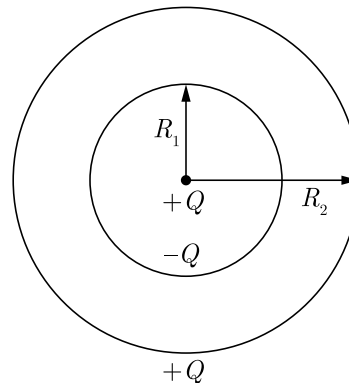
When a charge $+Q$ is placed at the centre of spherical cavity,

The charge induced on the inner surface $= -Q$

The charge induced on the outer surface $= +Q$

Surface charge density on the inner surface $= \frac{-Q}{4\pi R_1^2}$.

Surface charge density on the outer surface $= \frac{+Q}{4\pi R_2^2}$.



78. Plot a graph showing the variation of Coulomb force (F) versus $1/r^2$, where r is the distance between the two charges of each pair of charges ($1\text{ }\mu\text{C}, 2\text{ }\mu\text{C}$) and ($1\text{ }\mu\text{C} - 3\text{ }\mu\text{C}$). Interpret the graphs obtained.

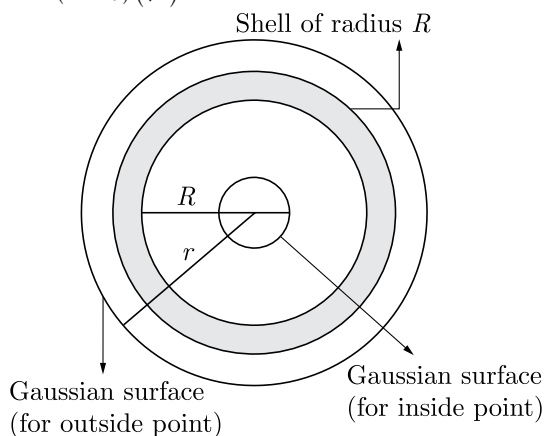
Ans :

Delhi 2011, OD 2009

According to Coulomb's law, the magnitude of force

acting between two stationary point charges is given

$$\text{by } F = \left(\frac{q_1 q_2}{4\pi\epsilon_0} \right) \left(\frac{1}{r^2} \right)$$

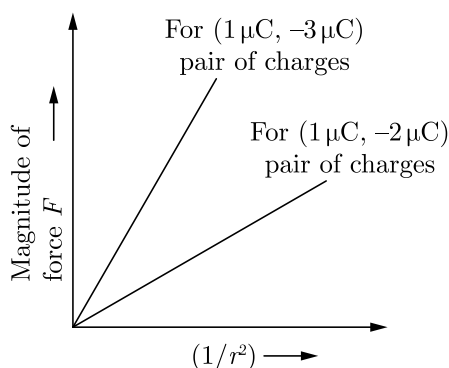


For given $q_1 q_2$ $F \propto \left(\frac{1}{r^2} \right)$

The slope of F versus $1/r^2$ graph depends on $q_1 q_2$.

Magnitude of $q_1 q_2$ is higher for second pair.

Hence Slope of F versus $1/r^2$ graph.



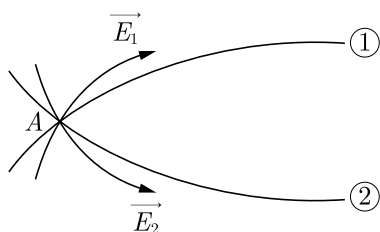
79. (a) An electrostatic field line is a continuous curve. That is a field line cannot have sudden breaks. Why is it so?
 (b) Explain why two field lines never cross each other at any point.

Ans :

OD 2017, Comp 2005

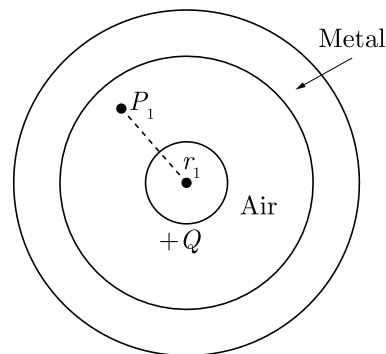
- (a) An electrostatic field line is the path of movement of a positive test charge ($q_0 \rightarrow 0$).

A moving charge experiences a continuous force in an electrostatic field, so an electrostatic field line is always a continuous curve.



- (b) Two electric lines of force can never cross each other because if they cross, there will be two directions of electric field at the point of intersection (say A); which is impossible.

80. A small metal sphere carrying a charge $+Q$ is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_1 .



Ans :

Foreign 2015

According to question,

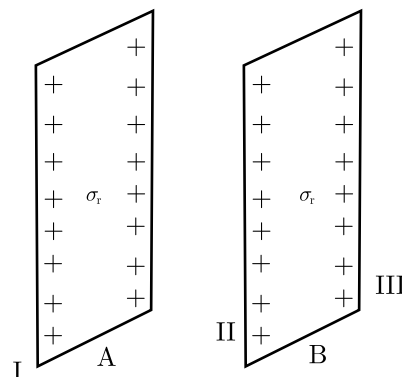
the charge on inner surface $= -Q$

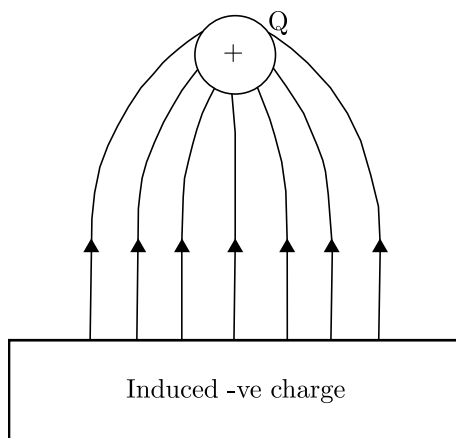
Charge on other surface $= +Q$

Electric field at point P_1 is given by

$$E = \frac{Q}{4\pi\epsilon_0 r_1^2}$$

81. (a) A point charge ($+Q$) is kept in the vicinity of uncharged conducting plate. Sketch electric field lines between the charge and the plate.
 (b) Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the net fields in the regions marked II and III.





Ans :

SQP 2014

(a) The lines of force start from $+Q$ and terminate at metal place inducing negative charge on it. The lines of force will be perpendicular to the metal surface.

(b) (i) Net electric field in regions

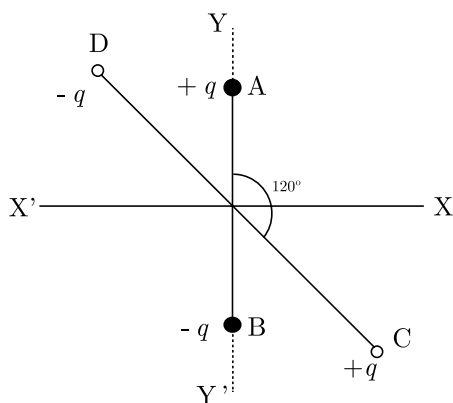
$$II = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$$

Direction of electric field is from sheet A to sheet B.

(ii) Net electric field in region

$$III = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$$

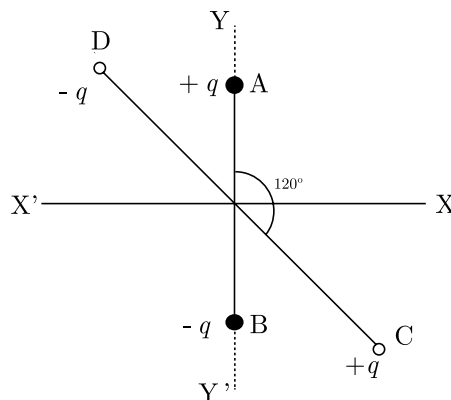
82. Two small identical electrical dipoles AB and CD each of dipole moment p are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field ($\vec{E}$) directed along $+X$ direction. What will be the magnitude and direction of the torque acting on this?



Ans :

Delhi 2018, OD 2010

The direction of dipole moment will be from negative to positive charge, so both dipoles form following structure :



So, resultant dipole moment,

$$\begin{aligned} \vec{p}' &= \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta} \quad (p_1 = p_2 = p) \\ &= \sqrt{p^2 + p^2 + 2p^2\cos 120^\circ} \\ &= \sqrt{2p^2 + 2p^2\left(\frac{1}{2}\right)} \end{aligned}$$

So, $\vec{p}' = p$

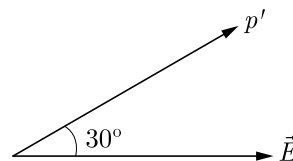
and direction of p'

$$\begin{aligned} \tan a &= \frac{p \sin 120^\circ}{p + p \cos 120^\circ} \\ &= \frac{p \frac{\sqrt{3}}{2}}{p + p\left(\frac{1}{2}\right)} \\ &= \frac{\sqrt{3} p/2}{p/2} \\ &= \sqrt{3} \end{aligned}$$

$$\tan a = \sqrt{3}$$

So, $a = 60^\circ$

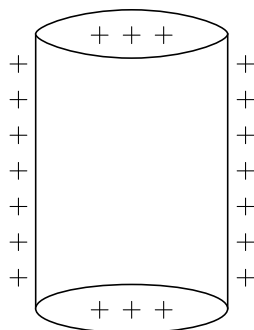
Now the system of dipole moment p' and E will be as follows :



So, torque on system

$$\begin{aligned} t &= pE \sin \theta \\ &= pE \sin 30^\circ \\ &= \frac{1}{2} pE \end{aligned}$$

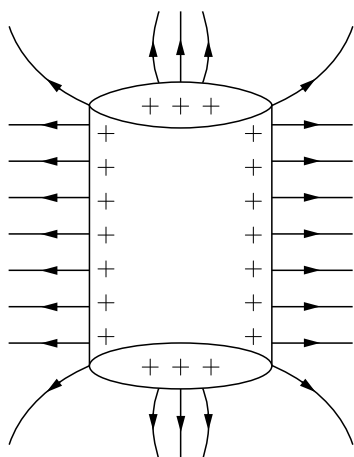
83. Sketch the electric field lines for a uniformly charged hollow cylinder shown in figure.



Ans :

OD 2012, Comp 2015

Here, the hollow cylinder is positively charged. We know that the electric lines of force appear to come out from the conductor. Thus, the lines of force of a uniformly positive charged hollow cylinder is shown in figure.



84. An electric dipole is held in a uniform electric field.

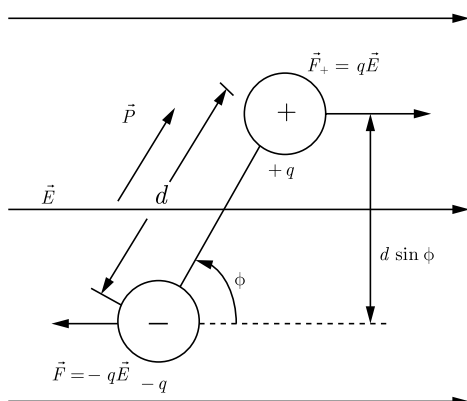
- Show that the net force acting on it is zero.
- The dipole is aligned parallel to the field.

Find the work done in rotating it through an angle of 180° .

Ans :

Delhi 2010

- Consider an electric dipole consisting of charges $-q$ and $+q$ and dipole length d placed in a uniform electric field E as shown in figure. Let the dipole moment make an angle θ with the direction of the electric field.



The two charges experience force qE each. These forces are equal, parallel and opposite. Therefore, the net force acting on the dipole is

$$E_n = qE - qE = 0$$

Thus, the net force acting on the dipole is zero.

$$\begin{aligned} \text{(ii)} \quad W &= -pE(\cos\theta_1 - \cos\theta_2) \\ &= -pE(\cos 0^\circ - \cos 180^\circ) \\ &= +2pE \end{aligned}$$

LONG ANSWER QUESTIONS

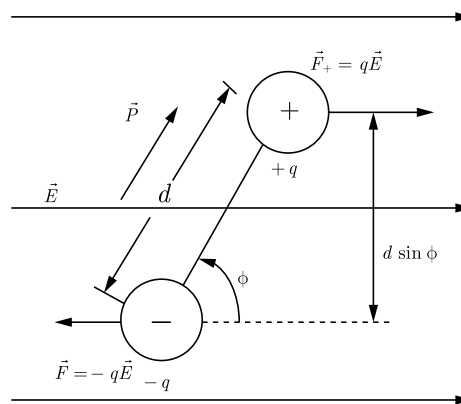
85. What is an electric dipole? Drive an expression for the torque acting on an electric dipole, when held in a uniform electric field. Hence, define the dipole moment.

Ans :

SQP 2013

An electric dipole is a pair of your charges with equal magnitude and opposite sign separated by a small distance.

Consider an electric dipole consisting of charges $-q$ and $+q$ and dipole length d placed in a uniform electric field as shown in figure below. Let the dipole moment make an angle ϕ with the direction of the electric field.



The two charges experience force qE each. These forces are equal, parallel and opposite. Therefore, the net force is zero. But these two forces constitute a couple. This applies a torque on the dipole given by

$$\tau = \text{Either force} \times \text{Arm of the couple}$$

$$\tau = qE \times d \sin \phi,$$

where $d \sin \phi$ is the are of the couple.

$$\tau = qdE \sin \phi,$$

$$\text{where, } p = qd,$$

$$\text{Dipole moment, } \tau = p E \sin \phi$$

In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$

The direction of torque is perpendicular to both $\vec{p}$ and $\vec{E}$.

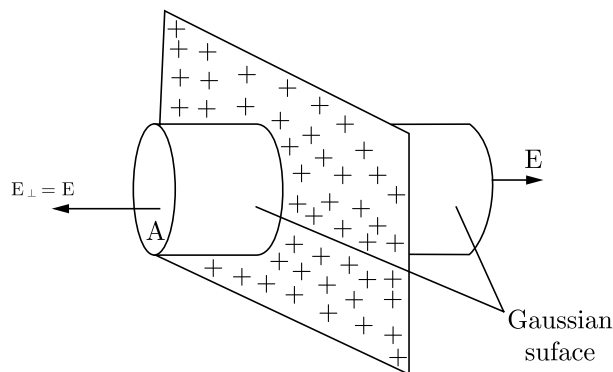
If $E = 1$ and $\phi = 90^\circ$, then $\tau = p$, thus, electric dipole moment is defined as numerically equal to the torque experienced by an electric dipole placed perpendicular to a unit electric field.

86. (a) Using Gauss's theorem prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.
(b) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged?

Ans :

Comp 2019, OD 2004

- (a) Consider an infinite plane sheet of charge. Let σ be the uniform surface charge density, i.e., the charge per unit surface area. From symmetry, we find that the electric field must be perpendicular to the plane of the sheet, and that the direction of $\vec{E}$ on one side of the plane must be opposite of its direction on the other side as shown in figure below. In such a case, let us choose a Gaussian surface in the form of a cylinder with its axis perpendicular to the sheet of charge, with ends of area A . The charged sheet passes through the middle of the cylinder's length, so that the cylinder's ends are equidistant from the sheet. The electric field has a normal component at each end of the cylinder and no normal component along the curved surface of the cylinder. As a result, the electric flux is linked with only the ends and not the curved surface.



Therefore, by the definition of electric flux, the flux linked with the Gaussian surface is given by

$$\phi = EA + EA = 2EA \quad \dots(1)$$

But by Gauss's law,

$$\phi = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \quad \dots(2)$$

From equations (1) and (2), we have

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

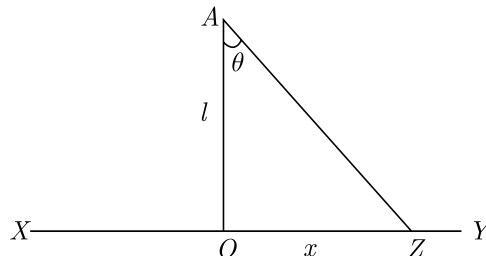
This gives the electric field due to an infinite plane sheet of charge which is independent of the distance from the sheet.

- (b) (i) Directed outwards.
(ii) Directed inwards.
87. (i) Use Gauss's law to obtain an expression for the electric field due to an infinitely long thin straight wire with uniform linear charge density.
(ii) An infinitely long positively charged straight wire has a linear charge density λ . An electron is revolving in a circle with a constant speed v such that the wire passes through the centre and is perpendicular to the plane of the circle. Find the kinetic energy of the electron in terms of magnitudes of the charge and linear charge density λ on the wire.
(iii) Draw a graph of kinetic energy as a function of linear charge density λ .

Ans :

OD 2023

- (i) Take a long thin wire of uniform linear charge density λ .



Consider a point A at a perpendicular distance l from the mid-point O of the wire. Let E be the electric field at point A due to the wire XY . Consider a small length element dx on the wire section with $OZ = x$.

Let q be charge on this piece.

$$q = \lambda \cdot dx$$

Electric field due to the piece ;

$$dE = \frac{1}{4\pi\epsilon_0(AZ)^2}$$

$$AZ = \sqrt{l^2 + x^2}$$

$$dE = \frac{\lambda dx}{4\pi\epsilon_0(l^2 + x^2)}$$

The electric field is resolved into rectangular components $dE \cos \theta$ and $dE \sin \theta$ when the whole wire is considered the component $dE \sin \theta$ is cancelled.

Only the perpendicular component $dE \cos \theta$ affects points.

Hence, effective electric field at point A due to element dx is dE_1 ,

$$dE_1 = \frac{\lambda dx \cos \theta}{4\pi\epsilon_0(x^2 + l^2)} \quad \dots(1)$$

In $\triangle AZO$, $\tan \theta = \frac{x}{l}$

$$x = l \tan \theta \quad \dots(2)$$

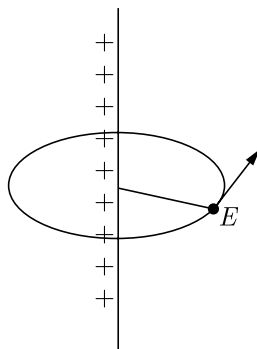
Differentiating equation (1), we obtain

$$\begin{aligned} dx &= l \sec^2 \theta d\theta \\ dE_1 &= \frac{\lambda \cos \theta d\theta}{4\pi\epsilon_0 l} \\ E_1 &= \int_{-\pi/2}^{\pi/2} \frac{\lambda \cos \theta d\theta}{4\pi\epsilon_0 l} = \frac{\lambda}{2\pi\epsilon_0 l} \end{aligned}$$

(ii) Infinitely long charged wire produces a radial electric field

$$E = \frac{\lambda}{2\pi\epsilon_0 l} \quad \dots(1)$$

The revolving electron experiences an electrostatic force and provides necessary centripetal force.



$$eE = \frac{mv^2}{r} \quad \dots(2)$$

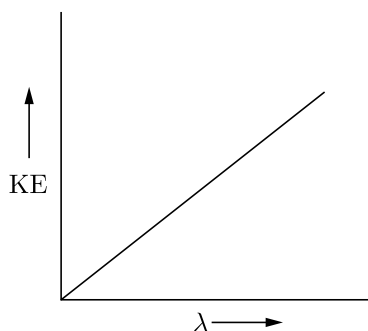
$$\frac{e\lambda}{2\pi\epsilon_0 r} = \frac{mv^2}{r}$$

$$mv^2 = \frac{e\lambda}{2\pi\epsilon_0}$$

Kinetic energy of the electron,

$$K = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\epsilon_0}$$

(iii)



88. (i) State and explain Superposition Principle.
(ii) Find an expression for the total force acting on a given charge due to a number of other charges, when the source charges are point charges.

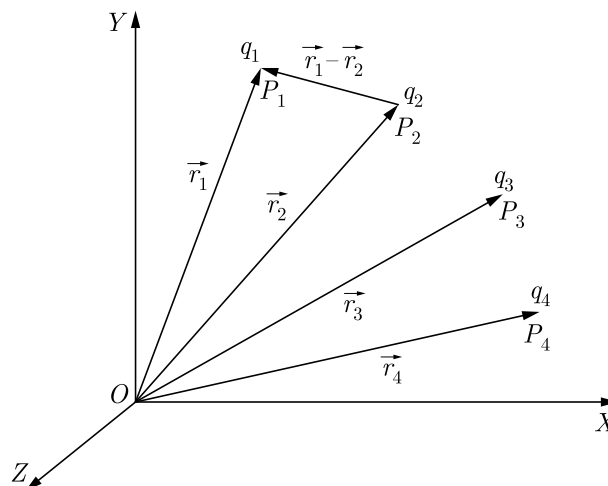
Ans :

Comp 2021, SQP 2005

- (i) **Principle of Superposition :** The resultant electric force at a point charge due to a number of charges in its neighbourhood is the vector sum of the electric forces produced by each charge individually in the absence of other charges.

If a charge q_1 experiences forces $\vec{F}_{12}, \vec{F}_{13}, \dots, \vec{F}_{1n}$ due to other charge $q_2, q_3, \dots$ respectively then the total force $\vec{F}$ experienced by the charge q_1 is given by,

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1n}$$



- (ii) Suppose we have many point charges $q_1, q_2, q_3, \dots$ at points $P_1, P_2, P_3, \dots$ in space, with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots$. The total force $\vec{F}_1$ on charge q_1 is the vector sum of the individual forces that each of the other charges $q_2, q_3, \dots$ exert on it.

Therefore, using Coulomb's law of electrostatics

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Similarly, force on charge q_1 due to charge q_2

$$\vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3} (\vec{r}_1 - \vec{r}_3)$$

$$\vec{F}_{1n} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_n}{|\vec{r}_1 - \vec{r}_n|^3} (\vec{r}_1 - \vec{r}_n)$$

Hence, total force on charge q_1 due to all other charges

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1n}$$

$$= \frac{q_1}{4\pi\epsilon_0} \left[\frac{q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) + \frac{q_3}{|\vec{r}_1 - \vec{r}_3|^3} (\vec{r}_1 - \vec{r}_3) + \dots + \frac{q_n}{|\vec{r}_1 - \vec{r}_n|^3} (\vec{r}_1 - \vec{r}_n) \right]$$

$$= \frac{q_1}{4\pi\epsilon_0} \sum_{i=1}^{i=n} \frac{q_i}{|\vec{r}_1 - \vec{r}_i|^3} (\vec{r}_1 - \vec{r}_i)$$

89. (i) Define electric intensity.
 (ii) Derive an expression for electric intensity at a point situated on the axis of electric dipole.

Ans :

Delhi 2020

(i) Electric Field Intensity

The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

It is expressed as,

$$E = \frac{F}{q_0}$$

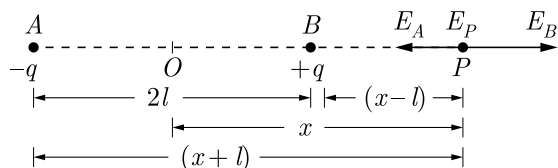
Where, E = electric field intensity and

and F = force experienced by the test charge q_0

It is a vector quantity and its SI unit is NC^{-1} .

(ii) Electric Field Intensity a Point Situated on the Axis of Electric Dipole

We have to calculate the field intensity E at a point P on the axial line of the dipole and at a distance $OP = x$ from the centre O of the dipole.



Electric field on axial line of an electric dipole.

Resultant electric field intensity at the point P,

$$\vec{E}_P = \vec{E}_A + \vec{E}_B$$

The vectors $\vec{E}_A$ and $\vec{E}_B$ are collinear and opposite.

$$E_P = E_B - E_A$$

Here,
$$\vec{E}_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x+l)^2}$$

and
$$\vec{E}_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x-l)^2}$$

Hence,
$$E_P = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{4qlx}{(x^2 - l^2)^2}$$

Hence,
$$E_P = \frac{1}{4\pi\epsilon_0} \cdot \frac{2px}{(x^2 - l^2)^2}$$

$$[\text{Since, } p = q \times 2l]$$

In vector form,
$$\vec{E}_P = \frac{1}{\pi\epsilon_0} \cdot \frac{2px}{(x^2 - l^2)^2}$$

If dipole is short, i.e., $2l \ll x$, then

$$E_P = \frac{2|p|}{4\pi\epsilon_0 x^3} \quad \dots(1)$$

The direction of E_P is along BP produced.

Clearly,
$$E_P \propto \frac{1}{x^3}$$

90. Define electric flux. State and prove Gauss theorem.

Ans :

Foreign 2015, Delhi 2012

Electric Flux

Electric flux linked with any surface is defined as the total number of electric field lines that normally pass through that surface.

Electric flux $d\phi$ through a small area element dS due to an electric field $\vec{E}$ at an angle θ with dS is,

$$d\phi = \vec{E} \cdot d\vec{S}$$

$$= E dS \cos \theta$$

Which is proportional to the number of field lines cutting the area element. Total electric flux ϕ over the whole surface S due to an electric field E ,

$$\phi = \oint_S \vec{E} \cdot d\vec{S}$$

$$= \oint_S E dS \cos \theta$$

Electric flux is a scalar quantity. But it is a property of vector field.

SI unit of electric flux is $\text{N-m}^2\text{C}^{-1}$.

Gauss' Theorem

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i = \frac{q}{\epsilon_0}$$

where, $\phi_E = \sum_{i=1}^n q_i$ is the algebraic sum of all the charges inside the closed surface.

Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\epsilon_0}$ times to total charge within the surface, regardless of how the charges may be distributed.

Proof of Gauss' Theorem

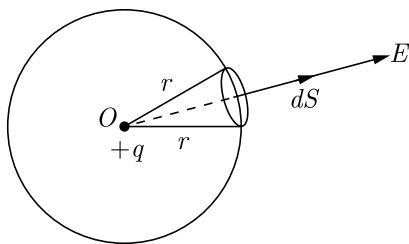
Electric flux through a surface element dS is given by,

$$d\phi_E = \vec{E} \cdot d\vec{S} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r} \cdot (dS \hat{n})$$

$$d\phi_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q dS}{r^2} \hat{r} \cdot \hat{n}$$

Here, $\hat{r} \cdot \hat{n} = 1 \cdot \cos 0^\circ = 1$

So,
$$d\phi_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q dS}{r^2}$$



Total electric flux through the spherical surface,

$$\begin{aligned}\phi_E &= \oint_S d\phi_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \oint_S dS \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 = \frac{q}{\epsilon_0} \\ \phi_E &= \frac{q}{\epsilon_0}\end{aligned}$$

91. Derive an expression for the electric field intensity at a point on the equatorial line of an electric dipole of dipole moment $\vec{p}$ and length $2l$. What is the direction of this field?

Ans :

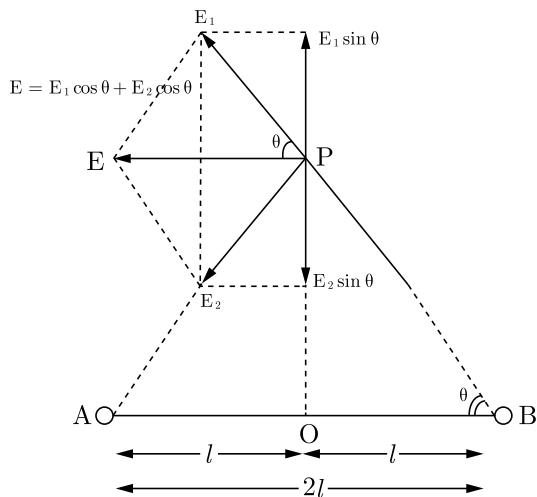
OD 2019

Consider an electric dipole AB . The charges $-q$ and $+q$ of dipole are situated at A and B respectively as shown in the figure. The separation between the charges is $2l$.

Electric dipole moment, $p = q \cdot 2l$

The direction of dipole moment is from $-q$ to $+q$.

At a point of equatorial line. Consider a point P on broad side on the position of dipole formed of charges $+q$ and $-q$ at separation $2l$. The distance of point P from mid point O of electric dipole is r . Let $\vec{E}_1$ and $\vec{E}_2$ be the electric field strengths due to charges $+q$ and $-q$ of electric dipole.



From figure, $AP = BP = \sqrt{r^2 + l^2}$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + l^2} \text{ (along } B \text{ to } P) \quad \dots(1)$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + l^2} \text{ (along } P \text{ to } A) \quad \dots(2)$$

Clearly $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude, i.e., $\vec{E}_1 = \vec{E}_2$ or $E_1 = E_2$

To find the resultant of $\vec{E}_1$ and $\vec{E}_2$ we resolve them into rectangular components.

Components of $\vec{E}_1$ parallel to $AB = E_1 \cos \theta$, in the direction of $\vec{BA}$

Component of $\vec{E}_1$ perpendicular to $AB = E_1 \sin \theta$ along OP component of $\vec{E}_2$ parallel to $AB = E_2 \cos \theta$ in the direction $\vec{BA}$.

Component of $\vec{E}_2$ perpendicular on $AB = E_2 \sin \theta$ along PO .

Clearly, components of $\vec{E}_1$ and $\vec{E}_2$ perpendicular to $AB : E_1 \sin \theta$ and $E_2 \sin \theta$ being equal and opposite cancel each other, while the components of $\vec{E}_1$ and $\vec{E}_2$ parallel to $AB : E_1 \cos \theta$ and $E_2 \cos \theta$, being in the same direction add up and give the resultant electric field whose direction is parallel to $\vec{BA}$.

Hence resultant electric field at p is

$$E = E_1 \cos \theta + E_2 \cos \theta$$

But

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + l^2)}$$

From the figure,

$$\begin{aligned}\cos \theta &= \frac{OB}{PB} = \frac{l}{\sqrt{r^2 + l^2}} = \frac{l}{(r^2 + l^2)^{1/2}} \\ E &= 2E_1 \cos \theta\end{aligned}$$

$$\begin{aligned}&= 2 \times \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + l^2)} \frac{l}{(r^2 + l^2)^{1/2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2ql}{(r^2 + l^2)^{3/2}}\end{aligned}$$

But,

$$q \cdot 2l = p = \text{electric dipole moment}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + l^2)^{3/2}} \quad \dots(3)$$

If dipole is infinitesimal and point P is far away, we have $l \ll r$, so l^2 may be neglected as compared to r^2 and so equation (3) gives

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

i.e., electric field strength due to a short dipole at broadside on position

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}, \quad \dots(4)$$

in the direction parallel to $\vec{BA}$.

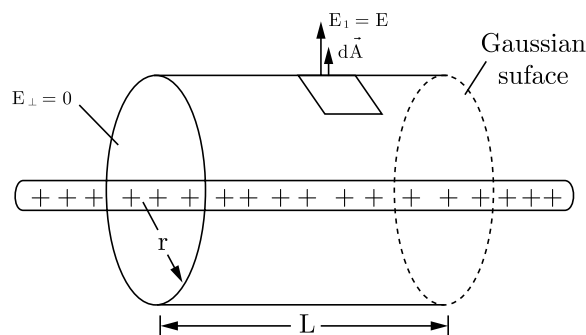
Its direction is parallel to the axis of dipole from positive to negative charge.

92. Using Gauss's theorem to obtain the expression for the electric field intensity at a point due to an infinitely long, thin, uniformly charged straight wire. Plot a graph showing the variation of electric field E with r .

Ans :

OD 2013, Comp 2008

Consider an infinitely long, thin wire charged positively and having uniform linear charge density λ . Symmetry of the charge distribution shows that $\vec{E}$ must be perpendicular to the line charge and directed outwards. As a result of this symmetry, we consider a Gaussian surface in the form of a cylinder with arbitrary radius r and arbitrary length L with its ends perpendicular to the wire as shown in figure below.



For the cylindrical part of this Gaussian surface, $\vec{E}$ is constant in magnitude and perpendicular to the surface at each point. Furthermore, the flux through the ends of the Gaussian cylinder is zero, since $\vec{E}$ is parallel to these surface, i.e., there is no normal component of electric field at these faces. Therefore, by the definition of electric flux, we have

$$\phi = E \times A \quad \dots(1)$$

where A is the curved surface area of the cylinder

$$\text{or} \quad \phi = E \times 2\pi rL \quad \dots(2)$$

By Gauss's law, the electric flux is given by

$$\phi = \frac{q}{\epsilon_0} = \frac{\lambda L}{\epsilon_0} \quad \dots(3)$$

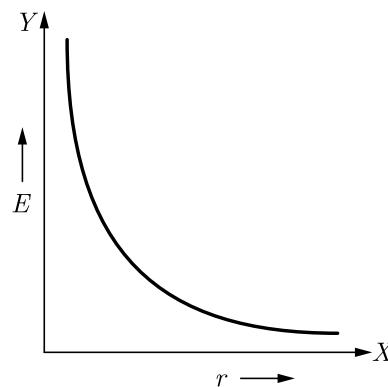
From equations (2) and (3) we have

$$E \times 2\pi rL = \frac{\lambda L}{\epsilon_0} \quad \dots(4)$$

$$\text{or} \quad E = \frac{\lambda}{2\pi\epsilon_0 r} = 2K\frac{\lambda}{r} \quad \dots(5)$$

This is the expression for the electric field due to an infinitely long thin wire.

The graph is shown as :



93. A charge is distributed uniformly over a ring of radius a . Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.

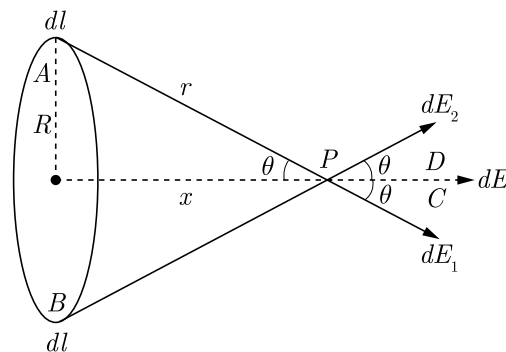
Ans :

Foreign 2018

Consider a point P on the axis of uniformly charged ring at a distance x from its centre O . Point P is at distance $r = \sqrt{R^2 + x^2}$ from each element dl of rings. If q is total charge on ring, then, charge per metre length,

$$\lambda = \frac{q}{2\pi R}$$

The ring may be supposed to be formed of a large number of ring elements.



Consider an element of length dl situated at A .

The charge on element, $dq = \lambda dl$

Hence the electric field at P due to this element

$$\begin{aligned} dE_1 &= \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2}, \text{ along } \vec{PC} \end{aligned}$$

The electric field strength due to opposite symmetrical element of length dl at B is

$$d\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{dg}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2}, \text{ along } \overrightarrow{PD}$$

If we resolve $\overrightarrow{dE}_1$ and $\overrightarrow{dE}_2$ along the axis and perpendicular on axis, we note that the components perpendicular to axis are oppositely directed and so get cancelled, while those along the axis are added up. Hence, due to symmetry of the ring, the electric field strength is directed along the axis.

The electric field strength due to charge element of length dl , situated at A , along the axis will be

$$\begin{aligned} dE &= dE_1 \cos \theta \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} \cos \theta \end{aligned}$$

But $\cos \theta = \frac{x}{r}$

$$\begin{aligned} dE &= \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{r^3} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2x}{r^3} dl \end{aligned}$$

The resultant electric field along the axis will be obtained by adding fields due to all elements of the ring, i.e.,

$$\begin{aligned} E &= \int \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{r^3} dl \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{r^3} \int dl \end{aligned}$$

But, $\int dl = \text{whole length of ring} = 2\pi R$

and

$$r = (R^2 + x^2)^{1/2}$$

Hence,

$$E = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(R^2 + x^2)^{3/2}} 2\pi R$$

As, $\lambda = \frac{q}{2\pi R}$ we have

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{q}{2\pi R}\right)x}{(R^2 + x^2)^{3/2}} 2\pi R \\ &= \frac{1}{4\pi\epsilon_0} \frac{qx}{(R^2 + x^2)^{3/2}} \text{ along the axis} \end{aligned}$$

At large distance, i.e., $x \gg R$,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

i.e., the electric field due to a point charge at a distance x .

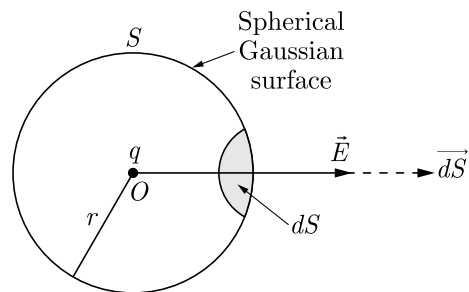
For points on the axis at distances much larger than the radius of ring, the ring behaves like a point charge.

94. Deduce Coulomb's law from Gauss law.

Ans :

OD 2017, SQP 2003

As shown in figure consider an isolated positive point charge q . We select a spherical surface S of radius r centred at charge q as the Gaussian surface.



By symmetry, $\vec{E}$ has same magnitude at all points on S . Also, $\vec{E}$ and $\vec{dS}$ at any point on S are directed radially outward. Hence, flux through area $\vec{dS}$ is

$$d\phi_E = \vec{E} \cdot \vec{dS} = E dS \cos 0^\circ = E dS$$

Net flux through closed surface S is,

$$\begin{aligned} \phi_E &= \oint_S \vec{E} \cdot \vec{dS} = \oint_S E dS = E \oint_S dS \\ &= E \times \text{total surface area of } S \\ &= E \times 4\pi r^2 \end{aligned} \quad \dots(1)$$

Using Gauss's theorem,

From Eq. (1), we get,

$$\begin{aligned} \phi_E &= \frac{q}{\epsilon_0} \\ E \times 4\pi r^2 &= \frac{q}{\epsilon_0} \\ E &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \end{aligned}$$

The force on the point charge q_0 if placed on surface S will be,

$$F = q_0 E = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2}$$

This proves the Coulomb's law.

95. (i) State Gauss theorem.
(ii) Apply this to obtain the expression for the electric field intensity at a point due to an infinitely long, thin, uniformly charged straight wire.

Ans :

Delhi 2009

(i) Gauss' Theorem

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $1/\epsilon_0$ times the net charge enclosed within the surface.

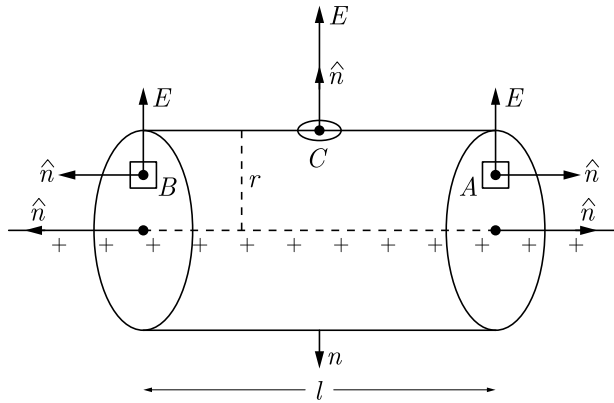
$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i = \frac{q}{\epsilon_0}$$

where, $q = \sum_{i=1}^n q_i$ is the algebraic sum of all the charges inside the closed surface.

Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\epsilon_0}$ times the total charge within the surface, regardless of how the charges may be distributed.

(ii) Electric Field Intensity Due to an Infinity Long Thin Straight Charged Wire

Consider an infinitely long with straight wire with uniform linear charge density λ .



From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance r .

From Gauss' law,

$$\phi_E = \oint_S E \cdot dS = \frac{q}{\epsilon_0}$$

$$\text{Now, } \phi_E = \oint_S E \cdot dS = \oint_S E \cdot \hat{n} dS$$

$$= \oint_A E \cdot \hat{n} dS + \oint_B E \cdot \hat{n} dS + \oint_C E \cdot \hat{n} dS$$

$$\begin{aligned} \text{Hence, } \oint_S E \cdot dS &= \oint_A E dS \cos 90^\circ + \oint_B E dS \cos 90^\circ \\ &\quad + \oint_C E dS \cos 0^\circ \\ &= \oint_C E dS = E(2\pi r l) \end{aligned}$$

Charge enclosed in the cylinder,

$$q = \lambda l$$

$$\text{Hence, } E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$\text{or } E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\text{Vectorially, } E = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$$

The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Thus, electric field E due to the linear charge is inversely proportional to the distance r from the linear charge.

96. (i) Define electric dipole moment. Is it a scalar or a vector?
 (ii) Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

Ans :

SQP 2013, Comp 2011

(i) Electric Dipole Moment

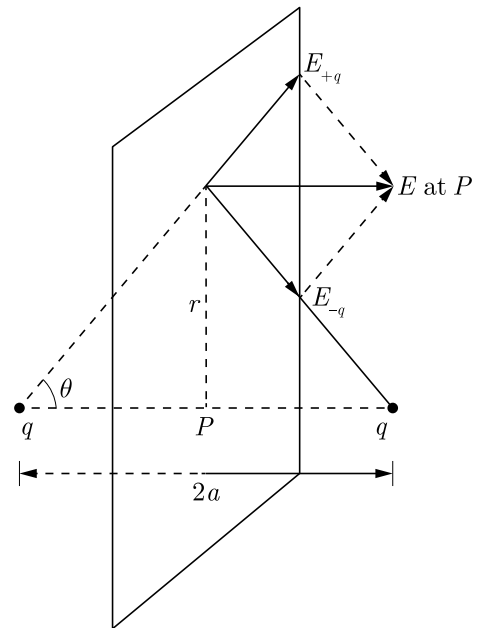
The strength of an electric dipole is measured by the quantity electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges.

Electric dipole moment,

$$p = q \times d$$

It is a vector quantity.

In vector form it is written as $p = \vec{q} \times \vec{d}$, where the direction of $\vec{d}$ is from negative charge to positive charge.

(ii) Electric field of dipole at points on the equatorial plane

The magnitudes of the electric field due to the two charges $+q$ and $-q$ are given by,

$$E_{+q} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r^2 + a^2} \quad \dots(1)$$

$$E_{-q} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r^2 + a^2} \quad \dots(2)$$

$$\text{Hence, } E_{+q} = E_{-q}$$

The direction of E_{+q} and E_{-q} are as shown in the figure.

The components normal to the dipole axis cancel away.

The components along the dipole axis add up.

Hence, Total electric field,

$$E = -(E_{+q} - E_{-q}) \cos \theta \hat{p}$$

[Negative sign shows that field is opposite to $\hat{p}$]

$$E = \frac{2qa}{4\pi\epsilon_0(r^2 + a^2)^{3/2}} \hat{p} \quad \dots(3)$$

At large distances ($r \gg a$), this reduces to

$$E = \frac{2qa}{4\pi\epsilon_0 r^3} \hat{p} \quad \dots(4)$$

Since,

$$\vec{p} = q \times 2a\hat{p}$$

Hence,

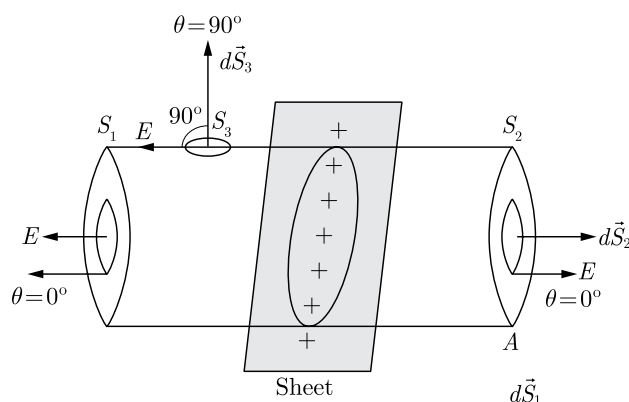
$$E = \frac{-\vec{p}}{4\pi\epsilon_0 r^3} (r \gg a)$$

97. Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

Ans :

Foreign 2016

Let electric charge be uniformly distributed over the surface of a thin, non-conducting infinite sheet. Let the surface charge density (i.e., charge per unit surface area) be σ . We need to calculate the electric field strength at any point distance r from the sheet of charge.



To calculate the electric field strength near the sheet, we now consider a cylindrical Gaussian surface bounded by two plane faces A and B lying on the opposite sides and parallel to the charged sheet and the cylindrical surface perpendicular to the sheet (fig). By symmetry the electric field strength at every point on the flat surface is the same and its direction is normal outwards at the points on the two plane surfaces and parallel to the curved surface, $d\vec{S}_1$

Total electric flux,

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_{S_1} \vec{E} \cdot d\vec{S}_1 + \oint_{S_2} \vec{E} \cdot d\vec{S}_2 + \oint_{S_3} \vec{E} \cdot d\vec{S}_3$$

$$\begin{aligned} \text{or } \oint_S \vec{E} \cdot d\vec{S} &= \int_{S_1} E dS_1 \cos 0^\circ + \int_{S_2} E dS_2 \cos 0^\circ \\ &\quad + \int_{S_3} E dS_3 \cos 90^\circ \\ &= E \int dS_1 + E \int dS_2 \\ &= Ea + Ea = 2Ea \end{aligned}$$

Hence, Total electric flux = $2Ea$.

As σ is charge per unit area of sheet and a is the intersecting area, the charge enclosed by Gaussian surface = σa .

According to Gauss's theorem.

Total electric flux

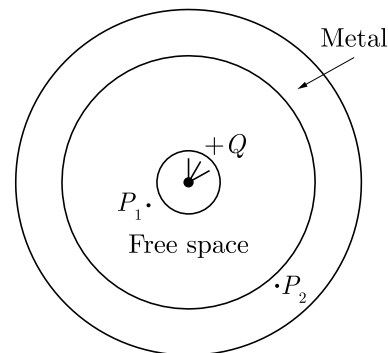
$$= \frac{1}{\epsilon_0} \times (\text{total charge enclosed by the surface})$$

$$\text{i.e., } 2Ea = \frac{1}{\epsilon_0} (\sigma a)$$

$$\text{Hence, } E = \frac{\sigma}{2\epsilon_0}$$

Thus, electric field strength due to an infinite flat sheet of charge is independent of the distance of the point and is directed normally away from the charge. If the surface charge density σ is negative the electric field is directed towards the surface charge.

98. (i) Define electric flux. Write its SI unit.
(ii) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2 .



- (iii) Draw the pattern of electric field lines in this arrangement.

Ans :

Delhi 2019, SQP 2011

- (i) Electric Flux over an area in an electric field represents the total number of electric field lines crossing this area and is given by the product of surface area and the component of electric field intensity normal to the area.

The SI unit of flux is NM^2C^{-2} .

- (iii) Let point P_1 is at distance R from the centre O . S_1 is the Gaussian surface, then according to Gauss's theorem

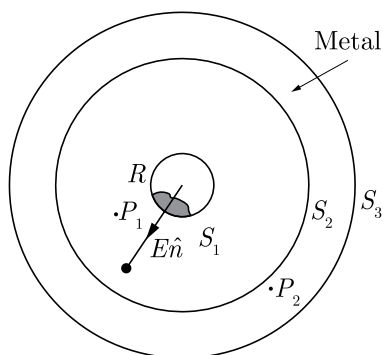
$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S \vec{E} \cdot \hat{n} ds = \frac{q}{\epsilon_0}$$

$$\text{or } E \oint_S dS = \frac{q}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 R^2}$$

$$\left[\text{As } \oint_S dS = 4\pi r^2 \right]$$

Inside the shell the charge is zero, so the field is also zero.

- (iii) The direction of electric field is shown in following figure.



99. (i) Using Gauss Theorem show mathematically that for any point outside the shell, the field due to a uniformly charged spherical shell is same as the entire charge on the shell concentrated at the centre.
- (ii) Why do you expect the electric field inside the shell to be zero according to this theorem?

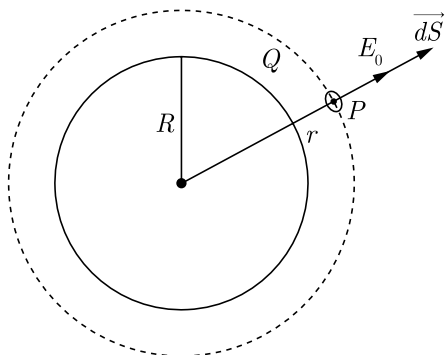
or

Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r , for $r > R$ and $r < R$.

Ans :

Comp 2019

- (i) Electric field intensity at a point outside a uniformly charged thin spherical shell : Consider a uniformly charged thin spherical shell of radius R carrying charge Q . To find the electric field outside the shell, we consider a spherical Gaussian surface of radius ($r > R$), concentric with given shell. If $\vec{E}$ is electric field outside the shell, then by symmetry electric field strength has same magnitude E_0 on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_1$ and $\vec{dS}$ is zero at each point. Hence, electric flux through Gaussian surface $\oint_s \vec{E}_0 \cdot \vec{dS}$



$$\oint_s = E_0 dS \cos 0$$

$$= E_0 \cdot 4\pi r^2$$

Now, Gaussian surface is outside the given charged shell, so charge enclosed by Gaussian surface is Q . Hence, by Gauss's theorem

$$\oint_s = \vec{E}_0 \cdot \vec{dS}$$

$$= \frac{1}{\epsilon_0} \times \text{charged enclosed}$$

$$E_0 4\pi r^2 = \frac{1}{\epsilon_0} \times Q \quad E_0 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Thus, electric field outside a charged thin spherical shell is the same as if the whole charge Q is concentrated at the centre.

If σ is the surface charge density of the spherical shell, then

$$Q = 4\pi R^2 \sigma \text{ coulomb}$$

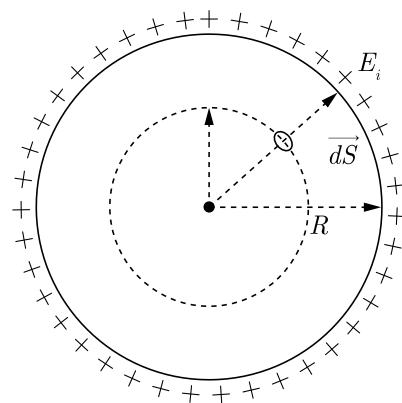
$$E_0 = \frac{1}{4\pi\epsilon_0} \frac{4\pi R^2 \sigma}{r^2}$$

$$= \frac{R^2 \sigma}{\epsilon_0 r^2}$$

- (ii) Electric field inside the shell (hollow charged conducting sphere) The charge resides on the surface of a conductor. Thus a hollow charged conductor is equivalent to a charged spherical shell. To find the electric field inside the shell, we consider a spherical Gaussian surface of radius ($r < R$), concentric with the given shell. If $\vec{E}$ is the electric field inside the shell, then by symmetry electric field strength has the same magnitude E_i on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_i$ and $\vec{dS}$ is zero at each point. Hence, electric flux through Gaussian surface

$$\int_s \vec{E}_i \cdot \vec{dS} = \int E_i dS \cos 0$$

$$= E_i \cdot 4\pi r^2$$



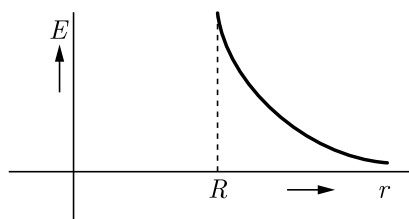
Now, Gaussian surface is inside the given charged shell, so charged shell, so enclosed by Gaussian surface is zero. Hence, by Gauss's theorem

$$\int_S \vec{E}_i \cdot d\vec{S} = \frac{1}{\epsilon_0} \times \text{charge enclosed}$$

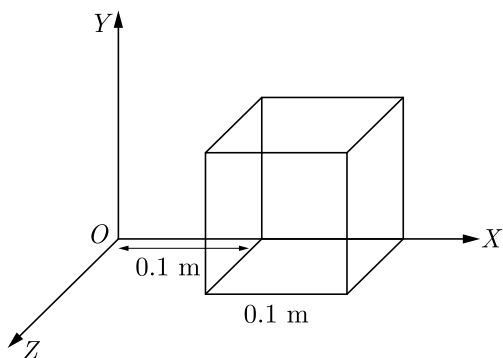
$$E_i 4\pi r^2 = \frac{1}{\epsilon_0} \times 0$$

$$E_i = 0$$

Thus, electric field at each point inside a charged thin and spherical shell is zero. The graph is shown in figure.



100. The electric field components due to a charge inside the cube of side 0.1 m are as shown :



$$E_x = ax, \text{ where } a = 500 \text{ N/C-m}$$

$$E_y = 0, E_z = 0$$

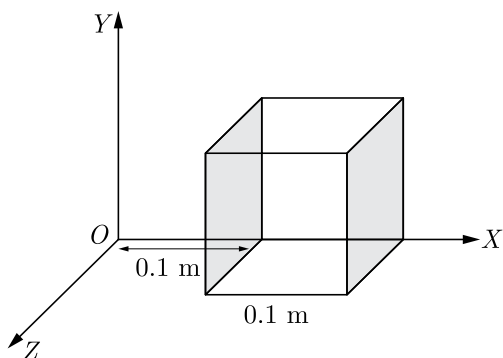
Calculate (i) the flux through the cube and (ii) the charge inside the cube.

Ans :

OD 2018, Comp 2006

- (i) Since the electric field has only an x - component for face perpendicular to x direction.

The angle between $\vec{E}$ and $\Delta \vec{S}$ is $\pm \pi/2$



Therefore, the flux $\phi = \vec{E} \cdot \Delta \vec{S}$ is separately zero and for each face of the cube except the two shaded ones. Now the magnitude of the electric field at the left face is

$$E_L = ax = \alpha(0.1) = 0.1\alpha$$

(as $x = 0.1$ at left face)

The magnitude of electric field at the right face is

$$E_R = ax = \alpha(2 \times 0.1) = 0.2\alpha$$

The corresponding flux are

$$\begin{aligned} \phi_L &= \vec{E}_L \cdot \Delta \vec{S} \\ &= \Delta S \vec{E}_L \cdot \vec{n}_L \\ &= E_L \Delta S \cos Q \quad (\text{as } \theta = 180^\circ) \\ &= -E_L \Delta S = -E_1(a^2) \\ &= -E_2(0.1)^2 \end{aligned}$$

Similarly ϕ_R for $\theta = 0^\circ$

$$\phi_R = E_R(a^2) = E_R(0.1)^2$$

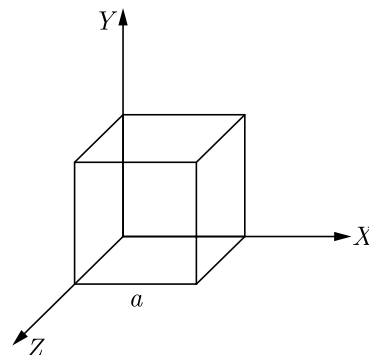
So, net flux through the cube

$$\begin{aligned} &= \phi_R + \phi_L \\ &= E_R(0.1)^2 - E_L(0.1)^2 \\ &= (0.1)^2 [0.2a - 0.1\alpha] \\ &= 0.01 \times 0.1 \times \alpha \\ &= 0.001 \times 500 \\ &= 0.5 \text{ Nm}^2\text{C}^{-1} \end{aligned}$$

101. (i) An electric dipole of dipole moment p consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field E due to the dipole at a distance x from the centre of the dipole on the axial line in terms of the dipole moment p . Hence, show that in the limit

$$x \gg a, E = \frac{2p}{(4\pi\epsilon_0 x^3)}$$

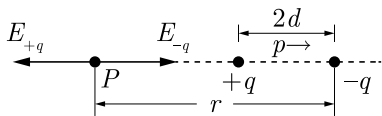
- (ii) Given the electric field in the region $E = 2xi$, find the net electric flux through the cube and the charge enclosed by it.



Ans :

OD 2015

(i) Electric field on an axial line of an electric dipole.



Let P be at distance r from the centre of the dipole on the side of charge $-q$.

Then, the electric field at point P due to charge $-q$ of the dipole is given by

$$E_{-q} = \frac{q}{4\pi\epsilon_0(r+a)^2} \hat{p}$$

where, $\hat{p}$ is the unit vector along the dipole axis (from $-q$ to $+q$)

Also, the electric field at point P due to charge $+q$ of the dipole is given by

$$E_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2} \hat{p}$$

The total field at point P is

$$\begin{aligned} E &= E_{+q} + E_{-q} \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} + \frac{1}{(r+a)^2} \right] \hat{p} \\ E &= \frac{q}{4\pi\epsilon_0} \cdot \frac{4ar}{(r^2 - a^2)^2} \hat{p} \\ r &= x \end{aligned} \quad (\text{given})$$

Since

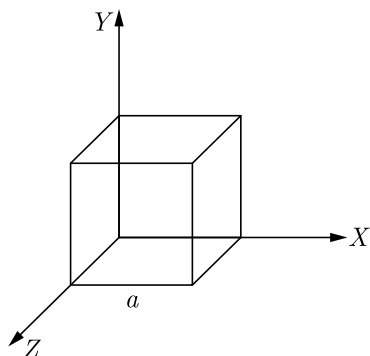
$$E = \frac{q}{4\pi\epsilon_0} \cdot \frac{4ax}{(x^2 - a^2)^2} \hat{p}$$

for $x \gg a$,

$$E = \frac{4qa}{4\pi\epsilon_0 x^3} \hat{p}$$

$$E = \frac{2p}{4\pi\epsilon_0 x^3}$$

(ii)



Since, the electrical field has only a component, for faces normal to x - direction, the angle between E and ΔS is $\pm \frac{\pi}{2}$.

Therefore, the flux is separately zero for each of the cube. The magnitude of the electric field at the left

face is

$$E_l = 0 \quad (\text{as, } x = 0 \text{ at the left face})$$

The magnitude of the electric field at the right face is

$$E_R = 3a \quad (\text{as, } x = 0 \text{ at the right face})$$

The corresponding fluxes are

$$\phi_L = \vec{E}_L \cdot \Delta \vec{S} = 0$$

$$\phi_R = \vec{E}_R \cdot \Delta \vec{S}$$

$$= E_R \Delta S \cos \theta$$

$$= E_R \Delta S \quad (\because \theta = 0^\circ)$$

$$\phi_R = E_R a^2$$

Net flux (ϕ) through the cube

$$= \phi_L + \phi_R$$

$$= 0 + E_R a^2 = E_R a^2$$

$$q = 2a(a)^2 = 2a^3$$

We can use Gauss's law to find the total charge q inside the cube.

$$\begin{aligned} \phi &= \frac{q}{\epsilon_0} \\ &= 2a^3 / \epsilon_0 \end{aligned}$$

102. (i) Derive an expression for the electric field E due to a dipole of length $2a$ at a point distance r from the centre of the dipole on the axial line.
(ii) Draw a graph of E versus r for $r \gg a$.

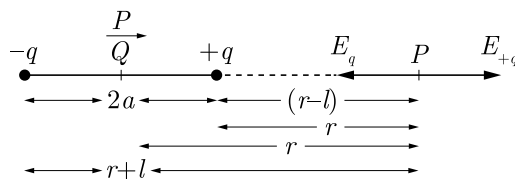
Ans :

Delhi 2017, OD 2007

(i) Expression for the electric field

Electric field at an axial point of a electric dipole. As shown in figure, consider an electric dipole consisting of charges $+q$ and $-q$ separated by distance $2a$ and placed in vacuum.

Let P be a point on the axial line at distance r from the centre O of the dipole on the side of the charge $+q$.



$$\vec{E}_{-q} = \frac{-q}{4\pi\epsilon_0(r+a)^2} \hat{p} \quad (\text{towards left})$$

where $\hat{p}$ is a unit vector along the dipole axis from $-q$ to $+q$. Electric field due to charge $+q$ at point p is

$$\vec{E}_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2} \hat{p} \quad (\text{towards right})$$

Hence, the resultant electric field at point p is

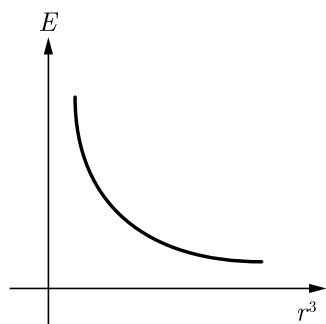
$$\begin{aligned}\vec{E}_{\text{axial}} &= \vec{E}_{+q} + \vec{E}_{-q} \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{p} \\ &= \frac{q}{4\pi\epsilon_0} \cdot \frac{4ar}{(r^2 - a^2)^2} \hat{p} \\ \vec{E}_{\text{axial}} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{4qar}{(r^2 - a^2)^2} \hat{p}\end{aligned}$$

Here, $p = q \times 2a = \text{dipole moment.}$

For $r \gg a$, a^2 can be neglected compared to r^2

Hence,
$$\vec{E}_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \hat{p}$$

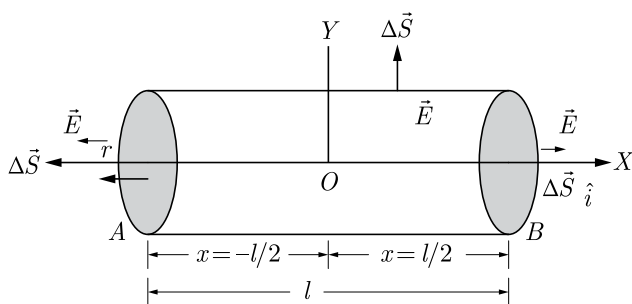
(ii)



- 103.** A uniform electric field $\vec{E} = E_x \hat{i}$ N/C for $x > 0$ and $\vec{E} = -E_x \hat{i}$ N/C for $x < 0$ are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the x -axis. Find out the net outward flux. Using Gauss's law write the expression for the net charge within the cylinder.

Ans :

SQP 2013



The ends of cylinder are at $-\frac{l}{2}$ and $\frac{l}{2}$ as clearly seen from figure.

We can see from figure, on the left face $\vec{E}$ and $\Delta \vec{S}$ are parallel. Therefore outward flux is

$$\begin{aligned}\phi_L &= \vec{E} \cdot \Delta \vec{S} = -E_x \hat{i} \cdot \Delta \vec{S} \\ &= +E_x \Delta S\end{aligned}$$

$$(\text{since } \hat{i} \cdot \Delta \vec{s} = -\Delta s)$$

On the right face, $\Delta \vec{E}$ and $\Delta \vec{S}$ are parallel, so

$$\begin{aligned}\phi_R &= \vec{E} \cdot \Delta \vec{S} = E_x \hat{i} \cdot \Delta \vec{S} \\ &= E_x \Delta S\end{aligned}$$

For any point on the side of the cylinder, $\vec{E}$ is perpendicular to $\Delta \vec{S}$, hence $\Delta \vec{E} \cdot \Delta \vec{S} = 0$. So, the flux out of the side of the cylinder is zero.

So net outward flux

$$\begin{aligned}\phi &= \phi_L + \phi_R \\ &= E_x \cdot \Delta S + E_x \cdot \Delta S \\ &= 2E_x \Delta S\end{aligned}$$

Here, $\Delta S = \pi r^2$

So, net outward flux $= 2E_x \pi r^2 = 2\pi E_x r^2$

(ii) The net charge within the cylinder can be found by using Gauss's law which gives

$$\begin{aligned}q &= \epsilon_0 \phi \\ &= 2\pi \epsilon_0 E_x r^2\end{aligned}$$

NUMERICAL QUESTIONS

- 104.** Consider two identical point charges located at points $(0, 0)$ and $(a, 0)$.

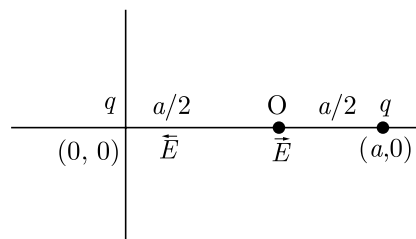
- Is there a point on the line joining them at which the electric field is zero.
- Is there a point on the line joining them at which the electric potential is zero?

Justify your answer for each case.

Ans :

Delhi 2023

If at a point electric field due to two charges are equal and opposite, then electric field at that point is zero.



Now, here both charges are equal or identical, then magnitude of the electric field at the mid-point of the line joining the charges is the same but opposite.

So, they will cancel out each other and the net electric field is zero.

- 105.** Find the electric dipole moment electron and a proton which distance is 4.3 nm apart.

Ans :

Delhi 2015

Charge, $q = 1.6 \times 10^{-19}$

$d = 4.3 \times 10^{-9} \text{ cm}$

Dipole moment, $p = q \times d$
 $= 1.6 \times 10^{-19} \times 4.3 \times 10^{-9}$
 $= 6.8 \times 10^{-28} \text{ C-m}$

- 106.** An infinite line charge produces a field of $9 \times 10^4 \text{ N/C}$ at a distance of 2 cm. Calculate the linear charge density.

Ans :

SQP 2008

Electric Field, $E = 9 \times 10^4 \text{ N/C}$

$r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

and $\lambda = ?$

As we know that,

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\lambda = 2\pi\epsilon_0 r E$$

$$= \frac{1}{2 \times 9 \times 10^9} \times 2 \times 10^{-2} \times 9 \times 10^4$$

$$= 10^{-7} \text{ C m}^{-1}$$

- 107.** What is the total charge of a system containing five charges +1, +2, -3, +4 and -5 in some arbitrary unit?

Ans :

Comp 2018, SQP 2009

As charges are additive in nature, i.e. the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system, i.e.

$$q_{\text{net}} = q_1 + q_2 + q_3 + q_4 + q_5$$

Hence, Total charge $= +1 + 2 - 3 + 4 - 5$

$= -1$ in the same unit

- 108.** How many electrons are there in one coulomb of negative charge?

Ans :

OD 2019

The negative charge is due to the presence of excess electrons. Because an electron has a charge whose magnitude is $e = 1.6 \times 10^{-19} \text{ C}$, the number of electrons is equal to the charge q divided by the charge e on each electron.

Therefore, the number of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{18} \text{ electrons}$$

- 109.** The dielectric constant of water is 80. What is its permittivity?

Ans :

SQP 2013

Given, $K = 80$

We have, $K = \frac{\epsilon_m}{\epsilon_0}$

Hence, $\epsilon_m = K\epsilon_0$

[Since, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$]

$= 80 \times 8.85 \times 10^{-12}$

$= 708 \times 10^{-12}$

$= 7.08 \times 10^{-10} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

- 110.** A metal sphere has a charge of $-6 \mu\text{C}$. When 5×10^{12} electrons are removed from the sphere, what would be net charge on it?

Ans :

Delhi 2015

Here, $q_1 = -6 \mu\text{C}$

and, $q_2 = ne = 5 \times 10^{12} \times (1.6 \times 10^{-19})$
 $= 8.0 \times 10^{-7} \text{ C} = 0.8 \times 10^{-6} \text{ C}$
 $= 0.8 \mu\text{C}$

- 111.** An infinite number of charges each equal to q are placed along X-axis at $x = 1, x = 2, x = 4, x = 8$ and so on. Find the electric field at the point $x = 0$ due to this set up of charges.

Ans :

Foreign 2016

At the point $x = 0$, the electric field due to all the charges are in the same negative x - direction and hence get added up, i.e.

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \dots \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{1 - \frac{1}{4}} \right] = \frac{q}{3\pi\epsilon_0}$$

This electric field is along negative X-axis.

- 112.** Calculate the amount of work done in turning an electric dipole of dipole moment $3 \times 10^{-8} \text{ cm}$ from its position of unstable equilibrium to the position of stable equilibrium in a uniform electric field of intensity 10^3 N/C .

Ans :

Delhi 2010

For unstable equilibrium, the angle between $\vec{p}$ and $\vec{E}$ is $\theta_1 = 180^\circ$

Finally, for stable equilibrium, $\theta_2 = 0^\circ$

Required work done

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

$$= 3 \times 10^{-8} \times 10^3 (\cos 180^\circ - \cos 0^\circ)$$

$$[\text{since } \cos 180^\circ = -1, \cos 0^\circ = +1]$$

$$W = -6 \times 10^{-5} \text{ J}$$

113. A charge $q = 1 \mu\text{C}$ is placed at point $(1 \text{ m}, 2 \text{ m}, 4 \text{ m})$. Find the electric field at point $P(0 \text{ m}, -4 \text{ m}, 3 \text{ m})$.

Ans :

OD 2011

Here, $r_q = \hat{i} + 2\hat{j} + 4\hat{k}$

and $r_p = -4\hat{j} + 3\hat{k}$

Hence, $r_p - r_q = -\hat{i} - 6\hat{j} - \hat{k}$

$$|r_p - r_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2}$$

$$= \sqrt{38} \text{ m}$$

Now, Electric field, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|r_p - r_q|^3} (r_p - r_q)$

Substituting the values, we get

$$E = \frac{(9.0 \times 10^9)(1.0 \times 10^{-6})}{(38)^{3/2}} (-\hat{i} - 6\hat{j} - \hat{k})$$

$$= (-38.42\hat{i} - 230.52\hat{j} - 38.42\hat{k}) \text{ N/C}$$

114. What is the electric dipole moment of an electric dipole made up of two opposite charges having magnitude $+3.2 \times 10^{-19} \text{ C}$ and $-3.2 \times 10^{-19} \text{ C}$ separates by a distance $2.4 \times 10^{-10} \text{ m}$?

Ans :

Foreign 2013, OD 2005

Magnitude of charge, $q = 3.2 \times 10^{-19} \text{ C}$

Length of dipole, $2l = 2.4 \times 10^{-10} \text{ m}$

We know that,

Electric dipole moment,

$$p = q \times 2l$$

$$= 3.2 \times 10^{-19} \times 2.4 \times 10^{-10}$$

$$= 7.68 \times 10^{-29} \text{ C-m}$$

115. Two point charges having equal charges separated by 1 m distance experience a force of 8 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given: $K_{\text{water}} = 80$)

Ans :

Delhi 2018

Two point charges system is taken from air to water keeping other variables (e.g. distance, magnitude of charge) unchanged. So, the only factor which may affect the interacting force is dielectric constant of medium.

Force acting between two point charges.

$$F = \frac{1}{4\pi\epsilon_0 K} \frac{|q_1 q_2|}{r^2}$$

or

$$F \propto \frac{1}{K}$$

$$\frac{F_{\text{air}}}{F_{\text{Medium}}} = K$$

$$\frac{8}{F_{\text{water}}} = 80$$

$$F_{\text{water}} = \frac{8}{80} = \frac{1}{10} \text{ N}$$

116. Two charges of $+25 \times 10^{-9} \text{ C}$ and $-25 \times 10^{-9} \text{ C}$ are placed 6 m apart. Find the electric field at a point 4 m from the centre of the electric dipole

- On axial line
- On equatorial line.

Ans :

OD 2019

Given, $q = 25 \times 10^{-9} \text{ C}$

$$2r = 6 \text{ m}, r = 4 \text{ m}$$

$$p = q(2r) = 25 \times 10^{-9} \times 6$$

$$= 1.5 \times 10^{-7} \text{ C-m}$$

Now, $E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$

$$= \frac{9 \times 10^9 \times 2 \times 1.5 \times 10^{-7} \times 4}{(4^2 - 3^2)^2}$$

$$= \frac{2700 \times 4}{49} = 220.4 \text{ NC}^{-1}$$

Hence, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$

$$= \frac{9 \times 10^9 \times 1.5 \times 10^{-7}}{(4^2 + 3^2)^{3/2}} = \frac{1350}{125}$$

$$= 10.8 \text{ N-C}^{-1}$$

117. Two charges q_1 and q_2 of $0.1 \mu\text{C}$ and $-0.1 \mu\text{C}$ respectively are $10 \text{ \AA}$ apart. What is the electric field at a point on the line joining them at a distance of 10 cm from their mid-point?

Ans :

Comp 2006, SQP 2015

Here, $q_1 = q_2 = q = 0.1 \mu\text{C}$

$$= 10^{-13} \text{ C} \quad [\text{in magnitude}]$$

Length of the electric dipole formed by these charges,

$$2a = 10^{-10} \text{ m} = 10^{-9} \text{ m}$$

Thus, electric dipole moment,

$$p = 2aq = 10^{-13} \times 10^{-9}$$

$$= 10^{-22} \text{ C-m}$$

Distance of the point under consideration on the axial line from the mid-point,

$$r = 10 \text{ cm} = 0.1 \text{ m}$$

Since, $a \ll r$, electric field at a point on the axial line,

$$E = k_e \frac{2p}{r^3}$$

$$= (9 \times 10^9) \frac{2 \times 10^{-22}}{(0.1)^3}$$

$$= 18 \times 10^{-10} \text{ N/C}$$

118. A sphere of lead of mass 10 g has net charge $-2.5 \times 10^{-9} \text{ C}$.

- (i) Find the number of excess electrons on the sphere.
 (ii) How many excess electrons are per lead atom?
 Atomic number of lead is 82 and its atomic mass is 207 g/mol.

Ans :

Foreign 2020

- (i) The charge of an electron

$$= -1.6 \times 10^{-19} \text{ C}$$

$$\text{Net charge on sphere} = -2.5 \times 10^{-9} \text{ C}$$

So, the number of excess electrons

$$= \frac{-2.5 \times 10^{-9} \text{ C}}{-1.6 \times 10^{-19} \text{ C}}$$

$$= 1.5 \times 10^{10} \text{ electrons}$$

- (ii) Atomic number of lead is 82.

Atomic mass of lead is 207 g/mol.

Hence, 10 g of lead will have

$$\frac{10 \text{ g}}{207 \text{ g/mol}} \times 6.02 \times 10^{23} \text{ atoms/mol}$$

$$= 2.91 \times 10^{22} \text{ atoms}$$

Hence, the number of excess electrons per atom

$$= \frac{1.56 \times 10^{10}}{2.91 \times 10^{22}}$$

$$= 5.36 \times 10^{-13} \text{ electrons}$$

119. Two insulated charged copper spheres A and B each having charge of $6.5 \times 10^{-7} \text{ C}$ are separated by a distance 50 cm. If they are placed in water of dielectric constant 80, then find the electrostatic force of repulsion between them.

Ans :

OD 2005, Comp 2014

Charge on sphere A, $q_1 = 6.5 \times 10^{-7} \text{ C}$

Charge on sphere B, $q_2 = 6.5 \times 10^{-7} \text{ C}$

Distance between the charges, $r = 50 \text{ cm}$
 $= 0.5 \text{ m}$

Dielectric constant, $K_m = 80$

Electrostatic force of repulsion (due to similar charges), between two spheres when they are placed in a dielectric medium,

$$F = \frac{k}{K_m} \cdot \frac{q_1 q_2}{r^2}$$

where, k = Electrostatic force constant

$$= 9 \times 10^9 \text{ N-m}^2\text{-C}^{-2}.$$

$$= \frac{(9 \times 10^9)}{80} \times \frac{(6.5 \times 10^{-7}) \times (6.5 \times 10^{-7})}{(0.5)^2}$$

$$= 1.9 \times 10^{-4} \text{ N}$$

120. The opposite corners of a square carry Q charge each and the other two opposite corners of the same square carry q charge each. If the resultant force on q is zero, how are Q and q related?

Ans :

SQP 2010

Let each side of square be x .

$$\text{Diagonal} = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$F_1 = F_2 = \frac{Qq}{4\pi\epsilon_0 x^2}$$

and

$$F_3 = \frac{qq}{4\pi\epsilon_0 (x\sqrt{2})^2} = \frac{q^2}{2 \times 4\pi\epsilon_0 x^2}$$

As, F_1 and F_2 are perpendicular to each other, their resultant force.

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{F_1^2 + F_1^2}$$

$$F = F_1 \sqrt{2}$$

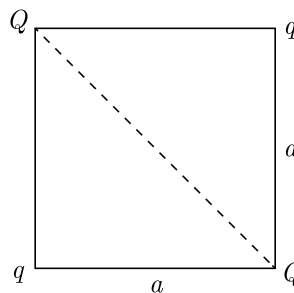
As, net force on q is zero. Therefore,

$$F_1 \sqrt{2} = -F_3$$

$$\frac{Qq\sqrt{2}}{4\pi\epsilon_0 x^2} = \frac{-q^2}{2 \times 4\pi\epsilon_0 x^2}$$

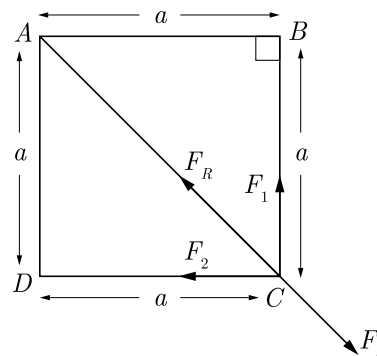
$$q = -2\sqrt{2} Q$$

121. In the given figure, if net force on Q is zero then find the value of $\frac{Q}{q}$.



Ans :

According to the question,



In ΔABC , According to Pythagorean theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{a^2 + a^2} = \sqrt{2} a$$

Resultant force, $F_R = \sqrt{F_1^2 + F_2^2} \quad \dots(1)$

According to coulomb's law,

$$F_1 = \frac{KQq}{a^2}$$

Similarly, $F_2 = \frac{KQq}{a^2}$

$$F_3 = \frac{KQQ}{(\sqrt{2} a)^2} = \frac{KQ^2}{2a^2}$$

From equation (1),

$$\begin{aligned} F_R &= \sqrt{\left(\frac{KQq}{a^2}\right)^2 + \left(\frac{KQq}{a^2}\right)^2} \\ &= \sqrt{\left(\frac{KQq}{a^2}\right)^2 (1+1)} = \frac{KQq\sqrt{2}}{a^2} \end{aligned}$$

Since, $F_R - F_3 = 0$

$$F_R = F_3$$

$$\frac{KQq\sqrt{2}}{a^2} = \frac{KQ^2}{2a^2}$$

$$\frac{Q}{q} = 2\sqrt{2}$$

- 122.** A particle of mass 10^{-3} kg and charge $5 \mu\text{C}$ enters into a uniform electric field of $2 \times 10^5 \text{ NC}^{-1}$, moving with a velocity of 20 ms^{-1} in a direction opposite to that of the field. Calculate the distance it would travel before coming to rest.

Ans :

Foreign 2012, OD 2002

Given, $m = 10^{-3} \text{ kg}$, $q = 5 \times 10^{-6} \text{ C}$,

$$E = 2 \times 10^5 \text{ N/C},$$

$$u = 20 \text{ m/s}, v = 0$$

As it enters opposite to the field, so particle will retard.

Acceleration, $a = \frac{qE}{m}$

$$= \frac{5 \times 10^{-6} \times 2 \times 10^5}{10^{-3}} = 10^2 \text{ m/s}^2$$

Using $v^2 = u^2 - 2as$, we get

$$0 = (20)^2 - 2 \times 1000 \times s$$

or $s = \frac{400}{2000} = \frac{1}{5} = 0.2 \text{ m}$

- 123.** An electric dipole of length 4 cm when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3} \text{ Nm}$. Calculate the potential energy of the dipole if it has charge $\pm 8 \text{ nC}$.

Ans :

OD 2014

Given, length $2a = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$

Angle, $\theta = 60^\circ$

Torque, $t = 4\sqrt{3} \text{ Nm}$

Charge, $Q = 8 \times 10^{-9} \text{ C}$

Electric field,

$$\begin{aligned} E &= \frac{t}{Q(2a)\sin\theta} \\ &= \frac{4\sqrt{3}}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60^\circ} \text{ N/C} \end{aligned}$$

Potential energy,

$$C = -pE\cos\theta = Q(2a)E\cos\theta$$

$$\begin{aligned} U &= -8 \times 10^{-9} \times 4 \times 10^{-2} \\ &\quad \times \frac{4\sqrt{3} \times \cos 60^\circ}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60^\circ} \\ &= \frac{-4\sqrt{3}}{\sqrt{3}} \text{ J} = -4 \text{ J} \end{aligned}$$

- 124.** Two point charge A and B of charge $+4 \mu\text{C}$ and $-1 \mu\text{C}$ are placed in air at a distance 1 m apart. What is the distance of the point from the charge B on the line joining the charges, where the resultant electric field is zero.

Ans :

Delhi 2001, OD 2012

Charge at A , $q_A = +4 \mu\text{C}$

Charge at B , $q_B = -1 \mu\text{C}$

Distance between the charges = 1 m.

If x is the distance between the point (where, resultant electric field is zero) and charge B , then distance between this point and charge

$$A = 1 + x$$

Electric field at this point due to charge A ,

$$\begin{aligned} E_1 &= k \cdot \frac{q_A}{r_1^2} = k \cdot \frac{4}{(1+x)^2} \\ &= \frac{4k}{(1+x)^2} \end{aligned}$$

Similarly, electric field at this point due to charge B ,

$$E_2 = k \cdot \frac{q_B}{r_2^2} = k \cdot \frac{(-1)}{x^2} = -\frac{k}{x^2}$$

Since the resultant electric field is zero. Therefore,

$$\begin{aligned} 0 &= \frac{4k}{(1+x)^2} - \frac{k}{x^2} \\ \frac{1}{x^2} &= \frac{4}{(1+x)^2} \\ \frac{1}{x} &= \frac{2}{1+x} \\ 2x &= 1+x = 1 \text{ m} \end{aligned}$$

125. An electric dipole is held in a uniform electric field.

- Show that the net force acting on it is zero.
- The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180° .

Ans :

Delhi 2018

- The dipole moment of dipole is

$$|\vec{p}| = q \times (2a)$$

$$\text{Force on } -q \text{ at } A = -q\vec{E}$$

$$\text{Force on } +q \text{ at } B = +q\vec{E}$$

$$\text{Net force on the dipole} = q\vec{E} - q\vec{E} = 0$$

- Work done on dipole

$$W = \Delta U$$

$$= pE(\cos\theta_1 - \cos\theta_2)$$

$$= pE(\cos 0^\circ - \cos 180^\circ) = 2pE$$

126. An electric dipole consists of two opposite charges each of 10^{-4} C separated by 0.5 cm. If dipole is placed in an external uniform field of 10^6 N-C $^{-1}$, how much work is done in rotating the dipole through 180° , (starting from the position $\theta = 0^\circ$)?

Ans :

Delhi 2019

Magnitude of each charge on dipole,

$$q = 10^{-4} \text{ C}$$

Distance between the charge,

$$2a = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$$

Electric field, $E = 10^6$ N-C $^{-1}$

Initial angle of dipole with electric field, $\theta_1 = 0^\circ$.

and final angle of dipole with electric field, $\theta_2 = 180^\circ$.

Electric dipole moment,

$$p = q \times 2a$$

$$= 10^{-4} \times (0.5 \times 10^{-2})$$

$$= 0.5 \times 10^{-6} \text{ C-m}$$

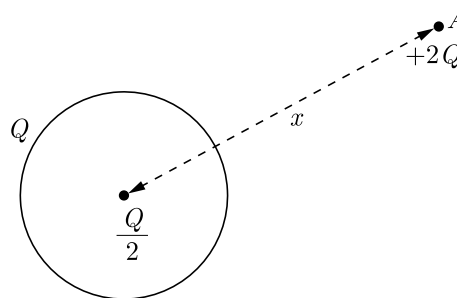
Therefore, work done in rotating the electric dipole in uniform electric field,

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

$$= (0.5 \times 10^{-6}) \times 10^6 \times (\cos 0^\circ - \cos 180^\circ)$$

$$= 0.5 \times [1 - (-1)] = 1$$

127. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at the centre C and other charge $+2Q$ is placed outside the shell at A at a distance X from the centre as shown in the figure.



- Find the electric flux through the shell.
- State the law used.
- Find the force on the charges at the centre C of the shell and at the point A .

Ans :

Foreign 2016

- Electric flux through a Gaussian surface,

$$\phi = \frac{\text{Total enclosed charge}}{\epsilon_0}$$

$$\text{Total enclosed charge} = Q + \frac{Q}{2} = \frac{3Q}{2}$$

Hence electric flux through the shell

$$\phi = \frac{3Q}{2\epsilon_0}$$

- Gauss's Law** : Electric flux through a Gaussian surface is $\frac{1}{\epsilon_0}$ times the net charge enclosed within it.

Mathematically,

$$\oint \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \times q$$

- Electric field or net charge inside the spherical conducting shell is zero. Hence, the force on charge $\frac{Q}{2}$ is zero.

Force on charge at A ,

$$F_A = \frac{1}{4\pi\epsilon_0} \frac{2Q \times \left(Q + \frac{Q}{2}\right)}{x^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{3Q^2}{x^2}$$

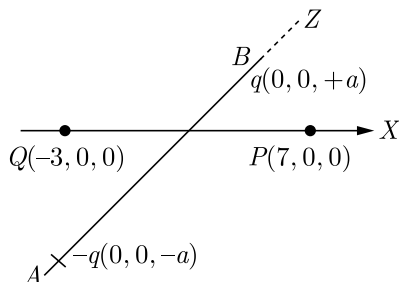
128. (i) Can two equipotential surfaces intersect each other? Give reasons.
- (ii) Two charges $-q$ and $+q$ are located at points $A(0,0,-a)$ and $B(0,0,+a)$ respectively. How much work is done in moving a test charge from point $P(7,0,0)$ to $Q(-3,0,0)$?

Ans :

Comp 2007, Foreign 2010

- No, if two equipotential surfaces (as these are normal to electric field) intersect each other, then at the point of intersection there will be two directions of electric field, which is impossible. Hence, two equipotential surfaces cannot intersect each other.

- (ii) Suppose W_1 and W_2 be the work done in moving a test charge q_0 from $P(7,0,0)$ to $Q(-3,0,0)$ in the fields of $+q(0,0,a)$ and $-q(0,0,-a)$ respectively.



Here,

$$\begin{aligned}\vec{BP} &= 7\hat{i} + a\hat{k}, \\ \vec{BQ} &= -3\hat{i} - a\hat{k} \\ \vec{AP} &= 7\hat{i} - a\hat{k}, \\ \vec{AQ} &= -3\hat{i} - a\hat{k} \\ BP &= \sqrt{49 + a^2}, \\ BQ &= \sqrt{9 + a^2} \\ AP &= \sqrt{49 + a^2}, \\ AQ &= \sqrt{9 + a^2}\end{aligned}$$

From the relation,

$$\begin{aligned}W_{AB} &= \frac{1}{4\pi\epsilon_0} \cdot q \left(\frac{1}{r_B} - \frac{1}{r_A} \right) \\ W_1 &= \frac{1}{4\pi\epsilon_0} \cdot q \left(\frac{1}{BQ} - \frac{1}{BP} \right) \\ &= \frac{1}{4\pi\epsilon_0} \cdot q \left(\frac{1}{\sqrt{9 + a^2}} - \frac{1}{\sqrt{49 + a^2}} \right)\end{aligned}$$

and $W_2 = \frac{1}{4\pi\epsilon_0} \cdot (-q) \left(\frac{1}{AQ} - \frac{1}{AP} \right)$

or $W_2 = \frac{-q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{9 + a^2}} - \frac{1}{\sqrt{49 + a^2}} \right)$

Hence, Total work done,

$$\begin{aligned}W &= W_1 + W_2 \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{9 + a^2}} - \frac{1}{\sqrt{49 + a^2}} \right) \\ &\quad - \frac{1}{\sqrt{9 + a^2}} + \frac{1}{\sqrt{49 + a^2}} \\ &= 0 \text{ (zero)}\end{aligned}$$

- 129.** Two charges of $\pm 1000 \mu\text{C}$ are separated by 2 mm. The dipole so formed is held at an angle of 30° with a uniform electric field of $8 \times 10^5 \text{ N-C}^{-1}$. Find out the value of torque acting on the dipole?

Ans :

OD 2009

Given,

Magnitude of each charge on dipole,

$$q = 1000 \mu\text{C} = 10^{-3} \text{C}$$

Distance between the charges,

$$2a = 2 \text{ mm} = 2 \times 10^{-3} \text{m}$$

Angle of dipole with direction of electric field, $\theta = 30^\circ$.

and Electric field, $E = 8 \times 10^5 \text{ N-C}^{-1}$

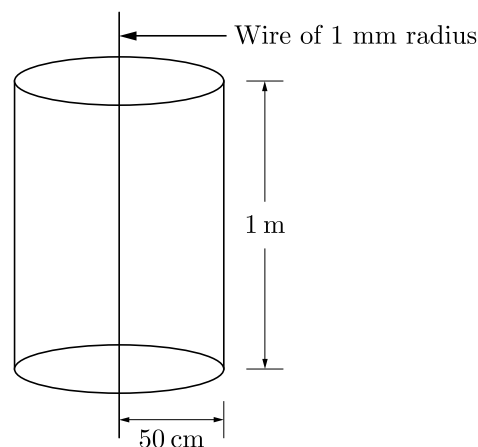
We know that,

$$\begin{aligned}\text{Dipole moment, } p &= q \times 2a \\ &= 10^{-3} \times (2 \times 10^{-3}) \\ &= 2 \times 10^{-6} \text{ C-m}\end{aligned}$$

Therefore, torque acting on the dipole in uniform electric field,

$$\begin{aligned}\tau &= pE \sin \theta \\ &= (2 \times 10^{-6}) \times (8 \times 10^5) \times \sin 30^\circ \\ &= 0.8 \text{ N-m}\end{aligned}$$

- 130.** Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wire is Q coulomb. A cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire as shown in the figure. Find out the total electric flux passing through the cylindrical surface?



Ans :

Comp 2009

Radius of wire, $r = 1 \text{ mm}$

Charge on the wire per cm length = Q .

Charge on 1 meter long wire

$$= Q \times 100 = 100Q \text{ coulomb}$$

Length of cylinder, $l = 1 \text{ m} = 100 \text{ cm}$

and radius of cylinder,

$$R = 50 \text{ cm}$$

We know from the Gauss's theorem that total electric flux passing through the cylindrical surface,

$$\phi = \frac{\text{Charge enclosed by the cylinder}}{\epsilon_0}$$

$$= \frac{\text{Charge on one metre length of wire}}{\varepsilon_0}$$

$$= \frac{100Q}{\varepsilon_0}$$

where ε_0 = Absolute electric permittivity of free space

- 131.** A regular hexagon of side 10 cm has charge $5\mu\text{C}$ at each of its vertices. What is the resultant potential at the centre of the hexagon?

Ans :

Delhi 2017

Side of hexagon, $a = 10\text{ cm} = 0.1\text{ m}$

and Charge at each vertex,

$$q = 5\mu\text{C} = 5 \times 10^{-6}\text{C}$$

We know that, distance of each charge from the centre of regular hexagon,

$$r = a = 0.1\text{ m}$$

We also know that, resultant potential at the centre of hexagon,

$$V = k \left[\frac{q_1 + q_2 + q_3 + q_4 + q_5 + q_6}{r} \right]$$

$$= k \left[\frac{q + q + q + q + q + q}{r} \right]$$

$$= k \cdot \frac{6q}{r}$$

$$= (9 \times 10^9) \times \frac{6 \times (5 \times 10^{-6})}{0.1}$$

$$= 27 \times 10^5\text{ V}$$

where, k = Electrostatic force constant

$$= 9 \times 10^9\text{ N-m}^2\text{-C}^{-2}$$

- 132.** A square of side 1 m, has four charges of $+2 \times 10^{-9}\text{ C}$, $+1 \times 10^{-9}\text{ C}$, $-2 \times 10^{-9}\text{ C}$ and $-3 \times 10^{-9}\text{ C}$ respectively at its corners. What is the resultant potential at the centre of the square?

Ans :

OD 2012, Delhi 2005

Side of square, $a = 1\text{ m}$

First charge, $q_1 = 2 \times 10^{-9}\text{ C}$

Second charge, $q_2 = 1 \times 10^{-9}\text{ C}$

Third charge, $q_3 = -2 \times 10^{-9}\text{ C}$

Fourth charge, $q_4 = -3 \times 10^{-9}\text{ C}$

Distance of each charge from the centre of the square,

$$r = \frac{\sqrt{a^2 + a^2}}{2}$$

$$= \frac{\sqrt{(1)^2 + (1)^2}}{2} = 0.707\text{ m}$$

Therefore, potential at the centre of square,

$$V = k \left[\frac{q_1 + q_2 + q_3 + q_4}{r} \right]$$

$$= k \left[\frac{[(2 \times 10^{-9}) + (1 \times 10^{-9}) + (-2 \times 10^{-9}) + (-3 \times 10^{-9})]}{0.707} \right]$$

$$= k \times (-2.83 \times 10^{-9})$$

$$= (9 \times 10^9) \times (-2.83 \times 10^{-9})$$

$$= 25.5\text{ V}$$

where, k = Electrostatic force constant

$$= 9 \times 10^9\text{ N-m}^2\text{-C}^{-2}$$

- 133.** Two spherical conductors of radii 4 cm and 5 cm are charged to the same potential. If σ_1 and σ_2 be the respective values of the surface density of charge on both the conductors, What is the ratio of $\frac{\sigma_1}{\sigma_2}$?

Ans :

Foreign 2016, OD 2009

Radius of first conductor, $r_1 = 4\text{ cm}$

Radius of second conductor, $r_2 = 5\text{ cm}$

Potential on first conductor, $V_1 = V_2$

where, V_2 = potential on second conductor

Surface density of charge on first conductor = σ_1 , and surface density of charge on second conductor = σ_2 .

We know that,

Electric potential, $V = k \cdot \frac{q}{r}$

$$q \propto r \quad (\text{as potential is same})$$

$$\text{Therefore, } \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

We also know that, surface charge density of a sphere,

$$\sigma = \frac{\text{Charge on sphere}}{\text{Surface area of sphere}}$$

$$= \frac{q}{4\pi r^2} \propto \frac{q}{r^2}$$

$$\text{Therefore, } \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2} \times \left[\frac{r_2^2}{r_1^2} \right]$$

$$= \frac{r_1}{r_2} \times \left[\frac{r_2^2}{r_1^2} \right] = \frac{r_2}{r_1} = \frac{5}{4}$$

- 134.** Two pith balls of mass 0.2 gm each are suspended by two nylon strings of length 50 cm each. They are charged with charge of equal magnitude and same nature. If they come to rest after repulsion at a distance of 4 cm, then find the charge on each ball.

Ans :

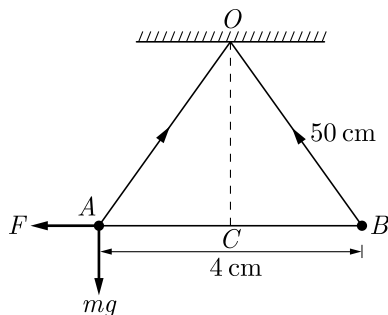
Comp 2020

Distance between ball,

$$r = 4\text{ cm} = 4 \times 10^{-2}\text{ m}$$

Let the charge on each ball is Q . Then force of repulsion between two ball is given by,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot Q}{(4 \times 10^{-2})^2}$$



$$F = \frac{Q^2 \times 9 \times 10^9}{16 \times 10^{-4}} \quad \dots(1)$$

Weight of each ball = mg

$$\begin{aligned} \text{Here, } m &= 0.2 \times 10^{-3} \text{ kg} \\ &= 0.2 \times 10^{-3} \times 9.8 \\ &= 1.96 \times 10^{-3} \text{ N} \quad \dots(2) \end{aligned}$$

If tension in string is T and ball are in equilibrium then algebraic sum of moments of force acting on it is zero.

$$F \times OC - Mg \times AC + T \times 0 = 0$$

$$\text{or } F \times OC = mg \times AC$$

$$\frac{F}{mg} = \frac{AC}{OC}$$

Since in ΔOAC ,

$$OA^2 = AC^2 + OC^2$$

$$OC = \sqrt{OA^2 - AC^2}$$

$$\begin{aligned} \text{Now, } \frac{F}{mg} &= \frac{AC}{\sqrt{OA^2 - AC^2}} \\ &= \frac{AB/2}{\sqrt{(OA)^2 - (\frac{AB}{2})^2}} \\ &= \frac{4 \times 10^{-2}/2}{\sqrt{(50 \times 10^{-2})^2 - (2 \times 10^{-2})^2}} \\ &= \frac{2 \times 10^{-2}}{\sqrt{(50 \times 10^{-2})^2 - (2 \times 10^{-2})^2}} \\ &= \frac{2 \times 10^{-2}}{10^{-2} \sqrt{2500 - 4}} \\ &= \frac{2 \times 10^{-2}}{49.95 \times 10^{-2}} = \frac{2}{49.95} \end{aligned}$$

From Eq. 1 and 2, we get

$$\begin{aligned} \frac{Q^2 \times 9 \times 10^9}{16 \times 10^{-4} \times 1.96 \times 10^{-3}} &= \frac{2}{49.95} \\ s &= \frac{2 \times 16 \times 10^{-4} \times 1.96 \times 10^{-3}}{9 \times 10^{-9} \times 49.95} \\ Q &= \sqrt{1.39 \times 10^{-17}} \\ &= 3.73 \times 10^{-9} = 3.73 \mu\text{C} \end{aligned}$$

CASE BASED QUESTIONS

135. Ramesh's aunt was housewife, she had a little knowledge of science. In a dark room when she pulled over a woollen sweater, she heard little crackles with tiny sparks. She became frightened and called Ramesh. Being a science student he explained the appropriate reason behind it with appropriate figure and then her aunt calmed down.



Transfer of charge

Due to a charge, there is an electric field produced around a sphere.

- A hollow conducting sphere of radius 8 cm is given a charge of $16 \mu\text{C}$. What is the electric field at the centre of the sphere?
- A hollow conducting sphere of radius 8 cm is given a charge of $16 \mu\text{C}$. What is the electric field on the outer surface of the sphere?

Ans :

- Electric field at its centre = 0, because electric field at any point inside a hollow sphere is always zero.
- Electric field on the outer surface of the sphere is

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \\ &= 9 \times 10^9 \times \frac{16 \times 10^{-6}}{(0.08)^2} \\ &= 9 \times 10^9 \times \frac{16 \times 10^{-6}}{64 \times 10^{-4}} \\ &= 2.25 \times 10^7 \text{ NC}^{-1} \end{aligned}$$

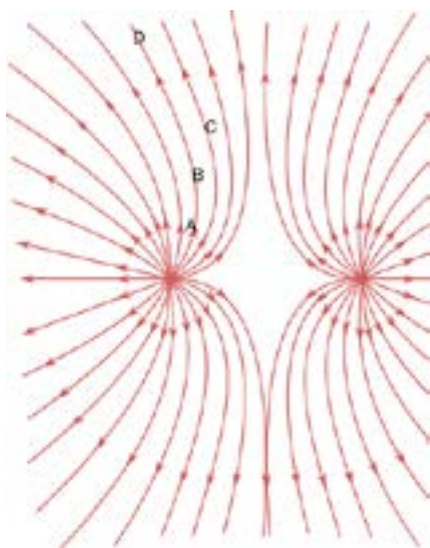
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136. While travelling back to his residence in the car, Dr. Pathak was caught up in a thunderstorm. It became very dark. He stopped driving the car and waited for the thunderstorm to stop. Suddenly he noticed a child walking alone on the road. He asked the boy to come inside the car till the thunderstorm stopped. Dr. Pathak dropped the boy at his residence. The boy insisted that Dr. Pathak should meet his parents. The parents expressed their gratitude to Dr. Pathak for his concern for safety of the child.



Answer the following questions based on the above information :

- Why is it safer to sit inside a car during a thunderstorm?
- Can charge be stored in a body?
- An electric field map is graphically expressed as shown in the figure. There are 4 indicated locations (A, B, C, and D) along one of the field lines. If a negative test charge is released at location B and free to move, in which way will it move toward?

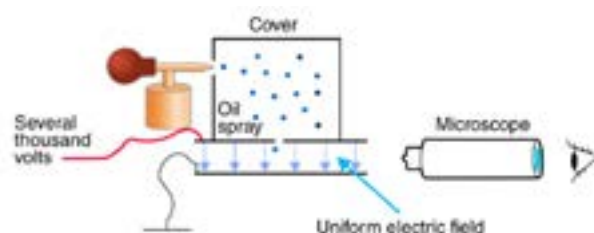


Ans :

- It is safer to be inside a car during thunderstorm because the car acts like a Faraday cage. The metal in the car will shield you from any external electric fields and thus prevent the lighting from the travelling within the car.
- Yes, Charge can be stored in a capacitor, which is a combination of two conductors with something insulating (a dielectric) between them.
- A negative electric charge always seeks to travel in against the electric field direction or toward the positive charge, whereas a positive electric charge always goes in the electric field direction or towards the negative charge. It can be concluded from the preceding explanation that a negative

test charge at location B in the figure attempts to move in against the electric field direction. Hence, the negative test charge moves towards point A.

- In 1909, Robert Millikan and Harvey Fletcher conducted the oil drop experiment to determine the charge of an electron. They suspended tiny, charged droplets of oil between two metal electrodes by balancing downward gravitational force with upward drag and electric forces. The density of the oil was known, so Millikan and Fletcher could determine the droplets' masses from their observed radii (since from the radii they could calculate the volume and thus, the mass). Using the known electric field and the values of gravity and mass, Millikan and Fletcher determined the charge on oil droplets in mechanical equilibrium. By repeating the experiment, they confirmed that the charges were all multiples of some fundamental value. They calculated this value to be 1.5924×10^{-19} Coulombs (C), which is within 1% of the currently accepted value of 1.602×10^{-19} C. They proposed that this was the charge of a single electron.

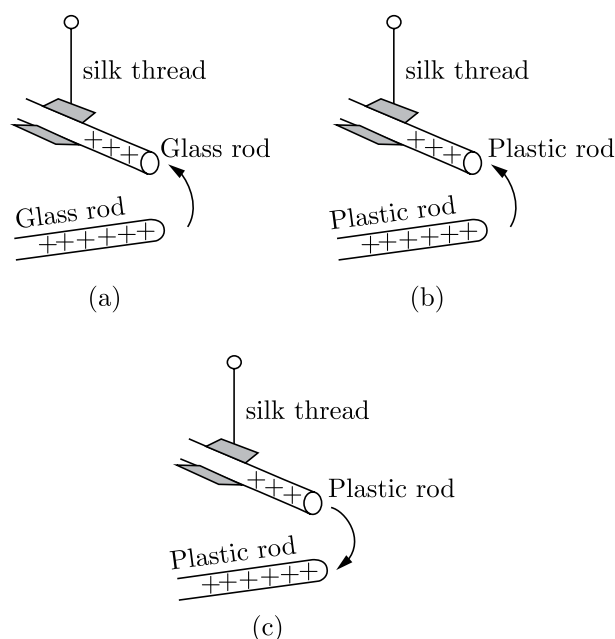


- What was determined from Millikan's oil drop experiment?
- What is the currently accepted value of electric charge of an electron?
- How was the mass of an electron determined?
- Does an electron have mass?
- What was the conclusion of Millikan's oil drop experiment?

Ans :

- In 1909, Robert Millikan and Harvey Fletcher developed an experiment to determine the fundamental charge of the electron.
 - 1.602×10^{-19} C.
 - The mass of an electron determined by the calculation of density and Volume.
 - Yes
 - Electric charge is integral multiple of fundamental charge
- When a glass rod is rubbed with silk, the rod acquires one kind of charge and the silk acquires the second kind of charge. This is true for any pair of objects that are rubbed to be electrified. Now if the electrified

glass rod is brought in contact with silk, with which it was rubbed, they no longer attract each other. They also do not attract or repel other light objects as they did on being electrified. Thus, the charges acquired after rubbing are lost when the charged bodies are brought in contact. What can you conclude from these observations? It just tells us that unlike charges acquired by the objects neutralise or nullify each other's effect. Therefore, the charges were named as positive and negative by the American scientist Benjamin Franklin. We know that when we add a positive number to a negative number of the same magnitude, the sum is zero. This might have been the philosophy in naming the charges as positive and negative. By convention, the charge on glass rod or cat's fur is called positive and that on plastic rod or silk is termed negative. If an object possesses an electric charge, it is said to be electrified or charged. When it has no charge it is said to be electrically neutral.



- When you charge a balloon by rubbing it on your hair this is an example of what method of charging?
- Which particle in an atom can you physically manipulate?
- If a negatively charged rod touches a conductor, the conductor will be charged by what method?

Ans :

- Friction
- Electrons
- Conduction

139. For electrostatics, the concept of electric field is convenient, but not really necessary. Electric field is an elegant way of characterizing the electrical environment of a system of charges. Electric field at a point in the space around a system of charges tells you the force a unit positive test charge would experience if placed at that point (without disturbing the system). Electric field is a characteristic of the system of charges and is independent of the test charge that you place at a point to determine the field. The term field in physics generally refers to a quantity that is defined at every point in space and may vary from point to point. Electric field is a vector field, since force is a vector quantity.

- What do you mean by electric field ?
- What is the SI unit of electric field ?

Ans :

- The force per unit charge is known as electric field
- N/C

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CHAPTER 2

Electrostatic Potential and Capacitance

SUMMARY

1. ELECTROSTATIC POTENTIAL

The electric potential at any point in an electric field is defined as the work done in bringing a unit positive test charge from infinity to that point without acceleration.

If W is the work done in bringing infinitesimal positive test charge q_0 from infinity to given point, then electric potential,

$$V = \frac{W}{q_0}$$

Its SI unit is Volt V and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

2. POTENTIAL DIFFERENCE

The potential difference between two points in an electric field is defined as the work done in bringing unit positive charge from one point to another.

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Where, W_{AB} is work done in taking charge q_0 from A to B against of electrostatic force.

3. ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

Electrostatic potential due to a point charge q at any p lying at a distance r from the origin, it is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

4. ELECTROSTATIC POTENTIAL DUE TO AN ELECTRIC DIPOLE AT ANY POINT

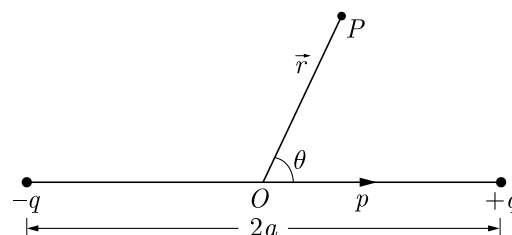
Electrostatic potential due to an electric dipole at any point P whose position vector is r w.r.t. mid-point of dipole is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos \theta}{r^2}$$

or

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{r}}{|\vec{r}|^2}$$

Where, θ is the angle between r and p .



Electrostatic potential due to a thin charged spherical shell carrying charge q and radius R respectively, at any point P lying.

$$\text{Inside the shell is Voltage} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

$$\text{On the surface of shell is Voltage} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

$$\text{Outside the shell is Voltage} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R} \text{ for } r > R$$

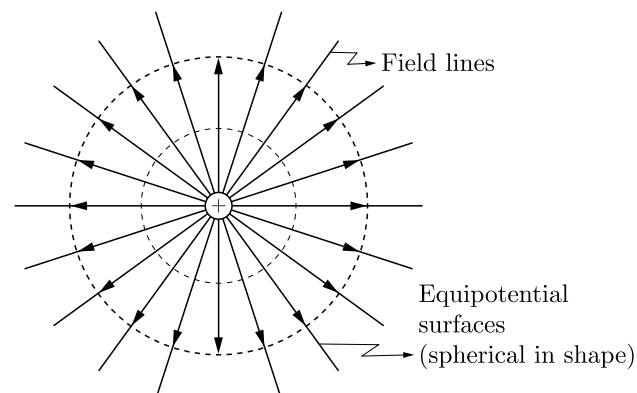
Where, r is the distance of point P from the centre of the shell.

5. EQUIPOTENTIAL SURFACE

A surface which have same electrostatic potential at every point on it is known as equipotential surface.

The shape of equipotential surface due to line charge is cylindrical.

Point charge is spherical as shown below:



Properties of Equipotential Surface

1. Equipotential surface do not intersect each other as it gives two directions of electric field E at intersecting point which is not possible.
2. Electric field is always normal to equipotential surface at every point of it and directly from one equipotential surface at higher potential to the equipotential surface at lower potential.
3. Work done in moving a test charge from one point of equipotential surface to other is zero.

The electric field at a point is related to the negative potential gradient as follows,

$$E = -\frac{dV}{dr}$$

i.e., $E_x = -\frac{\partial V}{\partial x}$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Where negative sign indicates that the direction of electric field is from higher potential to lower potential, i.e., in the direction of decreasing potential.

6. ELECTROSTATIC POTENTIAL ENERGY

The work done against electrostatic force gets stored as potential energy. This is called electrostatic potential energy.

i.e., $\Delta U = U_B - U_A = W_{AB}$

The work done in moving a unit positive test charge over a closed path in an electric field is zero.

Thus, electrostatic forces are conservative in nature.

1. Electrostatic potential energy of a system of two point charges is given by

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$$

Putting the values of charge with their signs.

2. Electrostatic potential of a system of n point charges is given by

$$U = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^n \sum_{i=1}^n \frac{q_j q_i}{r_{ji}} \quad j \neq i \text{ and } ij = ji$$

Potential Energy in an External Field

1. Potential energy of a single charge q at a point with position vector r , in an external field is $qV(r)$, where $V(r)$ is the potential at the point due to external electric field E .
2. Potential Energy of a system of two charges in an external field.

$$U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

where,

q_1 and q_2 = two point charges at position vectors r_1 and r_2 , respectively

$V(r_1)$ = potential at r_1 due to the external field

$V(r_2)$ = potential at r_2 due to the external field

Potential energy of an electric dipole in an uniform electric field

$$U = -pE \cos \theta$$

$$= \vec{p} \cdot \vec{E}, \text{ if } \cos \theta = 1$$

7. CAPACITANCE OF A CONDUCTOR

If charge q is given to an insulated conductor, it leads to increase its electric potential by V such that,

$$q \propto V \Rightarrow q = CV$$

where, C is known as capacitance of a conductor. The capacitance depends on the shape, size and geometry of conductor, nature of surrounding medium and presence of other conductor in the neighbourhood of it.

Its SI unit is farad F .

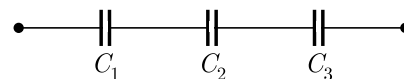
Here, $1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$

Farad is a very large unit of capacitance, So, μF is usually taken.

8. COMBINATION OF CAPACITORS

1. **Series Combination** : When capacitance are connected in series, then net capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Net charge $Q = q_1 = q_2 = q_3$ (remain same)

Net potential difference $V = V_1 + V_2 + V_3$

2. **Parallel Combination** : When capacitors are connected in parallel, then the net capacitance is given by

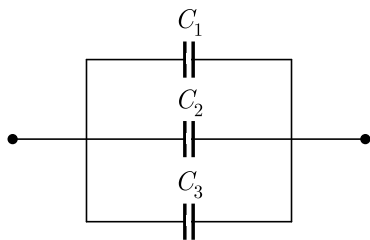
$$C = C_1 + C_2 + C_3$$

In parallel combination net charge,

$$Q = q_1 = q_2 = q_3$$

Net potential difference,

$$V = V_1 = V_2 = V_3 \text{ (remain same)}$$



9. CAPACITANCE OF PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two parallel metallic plates separated by a dielectric. The capacitance of parallel plate capacitor is given by,

$$C = \frac{k\epsilon_0 A}{d}$$

where, K = dielectric constant

A = area of each plate

and d = separation between the plates

Special Cases

- When there is no medium between the plates, then $K = 1$. So,

$$C_{\text{vacuum}} = \frac{\epsilon_0 A}{d} = C_0$$

- When space between the plates is partly filled with a medium of thickness t and dielectric constant K , then capacitance

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}} = \frac{\epsilon_0 A}{d - t(1 - \frac{1}{K})}$$

Clearly, $C > C_0$, i.e., on introduction slab between the plates of a parallel plate capacitor, its capacitance increases.

10. ENERGY STORED IN A CHARGED CAPACITOR

$$U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{1}{2} QV \text{ joule}$$

This energy resides in the medium between the plates. The energy stored per unit volume of a charged capacitor is given by

$$u = \frac{U}{V} = \frac{1}{2} \epsilon E^2 \text{ joule/m}^3$$

where, E is electric field strength.

OBJECTIVE QUESTIONS

- The electric potential due to a small electric dipole at a large distance r from the center of the dipole is proportional to

- (a) r (b) $\frac{1}{r}$
(c) $\frac{1}{r^2}$ (d) $\frac{1}{r^3}$

Ans :

OD 2018

Electric potential at an axial point of a dipole is given by,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2 - a^2}$$

where,

p = Electric dipole moment

r = Distance of axial point from the center of dipole

$2a$ = Length of the dipole

For large distance $r^2 \gg a^2$,

$$V = \frac{p}{4\pi\epsilon_0 r^2}$$

Hence, $V \propto \frac{1}{r^2}$

Thus (c) is correct option.

- If the uniform electric field exists along X-axis, then equipotential is along

- (a) XY-plane (b) XZ-plane
(c) YZ-plane (d) anywhere

Ans :

OD 2009

Equipotential surface are planes in uniform electric field. So it lies in plane XY-plane.

Thus (a) is correct option.

- If 125 water drops of equal radius and equal capacitance C , coalesce to form a single drop of capacitance C' the relation between C and C' is

- (a) $C' = C$ (b) $C' = 5C$
(c) $C' = 125C$ (d) $C' = 250C$

Ans :

SQP 2005

Number of water drops $n = 125$

Capacitance of each water drop = C

If n small drops, each of capacitance C , coalesce to form a big drop, then capacitance of the big drop,

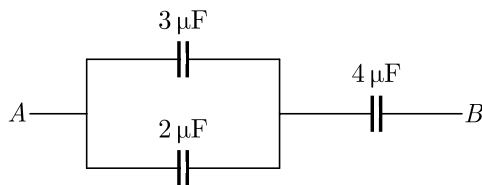
$$C' = n^{1/3} \times C = (125)^{1/3} \times C = 5C$$

Thus (b) is correct option.

In the capacitor charge on Y plate is $20\mu\text{C}$ and the charge on X -plate is $-20\mu\text{C}$.

Thus (b) is correct option.

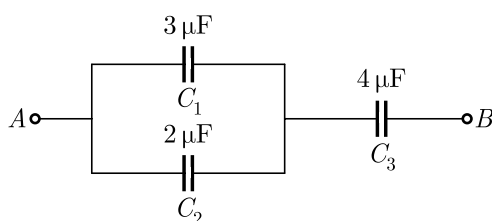
14. The equivalent capacity between A and B is



- (a) $\frac{20}{9}\mu\text{F}$ (b) $9\mu\text{F}$
(c) $1\mu\text{F}$ (d) $\frac{1}{9}\mu\text{F}$

Ans :

OD 2013, SQP 2010



In the given circuit C_1 and C_2 are connected in parallel combination. Hence, equivalent capacitance,

$$C' = C_1 + C_2 \\ = 3 + 2 = 5\mu\text{F}$$

C' and C_3 are connected in series combination. Now, equivalent capacitance between A and B ,

$$C_{AB} = \frac{C' \times C_3}{C' + C_3} \\ = \frac{5 \times 4}{5 + 4} = \frac{20}{9}\mu\text{F}$$

Thus (a) is correct option.

15. The energy stored in a capacitor is actually stored

- (a) between the plates
(b) on the positive plate
(c) on the negative plate
(d) on the outer surfaces of the plates

Ans :

Foreign 2006

Energy stored in a capacitor is in the form of electrostatic energy. It is actually stored between the plates of the capacitor.

Thus (a) is correct option.

16. Two capacitors of capacitance $3\mu\text{F}$ and $6\mu\text{F}$ are connected in a series combination. Their equivalent capacitance will be

- (a) $2\mu\text{F}$ (b) $3\mu\text{F}$
(c) $6\mu\text{F}$ (d) $9\mu\text{F}$

Ans :

Delhi 2016

Capacitance of first capacitor,

$$C_1 = 3\mu\text{F}$$

and capacitance of second capacitor,

$$C_2 = 6\mu\text{F}$$

Equivalent capacitance of the capacitors in series combination,

$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\mu\text{F}$$

Thus (a) is correct option.

17. Three capacitors of capacitances $2\mu\text{F}$, $4\mu\text{F}$ and $6\mu\text{F}$ are connected in a parallel combination. Their equivalent capacitance will be

- (a) $2\mu\text{F}$ (b) $4\mu\text{F}$
(c) $6\mu\text{F}$ (d) $12\mu\text{F}$

Ans :

SQP 2005

Capacitance of first capacitor, $C_1 = 2\mu\text{F}$

Capacitance of second capacitor, $C_2 = 4\mu\text{F}$

Capacitance of third capacitor $C_3 = 6\mu\text{F}$

Equivalent capacitance of the capacitors in parallel combination,

$$C_{eq} = C_1 + C_2 + C_3 \\ = 2 + 4 + 6 = 12\mu\text{F}$$

Thus (d) is correct option.

18. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If capacitance between any two adjacent plates is C , then the resultant capacitance is

- (a) C (b) nC
(c) $(n-1)C$ (d) $(n+1)C$

Ans :

Foreign 2014

Number of plates = n and capacitance between any two adjacent plates = C . We know that as n plates are joined alternatively.

Therefore, no. of capacitors = $(n-1)$.

Thus resultant capacitance of parallel plate capacitor,

$$C' = (n-1)C$$

Thus (c) is correct option.

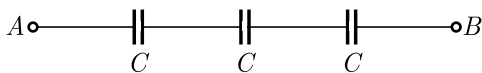
19. An infinite number of capacitors with capacitances $1\mu\text{F}$, $\frac{1}{2}\mu\text{F}$, $\frac{1}{4}\mu\text{F}$, $\frac{1}{8}\mu\text{F}$, ... etc. are connected in parallel combination. Their equivalent capacitance will be

9. Three capacitors each of capacity C are connected in series. The resultant capacity will be
 (a) $3C$ (b) $3/C$
 (c) $C/3$ (d) $1/3C$

Ans :

OD 2017

According to the question,



Resultant capacity is given by,

$$\begin{aligned}\frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \\ C_{eq} &= C/3\end{aligned}$$

Thus (c) is correct option.

10. A parallel plate air capacitor is charged to a potential difference of V . If distance between the plates is increased, then potential difference between the plates
 (a) decreases (b) increases
 (c) becomes zero (d) does not change

Ans :

Delhi 2008, OD 2014

Initial potential difference of capacitor,

$$V_1 = V$$

Charge on the plates of a parallel plate air capacitor always remains constant after increasing or decreasing the distance between the plate.

i.e., $Q = \text{Constant}$

Capacitance of a parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

We also know that charge on the plates of capacitor,

$$Q = CV = \frac{\epsilon_0 A}{d} \times V$$

or $V \propto d$

Since distance between the plates is increases, therefore potential difference V between the plates also increases.

Thus (b) is correct option.

11. The electrostatic capacity of a capacitor depends upon its
 (a) charge (b) potential
 (c) resistance (d) both a and b

Ans :

Foreign 2006

Electrostatic capacity of a capacitor,

$$C = \frac{q}{V}$$

It is also called capacitance of the capacitor.

where, $q = \text{charge}$,and $V = \text{potential of the capacitor}$

Thus (d) is correct option.

12. 64 identical drops each of capacity of $5\mu\text{F}$ combine to form a big drop. What is the capacity of big drop?
 (a) $25\mu\text{F}$ (b) $4\mu\text{F}$
 (c) $164\mu\text{F}$ (d) $20\mu\text{F}$

Ans :

OD 2016, SQP 2013

Capacitance of spherical conductor is given by,

$$C_s = 4\pi\epsilon_0 r$$

$$5 = 4\pi\epsilon_0 r \text{ (for small drop) } \dots(1)$$

When small drops combine to form a big drop than volume remains constant,

Big drop volume = $64 \times \text{One small drop volume}$

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$R = 4r \dots(2)$$

$$C_B = 4\pi\epsilon_0 R$$

where, $R = \text{radius of big drop}$

$$C_B = 4\pi\epsilon_0 (4r) \text{ [From equation (2)]}$$

$$C_B = 4(4\pi\epsilon_0 r) \dots(3)$$

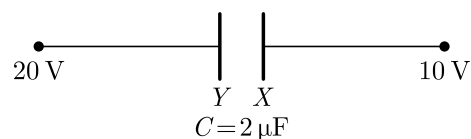
Dividing equation (1) by (2), we get

$$\frac{5}{C_B} = \frac{4\pi\epsilon_0 r}{4(4\pi\epsilon_0 r)}$$

$$C_B = 5 \times 4 = 20\mu\text{F}$$

Thus (d) is correct option.

13. The charge on plate X in the given figure



- (a) $20\mu\text{C}$ (b) $-20\mu\text{C}$
 (c) zero (d) $-10\mu\text{C}$

Ans :

Delhi 2011

Capacitance, $C = \frac{\text{Charge (Q)}}{\text{Potential (V)}}$

$$Q = CV$$

Here, $C = 2\mu\text{F} = 2 \times 10^{-6} \text{ F}$

$$V = 20 - 10 = 10 \text{ Volt}$$

Now, $Q = 2 \times 10^{-6} \times 10$

$$Q = 20 \times 10^{-6} = 20\mu\text{C}$$

4. The ratio of charge to potential of a capacitor is known as its

(a) capacitance (b) conductance
(c) inductance (d) resistance

Ans :

Delhi 2016

Capacitance of a capacitor,

$$C = \frac{q}{V}$$

It is also called electrostatic capacity of the capacitor.

where, q = charge,

and V = potential of the capacitor

Thus (a) is correct option.

5. Two plates of a parallel plate capacitor are 1 cm apart and potential difference between them is 10 V. The electric field between the plates is

(a) 10 N-C⁻¹ (b) 250 N-C⁻¹
(c) 500 N-C⁻¹ (d) 1000 N-C⁻¹

Ans :

Foreign 2017, Delhi 2002

Distance between plates,

$$d = 1\text{cm} = 0.01\text{ m}$$

and potential difference between them,

$$V = 10\text{ V}$$

Electric field between the plates in a parallel plate capacitor,

$$E = \frac{V}{d} = \frac{10}{0.01} = 1000\text{ N-C}^{-1}$$

Thus (d) is correct option.

6. The capacitance of a parallel plate capacitor by the introduction of a dielectric between the plates of capacitor.

(a) increases (b) decreases
(c) remains same (d) nothing can be said

Ans :

OD 2010

When vacuum is present between the plates of capacitor, then capacitance of parallel plate is given by,

$$C = \frac{\epsilon_0 A}{d} \quad \dots(1)$$

If a medium of dielectric strength K is introduced between the plate of capacitor then,

$$\text{Capacitance, } C' = \frac{K\epsilon_0 A}{d} \quad [\text{From equation (1)}]$$

$$C' = KC$$

i.e. capacitance increase K time

Thus (a) is correct option.

7. Which of the following ratios is constant for an isolated conductor?

(a) $\frac{\text{Total charge}}{\text{Potential}}$ (b) $\frac{\text{Charge added}}{\text{Potential difference}}$
(c) $\frac{(\text{Total charge})^2}{\text{Potential}}$ (d) none of these

Ans :

OD 2007

Capacitance of isolated conductor is given by,

$$C = \frac{\text{Total charge } (Q)}{\text{Potential } (V)}$$

Capacitance is constant for a conductor.

Thus (a) is correct option.

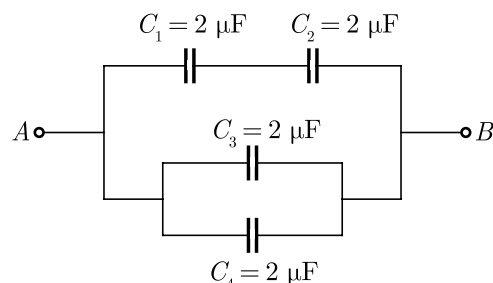
8. Minimum number of capacitors of 2 μF each required to obtain a capacitance of 5 μF will be

(a) 4 (b) 3
(c) 5 (d) 6

Ans :

SQP 2018

To obtain a equivalent capacitance of 5 μF , we will arranged the capacitor as shown in the figure.



In the circuit diagram C_1 and C_2 are connected in series combination.

Hence, Equivalent capacitance,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2}$$

$$C' = 1\mu\text{F}$$

C_2 and C_3 capacitors are connected in parallel combination.

Hence, Equivalent capacitance,

$$C'' = C_3 + C_4$$

$$= 2 + 2 = 4\mu\text{F}$$

C' and C'' are connected in parallel combination. Hence, Total equivalent capacitance,

$$C_{eq} = C' + C''$$

$$= 1 + 4 = 5\mu\text{F}$$

Hence, four capacitors are required to obtain equivalent capacitance of 5 μF .

Thus (a) is correct option.

- (a) $8\mu\text{F}$ (b) $6\mu\text{F}$
 (c) $4\mu\text{F}$ (d) $2\mu\text{F}$

Ans :

Delhi 2010

Capacitance of infinite number of capacitors,

$$C_1 = 1\mu\text{F}, C_2 = \frac{1}{2}\mu\text{F}, C_3 = \frac{1}{4}\mu\text{F},$$

$$C_4 = \frac{1}{8}\mu\text{F}, \dots \text{etc.}$$

Equivalent capacitance of infinite capacitors in parallel combination,

$$C_{eq} = C_1 + C_2 + C_3 + C_4 + \dots \text{upto } \infty$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{upto } \infty$$

$$= \frac{1}{1 - (\frac{1}{2})} = 2\mu\text{F}$$

Thus (d) is correct option.

20. The electrostatic energy stored in a capacitor is

- (a) $\frac{1}{2}QV$ (b) $\frac{1}{QV}$
 (c) $\frac{2}{QV}$ (d) QV

Ans :

OD 2016

Electrostatic energy stored in a capacitor,

$$U = \frac{1}{2} \times CV^2$$

$$= \frac{1}{2} \times \frac{Q}{V} \times V^2 = \frac{1}{2}QV$$

where, Q = Charge, V = Potentialand C = Capacity of the capacitor

Thus (a) is correct option.

21. A parallel plate capacitor is first charged and then a dielectric slab is introduced between the plates. The quantity that remains unchanged is

- (a) charge (b) energy
 (c) potential (d) capacitance

Ans :

SQP 2003, Delhi 2006

When a dielectric slab is introduced between the plates of a charged parallel plate capacitor, the charge remains unchanged.

Thus (a) is correct option.

22. A $10\mu\text{F}$ capacitor is charged by a battery of e.m.f. 100 V. The electrostatic energy stored in the capacitor is

- (a) 0.5 J (b) 0.05 J
 (c) 0.5 mJ (d) 0.05 mJ

Ans :

Foreign 2008

Capacitance of the capacitor,

$$C = 10\mu\text{F}$$

$$= 10 \times 10^{-6} \text{ F}$$

and voltage of the battery,

$$V = 100 \text{ Volt}$$

Electrostatic energy stored in the capacitor,

$$U = \frac{1}{2} \times CV^2$$

$$= \frac{1}{2} \times (10 \times 10^{-6}) \times (100)^2$$

$$= 0.05 \text{ J}$$

Thus (b) is correct option.

23. A $4\mu\text{F}$ capacitor is charged to 400 V. If its plates are joined through a resistance, then heat produced in the resistance will be

- (a) 0.16 J (b) 0.32 J
 (c) 0.64 J (d) 1.28 J

Ans :

OD 2008

Capacitance of the capacitor,

$$C = 4\mu\text{F} = 4 \times 10^{-6} \text{ F}$$

Voltage,

$$V = 400 \text{ Volt}$$

Heat produced in the resistance

$$= \text{Electrical energy stored in the capacitor}$$

$$= \frac{1}{2} \times CV^2$$

$$= \frac{1}{2} \times (4 \times 10^{-6}) \times (400)^2 = 0.32 \text{ J}$$

Thus (b) is correct option.

24. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If capacitance between any two adjacent plates is C , then the resultant capacitance is

- (a) C (b) nC
 (c) $(n-1)C$ (d) $(n+1)C$

Ans :

Delhi 2009

Number of plates = n and capacitance between any two adjacent plates = C . We know that as n plates are joined alternatively.Therefore, no. of capacitors = $(n-1)$.

Thus resultant capacitance of parallel plate capacitor,

$$C' = (n-1)C$$

Thus (c) is correct option.

25. A parallel plate capacitor having a plate separation of 2 mm is charged by connecting it to a 300 V supply. The energy density is

$$\text{Charge on } C_1, \quad Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q$$

$$\text{Charge on } C_2, \quad Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$$

$$\text{Charge on } C_3, \quad Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$$

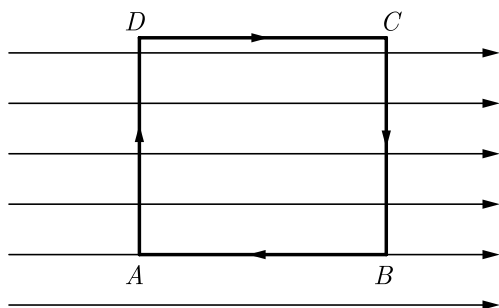
84. Can we create an electric field in which all the lines of force are parallel but their density increases continuously in a direction perpendicular to the lines of force?

Ans :

Foreign 2015

We cannot create such an electric field. Electric field is a conservative field, i.e. work done along a closed path in the field will be zero. In the field, that we are assuming, work done will not be equal to zero.

Work done along the paths CB and AD is zero. Work done along the path BA is more than that of along the path DC . Hence, net work done along the whole closed path is not zero.



85. 1. Draw equipotential surfaces due to a point $Q > 0$.
2. Are these surfaces equidistant from each other? If not, explain why.

Ans :

Foreign 2011, Delhi 2014

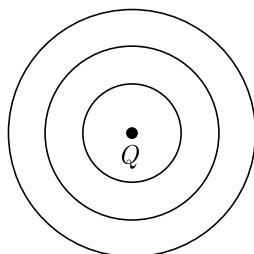
The equipotential surfaces due to a charge Q are as shown below :

1. The equipotential surfaces are spherical concentric spheres.
2. The equipotential surfaces are not equidistant.

Since,

$$E = -\frac{dV}{dr}$$

$$dr = -\frac{dV}{E}$$



Clearly, as E decreases, the distance between the equipotential surfaces goes on increasing as shown in the above figure.

86. Two point charges q_1 and q_2 are located at r_1 and r_2 , respectively in an external electric field E . Obtain the expression for the total work done in assembling this configuration.

Ans :

Delhi 2018

Work done in bringing the charge q_1 from infinity to position r_1

$$W_1 = q_1 V(r_1) \quad \dots(1)$$

Work done in bringing charge q_2 to the position r_2

$$W_2 = q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad \dots(2)$$

Hence, total work done in assembling the two charges.

$$W = W_1 + W_2$$

From equations (1) and (2), we get

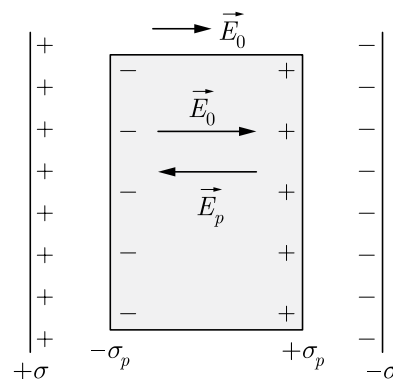
$$W = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

87. What is polarization of charge? Which the help of a diagram show why the electric field between the plates of capacitor reduces on introducing a dielectric slab. Define the dielectric constant on the basis of these fields.

Ans :

Foreign 2011

The induced dipole moment developed per unit volume in a dielectric slab on placing it inside the electric field is called polarisation.



Let $\vec{E}_0$ be the uniform external electric field. When a dielectric slab is placed in uniform electric field, then the molecules get polarised, due to which $-\sigma_p$ (charge density due to polarisation) will appear near the positive plate and $+\sigma_p$ will appear in the dielectric near the negative plate.

Therefore, due to polarization of molecules, electric field will appear in the opposite direction, and the net electric field inside the dielectric will be

31. If an uncharged capacitor is charged by connecting it to a battery, then the amount of energy lost as heat is
 (a) QV (b) QV^2
 (c) $\frac{1}{2}QV$ (d) $\frac{1}{2}QV^2$

Ans :

OD 2000, Foreign 2014

Charge given to the capacitor by a battery,

$$Q = CV$$

and work done by the battery,

$$W = QV$$

Energy stored in the capacitor,

$$\begin{aligned} U &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} VC \cdot V \\ &= \frac{1}{2} QV \end{aligned}$$

Therefore,

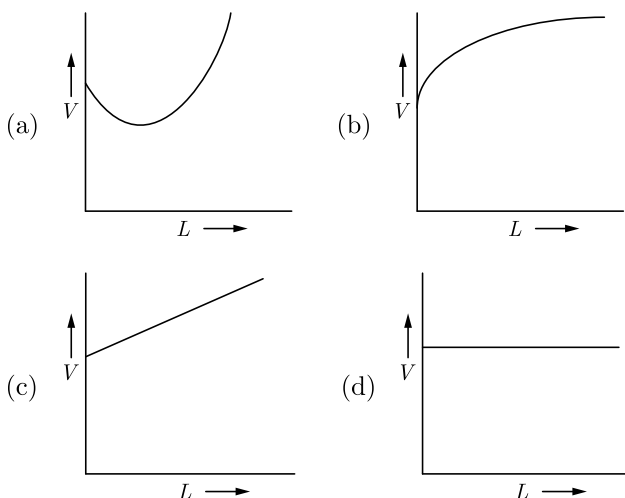
Energy lost as heat = Work done – Energy stored

$$\begin{aligned} U' &= QV - \frac{1}{2} QV \\ &= \frac{1}{2} QV \end{aligned}$$

where, Q = Charge, V = Potentialand C = Capacity of the capacitor

Thus (c) is correct option.

32. A dielectric plate is inserted between plates of a parallel plate capacitor, to fill the space between the plates. The capacitor is charged and later disconnected from the battery. Now the dielectric plate is slowly withdrawn from the capacitor. The graph of potential difference V across the plates and the length of the dielectric plate L with drawn is



Ans :

Delhi 2004

Potential difference across a parallel plate capacitor,

$$V = \frac{qd}{\epsilon_0 \kappa A} \propto \frac{1}{\kappa}$$

Therefore as the dielectric plate is taken out, this potential difference increases.

And it becomes constant when the dielectric plate is completely removed.

Therefore the graph of potential difference V across the plates and length of the dielectric plate L will be as shown in option (b).

Thus (b) is correct option.

33. A spherical drop of capacitance $1\mu\text{F}$ is broken into eight drops of equal radius. The capacitance of each small drop is

- (a) $\frac{1}{2}\mu\text{F}$ (b) $\frac{1}{4}\mu\text{F}$
 (c) $\frac{1}{8}\mu\text{F}$ (d) $\frac{1}{16}\mu\text{F}$

Ans :

SQP 2013

Capacitance of big drop, $C_1 = 1\mu\text{F}$ Number of small drops, $n = 8$

Since volume of big drop remains the same after it is broken into eight small drops,

$$\text{Therefore, } \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$R = 2r$$

where, R = Radius of big dropand r = Radius of small drop

Capacitance of the spherical drop,

$$C = 4\pi\epsilon_0 r$$

$$C \propto r$$

$$\text{Therefore, } \frac{C_1}{C_2} = \frac{R}{r} = \frac{2r}{r} = 2$$

$$\text{or } C_2 = \frac{C_1}{2} = \frac{1}{2}\mu\text{F}$$

where, C_2 = Capacitor of each small drop

Thus (a) is correct option.

34. The capacitance of an isolated spherical conductor of radius 15 cm is

- (a) $1.7 \times 10^{-11}\text{ F}$ (b) $3.4 \times 10^{-11}\text{ F}$
 (c) $5.1 \times 10^{-11}\text{ F}$ (d) $6.8 \times 10^{-11}\text{ F}$

Ans :

OD 2007

Radius of spherical conductor,

$$r = 15\text{ cm} = 0.15\text{ m}$$

Capacitance of an isolated spherical conductor,

$$C = 4\pi\epsilon_0 r$$

- (a) 0.01 J-m^{-3} (b) 0.1 J-m^{-3}
 (c) 1.0 J-m^{-3} (d) 10 J-m^{-3}

Ans :

SQP 2016

Distance between the plates,

$$d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

Supply voltage, $V = 300 \text{ Volt}$

Energy density of a parallel plate capacitor,

$$\begin{aligned} E &= \frac{1}{2} \epsilon_0 \frac{V^2}{d^2} \\ &= \frac{1}{2} \times (8.854 \times 10^{-12}) \times \frac{(300)^2}{(2 \times 10^{-3})^2} \\ &= 0.1 \text{ J-m}^{-3} \end{aligned}$$

where, ϵ_0 = Absolute electric permittivity of free space equal to $8.854 \times 10^{-12} \text{ C}^2\text{-N}^{-1}\text{-m}^{-2}$

Thus (b) is correct option.

26. When two charged capacitors having capacitance and potential C_1 , V_1 and C_2 , V_2 respectively, are joined with the help of a wire, the common potential will be

- (a) $C_1 + C_2$ (b) $\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$
 (c) $\frac{C_1 V_1 + C_2 V_2}{V_1 + V_2}$ (d) $\frac{C_1 V_1^2 + C_2 V_2^2}{V_1^2 + V_2^2}$

Ans :

Foreign 2011, OD 2007

Capacitance of first capacitor = C_1 Potential of first capacitor = V_1 Capacitance of second capacitor = C_2 and Potential of second capacitor = V_2

When two charged capacitors are joined with the help of a wire, then the common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Thus (b) is correct option.

27. Two capacitors of capacitances $3 \mu\text{F}$ and $6 \mu\text{F}$ are charged to a potential of 12 V each. They are now connected to each other, with the positive plate of each joined to the negative plate of the other. The potential difference across each capacitor will be

- (a) 6 V (b) 4 V
 (c) 3 V (d) Zero

Ans :

OD 2002

Capacitance of first capacitor, $C_1 = 3 \mu\text{F}$ Capacitance of second capacitor, $C_2 = 6 \mu\text{F}$ Potential difference, $V = 12 \text{ Volt}$

When the capacitors are connected to each other with the positive plate of each joined to the negative plate

of the other, then the potential difference across each capacitor will be zero.

Thus (d) is correct option.

28. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

- (a) increases (b) decreases
 (c) becomes infinite (d) remains unchanged

Ans :

Delhi 2013

When a sheet of conducting metal of negligible thickness is introduced between the plates of a capacitor, the capacitance of the capacitor remains unchanged.

Since the sheet of aluminium foil of negligible thickness is a sheet of the conducting metal, therefore the capacitance of the capacitor remains unchanged.

Thus (d) is correct option.

29. A parallel plate capacitor with oil between the plates (dielectric constant of oil $K = 2$) has a capacitance C . If the oil is removed, then capacitance of the capacitor becomes

- (a) $\sqrt{2} C$ (b) $2C$
 (c) $\frac{C}{\sqrt{2}}$ (d) $\frac{C}{2}$

Ans :

Foreign 2001

Dielectric constant of oil, $K = 2$ and Capacitance with oil = C

Capacitance of the capacitor with oil,

$$C = KC_0 = 2C_0$$

$$C_0 = \frac{C}{2}$$

where, C_0 = Capacitance of the capacitor after oil is removed.

Thus (d) is correct option.

30. If a copper plate of thickness b is inserted a parallel plate capacitor, then its new capacity will be (where d = Distance between plates)

- (a) $\frac{\epsilon_0 A}{d+b}$ (b) $\frac{\epsilon_0 A}{d-b}$
 (c) $\frac{\epsilon_0 A}{2d-b}$ (d) $\frac{2\epsilon_0 A}{2d-b}$

Ans :

SQP 2011

Thickness of copper plate = b and Distance between the plates = d

As the copper plate is a conducting slab, therefore capacity of the capacitor with copper plate,

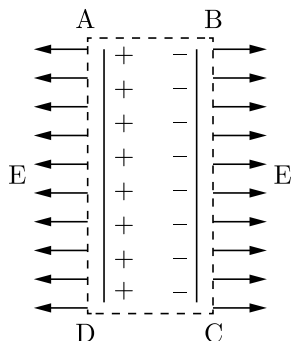
$$C = \frac{\epsilon_0 A}{d-t} = \frac{\epsilon_0 A}{d-b}$$

Thus (b) is correct option.

- (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Charge stored on the two plates are $+Q$ and $-Q \Rightarrow Q + (-Q) = 0$ and hence Assertion is correct. The field however, outside the plates is zero.



Draw a Gaussian surface $ABCD$ as shown. The field $\vec{E}$ is uniform on faces AD and BC

$$\oint \vec{E} \cdot d\vec{s} = 0 \text{ yields } \vec{E} = 0$$

Thus (c) is correct option.

- 40. Assertion :** Two equipotential surfaces cannot cut each other.

Reason : Two equipotential surfaces are parallel to each other.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Two equipotential surfaces are not necessarily parallel to each other.

Thus (c) is correct option.

- 41. Assertion :** Electric potential and electric potential energy are different quantities.

Reason : For a system of positive test charge and point charge electric potential energy = electric potential.

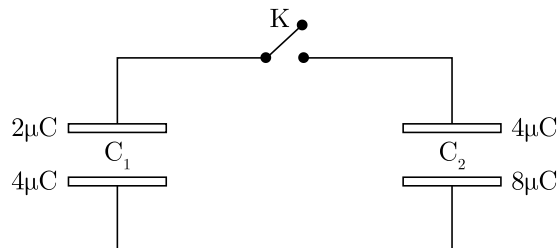
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Potential and potential energy are different quantities and cannot be equated.

Thus (c) is correct option.

- 42. Assertion :** Charges are given to plates of two plane parallel plate capacitors C_1 and C_2 (such that $C_2 = 2C_1$) as shown in figure. Then the key K is pressed to complete the circuit. Finally the net charge on upper plate and net charge the circuit. Finally the net charge on upper plate and net charge on lower plate of capacitor C_1 is positive.



Reason : In a parallel plate capacitor both plates always carry equal and opposite charge.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Charge distribution on each surfaces makes both capacitor of same potential difference hence charge will not flow.

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

- 43.** A hollow metal sphere of radius 5 cm is charged such that potential on its surface is 10 V. What is the potential at the centre of the sphere ?

Ans :

Comp 2021

The electric potential at every point inside the charged spherical shell is same and equal to the electric potential on its surface. The electric potential at the centre of sphere is 10 V.

- 44.** Can two equipotential surfaces intersect each other ? Justify your answer.

Ans :

OD 2021

No, two equipotential surfaces cannot intersect each other because :

- (i) Two normals can be drawn at intersecting point on two surfaces which gives two directions of $\vec{E}$ at the same point which is impossible.
 (ii) Also two values of potential at the same point is not possible.

$$= \frac{1}{9 \times 10^9} \times 0.15$$

$$= 1.7 \times 10^{-11} \text{ F}$$

where, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^{-2}\text{-C}^{-2}$

Thus (a) is correct option.

35. The earth is a spherical conductor of radius $6.4 \times 10^6 \text{ m}$. Its capacitance is

- (a) $7.11 \times 10^4 \text{ F}$ (b) $7.11 \times 10^{-4} \text{ F}$
 (c) $7.11 \times 10^5 \text{ F}$ (d) $7.11 \times 10^{-5} \text{ F}$

Ans :

Delhi 2003, Comp 2017

Radius of earth, $r = 6.4 \times 10^6 \text{ m}$

Capacitance of earth as spherical conductor,

$$C = 4\pi\epsilon_0 r$$

$$= \frac{1}{9 \times 10^9} \times (6.4 \times 10^6)$$

$$= 7.11 \times 10^{-4} \text{ F}$$

where, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^{-2}\text{-C}^{-2}$

Thus (b) is correct option.

ASSERTION AND REASON

36. **Assertion :** If the distance between parallel plates of a capacitor is halved and dielectric constant is three times, then the capacitance becomes 6 times.

Reason : Capacity of the capacitor does not depend upon the nature of the material.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Capacity of capacitor is directly proportional to dielectric constant and inversely proportional to distance. So, the net effect of making distance halved and making dielectric constant three times will be capacity becoming six times.

As nature of the material (dielectric constant) is a factor influencing the capacity, therefore, Reason is incorrect.

Thus (c) is correct option.

37. **Assertion :** If three capacitors of capacitances $C_1 < C_2 < C_3$ are connected in parallel then their equivalent capacitance $C_P > C_S$.

Reason : $\frac{1}{C_P} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

If three capacitors are joined in parallel then their equivalent capacitor will be less than the least value of capacitor so

$$C_P > C_S$$

$$\frac{1}{C_P} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ is incorrect.}$$

Thus (c) is correct option.

38. **Assertion :** A parallel plate capacitor is connected across battery through a key. A dielectric slab of dielectric constant K is introduced between the plates. The energy which is stored becomes K times.

Reason : The surface density of charge on the plate remains constant or unchanged.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

In the given cases, $V = V_0$ (remains constant).

Energy stored in the capacitor $U = \frac{1}{2} CV^2$

When a dielectric slab of dielectric constant K is introduced between the plates of the condenser, then $C' \rightarrow KC$

So energy stored will become K times.

Since $Q = CV$, So Q will become K times

Hence Surface charge density

$$\sigma' = \frac{KQ}{A} = K\sigma_0$$

Thus (c) is correct option.

39. **Assertion :** The total charge stored in a capacitor is zero.

Reason : The field just outside the capacitor is $\frac{\sigma}{\epsilon_0}$. (σ is the charge density).

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

54. Why is the dielectric constant of conductors taken as ∞ ?

Ans :

Foreign 2013

When the conductors are placed in the external field, then the induced electric field is equal and opposite to the external field E_0 .

Hence, Net field,

$$E_{\text{Net}} = E_0 - E_m = E_0 - E_0 = 0$$

Since, Dielectric constant,

$$K = \frac{E_0}{E_{\text{net}}} = \frac{E_0}{0} = \infty$$

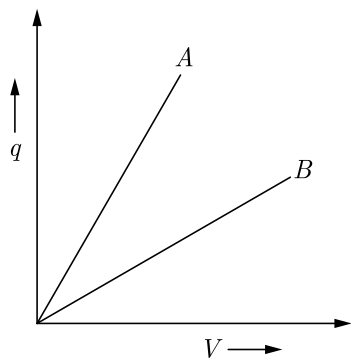
55. What is the amount of work done in moving a point charge Q around a circular arc of radius r at the centre of which another point charge q is located?

Ans :

Foreign 2014

We know that, work done $W = qV$. Since potential difference between any two point on circumference of circle is zero, so work done will be zero.

56. The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2 . Both the capacitors have same plate separation but plate area of C_2 is greater than that C_1 . Which line (A or B) corresponds to C_1 and why ?



Ans :

Foreign 2019, OD 2003

Line B corresponds to C_1 because slope (q/v) of B is less than slope of A .

57. Why is there no work done in moving a charge from one point to another on an equipotential surface?

Ans :

Delhi 2018

The potential difference between any two points of equipotential surface is zero. We have,

$$V_1 - V_2 = \frac{W}{q} = 0$$

$$W = 0$$

Therefore, the work done in moving a charge on an equipotential surface is zero.

58. What is the function of second plate in a parallel plate capacitor?

Ans :

Comp 2010

A parallel plate capacitor of two large plane parallel conducting metal plates separated by a small distance. The second plate acts as a neighbouring conductor due to which the potential of first plate is reduced, keeping it same size, due to which the capacitance increase.

59. Why are equipotential surfaces perpendicular to field lines?

Ans :

OD 2017

Electric field should not have any component along the equipotential surface otherwise it will cause the charges on the surface of the conductor to move, i.e. work is done. But no work is done in moving a charge on an equipotential surface. Hence, the equipotential surface is perpendicular to field lines.

60. Define Dielectric strength and Relative permittivity.

Ans :

OD 2021

Dielectric Strength : The maximum electric field that a dielectric can withstand without breakdown (for its insulating property) is called its dielectric strength.

Relative Permittivity : The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two plates is called relative permittivity.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

61. Why do electric lines of force never intersect each other? Can two equipotential surfaces intersect?

Ans :

Delhi 2021, Comp 2016

Two electric field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point which is not possible. Two equipotential surface cannot intersect each other because two different equipotential surface have different electric potential, so if they intersect then the point of intersection will have two different potentials at the same point which is not possible.

62. Why electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface?

Ans :

OD 2019

Since, electric field intensity inside the conductor is zero. So, electrostatic potential is a constant.

But,
$$E = -\frac{\Delta V}{\Delta r}$$

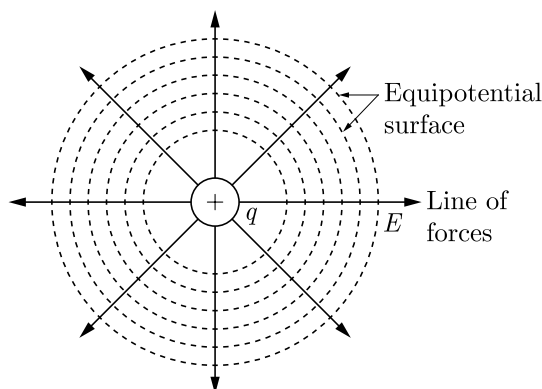
Since,
$$E = 0, \Delta V = 0$$

45. Draw equipotential surfaces due to a single point charge.

Ans :

Delhi 2020

Equipotential surfaces due to a single point charge are concentric sphere having charge at the centre.



46. Why there is no work done in moving a charge from one point to another on an equipotential surface ?

Ans :

Foreign 2015

On an equipotential surface, the potential remains constant and thus potential difference (ΔV) is zero.

The work done on a charge q is given as

$$W = q\Delta V$$

Now, as $\Delta V = 0$

So, the work done in moving a charge from one point to another on an equipotential surface is zero.

47. In a certain 0.5 cm^3 of space, electric potential is found to be 7 V throughout. What is the electric field in this region ?

Ans :

OD 2011, Comp 2005

Zero, because electric potential is same throughout as

$$E = -\frac{dV}{dr}$$

48. In which situation is there a displacement current but no conduction current ?

Ans :

SQP 2016

During charging or discharging there is a displacement current but no conduction current between plates of capacitor.

49. Distinguish between a dielectric and a conductor.

Ans :

Comp 2019

Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons in its any volume which makes it able to pass the electricity through it.

50. Give two factors which affect capacitance of a conductor.

Ans :

OD 2012

1. Material of conductor.
2. Cross-section area of conductor.

51. How much work is done in moving a $500\text{ }\mu\text{C}$ charge between two points on an equipotential surface?

Ans :

Delhi 2011

For equipotential surface each point is at the same potential, so

$$V_{\text{final}} = V_{\text{initial}}$$

So,

$$W = q(V_{\text{final}} - V_{\text{initial}}) = 0$$

52. Define the dielectric constant of a medium. What is its unit ?

Ans :

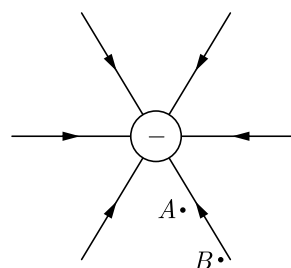
Delhi 2017

When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_P induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric gets reduced to $E_0 - E_P$, where E_0 is external electric field. The ratio of applied external electric field and reduced electric field is known as dielectric constant K of dielectric medium, i.e.,

$$K = \frac{E_0}{E_0 - E_P}$$

It is dimensionless quantity.

53. Figure shows the field lines due to a negative point charge. Give the sign of the potential energy difference of a small negative charge between the points A and B.



Ans :

Delhi 2013

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$$

$$U \propto \frac{1}{r}$$

Hence,

$$U_A > U_B$$

Therefore, $U_A - U_B$ is positive.

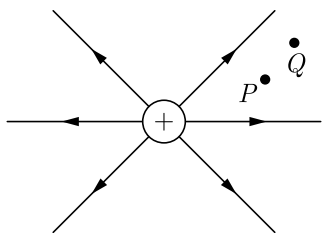
and $r_A < r_B, \frac{1}{r_A} > \frac{1}{r_B}$

Therefore, $\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$ has positive value.

$(V_A - V_B)$ depends on the nature of charge Q ,

1. $(V_A - V_B)$ is positive when $Q > 0$ as.
2. $(V_A - V_B)$ is negative when $Q < 0$.

70. The figure shows the field lines of a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative?



Ans :

Comp 2020

Work done by charge is given by,

$$W = q (\text{potential at } Q - \text{potential at } P)$$

where, q = small positive charge

The electric potential at a point distant r due to the field created by a positive charge Q is given by,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

Since,

$$r_p < r_Q$$

$$V_p > V_Q$$

So, work done will be negative.

71. How can a positively charged body be at zero or negative potential?

Ans :

SQP 2013, Foreign 2002

If a positively charged body B is moved gradually towards a small positively charged body A , at a particular distance, the negative charge induced on A due to B will become equal to positive charge initially present on body A . Hence the body A will be at zero potential. If body B is further moved towards A , a negative charge will be induced on body A . Hence body A will be positive charged but at negative potential. So a positively charged body can be at zero or negative potential.

72. What is the importance of equipotential surfaces?

Ans :

Delhi 2010

The drawings of equipotential surface give us a visual picture of both the magnitude as well as direction of electric field intensity in region of space.

In the region of strong electric fields, equipotential surfaces are close together. The reverse is also true. This follows from,

$$E = -\frac{dV}{dr}$$

i.e. $dr = -\frac{dV}{E}$

or $dr \propto \frac{1}{E}$

73. Define conductors and insulators. Why conductors were called non-electrics and insulators were called dielectrics?

Ans :

OD 2017

Conductors and Insulators : A substance, which allows an electric current to flow through itself, is called a conductor. It has free electrons inside itself. Examples are metals, human body, earth etc.

A substance, which does not allow an electric current to flow through itself, is called an insulator. It does not have free electrons inside itself. Examples are ebonite, glass, quartz, rubber, plastic etc.

Non-electric and Dielectric : When a conductor was rubbed, electric charge produced on it was conducted away through body to earth. As no charge was found present of the rubbed conductor, it was supposed to have no charge. Hence it was called non-electric.

On the other hand, charge stays in an insulator when it is rubbed. It was supposed to have charge. Hence is was called dielectric.

74. Discuss the cause of increase in capacitance of a parallel plate capacitor when
1. A conducting slab is introduced
 2. A dielectric slab is introduced.

Ans :

Foreign 2016

In a conducting slab, large scale movement of free charges reduces the field in the interior of the conductor slab to zero. This decreases the potential difference between the plates resulting in increase in the capacitance.

In a dielectric slab, the applied field polarizes the dielectric on account of small scale alignment of dipoles. The field inside the dielectric decreases, decreasing thereby the potential difference between the plates. Thus the capacitance of the capacitors is increased.

75. Two copper spheres of radii r_1 and r_2 having charges q_1 and q_2 are connected by means of a wire. What is the electric conditions that no charge flows between them?

Hence, $V_2 - V_1 = 0$,
 $V_2 = V_1$

The potential at every point inside the conductor remains same.

63. What are the characteristics of electric potential?

Ans :

Delhi 2018

Characteristics of electric potential

1. Potential is a scalar quantity.
 2. Its value at infinity is zero.
 3. It depends only on the distance from the source charge.
 4. Potential is positive if $q > 0$ and negative if $q < 0$.
64. For any charge configuration, equipotential surface through a point is a normal to the electric field. Justify.

Ans :

OD 2016

No work is done in moving the test charge from one point of an equipotential surface to the other.

$$W_B - W_A = 0 - \int E \cdot dl$$

$$E \cdot dl = 0$$

Hence, $E \perp dl$

65. Give the dimensional formula and unit of electric potential.

Ans :

OD 2016

Dimensional formula of electric potential

Since, $V = \frac{W}{q_0}$

$$= \frac{[ML^2T^{-2}]}{[AT]}$$

$$= [M^1L^2T^{-3}A^{-1}]$$

Units of V

If $W = 1 \text{ J}$, $q_0 = 1 \text{ C}$, then $V = 1 \text{ volt}$. So potential at a point in an electric field is said to be 1 V if 1 J work is done in bringing 1 C charge from infinity to that point.

And $1 \text{ V} = 1 \text{ JC}^{-1}$

66. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface?

Ans :

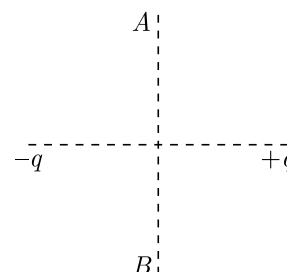
Delhi 2015

Electric field intensity is zero inside the hollow spherical charged conductor. So, no work is done in moving a test charge inside the conductor and on its surface. Therefore, there is no potential difference between any two points inside or on the surface of the conductor.

$$V_A - V_B = - \int \vec{E} \cdot d\vec{l} = 0$$

$$V_A = V_B = \text{constant}$$

67. A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole in equatorial plane without acceleration. Find the work done in the process.



Ans :

SQP 2017

Work done in the process is zero. Because, equatorial plane of a dipole is equipotential surface and work done in moving charge on equipotential surface is zero.

$$W = qW_{AB} = q \times 0 = 0$$

68. Why water has very high dielectric constant?

Ans :

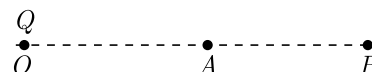
Delhi 2009

Due to unsymmetrical placement of the atoms of water molecule, it possess permanent electric dipole moment which is about $0.6 \times 10^{-20} \text{ Cm}$. This magnitude of dipole moment is about 10 times more than the induced dipole moment acquired by the molecule. Hence due to this large value of permanent dipole moment, the dielectric constant of water molecules is very high.

SHORT ANSWER QUESTIONS

69. A point charge Q is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero if Q is

1. Positive,
2. Negative



Ans :

Delhi 2020

Let the distance of points A and B from charge Q be r_A and r_B respectively.

Hence, Potential difference between points A and B ,

$$V_A - V_B = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

As

$$r_A = OA, r_B = OB$$

$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right) \quad \dots(2)$$

Dividing equation (2) by (1), we get

$$\begin{aligned} \frac{C'}{C} &= 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right) \times \frac{1}{4\pi\epsilon_0 a} \\ &= \frac{b}{b-a} \end{aligned}$$

Since, $\frac{b}{b-a} > 1$

Hence, $\frac{C'}{C} > 1$

or $C' > C$

80. Write the working principle of a parallel plate capacitor. On what factors, the capacitance of a parallel plate capacitor depends?

Ans :

SQP 2008

When an uncharged, earthed conductor is brought near to a charged conductor, then the potential of later decreases and its charge holding capacity increases.

The capacitance depends on :

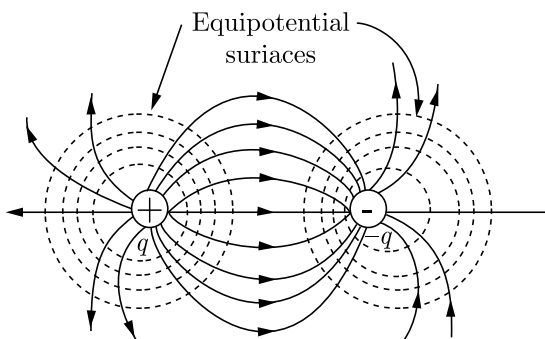
1. Geometrical configuration (shape, size and separation) of the system of two conductors.
 2. Nature of the dielectric separating two conductors.
81. Two point charges $3\mu\text{C}$ and $-3\mu\text{C}$ are placed at points A and B, 5 cm apart.

- (i) Draw the equipotential surfaces of the system.
- (ii) Why do equipotential surfaces get close to each other near the point charge ?

Ans :

SQP 2010, OD 2015

- (i) Equipotential surfaces



- (ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

$$E = -\frac{\Delta V}{\Delta r}$$

and $E \propto -\frac{1}{\Delta r}$

For a given equipotential surface, small Δr represents strong electric field and vice-versa.

82. How does electric potential vary from point to point due to a thin charged spherical shell? Draw a graph showing variation of potential with distance.

Ans :

SQP 2008

For a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the shell is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad (r \geq R)$$

where q is the total charge on the shell and R is its radius.

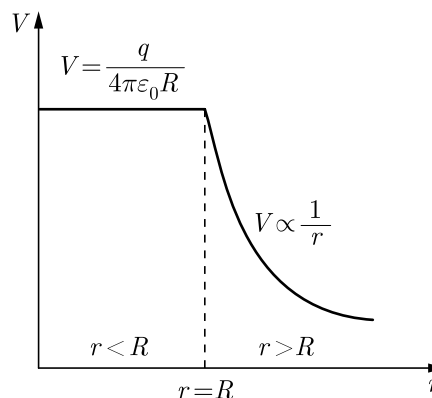
As the electric field inside the shell is zero.

Hence, $E = -\frac{dV}{dr} = 0$

$$V = \text{constant}$$

Therefore, the potential inside the shell is constant (as no work is done in moving a charge inside the shell), and hence, it equals to its value at the surface, which is

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$



A graph showing the variation of potential with distance is given above.

83. A charge Q is given to three capacitors C_1 , C_2 and C_3 connected in parallel. Determine the charge on each.

Ans :

Delhi 2013

In parallel combination of capacitors, the total charge,

$$Q = Q_1 + Q_2 + Q_3$$

As potential difference across each capacitor is same, therefore

Since, $C_{eq} = C_1 + C_2 + C_3$

Hence, P.D. = $V = \frac{Q}{C_1 + C_2 + C_3}$

$$= \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

Hence,

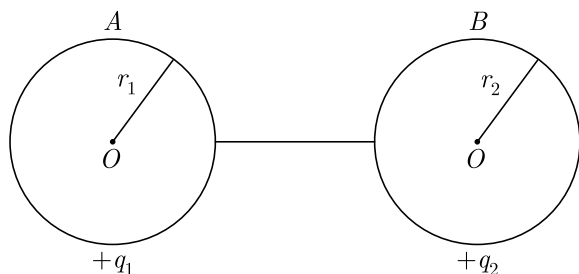
Ans :

Delhi 2019

In figure (as shown below), we have two copper spheres A and B of radii r_1 and r_2 having charges q_1 and q_2 respectively.

Potential of A , $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1}$

Potential of B , $V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$



On connecting these two spheres by a wire, the charge will flow from a body at higher potential to the body at low potential, till the potential of both spheres becomes equal.

So the electrical condition that no charge flows between them is that when,

$$\text{Potential of sphere } A = \text{Potential of sphere } B$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$$

$$q_1 r_2 = q_2 r_1$$

This is the condition when flow of charge will stop.

76. Show that the potential is constant within and at the surface of a charged conductor.

Ans :

Foreign 2016, OD 2010

Since the electric field is equal to negative of potential gradient,

i.e. $E = -\frac{dV}{dx}$

Inside the conductor, $E = 0$

or $V = \text{constant}$

Hence the potential within and at the surface of a charged conductor is constant i.e. surface of a conductor is an equipotential surface.

77. Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

Ans :

Comp 2016

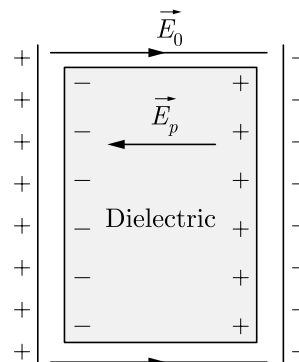
When a dielectric is placed inside an electric field $\vec{E}_0$, it gets polarised as shown in the figure and electric field $\vec{E}_p$ due to polarisation is produced inside the dielectric in a direction opposite to the electric field $\vec{E}_0$.

Hence, resultant electric field between the plates is,

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

Hence,

$$\vec{E} < \vec{E}_0$$



78. Show that the capacitance of an insulated spherical conductor is directly proportional to the radius of the spherical conductor.

Ans :

OD 2014

Consider an isolated spherical conductor of radius r having charge $+q$, which reside at its surface only. The charge behaves as if it is concentrated at its centre O . If V is the potential at the surface of the spherical conductor, then

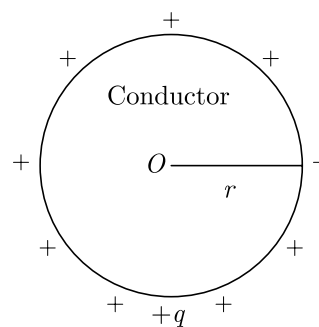
$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad \dots(1)$$

Hence, capacitance of the spherical conductor is given by,

$$C = \frac{q}{V} = \frac{q \cdot 4\pi\epsilon_0 r}{q} = 4\pi\epsilon_0 r$$

or

$$C \propto r$$



79. Show that the capacitance of spherical capacitor is more than that of a spherical conductor.

Ans :

OD 2019

The capacitance of a spherical conductor of radius a is given by,

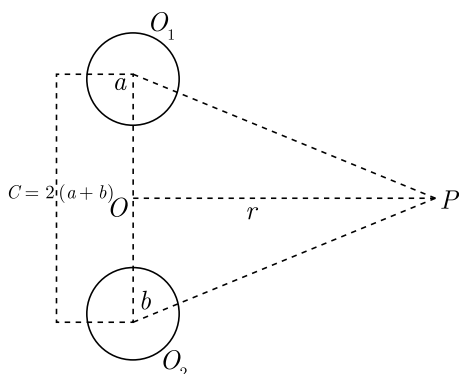
$$C = 4\pi\epsilon_0 a \quad \dots(1)$$

The capacitance of a spherical capacitor of inner radius a and outer radius b is given by,

$$\vec{E} = \vec{E}_0 - \vec{E}_p < \vec{E}_0$$

So, the dielectric constant is defined as the ratio of the electric field in vacuum to the electric field in medium.

88. Find the P.E. associated with a charge q if it were present at the point P with respect to the 'set up' of two charged spheres, arranged as shown. Here O is the mid-point of the line $O_1 O_2$.



Ans :

Delhi 2013

We have

$$r_1 = O_1 P = \sqrt{r^2 + (2a + b)^2}$$

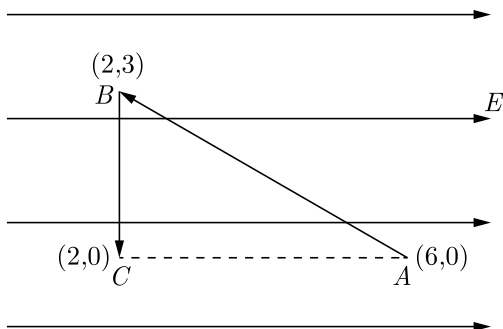
$$r_2 = O_2 P = \sqrt{r^2 + (a + 2b)^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

P.E. of charge, q at $P = qV$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{Q_1}{[r^2 + (2a + b)^2]^{1/2}} + \frac{Q_2}{[r^2 + (a + 2b)^2]^{1/2}} \right]$$

89. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.
- Calculate the potential difference between A to C .
 - At which point (of the two) is the electric potential more and why ?



Ans :

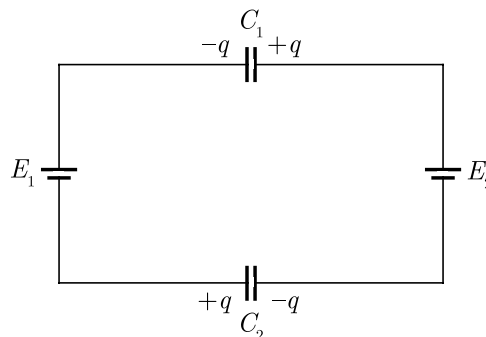
OD 2017, SQP 2001

(i)

$$\begin{aligned} dV &= -E dr \\ &= -E(6 - 2) = -4E \end{aligned}$$

- Electric potential is more at point C as $dV = -E dr$, i.e., the electric potential decreases in the direction of the electric field.

90. Determine the potential difference across the plates of the capacitor C_1 of the network shown in the figure. [Assume $E_2 > E_1$].



Ans :

Foreign 2012

Let $E_2 > E_1$

Therefore, right plate of C_1 has charge $+q$ and left plate of C_1 has charge $-q$. On the same basis, left plate of C_2 has charge $+q$ and right plate of C_2 has charge $-q$.

In closed circuit, potential difference $dV = 0$

$$\frac{q}{C_1} + E_1 + \frac{q}{C_2} - E_2 = 0$$

or

$$q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = E_2 - E_1$$

$$q \left(\frac{C_2 + C_1}{C_1 C_2} \right) = E_2 - E_1$$

$$q = \frac{(E_2 - E_1) C_1 C_2}{C_1 + C_2}$$

Hence, potential difference across left and right plates of C_1 is

$$V = \frac{q}{C_1} = \frac{(E_2 - E_1) C_2}{C_1 + C_2}$$

91. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

Ans :

Comp 2017

Initially when there is vacuum between the two plates, the capacitance of the two-parallel plates is,

$$C_0 = \frac{\epsilon_0 A}{d}$$

where A is the area of parallel plates.

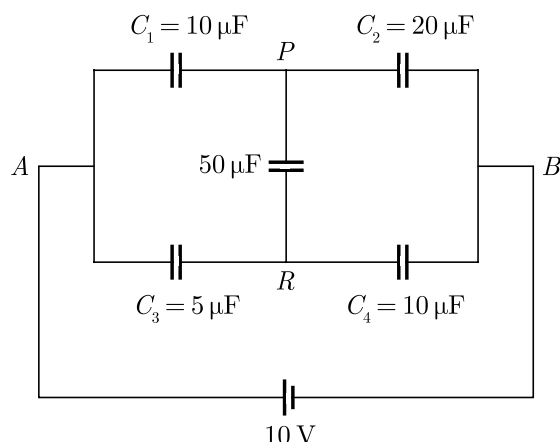
Suppose that the capacitor is connected to a battery, an electric field E_0 is produced.

Ans :

Delhi 2016, OD 2012

$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

This is the condition of balance so there will be no current across PR ($50 \mu\text{F}$ capacitor)

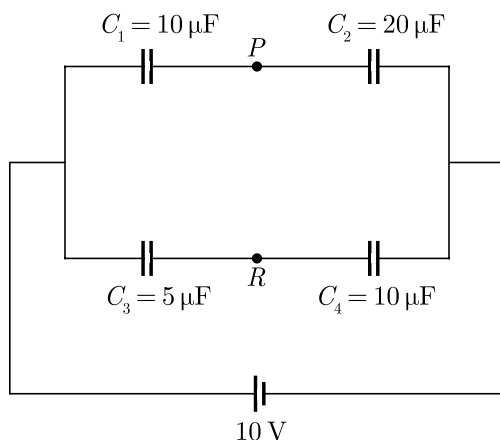


Now, C_1 and C_2 are in series,

$$\begin{aligned} C_{12} &= \frac{C_1 C_2}{C_1 + C_2} \\ &= \frac{10 \times 20}{10 + 20} = \frac{200}{30} = \frac{20}{3} \mu\text{F} \end{aligned}$$

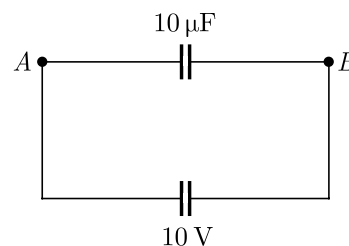
C_3 and C_4 are in series,

$$\begin{aligned} C_{34} &= \frac{C_3 C_4}{C_3 + C_4} \\ &= \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3} \mu\text{F} \end{aligned}$$



Equivalent capacitance between A and B is

$$\begin{aligned} C_{AB} &= C_{12} + C_{34} \\ &= \frac{20}{3} + \frac{10}{3} = 10 \mu\text{F} \end{aligned}$$



$$\begin{aligned} \text{Hence, charge drawn from battery, } Q &= CV \\ &= 10 \times 10 \mu\text{C} \\ &= 100 \mu\text{C} = 10^{-4} \text{ C} \end{aligned}$$

155. Three capacitors each of capacitance 9 pF are Net capacitance of three identical capacitors in series is $1 \mu\text{F}$. What will be their net capacitance if connected in parallel ?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

Ans :

Foreign 2018

Let C be the capacitance of each capacitor, then in series,

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

or

$$C = 3C_s = 3 \times 1 \mu\text{F} = 3 \mu\text{F}$$

When these capacitors are connected in parallel, net capacitance,

$$C_p = 3C = 3 \times 3 = 9 \mu\text{F}$$

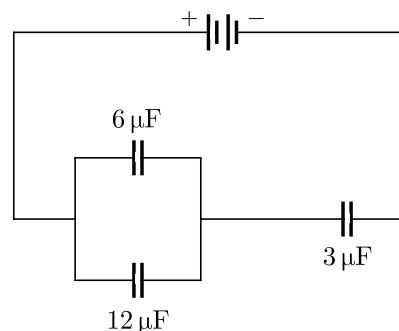
When these two combinations are connected to same source the potential difference across each combination is same.

Ratio of energy stored,

$$\frac{U_s}{U_p} = \frac{\frac{1}{2} C_s V^2}{\frac{1}{2} C_p V^2} = \frac{C_s}{C_p} = \frac{1 \mu\text{F}}{9 \mu\text{F}} = \frac{1}{9}$$

$$U_s : U_p = 1 : 9$$

156. In the following arrangement of capacitors, the energy stored in $6 \mu\text{F}$ capacitor is E . Find the value of the following :



$$E = \frac{-\delta V}{\delta l}$$

- (i) Electric field is in the direction in which the potential decreases steepest.
- (ii) Magnitude of electric field is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.
- 94.** What is electrostatic shielding ? How is this property used in actual practice ? Is the potential in the cavity of a charged conductor zero ?

Ans :

OD 2016

Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence. The field inside a conductor is zero. This is known as electrostatic shielding.

- (i) Sensitive instruments are shielded from outside electrical influences by enclosing them in a hollow conductor.
- (ii) During lightning it is safest to sit inside a car, rather than near a tree. The metallic body of a car becomes an electrostatic shielding from lightning.
- Potential inside the cavity is not zero. Potential is constant.

- 95.** An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q , from infinity to a point distant r , in front of the charged plane sheet.

Ans :

OD 2017, Foreign 2009

Let P be a point at distance r from the sheet.

The required work done to bring point charge q from infinity to P is

$$W = q \cdot (V_P - V_\infty) \quad \dots(1)$$

$$\begin{aligned} \text{Now, } V_P - V_\infty &= -\int_\infty^r \vec{E} \cdot \vec{dr} \\ &= -\int_\infty^r E \cdot dr = -\int_\infty^r \left(\frac{\sigma}{2\epsilon_0} \right) \cdot dr \\ \frac{-\sigma}{2\epsilon_0} \int_\infty^r dr &= -\frac{\sigma}{2\epsilon_0} \cdot [r]_\infty^r \end{aligned}$$

[Since, field in front of an infinity large plane sheet of charge is uniform and is given by $\frac{\sigma}{2\epsilon_0}$.]

$$\begin{aligned} -\frac{\sigma}{2\epsilon_0}(r - \infty) &= 0 \\ V_P - V_\infty &= 0 \end{aligned}$$

From equations (1), $W = 0$

LONG ANSWER QUESTIONS

- 96.** (i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d .
- (ii) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting plate respectively. Find the ratio of their surface charge densities in terms of their radii.

Ans :

Comp 2022

- (i) Expression for the capacitance of a parallel plate capacitor

Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d . One of the plates is insulated and other is earthed. There is a vacuum between the plates.

Suppose, the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges.

Suppose the surface density of charge on each plate is σ . We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is σ/ϵ_0 , where ϵ_0 is the permittivity of free space.

The intensity of electric field between the plates will be given by $E = \frac{\sigma}{\epsilon_0}$. The charge on each plate is q and the area of each plate is A . Thus,

$$\sigma = \frac{q}{A}$$

$$\text{and } E = \frac{q}{\epsilon_0 A} \quad \dots(1)$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d}$$

$$\text{or } V = Ed$$

Substituting the value of E from Equation (1), we get

$$V = \frac{qd}{\epsilon_0 A}$$

Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{\frac{qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

It is clear from this formula that in order to obtain high capacitance,

Now if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E .

Now the gap between plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0 .

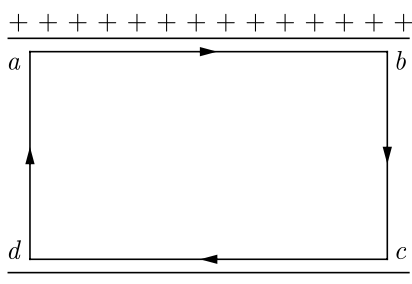
If V be the potential difference between the plates of the capacitor, then

$$\begin{aligned} V &= E_t + E_0(d - t) \\ V &= \frac{Ed}{2} + \frac{E_0 d}{2} \\ &= \frac{d}{2}(E + E_0) \quad \left(\text{since } t = \frac{d}{2}\right) \\ V &= \frac{d}{2}\left(\frac{E_0}{K} + E_0\right) \\ &= \frac{dE_0}{2K}(K + 1) \quad \left(\text{As, } \frac{E_0}{E} = K\right) \end{aligned}$$

Now,

$$\begin{aligned} E_0 &= \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} \\ V &= \frac{d}{2K} \cdot \frac{q}{\epsilon_0 A}(K + 1) \\ C &= \frac{q}{V} = \frac{2K\epsilon_0 A}{d(K + 1)} \end{aligned}$$

92. (i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.
(ii) The electric field inside a parallel plate capacitor is E . Find the amount of work done in moving a charge q over a closed rectangular loop $abcd$.



Ans :

Delhi 2007, Foreign 2009

- (i) The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

Let us assume that initially both the plates are uncharged. Now, we have to repeatedly remove small positive charges from one plate and transfer them to other plate.

Now, when an additional small charge (dq) is transferred from one plate to another, the small work done is given by

$$dW = V' dq = \frac{q'}{C} dq$$

[Let charge on plate, when dq charge is transferred is q']

The total work done in transferring charge Q is given by

$$\begin{aligned} W &= \int_0^Q \frac{q'}{C} dq = \frac{1}{C} \int_0^Q q' dq \\ &= \frac{1}{C} \left[\frac{(q')^2}{2} \right]_0^Q = \frac{Q^2}{2C} \end{aligned}$$

This work is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} \quad [\text{since } Q = CV]$$

$$U = \frac{1}{2} CV^2$$

The energy stored per unit volume of space in a capacitor is called energy density.

$$u = \frac{\frac{1}{2} CV^2}{A \cdot d} = \frac{1}{2} \frac{\epsilon_0 A V^2}{d^2 A}$$

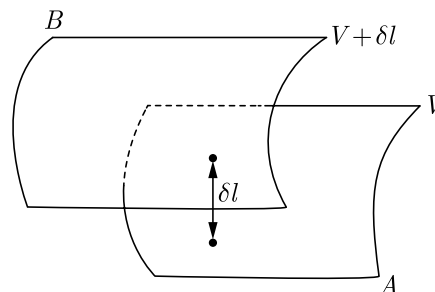
Energy density, $u = \frac{1}{2} \epsilon_0 E^2$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitor.

i.e., $U = U_1 + U_2 + U_3 + \dots$

- (ii) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero.

93. Two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, (where δV is the change in V) are kept δl distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potential.



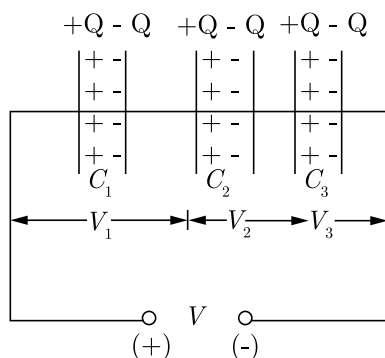
Ans :

OD 2014

Work done in moving a unit positive charge along distance δl ,

$$\begin{aligned} |E_l| \delta l &= V_A - V_B \\ &= V - (V + \delta V) = -\delta V \end{aligned}$$

- (ii) In series combination of capacitors, same charge lie on each capacitor for any value of capacitances.



Also, potential difference across the combination is equal to the algebraic sum of potential differences across each capacitor.

$$\text{i.e., } V = V_1 + V_2 + V_3$$

where, V_1, V_2, V_3 s and V are the potential differences across C_1, C_2, C_3 and equivalent capacitor, respectively.

$$\text{Hence, } q = C_1 V_1$$

$$V_1 = \frac{q}{C_1}$$

$$\text{Similarly, } V_2 = \frac{q}{C_2}$$

$$\text{and, } V_3 = \frac{q}{C_3}$$

Total potential difference,

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

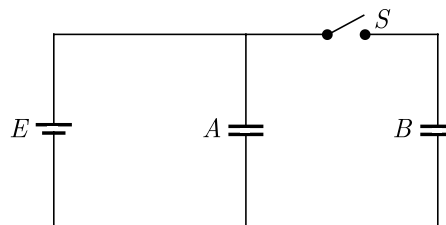
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Since, $\frac{V}{q} = \frac{1}{C}$ s, where C is equivalent capacitance of combination

$$\text{or } \frac{1}{C} = \frac{C_2 C_3 + C_3 C_1 + C_1 C_2}{C_1 C_2 C_3}$$

$$C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

99. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K . Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



or

A parallel plate capacitor (A) of capacitance C is charged by a battery to voltage V . The battery is disconnected and an uncharged capacitor (B) of capacitance $2C$ is connected across A . Find the ratio of

- final charges on A and B .
- total electrostatic energy stored in A and B finally and that stored in A initially.

Ans :

OD 2023

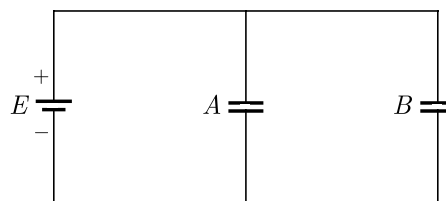
Let C be the capacitance of each capacitor.

With switch S closed, the two capacitor are in parallel.

Hence, Equivalent capacitance is $2C$.

$$\text{Hence, Energy stored} = \frac{1}{2} (2C) V^2$$

$$U_1 = CV^2 \quad \dots(1)$$



Now, when switch is opened and then free space of capacitors are filled with dielectric, the capacitance of each capacitor will be KC . For capacitor B , the charge will remain as before and for A , the potential difference will remain same.

Charge on each capacitor in the previous case will be CV .

Energy stored in capacitor A in circuit case is

$$U_A = \frac{1}{2} (KC) V^2 = \frac{1}{2} KCV^2$$

and that in capacitor B , is

$$U_B = \frac{Q^2}{2KC} = \frac{(CV)^2}{2KC} = \frac{1}{2K} CV^2$$

Total energy stored, $U_2 = U_A + U_B$

$$U_2 = \frac{1}{2} KCV^2 + \frac{1}{2K} CV^2$$

$$U_2 = \frac{1}{2} \left(K + \frac{1}{K} \right) CV^2$$

- (a) A should be large, i.e., the plates of large area should be taken.
 (b) d should be small, i.e., the plates should be kept closer to each other.
 (ii) Ratio of their surface charge densities

Surface charge density is given by

$$\sigma = \frac{q}{4\pi R^2}$$

After connecting both the conductors, their potentials will become equal.

$$V_1 = V_2$$

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$

[Since for spherical conductors]

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

$$V = \frac{Kq}{R}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \frac{q_1}{q_2} \left(\frac{R_2}{R_1} \right)^2 = \frac{R_2}{R_1}$$

97. A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates.

How will

- (a) its capacitance
 (b) electric field between the plates and
 (c) energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expression for each case.

Ans :

Comp 2019, OD 2004

On introduction of dielectric slab in a isolated charged capacitor.

- (a) The capacitance (C') becomes K s times of original capacitor as

$$C = \frac{\epsilon_0 A}{d}$$

and
$$C' = \frac{K\epsilon_0 A}{d}$$

- (b) The total charge on the capacitor remains conserved on introduction of dielectric slab. Also, the capacitance of capacitor increases to K times of original values.

$$CV = C' V'$$

$$CV = (KC) V'$$

$$V' = \frac{V}{K}$$

New electric field,

$$E' = \frac{V'}{d} = \left(\frac{V/K}{d} \right) = \left(\frac{V}{d} \right) \frac{1}{K} = \frac{E}{K}$$

On introduction of dielectric medium new electric field E' becomes $\frac{1}{K}$ times of its original value.

- (c) Energy stored initially,

$$U = \frac{q^2}{2C}$$

Energy stored later,

$$U' = \frac{q^2}{2(KC)} \quad [\text{Since, } C' = KC]$$

where, K = dielectric constant of medium

$$\begin{aligned} U' &= \frac{1}{K} \left(\frac{q^2}{2C} \right) \\ &= \frac{1}{K} (U) \\ &= \frac{1}{K} \times U \end{aligned}$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy.

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98. (i) Deduce the expression for the energy stored in a charged capacitor.
 (ii) Show that the effective capacitances C of a series combination of three capacitors C_1 , C_2 and C_3 is given by $C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$.

Ans :

Foreign 2005

- (i) Expression for the energy stored in a charged capacitor

We have

$$q = CV$$

$$V = q/C$$

$$dW = Vdq = \frac{q}{C} dq$$

where, q = instantaneous charge,

C = instantaneous capacitance and

V = instantaneous voltage

Total work done in storing charge from 0 to q ,

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$\begin{aligned}
 &= \frac{d}{2} \left(\frac{E_0}{K} + E_0 \right) \\
 &= \frac{dE_0}{2K} (K+1) \quad \left(\text{As, } \frac{E_0}{E} = K \right) \\
 E_0 &= \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} \\
 V &= \frac{d}{2K} \cdot \frac{q}{\epsilon_0 A} (K+1) \\
 C &= \frac{q}{V} = \frac{2K\epsilon_0 A}{d(K+1)}
 \end{aligned}$$

- 102.** Find an expression for capacity of a parallel plate capacitor with compound dielectric.

Ans :

Comp 2016

Consider a parallel plate capacitor of plate area A and plate separation d . If the space between the plates is vacuum, its capacitance is given by,

$$C_0 = \frac{\epsilon_0 A}{d}$$

Suppose initially the charge on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is,

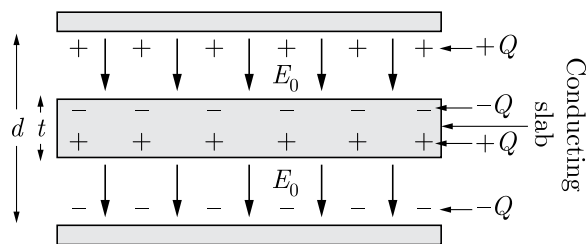
$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

where σ is the surface charge density. The potential difference between the capacitor plates will be,

$$V_0 = E_0 d = \frac{Qd}{A\epsilon_0}$$

When a conducting slab of thickness $t < d$ is placed between the capacitor plates, free electrons flow inside it so as to reduce the field to zero inside the slab, as shown in Figure. Charges $-Q$ and $+Q$ appear on the upper and lower faces of the slab. Now the electric field exists only in the vacuum region between the plates of the capacitor on the either side of the slab, i.e. the field exists only in thickness $d - t$, therefore, potential difference between the plates of the capacitor is,

$$\begin{aligned}
 V &= E_0(d-t) \\
 &= \frac{Q}{A\epsilon_0}(d-t)
 \end{aligned}$$



Hence, Capacitance of the capacitor in the presence of conducting slab becomes,

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{(d-t)} = \frac{\epsilon_0 A}{d} \cdot \frac{d}{d-t}$$

or

$$C = \left(\frac{d}{d-t} \right) \cdot C_0$$

Clearly, $C > C_0$. Thus the introduction of a conducting slab of thickness t in a parallel plate capacitor increases its capacitance by a factor of $\frac{d}{d-t}$.

- 103.**
1. Define capacitance of a capacitor.
 2. Derive expression for stored energy between plates of parallel plate capacitor. Show that energy-density between plates of the capacitor can be expressed as $\frac{1}{2} \epsilon_0 E^2$, when E = Electric field between plates.

or

1. Define capacitance and gives its SI units.
2. Prove that the total electrostatic energy stored in a parallel plate capacitor is $\frac{1}{2} CV^2$. Hence derive the expression for the energy density of the electric field at its base.

Ans :

OD 2020

1. **Capacitance :** The capacitance of capacitor may be defined as the charge required to be supplied to either of the conductors of the capacitor so as to increase the potential difference between them by unit amount.

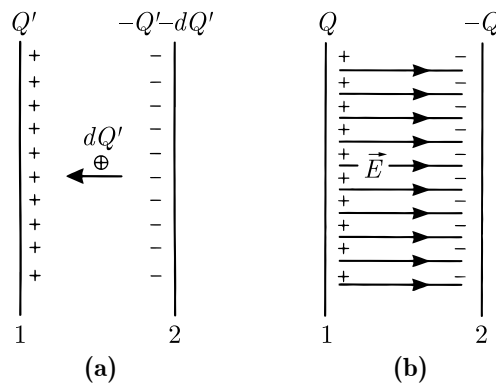
Capacitance,

$$C = \frac{\text{Charge on either plate (Q)}}{\text{Potential difference between the two plates (V)}}$$

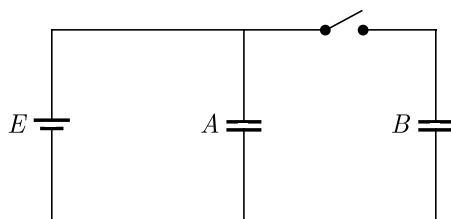
SI unit of capacitance is farad (F).

2. **Expression for the Energy Stored in a Capacitor :** Consider a capacitor of capacitance C . Initially, its two plates are uncharged. Suppose the positive charge is transferred from plate 2 to plate 1 bit by bit. In this process, external work has to be done because at any stage plate 1 is at higher potential than the plate 2. Suppose at any instant the plates 1 and 2 have charges Q' and $-Q'$ respectively, as shown in Figure a. Then the potential difference between the two plates will be

$$V' = \frac{Q'}{C}$$



$$U_2 = \left(\frac{K^2 + 1}{2K} \right) CV^2 \quad \dots(2)$$



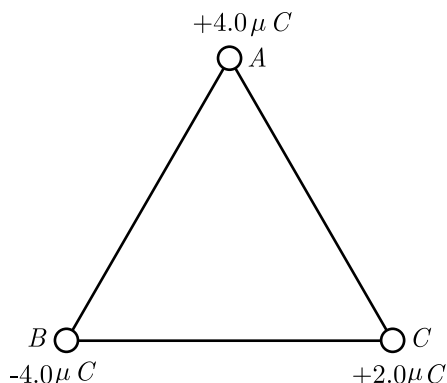
From equations (1) and (2), we get

$$\frac{U_1}{U_2} = \frac{CV^2}{\left(\frac{K^2 + 1}{2K} \right) CV^2}$$

$$\frac{U_1}{U_2} = \frac{2K}{K^2 + 1}$$

- 100.** State the significance of negative value of electrostatic potential energy of a system of charges.

Three charges are placed at the corners of an equilateral triangle ABC of side 2.0 m as shown in figure. Calculate the electric potential energy of the system of three charges.



Ans :

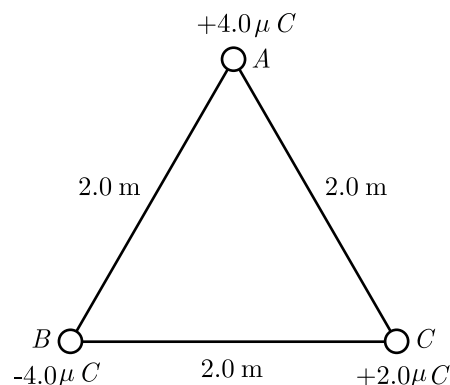
Comp 2023, OD 2011

The expression for electric potential due to a point charge is,

$$V = k \frac{q}{r}$$

The electric potential is a scalar quantity. Since, both the charges are positive, therefore, the electric potential at any point is the sum of the electric potentials of both the charges, which is a positive quantity. Thus, the electric potential along the line joining of two equal positive charges cannot be zero.

A negative value of electrostatic potential energy means that work must be done against the electric field in moving the charges apart.



Formula of electrostatic potential energy,

$$U_E = k \frac{q_1 q_2}{r}$$

Now, total P.E. = $RE_{AB} + PE_{BC} + PE_{AC}$

$$\begin{aligned} k &= \left[\frac{4 \times (-4)}{2} + \frac{(-4) \times 2}{2} + \frac{(4) \times (2)}{2} \right] \times 10^{-12} \\ &= k \left[\frac{-16}{2} + \frac{-8}{2} + \frac{8}{2} \right] \times 10^{-12} \\ &= -8k \times 10^{-12} \\ &= -8 \times 9 \times 10^9 \times 10^{-12} \\ &= -72 \times 10^{-3} \\ &= -7.2 \times 10^{-2} \text{ J} \end{aligned}$$

- 101.** A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

Ans :

Delhi 2021

Initially when there is vacuum between the two plates, the capacitance of the two-parallel plates is,

$$C_0 = \frac{\epsilon_0 A}{d}$$

where, A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced.

Now, if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E .

Now, the gap between plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0 .

If V be the potential difference between the plates of the capacitor, then

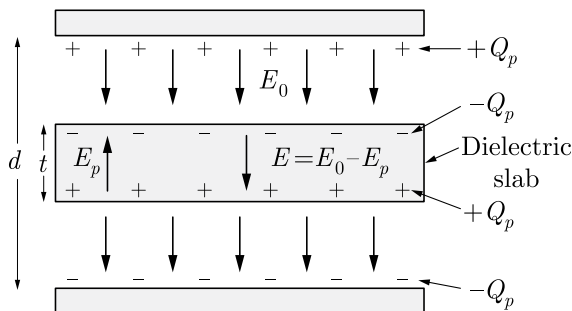
$$\begin{aligned} V &= E_t + E_0(d - t) \\ &= \frac{Ed}{2} + \frac{E_0 d}{2} \\ &= \frac{d}{2}(E + E_0) \quad \left(\because t = \frac{d}{2} \right) \end{aligned}$$

$$C_0 = \frac{\epsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is,

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

When, a dielectric slab of thickness $t < d$ is placed between the plates, the field E_0 polarises the dielectric. This induces charge $-Q_p$ on the upper surface and



$+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by,

$$E_p = \frac{\sigma_p}{\epsilon_0} = \frac{P}{\epsilon_0}$$

$$\left[\sigma_p = \frac{Q}{A} = P, \text{Polarisation density} \right]$$

The net field inside the dielectric is,

$$E = E_0 - E_p = \frac{E_0}{k} \quad \left[\text{Since, } \frac{E_0}{E_0 - E_p} = k \right]$$

where k is the dielectric constant of the slab. So between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance $(d - t)$. Hence the potential difference between the capacitor plates is,

$$\begin{aligned} V &= E_0(d - t) + Et \\ &= E_0(d - t) + \frac{E_0}{k}t \quad \left[\text{Since, } \frac{E_0}{E} = k \right] \\ &= E_0 \left(d - t + \frac{t}{k} \right) \\ &= \frac{Q}{\epsilon_0 A} \left(d - t + \frac{t}{k} \right) \end{aligned}$$

The capacitance of the capacitor on introduction of dielectric slab becomes,

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$$

- 105.** Find the electric potential and then electric field due to an electric dipole by differential relationship between field and potential.

Ans :

SQP 2009

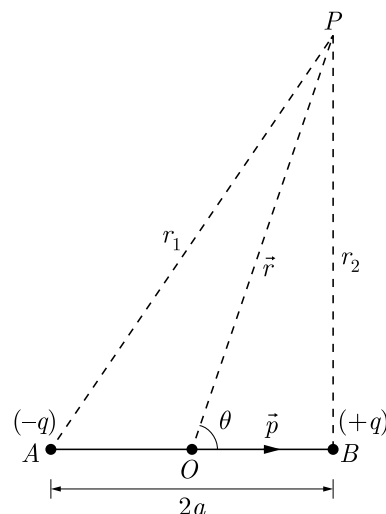
Let an electric dipole consist of two equal and unlike point charges $-q$ at A and $+q$ at B , separated by a small distance $AB = 2a$, with centre at O . The dipole moment,

$$|\vec{p}| = q \times 2a$$

Let us take the origin at the centre of the dipole. We have to calculate electric potential at any point P .

where, $\vec{OP} = \vec{r}$

and $\angle BOP = \theta$



Let the distance of P from charge $-q$ at A be r_1 , i.e., $AP = r_1$

and distance of P from charge $+q$ at B be r_2 , i.e., $BP = r_2$.

Electrostatic potential at P due to $-q$ charge at A ,

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1} \quad \dots(1)$$

Electrostatic potential at P due to q charge at B ,

$$V_2 = \frac{q}{4\pi\epsilon_0 r_2} \quad \dots(2)$$

Therefore, Potential at P due to the dipole,

$$V = V_2 + V_1$$

From equation (1) and (2), we get

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_2} - \frac{1}{r_1} \right] \quad \dots(3)$$

Now, by geometry,

$$\begin{aligned} r_1^2 &= r^2 + a^2 + 2ar \cos(180^\circ - \theta) \\ &= r^2 + a^2 - 2ar \cos \theta \end{aligned}$$

We may rewrite, $r_2^2 = r^2 + a^2 + 2ar \cos(180^\circ - \theta)$

$$= r^2 + a^2 - 2ar \cos \theta$$

We may rewrite, $r_1^2 = r^2 \left(1 + \frac{a^2}{r^2} + \frac{2a}{r} \cos \theta \right)$

Suppose now a small additional charged dQ' be transferred from plate 2 to plate 1. The work done will be,

$$dW = V' \cdot dQ' = \frac{Q'}{C} \cdot dQ'$$

The total work done in transferring a charge Q from plate 2 to plate 1 (Figure b) will be,

$$\begin{aligned} W &= \int dW = \int_0^Q \frac{Q'}{C} \cdot dQ' \\ &= \left[\frac{Q'^2}{2C} \right]_0^Q = \frac{1}{2} \cdot \frac{Q^2}{C} \end{aligned}$$

This work done is stored as electrical potential energy U of the capacitor.

$$\begin{aligned} U &= \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} \cdot CV^2 \\ &= \frac{1}{2} QV \quad (\text{Since, } Q = CV) \end{aligned}$$

Energy Density between Plates of Capacitor

Capacitance of the parallel plate capacitor is given by,

$$C = \frac{\epsilon_0 A}{d} \quad \dots(1)$$

If σ is the surface charge density on the capacitor plates, then electric field between the capacitor plates will be,

$$\begin{aligned} E &= \frac{\sigma}{\epsilon_0} \\ \sigma &= \epsilon_0 E \end{aligned}$$

Charge on either plate of capacitor is,

$$Q = \sigma A = \epsilon_0 EA \quad \dots(2)$$

Hence, Energy stored in the capacitor is,

$$\begin{aligned} U &= \frac{Q^2}{2C} = \frac{(\epsilon_0 EA)^2}{2 \cdot \frac{\epsilon_0 A}{d}} \\ &\quad [\text{From Eq. (1) and (2)}] \\ &= \frac{1}{2} \epsilon_0 E^2 Ad \end{aligned}$$

But Ad = volume of the capacitor between its two plates. Therefore, the energy stored per unit volume or the energy density of the electric field is given by,

$$u = \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2$$

- 104.** Derive an expression for the capacitance of a parallel plate capacitor. If a compound dielectric medium is introduced between the plates of the capacitor, how will the capacitance of the capacitor change?

Ans :

OD 2015, Comp 2002

Capacitance of Parallel Plate Capacitor

The simplest and the most widely used capacitor is the parallel plate capacitor. It consists of two large

plane parallel conducting plates, separated by a small distance.

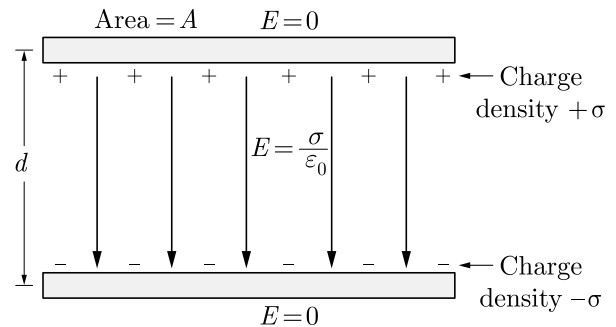
Let, A = area of each plate,

d = distance between the two plates

$\pm \sigma$ = uniform surface charge densities on the two plates

$$\pm Q = \pm \sigma A$$

= total charge on each plate



Parallel plate capacitor

In the outer regions above the upper plate and below the lower plate, the electric field due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

In the inner region between the two capacitor plates, the electric fields due to the two charged plates add up. The net field is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

The direction of the electric field is from the positive to the negative plate and the field is uniform throughout. For plates with finite area, the field lines bend at the edges. This effect is called fringing of the field. But for large plates separated by small distance ($A \gg d^2$), the field is almost uniform in the regions far from the edges. For a uniform electric field,

P.D. between the plates

$$= \text{Electric field} \times \text{distance between the plates}$$

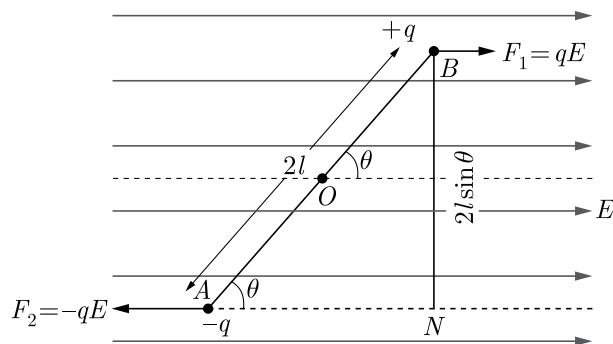
$$\text{or } V = Ed = \frac{\sigma d}{\epsilon_0}$$

Capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d / \epsilon_0} \text{ or } C = \frac{\epsilon_0 A}{d}$$

Capacitance of a Parallel Plate Capacitor with a Dielectric Slab

The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum between its plates is given by,



Now, the dipole is rotated and brought to orientation making an angle θ with the field direction (i.e., $\theta_0 = 90^\circ$ and $\theta_1 = \theta^\circ$). Therefore, work done

$$\begin{aligned} W_2 &= pE(\cos\theta_0 - \cos\theta_1) \\ &= pE(\cos 90^\circ - \cos\theta) \\ &= -pE\cos\theta \end{aligned}$$

Hence, Total work done in bringing the electric dipole from infinity, i.e., electric potential energy of electric dipole. Thus, work done by external torque in rotating a dipole in uniform electric field is stored as the potential energy of the system.

$$\begin{aligned} U &= W_1 + W_2 \\ &= 0 - pE\cos\theta \\ &= -pE\cos\theta \end{aligned}$$

In vector form, $U = -\vec{p} \cdot \vec{E}$

For rotating dipole from position of unstable equilibrium ($\theta_0 = 180^\circ$) to the stable equilibrium ($\theta = 0^\circ$).

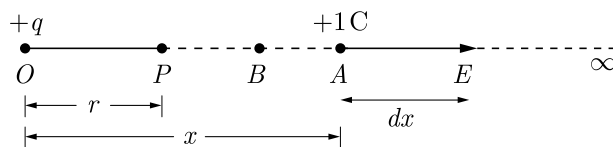
$$\begin{aligned} \text{Hence, } W_{\text{req}} &= pE(\cos 180^\circ - \cos 0^\circ) \\ &= pE(-1 - 1) = -2pE \end{aligned}$$

107. Find out the expression electrostatic potential due to a point charge?

Ans :

OD 2019

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required.



The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to P . As, work done is independent of the path, we choose a convenient path along the radial direction

from infinity to the point P without acceleration. Let A be an intermediate point on this path where $OA = x$. The electrostatic force on a unit positive charge at A is given by,

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \times 1}{x^2} \quad [\text{along } OA]$$

Small work done in moving the charge through a distance dx from A to B is given by,

$$\begin{aligned} dW &= F \cdot dx = Fdx \cos 180^\circ \\ &= -Fdx \quad [\text{Since, } \cos 180^\circ = -1] \\ &= -Fdx \end{aligned}$$

Total work done in moving a unit positive charge from ∞ to the point P is given by,

$$\begin{aligned} W &= \int_{\infty}^r -Fdx \\ &= \int_{\infty}^r -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} dx \\ &= -\frac{q}{4\pi\epsilon_0} \int_{\infty}^r x^{-2} dx \\ &= -\frac{q}{4\pi\epsilon_0} \left[\frac{-1}{x} \right]_{\infty}^r \quad \left[\text{since } \int x^{-2} dx = -\frac{1}{x} \right] \\ &= -\frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] = \frac{q}{4\pi\epsilon_0 r} \end{aligned}$$

From the definition of electric potential, this work is equal to the potential at point P .

$$V = \frac{q}{4\pi\epsilon_0 r}$$

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108. What do you mean by polar molecules? What happens to them when electric field is applied?

Ans :

Foreign 2012

A polar molecule is one in which the centre of gravity of the positive charge (i.e. protons) does not coincide with the centre of gravity of the negative charges (i.e. electrons). Due to finite separation between the positive and negative charge, polar molecules are permanent electric dipoles and have permanent electric dipole moments e.g. HCl , NH_3 , H_2O , CO_2 are polar molecules. In the absence of electric field, the dipole moment of these polar molecules point in random direction and arrange themselves in closed chains [Figure (a)] and net dipole moment is zero.

If $a \ll r$, $\frac{a}{r}$ is small, $\frac{a^2}{r^2}$ can be neglected.

Hence, $r_1^2 = r^2 \left(1 + \frac{2a}{r} \cos \theta\right)$

$$r_1 = r \left(1 + \frac{2a}{r} \cos \theta\right)^{1/2}$$

$$\frac{1}{r_1} = \frac{1}{r} \left(1 + \frac{2a}{r} \cos \theta\right)^{-1/2}$$

Similarly, $\frac{1}{r_2} = \frac{1}{r} \left(1 - \frac{2a}{r} \cos \theta\right)^{-1/2}$

Putting these values in (3), we get

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \left(1 - \frac{2a}{r} \cos \theta\right)^{-1/2} - \frac{1}{r} \left(1 + \frac{2a}{r} \cos \theta\right)^{-1/2} \right]$$

Using Binomial theorem and retaining terms upto the first order in a/r , we get

$$\begin{aligned} V &= \frac{q}{4\pi\epsilon_0 r} \left[1 + \frac{a}{r} \cos \theta - \left(1 - \frac{a}{r} \cos \theta\right) \right] \\ &= \frac{q}{4\pi\epsilon_0 r} \left[1 + \frac{a}{r} \cos \theta - 1 + \frac{a}{r} \cos \theta \right] \\ &= \frac{q \times 2a \cos \theta}{4\pi\epsilon_0 r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} \end{aligned}$$

As $p \cos \theta = \vec{p} \cdot \vec{r}$

where, $\hat{r}$ = is unit vector along the position vector

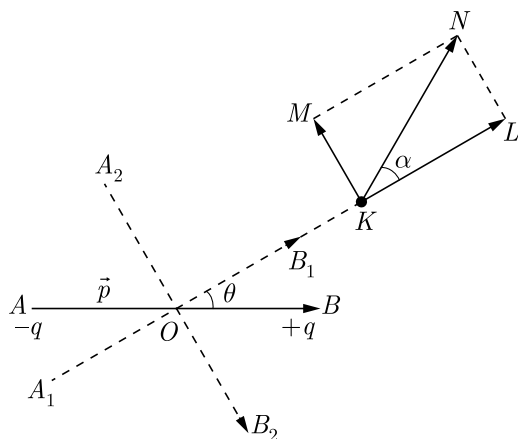
$$\vec{OP} = \vec{r}$$

Hence, Electrostatic potential at P due to a short dipole ($a \ll r$) is,

$$V = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^2}$$

Electric Field Intensity at any Point due to a Short Electric Dipole

In figure (as shown below), AB represents a short electric dipole of moment $\vec{p}$ along $\vec{AB}$. O is the centre of dipole. We have to calculate electric field intensity $\vec{E}$ at any point K ,



where, $OK = r$

$$\angle BOK = \theta$$

The dipole moment p can be resolved into two rectangular components:

$(p \cos \theta)$ along $A_1 B_1$ and $(p \sin \theta)$ along $A_2 B_2 \perp A_1 B_1$.

Field intensity at K on the axial line of $A_1 B_1$,

$$|\vec{E}_1| = \frac{2p \cos \theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\vec{KL}$ along OK .

Field intensity at K on equatorial line of $A_2 B_2$,

$$|\vec{E}_2| = \frac{p \sin \theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\vec{KM} \parallel B_2 A_2$ and $\perp \vec{KL}$.

Complete the rectangle $KLMN$. Join $\vec{KN}$.

According to Parallelogram law, $\vec{KN}$ represents resultant intensity ($\vec{E}$) at K due to the short dipole.

$$\text{As, } KN = \sqrt{KL^2 + KM^2}$$

$$\begin{aligned} \text{Hence, } |\vec{E}| &= \sqrt{E_1^2 + E_2^2} \\ &= \sqrt{\left(\frac{2p \cos \theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p \sin \theta}{4\pi\epsilon_0 r^3}\right)^2} \\ &= \frac{p}{4\pi\epsilon_0 r^3} \sqrt{4 \cos^2 \theta + \sin^2 \theta} \end{aligned}$$

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3} \sqrt{3 \cos^2 \theta + (\cos^2 \theta + \sin^2 \theta)}$$

$$\text{i.e., } |\vec{E}| = \frac{P \sqrt{3 \cos^2 \theta + 1}}{4\pi\epsilon_0 r^3} \quad \dots(1)$$

Let, $\angle LKN = \alpha$

$$\text{In } \triangle KLN, \quad \tan \alpha = \frac{LN}{KL} = \frac{KM}{KL}$$

$$= \frac{p \sin \theta}{4\pi\epsilon_0 r^3} \cdot \frac{4\pi\epsilon_0 r^3}{2p \cos \theta} = \frac{1}{2} \tan \theta$$

Hence, α can be calculated.

- 106.** Show that the potential energy of a dipole making angle θ with the direction of the field is given by $u(\theta) = -\vec{p} \cdot \vec{E}$. Hence, find out the amount of work done in rotating it from the position of unstable equilibrium to the stable equilibrium.

Ans :

OD 2017, Comp 2012

As charges $+q$ and $-q$ traverse equal distance under equal and opposite forces; therefore, net work done in bringing the dipole in the region of electric field perpendicular to field-direction will be zero, i.e., $W_1 = 0$.

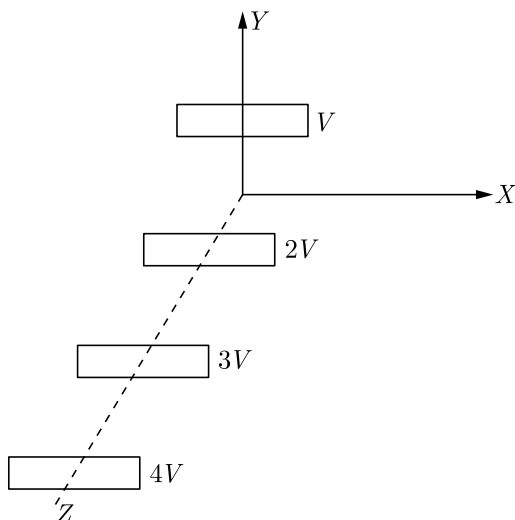
1. A constant electric field in Z -direction.
2. A field that uniformly increases in magnitude but remains in a constant (say Z) direction.
3. A single positive charge at the origin.
4. A uniform grid consisting of long equally spaced parallel charged wires in a plane.

Ans :

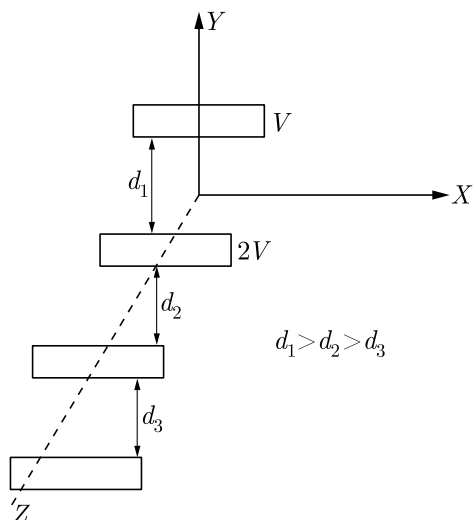
Delhi 2013

Equipotential surface is a surface having the same potential at each of its points. In the given cases the equipotential surface are

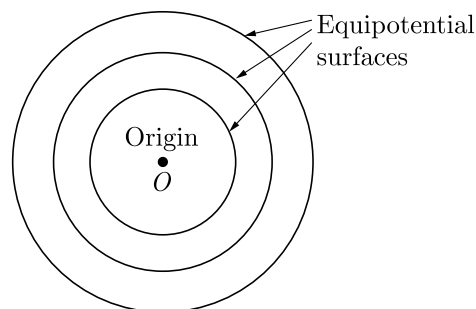
1. The planes parallel to XY plane. For some potential difference, the planes are equidistant.



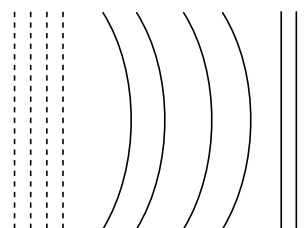
2. The planes are parallel to XY plane, but for the same potential difference, the separation between the planes decreases.



3. Concentric spheres centred at the origin.



4. A periodically varying shape near the grid which gradually attains the shape of planes parallel to grid at far distance.



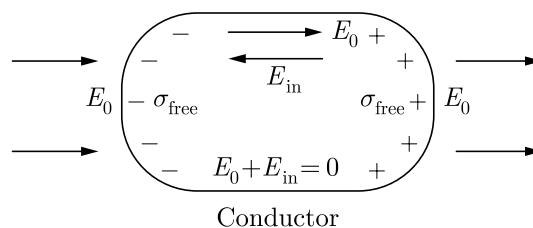
112. Explain, using suitable diagram, the difference in the behaviour of a

1. Conductor
2. Dielectric in the presence of external electric field. Define the terms polarisation of a dielectric and write its relation with susceptibility.

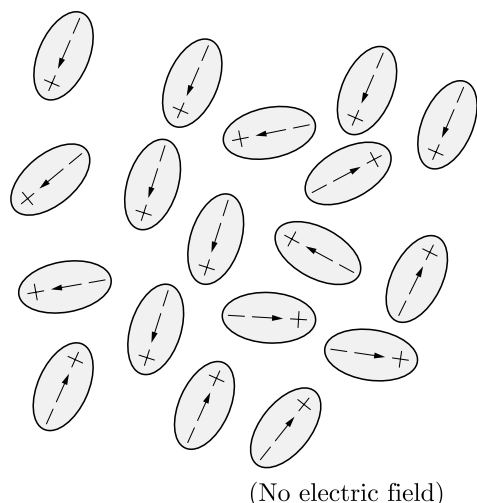
Ans :

Foreign 2017

1. When a capacitor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved, i.e. When the two fields cancel each other and the net electrostatic field in the conductor becomes zero.



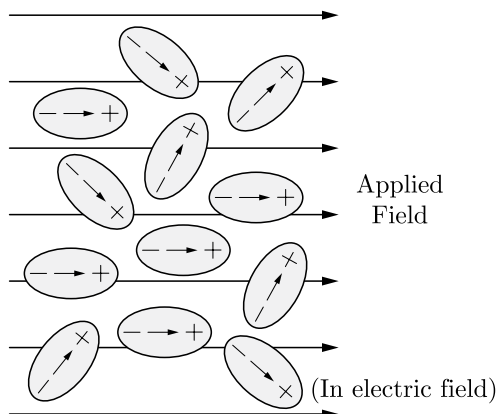
2. In contrast to conductors, dielectrics are non-conducting substances, i.e. they have no charge carriers. Thus, in a dielectric, free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching molecular of the dielectric. The collective effect of all the molecular dipole moments is the net charge



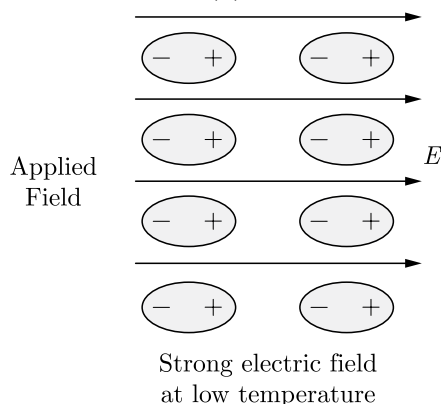
(a)

Effect of Applied Electric Field on Polar Molecules

An applied electric field merely aligns the polar molecules parallel to itself. Because the molecules are in constant thermal agitation, the alignment is not complete as shown in Figure (b). However, the alignment increases as the applied electric field is increased or as the temperature is decreased Figure (c). The dipole moment may also be increased by the applied electric field.



(b)



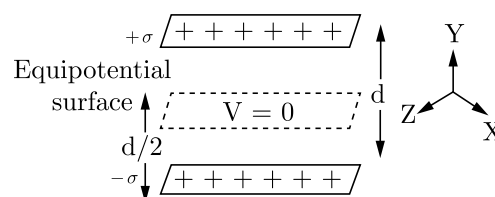
(c)

109. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the $X-Z$ plane at a distance d apart. Sketch and equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates, what is the magnitude and direction of this field ?

Ans :

Comp 2013, OD 2011

The equipotential surface is at a distance $d/2$ from either plate in $X-Z$ plane. For a particle of charge $(-q)$ at rest between the plates, then



- (i) weight mg acts vertically downward
(ii) electric force qEs acts vertically upwards.

$$\text{So, } mg = qE$$

$$E = \frac{mg}{q},$$

vertically downward, i.e. along $(-)$ Y -axis.

110. Find the ratio of the potential difference that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio $1 : 2$ so that the energy stored in the two cases becomes the same.

Ans :

OD 2019

$$\text{As we know that, } U_s = \frac{1}{2} C_s V_s^2$$

$$U_p = \frac{1}{2} C_p V_p^2$$

$$\text{Also, } \frac{C_1}{C_2} = \frac{1}{2} \quad (\text{given})$$

$$C_2 = 2C_1$$

$$\frac{V_{\text{series}}}{V_{\text{parallel}}} = \sqrt{\frac{C_{\text{equivalent parallel}}}{C_{\text{equivalent series}}}}$$

$$= \sqrt{\frac{C_1 + C_2}{\frac{C_1 C_2}{C_1 + C_2}}}$$

$$= \frac{C_1 + C_2}{\sqrt{C_1 C_2}}$$

$$= \frac{3C_1}{\sqrt{2C_1^2}} = \frac{3}{\sqrt{2}}$$

111. Draw schematically equipotential surfaces corresponding to

redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static-situation is achieved, i.e., when the two fields cancel each other and the net electrostatic field in the conductor becomes zero.

In contrast to conductors, dielectrics are non-conducting substances, i.e., they have no charge carriers. Thus, in a dielectric, free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produces a field that opposes the external field. However, the opposing field is so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of dielectric.

Both polar and non-polar dielectric develop net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P for linear isotropic dielectrics.

$$P = \chi E$$

- (ii) (a) At point C , inside the shell.

The electric field inside a spherical shell is zero. Thus, the force experienced by charge at the centre C will also be zero.

$$F_C = qE \quad (E_{\text{inside the shell}} = 0)$$

$$F_C = 0$$

At point A ,

$$|F_A| = 2Q \left(\frac{1}{4\pi\epsilon_0} \frac{3Q/2}{x^2} \right)$$

$$F = \frac{3Q^2}{4\pi\epsilon_0 x^2}, \text{ away from shell}$$

- (b) Electric flux through the shell

$\phi = \frac{1}{\epsilon_0} x$ magnitude of the charge enclosed by the shell.

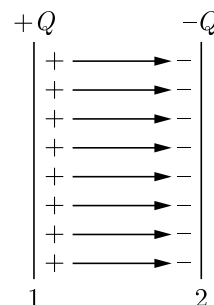
$$\phi = \frac{1}{\epsilon_0} \times \frac{Q}{2} = \frac{Q}{2\epsilon_0}$$

115. (i) Derive the expression for the energy stored in parallel plate capacitor. Hence, obtain the expression for the energy density of the electric field.
- (ii) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than stored initially in the single capacitor.

Ans :

(i)

Foreign 2015, OD 2001



In order to calculate the energy stored in this charge configuration, suppose the conductors 1 and 2 are initially uncharged. Let positive charge be transferred from conductor 2 to conductor 1 in very small instalments of dq each till conductor 1 get charge $+Q$. By charge conservation, conductor 2 would get charge $-Q$.

At every stage of charging, conductor 1 is at higher potential than conductor 2. Therefore, work is done externally in transferring each instalment of charge.

Hence, Potential difference between conductor 1 and 2 is $\frac{q}{C}$

Hence, Potential of condenser = $\frac{q}{C}$

Small amount of work done in giving an additional charge dq to the condenser is $dW = \frac{q}{C} \times dq$

Hence, Total work done in giving a charge Q to the condenser,

$$W = \int_{q=0}^{q=Q} \frac{q}{C} = \frac{1}{C} \left[\frac{q^2}{2} \right]_{q=0}^{q=Q}$$

$$W = \frac{1}{C} \frac{Q^2}{2}$$

As, electrostatic force is conservative, this work is stored in the form of potential energy (U) of the condenser.

$$U = W = \frac{1}{2} \frac{Q^2}{C}$$

$$Q = CV$$

$$U = \frac{1}{2} \frac{(CV)^2}{C} = \frac{1}{2} CV^2$$

$$CV = Q$$

$$U = \frac{1}{2} QV$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

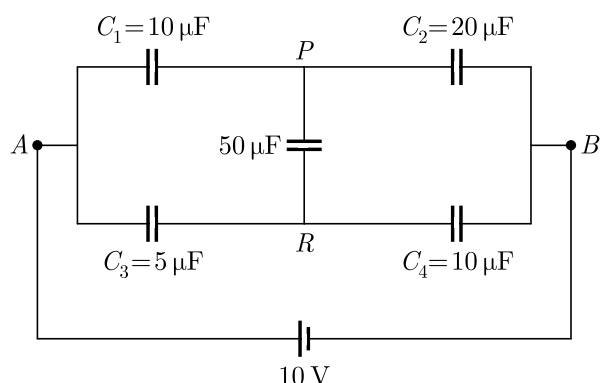
and energy stored in parallel combination,

$$E_P = \frac{1}{2} C_P V^2$$

Ratio,
$$\frac{E_S}{E_P} = \frac{\frac{1}{2} C_S V^2}{\frac{1}{2} C_P V^2} = \frac{C_S}{C_P} = \frac{1}{9}$$

Required ratio = 1:9

- 137.** Calculate the equivalent capacitance between point A and B in the circuit below. If a battery of 10 V is connected across A and B , calculate the charge drawn from the battery by the circuit.

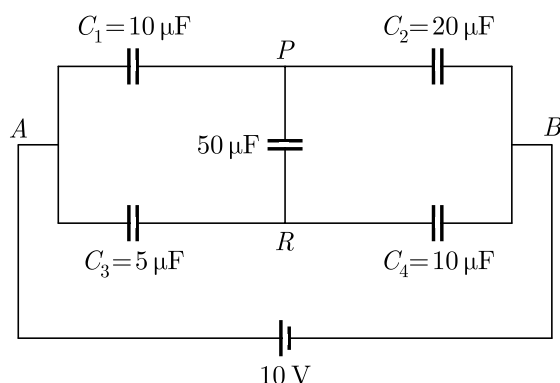


Ans :

Delhi 2013, Comp 2010

Since,
$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

This is the condition of balance so there will be no current across PR ($50 \mu\text{F}$ capacitor).

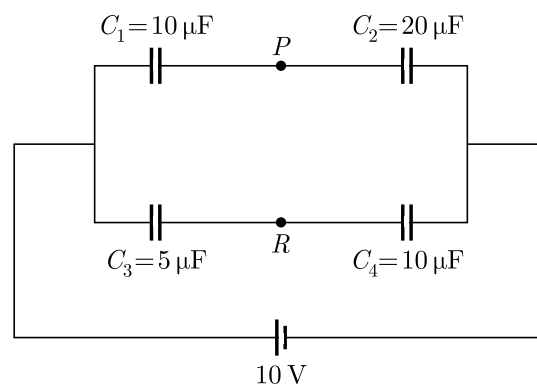


Now, C_1 and C_2 are in series,

$$\begin{aligned} C_{12} &= \frac{C_1 C_2}{C_1 + C_2} \\ &= \frac{10 \times 20}{10 + 20} = \frac{200}{30} = \frac{20}{3} \mu\text{F} \end{aligned}$$

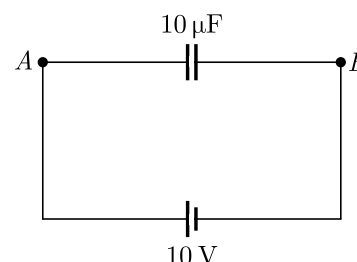
Since, C_3 and C_4 are in series,

$$\begin{aligned} C_{34} &= \frac{C_3 C_4}{C_3 + C_4} \\ &= \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3} \mu\text{F} \end{aligned}$$



Equivalent capacitance between A and B is,

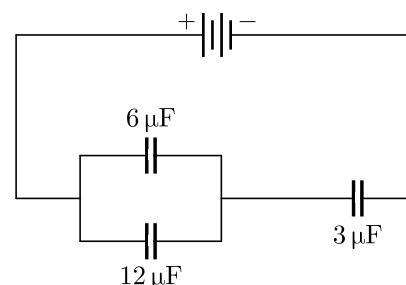
$$C_{AB} = C_{12} + C_{34} = \frac{20}{3} + \frac{10}{3} = 10 \mu\text{F}$$



Hence, charge drawn from battery,

$$\begin{aligned} Q &= CV \\ &= 10 \times 10 \mu\text{C} \\ &= 100 \mu\text{C} = 10^{-4} \text{ C} \end{aligned}$$

- 138.** In the following arrangement of capacitors, the energy stored in $6 \mu\text{F}$ capacitor is E . Find the value of the following.



1. Energy stored in $12 \mu\text{F}$ capacitor.
2. Energy stored in $3 \mu\text{F}$ capacitor.
3. Total energy drawn from the battery.

Ans :

OD 2012

Given that energy stored in $6 \mu\text{F}$ capacitor.

1. Let V be the voltage across $6 \mu\text{F}$ is E .
Also $6 \mu\text{F}$ and $12 \mu\text{F}$ capacitors are in parallel.
Therefore,

Voltage across $12 \mu\text{F}$ = Voltage across $6 \mu\text{F}$ capacitor

NUMERICAL QUESTIONS

- 117.** Evaluate the capacitance of a parallel plate capacitor, having parallel plates of area 6 cm^2 placed at a separation of 2 mm . Consider air between plates as a dielectric medium. If the capacitor is connected to 200 V power supply. What will be the charge on each plate?

Ans :

Delhi 2018

Area of Plate, $A = 6 \text{ cm}^2 = 6 \times 10^{-4} \text{ m}^2$ Distance between plates, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

We know that,

Capacitance of parallel plate capacitor,

$$\begin{aligned} C &= \frac{\epsilon_0 A}{d} \\ &= \frac{8.85 \times 10^{-12} \times 6 \times 10^{-4}}{2 \times 10^{-3}} \\ &= 2.655 \times 10^{-12} \text{ F} \end{aligned}$$

Charge,

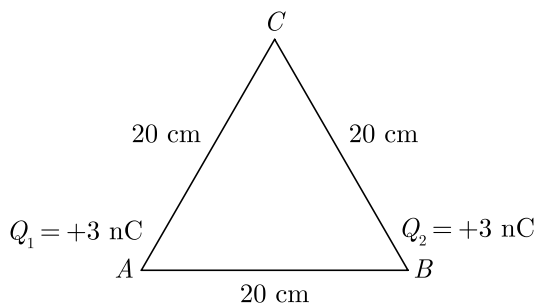
$$\begin{aligned} Q &= CV \\ &= 2.655 \times 10^{-12} \times 200 \\ &= 5.31 \times 10^{-10} \text{ C} \\ &= 0.531 \times 10^{-9} \text{ C} \\ &= 0.531 \text{ nC} \end{aligned}$$

- 118.** The side of an equilateral triangle is 20 cm . Two equal point charges $(+)$ 3 nC are placed at its two corners. What will be the amount of work done in bringing a $(+)$ 1 nC test charge from infinity to third corner of the triangle.

Ans :

OD 2020, Foreign 2006

According to the question,



The amount of work done in bringing a test charge $= +1 \text{ nC}$ from infinity is given by,

$$W = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{r} + \frac{q_2 q_3}{r} \right]$$

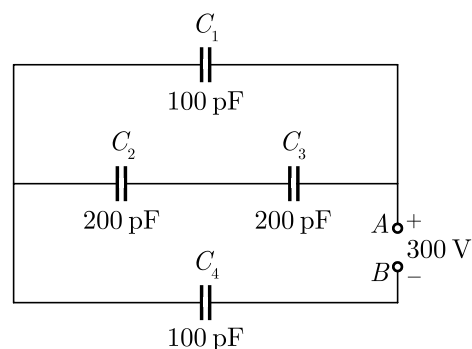
Here, $q_1 = 3 \text{ nC} = 3 \times 10^{-9} \text{ C}$ $q_2 = 3 \text{ nC} = 3 \times 10^{-9} \text{ C}$ $q_3 = 1 \text{ nC} = 1 \times 10^{-9} \text{ C}$

$$r = 20 \text{ cm} = 0.2 \text{ m}$$

$$\begin{aligned} W &= \frac{1}{4\pi\epsilon_0} \left[\frac{3 \times 10^{-9} \times 1 \times 10^{-9}}{0.2} + \frac{3 \times 10^{-9} \times 1 \times 10^{-9}}{0.2} \right] \\ &= \frac{1}{4\pi\epsilon_0} [15 \times 10^{-18} + 15 \times 10^{-18}] \\ &= 9 \times 10^9 \times 30 \times 10^{-18} \\ &= 2.7 \times 10^{-7} \text{ J} \end{aligned}$$

- 119.** Obtain the equivalent capacitance of the network in figure.

For a 300 V supply, determine the charge and voltage across each capacitor.



Ans :

OD 2011

Given,

$$C_1 = C_4 = 100 \text{ pF}$$

$$C_2 = C_3 = 200 \text{ pF}$$

The capacitors C_2 and C_3 are connected in series. Their equivalent capacitance,

$$\begin{aligned} C' &= \frac{C_2 C_3}{C_2 + C_3} \\ &= \frac{200 \times 200}{200 + 200} = 100 \text{ pF} \end{aligned}$$

The combination of C_2 and C_3 (i.e., C') is connected in parallel with C_1 , therefore, equivalent capacitance of C_1 and C' .

$$\begin{aligned} C'' &= C_1 + C' \\ &= 100 + 100 = 200 \text{ pF} \end{aligned}$$

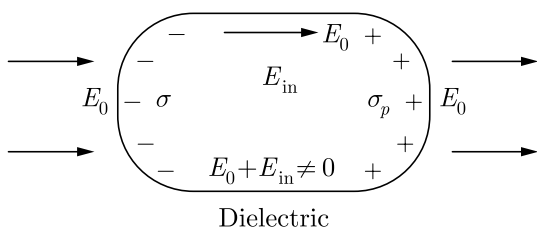
The capacitance C'' is in series with C_4 , hence equivalent capacitance between A and B

$$\begin{aligned} C &= \frac{C'' C_4}{C'' + C_4} \\ &= \frac{200 \times 100}{200 + 100} \\ &= \frac{200}{3} \text{ pF} = 66.7 \text{ pF} \end{aligned}$$

Total charge,

$$\begin{aligned} Q &= CV \\ &= \left(\frac{200}{3} \times 10^{-12} \text{ F} \right) \times (300 \text{ V}) \end{aligned}$$

on the surface of the dielectric which produces a field that opposes the external field. However, the opposing field is so induced, that does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of dielectric.



Both polar and non-polar dielectrics develop net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P for linear isotropic dielectrics.

$$p = \chi E$$

Where χ is constant of proportionality and is called electric susceptibility of the electric slab.

113. (a) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E . It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change ?
- charge stored by the capacitor.
 - field strength between the plates.
 - energy stored by the capacitor.
- (b) Explain why, for any configuration, the equipotential surface through a point is normal to the electric field at the point.

Draw a sketch of equipotential surfaces due to a single charge ($-q$), depicting the electric field lines due to the charge.

Ans :

Delhi 2016, Comp 2003

- (a) (i) Charge remains same, as after disconnecting capacitor no transfer of charge take place.
- (ii) Electric field, $E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$ remain same, as there is no change in charge.
- (iii) Energy stored $= \frac{q^2}{2C} = \frac{q^2}{2\left(\frac{\epsilon_0 A}{d}\right)} = \frac{q^2 d}{2\epsilon_0 A}$

Energy will be doubled as separation between the plates (d) is doubled.

- (b) The work done in moving a charge from one point to another on an equipotential surface is zero. If the field is not normal to an equipotential surface, it would have a non zero component along the

surface. This would imply that work would have to be done to move a charge on the surface which is contradictory to the definition of equipotential surface.

Mathematically

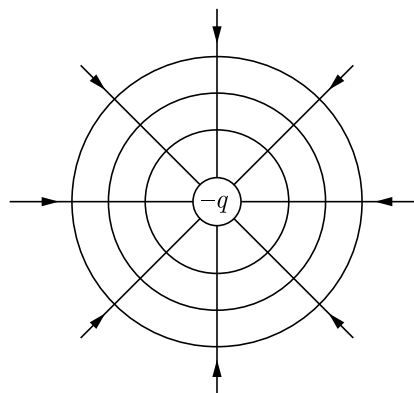
Work done to move a charge dq , on a surface, can expressed as

$$dW = dq(\vec{E} \cdot \vec{dr})$$

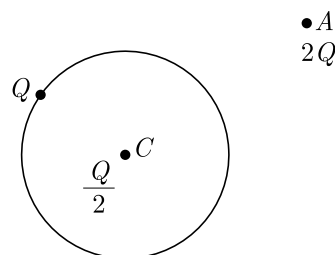
But $dW = 0$ on an equipotential surface

Hence, $\vec{E} \perp \vec{dr}$

Equipotential surfaces for a charges $-q$



114. (i) Explain using suitable diagrams, the difference in the behaviour of a
- conductor and
 - dielectric in the presence of external electric field. Define the terms polarisation of a dielectric and write its relation with susceptibility.
- (ii) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and an other charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in the figure. Find (a) the force on the charge at the centre of shell and at the point A , (b) the electric flux through the shell.



Ans :

Delhi 2014

- (i) When a capacitor is placed in an external electric field, the free charges present inside the conductor

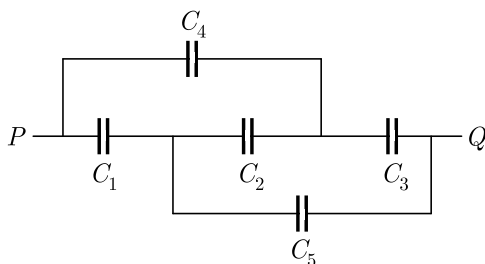
So, $E = \frac{1}{2} q \times \left(\frac{1}{4\pi\epsilon_0} \times \frac{q}{r} \right)$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2}$$

$$E = \frac{1}{2} \times 4 \times 10^{-6} \times 9 \times 10^9 \times \frac{4 \times 10^{-6}}{9 \times 10^{-2}}$$

$$= 0.8 \text{ J}$$

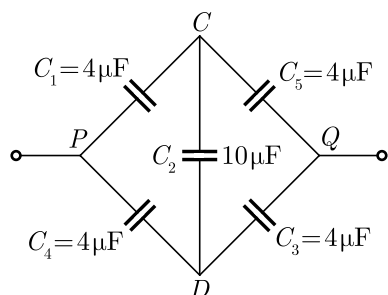
- 122.** In the given figure, find the capacitance resistance between P and Q . If C_1 , C_3 , C_4 and C_5 are each of $4 \mu\text{F}$ and $C_2 = 10 \mu\text{F}$.



Ans :

Comp 2019, OD 2001

The equivalent circuit of given circuit is drawn below,



The above circuit is formed a balanced Wheatstone bridge. Because,

$$\frac{C_1}{C_5} = \frac{C_4}{C_3}$$

Hence, C_2 is ineffective.

In the circuit C_1 and C_5 in series combination.

Hence,

$$C_{PCQ} = \frac{C_1 \times C_5}{C_1 + C_5}$$

$$= \frac{4 \times 4}{4 + 4} = 2 \mu\text{F}$$

Similarly,

$$C_{POQ} = \frac{C_3 \times C_4}{C_3 + C_4}$$

$$= \frac{4 \times 4}{4 + 4} = 2 \mu\text{F}$$

C_{PCQ} and C_{POQ} are arranged in parallel combination.

Hence,

$$C_{eq} = C_{PCQ} + C_{POQ}$$

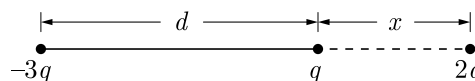
$$= 2 + 2 = 4 \mu\text{F}$$

- 123.** Two charges q and $-3q$ are placed on X -axis at a separation d . Where should a third charge $2q$ be placed such that it will not experience any force? Also calculate potential energy of the system.

Ans :

Foreign 2016

According to the question,



The forces need to be balanced so that the force that $2q$ experiences is cancelled out. So assuming on the line joining q and $-3q$, $-2q$ is x distance from q .

$$\frac{k(q)(2q)}{x^2} = \frac{4}{4\pi\epsilon_0} \frac{(3q)(2q)}{(x+d)^2}$$

$$\frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$$x^2 + d^2 + 2xd = 3x^2$$

$$2x^2 - 2xd - d^2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2d \pm \sqrt{4d^2 + 8d^2}}{2 \times 2}$$

$$x = \frac{2d \pm 2\sqrt{3}d}{2 \times 2}$$

$$= \frac{d(1 \pm \sqrt{3})}{2}$$

Potential energy of the system is given by,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{2q^3}{\left(\frac{1+\sqrt{3}}{2}\right)d} - \frac{3d^2}{d} - \frac{6q^2}{\left(\frac{3+\sqrt{3}}{2}\right)d} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} \left(\frac{4}{1+\sqrt{3}} - \frac{3}{1} - \frac{12}{3+\sqrt{3}} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} \left(\frac{4}{1+\sqrt{3}} - \frac{3}{1} - \frac{4\sqrt{3}}{\sqrt{3}+1} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a} \left(\frac{4-3\sqrt{3}-3-4\sqrt{3}}{\sqrt{3}+1} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} \left(\frac{1-7\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} \left(\frac{\sqrt{3}-21-1+7\sqrt{3}}{2} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{d} \left(\frac{8\sqrt{3}-22}{2} \right)$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{q^2}{d} (11-4\sqrt{3})$$

- 124.** Net capacitance of three identical capacitors in series is $1 \mu\text{F}$. What will be their net capacitance if connected in parallel? Find the ratio of energy stored

Energy density (U) is defined as the total energy per unit volume of the condenser.

$$\begin{aligned} \text{i.e., } u &= \frac{\text{Total energy (U)}}{\text{Volume (V)}} \\ &= \frac{\frac{1}{2} CV^2}{Ad} \end{aligned}$$

$$\text{Using, } C = \frac{\epsilon_0 A}{d}$$

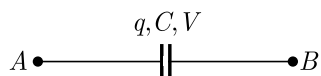
$$\text{and } V = Ed$$

$$\begin{aligned} \text{We get, } U &= \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) \left(\frac{E^2 d^2}{Ad} \right) \\ &= \frac{1}{2} \epsilon_0 E^2 \end{aligned}$$

Hence, E is the strength of electric field in the space between the plates of the capacitor.

(ii) Initial condition :

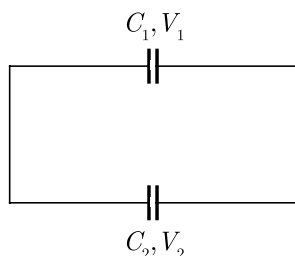
If we consider a charge capacitor, then its charge would be given $q = CV$.



and energy stored in it is given by

$$U_1 = \frac{1}{2} CV^2 \quad \dots(1)$$

When this charged capacitor is connected to uncharged capacitor,



Let the common potential be V_1 , the charge flow from first capacitor to the other capacitor unless both the capacitor attained the common potential.

$$Q_1 = CV_1$$

and

$$Q_2 = CV_2$$

Applying conservation of charge,

$$Q = Q_1 + Q_2$$

$$CV = CV_1 + CV_2$$

$$V = V_1 + V_2$$

$$V_1 = \frac{V}{2}$$

Total energy stored on both the capacitor

$$U_2 = \frac{1}{2} CV_1^2 + \frac{1}{2} CV_1^2$$

$$U_2 = \frac{1}{2} C \left(\frac{V}{2} \right)^2 + \frac{1}{2} C \left(\frac{V}{2} \right)^2$$

$$U_2 = \frac{2CV^2}{8} = \frac{1}{4} CV^2 \quad \dots(2)$$

From equations (1) and (2), we get

$$U_2 < U_1$$

It means that energy stored in the combination is less than that stored initially in the single capacitor.

116. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q .

Ans :

Delhi 2009

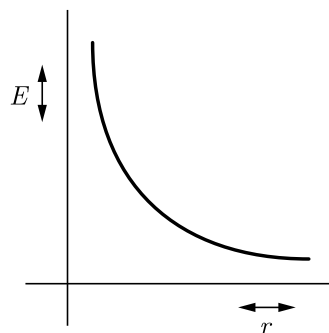
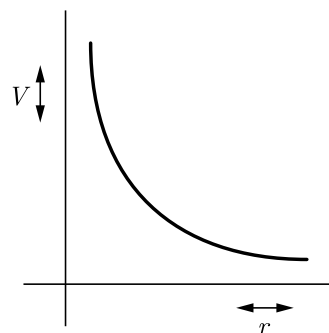
For point charge Q ,

$$\text{Electric potential, } V = \frac{Q}{4\pi\epsilon_0 r} \text{ or } V \propto \frac{1}{r}$$

$$\text{and electric field, } E = \frac{Q}{4\pi\epsilon_0 r^2} \text{ or } E \propto \frac{1}{r^2}$$

Thus, electric potential shows an inverse relationship while electric field shows an inverse square relationship with r .

So, their corresponding plots would be



Keeping charge constant, final voltage $V' = 2\text{ V}$ and capacitance of combined system $= C_s$.

$$U_f = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} \frac{q^2}{C} = \frac{q^2}{C}$$

$$U_f : U_i = \frac{q^2}{C} : \frac{q^2}{2C} = 2 : 1$$

Alternately, $U_f : U_i = \frac{1}{2} C_s \cdot V'^2 : \frac{1}{2} C V^2$

$$= \frac{1}{2} \times \frac{C}{2} \times (2V)^2 : \frac{1}{2} C V^2$$

$$= C V^2 : \frac{C V^2}{2} = 2 : 1$$

- 128.** Net capacitance of three identical capacitors in series is $1\text{ }\mu\text{F}$. What will be their net capacitance if connected in parallel ?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

Ans :

Foreign 2010, OD 2008

Let the capacitance of each capacitor is $C\text{ }\mu\text{F}$. Then for series,

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

Given, $C_s = 1\text{ }\mu\text{F}$

So, $\frac{1}{1} = \frac{3}{C}$

$$C = 3\text{ }\mu\text{F}$$

Now all three capacitors are arranged in parallel, net capacitance in parallel,

$$C_P = C + C + C$$

$$= 3 + 3 + 3 = 9\text{ }\mu\text{F}$$

Now, energy stored in series combination,

$$E_s = \frac{1}{2} C_s V^2$$

(V = Potential of source)

and energy stored in parallel combination,

$$E_P = \frac{1}{2} C_P V^2$$

Ratio, $\frac{E_s}{E_P} = \frac{\frac{1}{2} C_s V^2}{\frac{1}{2} C_P V^2} = \frac{C_s}{C_P} = \frac{1}{9}$

Hence, Required ratio is $1 : 9$.

- 129.** Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V . If the energy stored in the two combination is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2 . Also calculate the charge on each capacitor in parallel combination.

Ans :

OD 2008

Energy stored in a capacitor,

$$E = \frac{1}{2} C V^2$$

In parallel, $0.25 = \frac{1}{2} (C_1 + C_2) (100)^2 \quad \dots(1)$

In series, $0.045 = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) (100)^2 \quad \dots(2)$

From equation (1),

$$C_1 + C_2 = 0.25 \times 2 \times 10^{-4}$$

$$C_1 + C_2 = 5 \times 10^{-5} \quad \dots(3)$$

From equation (2),

$$\frac{C_1 C_2}{C_1 + C_2} = 0.45 \times 2 \times 10^{-4} \quad \dots(4)$$

$$\frac{C_1 C_2}{C_1 + C_2} = 0.09 \times 10^{-4} = 9 \times 10^{-6}$$

From equation (4),

$$C_1 C_2 = \frac{2 \times 0.045 \times 5 \times 10^{-5}}{10^4}$$

$$= 4.5 \times 10^{-10}$$

$$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4 C_1 C_2}$$

$$C_1 - C_2 = 2.64 \times 10^{-5} \quad \dots(5)$$

Solving equation (3) and (5),

$$C_1 = 38.2\text{ }\mu\text{F}$$

$$C_2 = 11.8\text{ }\mu\text{F}$$

In parallel, $Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100$

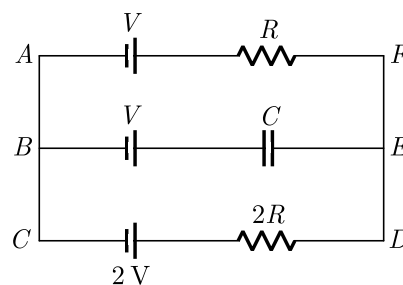
$$= 38.2 \times 10^{-4}\text{ C}$$

and $Q_2 = C_2 V = 11.8 \times 10^{-6} \times 100$

$$= 11.8 \times 10^{-4}\text{ C}$$

- 130.** In the given circuit in the steady state, obtain the expression for:

1. The potential drop
2. The charge
3. The energy stored in the capacitor, C .



Ans :

Delhi 2016

1. In steady state, current in the circuit $ACDFA$,

$$I = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

$$= 2 \times 10^{-8} \text{ coulomb}$$

As C_4 is connected in series with battery, charge on C_4 is

$$Q_4 = 2 \times 10^{-8} \text{ C}$$

Potential difference across C_4 is

$$V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8} \text{ C}}{100 \times 10^{-12} \text{ F}} = 200 \text{ V}$$

As C_2 and C_3 have resultant capacitance C' equal to $C_1 = 100 \text{ pF}$, so the charge Q is equally divided among two branches; charge on C_1 is

$$Q_1 = \frac{Q}{2} = \frac{2 \times 10^{-8}}{2} = 1 \times 10^{-8} \text{ C} = 10^{-8} \text{ C}$$

Charge in branch C_2 and C_3 is also $1 \times 10^{-8} \text{ C}$. As charge in series remains same, so charge on C_2 and C_3 are equal to $1 \times 10^{-8} \text{ C}$.

$$Q_2 = Q_3 = 10^{-8} \text{ C}$$

Potential difference across,

$$C_1 = V_1 = \frac{Q_1}{C_1} = \frac{10^{-8}}{100 \times 10^{-12}} = 100 \text{ V}$$

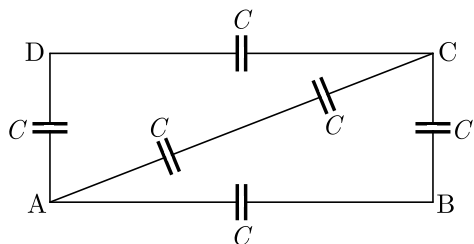
Potential difference across,

$$C_2 = \frac{Q_2}{C_2} = \frac{10^{-8}}{200 \times 10^{-12}} = 50 \text{ V}$$

Potential difference across,

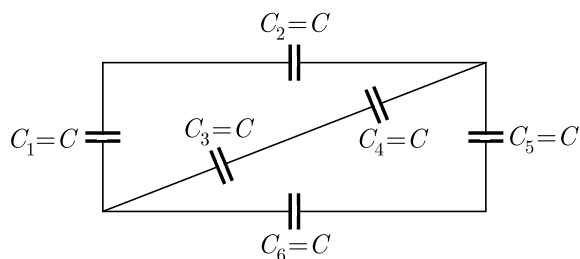
$$C_3 = \frac{Q_3}{C_3} = \frac{10^{-8}}{200 \times 10^{-12}} = 50 \text{ V}$$

- 120.** Find equivalent capacitance between A and B.



Ans :

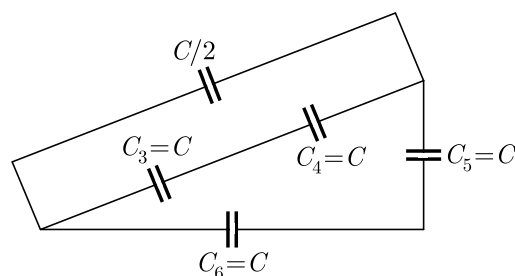
Delhi 2011, OD 2017



In the above circuit C_1 and C_2 are connected in series combination. Now equivalent capacitance is given by,

$$\begin{aligned} \frac{1}{C'} &= \frac{1}{C_1} + \frac{1}{C_2} \\ \frac{1}{C'} &= \frac{1}{C} + \frac{1}{C} \\ C' &= \frac{C}{2} \end{aligned}$$

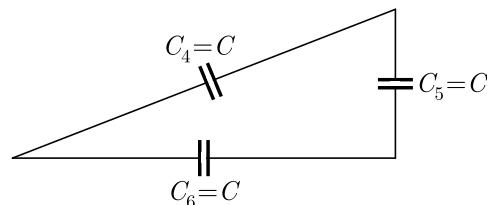
Circuit can be made as follows :



In the circuit C_3 and C_4 are connected in series combination and $C/2$ is in the parallel combination with C_3 and C_4 .

$$\begin{aligned} \text{Now, } C'' &= \frac{C}{2} + \frac{C \times C}{C + C} \\ &= \frac{C}{2} + \frac{C}{2} = C \end{aligned}$$

Now, circuit can be made as follows :



$$\begin{aligned} C_{eq} &= C + \frac{C \times C}{C + C} \\ &= C + \frac{C}{2} = \frac{3C}{2} \end{aligned}$$

- 121.** A metallic sphere of radius 9 cm has been given a charge of $4 \times 10^{-6} \text{ C}$. Calculate energy of charged conductor.

Ans :

OD 2012

Radius of metallic sphere,

$$\begin{aligned} r_s &= 9 \text{ cm} \\ &= 9 \times 10^{-2} \text{ m} \end{aligned}$$

Charge on sphere, $q = 4 \times 10^{-6} \text{ C}$

Now, Energy of charged conductor is given by,

$$\text{Energy, } E = \frac{1}{2} qv$$

$$\text{Potential, } V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

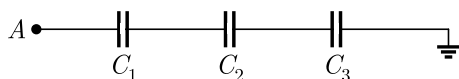
Ratio of energy stored,

$$\frac{U_S}{U_P} = \frac{\frac{1}{2} C_S V^2}{\frac{1}{2} C_P V^2} = \frac{C_S}{C_P}$$

$$= \frac{1 \mu\text{F}}{9 \mu\text{F}} = \frac{1}{9}$$

$$U_S : U_P = 1 : 9$$

- 134.** Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, $C_1 = 20 \mu\text{F}$, $C_2 = 30 \mu\text{F}$, $C_3 = 15 \mu\text{F}$.



Ans :

Comp 2007, OD 2016

Capacitors C_1 , C_2 and C_3 are in series. So its net capacitance is,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$C_S = \frac{20}{3} \mu\text{F}$$

Net charge on the capacitors C_1 , C_2 and C_3 remain same.

$$q = C_S (V_A - V_E)$$

$$= \frac{20}{3} \mu\text{F} \times (90 - 0)$$

$$= 600 \mu\text{C}$$

The potential difference across C_2 due to charge $600 \mu\text{C}$ is,

$$V_2 = \frac{q}{C_2}$$

$$= \frac{600}{30} = 20 \text{ Volt}$$

Energy stored in the capacitor C_2

$$U_2 = \frac{1}{2} \frac{q^2}{C_2} \text{ or } \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2} \times 30 \mu\text{F} \times (20)^2$$

$$= 6000 \mu\text{J} = 6 \times 10^{-3} \text{ J}$$

- 135.** In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the separation between the plate is 3 mm.

- Calculate the capacitance of the capacitor.
- If this capacitor is connected to 100 V supply, what would be the charge on each plate?
- How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected?

Ans :

Foreign 2016

Area of each plate,

$$A = 6 \times 10^{-3} \text{ m}^2$$

Distance between plates,

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

1. Capacitance of parallel plate capacitor is given by,

$$C = \frac{\epsilon_0 A}{d}$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$C = 1.77 \times 10^{-11} \text{ F}$$

2. Charge on parallel plate capacitor is given by

$$Q = CV$$

Given, $V = 100 \text{ Volt}$

Now, $Q = 1.77 \times 10^{-11} \times 100$

$$Q = 1 \times 10^{-10} \text{ C} \quad \dots(1)$$

3. Given,

$$K = 6$$

Now, $C' = KC$

$$Q' = KQ \quad [\text{From equation (1)}]$$

$$= 6 \times 1.77 \times 10^{-10}$$

$$= 10.62 \times 10^{-10} \text{ C}$$

- 136.** Net capacitance of three identical capacitors in series is $1 \mu\text{F}$. What will be their net capacitance if connected in parallel ?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

Ans :

OD 2007

Let the capacitance of each capacitor is $C \mu\text{F}$. Then for series,

$$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

Given, $C_S = 1 \mu\text{F}$

So, $\frac{1}{1} = \frac{3}{C}$

$$C = 3 \mu\text{F}$$

Now all three capacitors are arranged in parallel, net capacitance in parallel,

$$C_P = C + C + C$$

$$= 3 + 3 + 3 = 9 \mu\text{F}$$

Now, energy stored in series combination,

$$E_S = \frac{1}{2} C_S V^2 \quad (V = \text{potential of source})$$

in the two configurations if they are both connected to the same source.

Ans :

Delhi 2015, Foreign 2001

Let each capacitor have a capacitance C . Then in series, we have,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{3}{C} = 1\mu\text{F}$$

$$C_s = 3\mu\text{F}$$

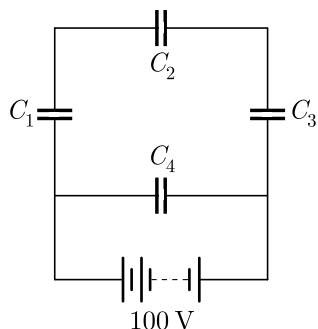
Hence, in parallel, we have

$$C_P = C_1 + C_2 + C_3$$

$$= 3 + 3 + 3 = 9\mu\text{F}$$

$$\frac{E_s}{E_P} = \frac{\frac{1}{2} C_s V^2}{\frac{1}{2} C_P V^2} = \frac{C_s}{C_P} = \frac{1}{9}$$

- 125.** A network of four capacitors, each of capacitance $15\mu\text{F}$, is connected across a battery of 100 V , as shown in the figure. Find the net capacitance and the charge on the capacitor C_4 .



Ans :

OD 2019

In the given network, C_1 , C_2 and C_3 are connected in series, the net capacitance of this series is,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

As,

$$C_1 = C_2 = C_3 = C_4 = 15\mu\text{F}$$

Hence,

$$\frac{1}{C'} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

$$= \frac{3}{15} = \frac{1}{5}$$

Hence,

$$C' = 5\mu\text{F}$$

Now, C' and C_4 are connected in parallel. So total capacitance of combination

$$= C' + C_4$$

$$= 5 + 15 = 20\mu\text{F}$$

For capacitor C_4 , the net charge is Q (say), then

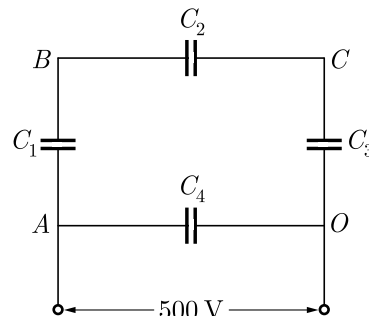
$$\frac{Q}{C_4} = 100\text{ V}$$

$$Q = C_4 \times 100$$

$$= 15 \times 10^{-6} \times 100$$

$$= 1.5 \times 10^{-3}\text{ C}$$

- 126.** A network of four capacitors each of $12\mu\text{F}$ capacitance is connected a 500 V supply as shown in the figure. Determine equivalent capacitance of the network.



Ans :

SQP 2012

The equivalent circuit is shown in figure.

$$C_1 = C_2 = C_3 = C_4 = 12\mu\text{F}$$

C_1 , C_2 and C_3 are in series, so their equivalent capacitance is,

$$\frac{1}{C'} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$= \frac{3}{12}$$

$$C' = \frac{12}{3} = 4\mu\text{F}$$

Now C' and C_4 are parallel, so equivalent capacitance is,

$$C = C' + C_4$$

$$= 4 + 12 = 16\mu\text{F}$$

- 127.** A parallel plate capacitor of capacitance C is charged to a potential V . It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor.

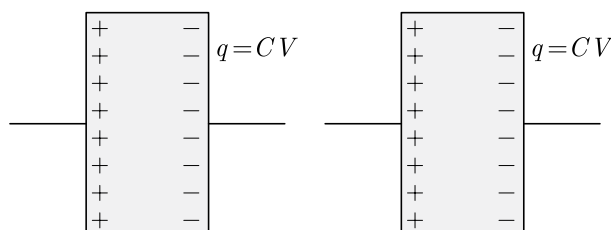
Ans :

Delhi 2014

The charge on the capacitor $q = CV$ and initial energy stored in the capacitor.

$$U_1 = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} CV^2$$

If another uncharged capacitor is connected in series then the same amount of the charge will transfer as shown in figure.



$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 6 \times V^2$$

$$V = \sqrt{\frac{E}{3}} \text{ (Energy stored in } 12\mu\text{F)}$$

$$= \frac{1}{2} \times 12 \left(\sqrt{\frac{E}{3}} \right)^2 = 2E$$

2. Since charge remains constant in series. Sum of charge on $6\mu\text{F}$ capacitor and $12\mu\text{F}$ capacitor is equal to charge on $3\mu\text{F}$ capacitor (Using $Q = CV$).

Charge on $3\mu\text{F}$ capacitor,

$$E = \frac{Q^2}{2C} = \frac{(18V)^2}{2 \times 3}$$

$$= \frac{18 \times 18}{6} \left(\frac{\sqrt{E}}{3} \right)^2 = 18E$$

3. Total energy drawn from battery

$$= E + 2E + 18E = 21E$$

- 139.** A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF . Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor.

Ans :

OD 2018, Comp 2012

We have $C = 200\text{ pF} = 200 \times 10^{-12}\text{ F}$

$$V = 300\text{ Volt}$$

The energy (initially) stored by the capacitor is,

$$U_i = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 200 \times 10^{-12} \times 300 \times 300$$

$$= 9 \times 10^{-6}\text{ J}$$

The charge on the capacitor when charge through 300V battery is,

$$Q = CV$$

$$= 200 \times 10^{-12} \times 300$$

$$= 6 \times 10^{-8}\text{ C}$$

When two capacitor are connected, they have their positive plates and negative plates at the same potential. Let V be the common potential difference. By charge conservation, charge would distribute but total charge would remain constant.

Thus,

$$Q = q + q'$$

$$\frac{q}{C} = \frac{q'}{C'}$$

$$\frac{q}{200} = \frac{q'}{100}$$

$$q = 2q'$$

$$\text{Thus, } Q = 2q' + q' = 3q'$$

$$\text{So, } q' = \frac{Q}{3} = \frac{60\text{ nC}}{3} = 20\text{ nC}$$

$$\text{and } q = 2q' = 40\text{ nC}$$

Thus, final energy,

$$U_f = \frac{q^2}{2C} + \frac{q'^2}{2C'}$$

$$= \frac{1}{2} \times \frac{(40 \times 10^{-9})^2}{200 \times 10^{-12}} + \frac{1}{2} \times \frac{(20 \times 10^{-9})^2}{100 \times 10^{-12}}$$

$$= 4 \times 10^{-6} + 2 \times 10^{-6}$$

$$= 6 \times 10^{-6}\text{ J}$$

Difference in energy = Final energy – Initial energy

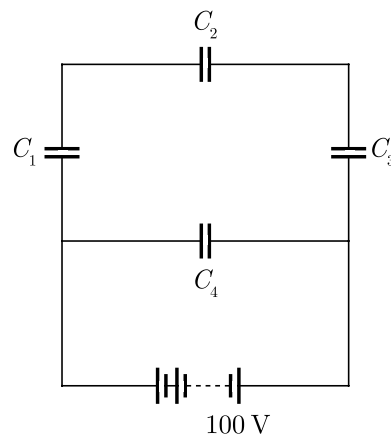
$$= U_f - U_i$$

$$= 6 \times 10^{-6} - 9 \times 10^{-6}$$

$$= -3 \times 10^{-6}$$

Thus, difference in energy is $-3 \times 10^{-6}\text{ J}$.

- 140.** A network of four capacitors, each of capacitance $15\mu\text{F}$ is connected across a battery of 100 V , as shown in the figure. Find the net capacitance and the charge on the capacitor C_4 .



Ans :

OD 2006

In the given network, C_1, C_2 and C_3 are connected in series, the net capacitance of this series is

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\text{As, } C_1 = C_2 = C_3 = C_4 = 15\mu\text{F}$$

$$\text{Hence, } \frac{1}{C'} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

$$= \frac{3}{15} = \frac{1}{5}$$

$$\text{Hence, } C' = 5\mu\text{F}$$

Now C' and C_4 are connected in parallel, so total capacitance of combination = $C' + C_4$
 $= 5 + 15 = 20\mu\text{F}$

Potential at point E , assuming $V_E = 0$

$$\begin{aligned} V_E &= 2V - 2IR \\ &= 2V - 2R \times \frac{V}{3R} = \frac{4V}{3} \end{aligned}$$

Potential difference,

$$\begin{aligned} V_{BE} &= V_E - V \\ &= \frac{4V}{3} - V = \frac{V}{3} \end{aligned}$$

Hence, potential drop across the capacitor is $\frac{V}{3}$.

2. Charge on the capacitor $q = C(\frac{V}{3}) = \frac{1}{3} CV$.

3. Energy stored in the capacitor,

$$U = \frac{1}{2} C \left(\frac{V}{3} \right)^2 = \frac{1}{18} CV^2$$

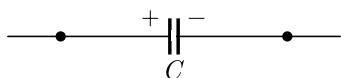
- 131.** A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $360 \mu\text{C}$. When potential across the capacitor is reduced by 120 V , the charge stored in it becomes $120 \mu\text{C}$.

Calculate:

- The potential V and the unknown capacitance C .
- What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ?

Ans :

OD 2009, Foreign 2008



- The potential V and the unknown capacitance C .
If unknown capacitor of capacitance C is connected to a battery of V volts,

$$Q = CV$$

$$CV = 360 \mu\text{C} \quad \dots(1)$$

Reducing the potential/voltage by 120 V .

$$\text{So,} \quad Q' = C(V - 120)$$

$$C(V - 120) = 120 \mu\text{C} \quad \dots(2)$$

Solving equation (1) and (2)

$$\frac{360 \mu\text{C}}{V} = \frac{120 \mu\text{C}}{V - 120}$$

$$V = 180 \text{ Volt}$$

Unknown capacitance from equation (1),

$$Q = CV$$

$$360 \mu\text{C} = C \times 180 \text{ V}$$

$$C = \frac{360 \mu\text{C}}{180 \text{ V}} = 2$$

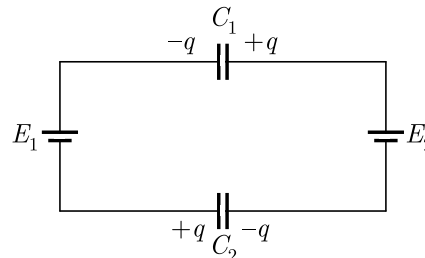
$$C = 2 \mu\text{F}$$

- Charge stored in the capacitor

Charge on the capacitor, if voltage is increased by 120 V .

$$\begin{aligned} Q &= C(V + 120) \\ &= 2(180 + 120) = 600 \mu\text{C} \end{aligned}$$

- 132.** Determine the potential difference across the plates of the capacitor C_1 of the network shown in the figure. [Assume $E_2 > E_1$].



Ans :

Comp 2013

Let,

$$E_2 > E_1$$

Therefore, right plate of C_1 has charge $+q$ and left plate of C_1 has charge $-q$. On the same basis, left plate of C_2 has charge $+q$ and right plate of C_2 has charge $-q$.

In closed circuit, potential difference $dV = 0$.

$$\frac{q}{C_1} + E_1 + \frac{q}{C_2} - E_2 = 0$$

$$q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = E_2 - E_1$$

$$q \left(\frac{C_2 + C_1}{C_1 C_2} \right) = E_2 - E_1$$

$$q = \frac{(E_2 - E_1) C_1 C_2}{C_1 + C_2}$$

Hence, potential difference across left and right plate of C_1 is

$$V = \frac{q}{C_1} = \frac{(E_2 - E_1) C_2}{C_1 + C_2}$$

- 133.** Net capacitance of three identical capacitors in series is $1 \mu\text{F}$. What will be their net capacitance if connected in parallel? Find the ratio of energy stored in the two configurations if they are both connected to the same source.

Ans :

Delhi 2017

Let C be the capacitance of each capacitor, then in series,

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C = 3C_s = 3 \times 1 \mu\text{F} = 3 \mu\text{F}$$

When these capacitors are connected in parallel, net capacitance,

$$C_p = 3C = 3 \times 3 = 9 \mu\text{F}$$

When these two combinations are connected to same source the potential difference across each combination is same.

- 145.** A $4\mu\text{F}$ capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged $2\mu\text{F}$ capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation ?

Ans :

Foreign 2015

We have

$$C_1 = 4\mu\text{F} = 4 \times 10^{-6}$$

$$V_1 = 200\text{ Volt}$$

$$C_2 = 2\mu\text{F} = 2 \times 10^{-6}\text{ F}$$

Initial energy of first capacitor,

$$\begin{aligned} U_1 &= \frac{1}{2} C_1 V_1^2 \\ &= \frac{1}{2} \times (4 \times 10^{-6}) \times (200)^2 \\ &= 8 \times 10^{-2}\text{ J} \end{aligned}$$

When another uncharged capacitor $C_2 = 2\mu\text{F}$, is connected across capacitor C_1 then common potential difference

$$\begin{aligned} V &= \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + 0}{C_1 + C_2} \\ &= \frac{4 \times 10^{-6} \times 200}{(4 + 2) \times 10^{-6}} \\ &= \frac{400}{3}\text{ volt} \end{aligned}$$

Final electrostatic energy,

$$\begin{aligned} U_2 &= \frac{1}{2} (C_1 + C_2) V^2 \\ &= \frac{1}{2} \times (4 \times 2) \times 10^{-6} \times \left(\frac{400}{3}\right)^2 \\ &= \frac{16}{3} \times 10^{-2}\text{ J} \\ &= 5.33 \times 10^{-2}\text{ J} \end{aligned}$$

Energy loss,

$$\begin{aligned} \Delta U &= U_1 - U_2 \\ &= 8 \times 10^{-2} - 5.33 \times 10^{-2} \\ &= 2.67 \times 10^{-2}\text{ J} \end{aligned}$$

- 146.** Two capacitor of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V . If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2 . Also calculate the charge on each capacitor in parallel combination.

Ans :

Comp 2005

Energy stored in a capacitor,

$$E = \frac{1}{2} CV^2$$

In parallel, $0.25 = \frac{1}{2} (C_1 + C_2) (100)^2 \quad \dots(1)$

In series, $0.045 = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) (100)^2 \quad \dots(2)$

From (1), $C_1 + C_2 = 0.25 \times 2 \times 10^{-4}$
 $C_1 + C_2 = 5 \times 10^{-5} \quad \dots(3)$

From (2), $\frac{C_1 C_2}{C_1 + C_2} = 0.45 \times 2 \times 10^{-4} \quad \dots(4)$

$$\frac{C_1 C_2}{C_1 + C_2} = 0.09 \times 10^{-4} = 9 \times 10^{-6}$$

From (4), $C_1 + C_2 = \frac{2 \times 0.045 \times 5 \times 10^{-5}}{10^4}$
 $= 4.5 \times 10^{-10}$

$$C_1 + C_2 = \sqrt{(C_1 + C_2)^2 - 4 C_1 C_2}$$

$$C_1 + C_2 = 2.64 \times 10^{-5} \quad \dots(5)$$

Solving (3) and (5),

$$C_1 = 38.2\mu\text{F}$$

$$C_2 = 11.8\mu\text{F}$$

In parallel, $Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100$
 $= 38.2 \times 10^{-4}\text{ C}$

$$\begin{aligned} Q_2 &= C_2 V = 11.8 \times 10^{-6} \times 100 \\ &= 11.8 \times 10^{-4}\text{ C} \end{aligned}$$

- 147.** A parallel plate capacitor of capacitance C is charged to a potential V . It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor.

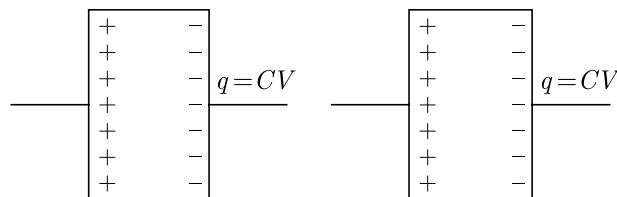
Ans :

OD 2009, SQP 2013

The charge on the capacitor $q = CV$ and initial energy stored in the capacitor

$$U_i = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} CV^2 \quad \dots(1)$$

If another uncharged capacitor is connected in series then the same amount of the charge will transfer as shown in figure.



Keeping charge constant, final voltage $V' = 2V$ and capacitance of combined system $= C_s$

$$U_f = \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} \frac{q^2}{C} = \frac{q^2}{C} \quad \dots(2)$$

$$U_f : U_i = \frac{q^2}{C} : \frac{q^2}{2C} = 2 : 1$$

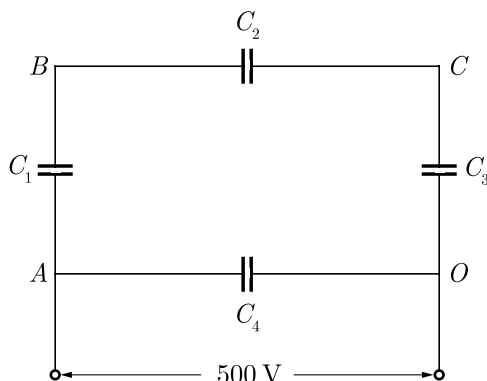
For capacitor C_4 , the net charge is Q (say), then

$$\frac{Q}{C_4} = 100 \text{ V}$$

or

$$\begin{aligned} Q &= C_4 \times 100 \\ &= 15 \times 10^{-6} \times 100 \\ &= 1.5 \times 10^{-3} \text{ C} \end{aligned}$$

- 141.** A network of four capacitors each of $12 \mu\text{F}$ capacitance is connected a 500 V supply as shown in the figure. Determine equivalent capacitance of the network.



Ans :

Delhi 2013, Foreign 2002

The equivalent circuit is shown in figure.

$$C_1 = C_2 = C_3 = C_4 = 12 \mu\text{F}$$

C_1, C_2 and C_3 are in series, so their equivalent capacitance is

$$\begin{aligned} \frac{1}{C'} &= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} \\ C' &= \frac{12}{3} = 4 \mu\text{F} \end{aligned}$$

Now C' and C_4 are parallel, so equivalent capacitance is

$$\begin{aligned} C &= C' + C_4 \\ C &= 4 + 12 = 16 \mu\text{F} \end{aligned}$$

- 142.** A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ?

Ans :

OD 2013

Given, $C = 12 \text{ pF} = 12 \times 10^{-12} \text{ F}$

and $V = 50 \text{ V}$, $U = ?$

Using the relation, $U = \frac{1}{2} CV^2$

$$\begin{aligned} &= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 \\ &= 1.5 \times 10^{-8} \text{ J} \end{aligned}$$

- 143.** A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ?

Ans :

Delhi 2017

$$\begin{aligned} \text{Given, } C_1 &= C_2 = 600 \text{ pF} \\ &= 600 \times 10^{-12} \text{ F} \\ V &= 200 \text{ Volt} \\ \Delta U &= ? \end{aligned}$$

Using the relation

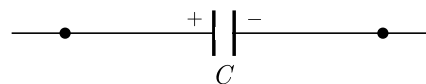
$$\begin{aligned} \Delta U &= \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}, \text{ we get} \\ \Delta U &= \frac{600 \times 600 \times 10^{-24} (200 - 0)^2}{2(600 + 600) \times 10^{-12}} \\ &= 6 \times 10^{-6} \text{ J} \end{aligned}$$

- 144.** A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $360 \mu\text{C}$. When potential across the capacitor is reduced by 120 V , the charge stored in it becomes $120 \mu\text{C}$. Calculate :

- The potential V and the unknown capacitance C .
- What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ?

Ans :

Delhi 2013



- If unknown capacitor of capacitance C is connected to a battery of V volts,

$$\begin{aligned} Q &= CV \\ CV &= 360 \mu\text{C} \end{aligned} \quad \dots(1)$$

On reducing the potential/voltage by 120 Volt

$$\begin{aligned} \text{So, } Q' &= C(V - 120) \\ C(V - 120) &= 120 \mu\text{C} \end{aligned} \quad \dots(2)$$

On solving equation (1) and (2),

$$\begin{aligned} \frac{360 \mu\text{C}}{V} &= \frac{120 \mu\text{C}}{V - 120} \\ V &= 180 \text{ V} \end{aligned}$$

Unknown capacitance from equation (1),

$$\begin{aligned} Q &= CV \\ 360 \mu\text{C} &= C \times 180 \text{ V} \\ C &= \frac{360 \mu\text{C}}{180 \text{ V}} = 2 \\ C &= 2 \mu\text{F} \end{aligned}$$

- Charge on the capacitor, if voltage is increased by 120 Volt ,

$$\begin{aligned} Q &= C(V + 120) = 2(180 + 120) \\ &= 600 \mu\text{C} \end{aligned}$$

- 151.** The two plates of a parallel plate capacitor are 5 mm apart. A slab of a dielectric, of thickness 4 mm is introduced between the plates with its faces parallel to them. The distance between the plates is adjusted so that the capacitance of the capacitor becomes equal to its original value. If the new distance between the plates equals 8 mm, what is the dielectric constant of the dielectric used ?

Ans :

OD 2006, Comp 2014

Let original capacitance $C_0 = \frac{\epsilon_0 A}{d}$

Here, $d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

So, $\epsilon_0 A = 5 \times 10^{-3} C_0$

Now, $d = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$

$t = 4 \text{ mm} = 4 \times 10^{-3}$

and $C = C_0$

So, $C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$

where $K = \text{dielectric constant}$

Hence, $C_0 = \frac{5 \times 10^{-3} \times C_0}{d - t\left[1 - \frac{1}{K}\right]}$

or $d - t\left(1 - \frac{1}{K}\right) = 5 \times 10^{-3}$

$8 \times 10^{-3} - 5 \times 10^{-3} = t\left(1 - \frac{1}{K}\right)$

$t\left(1 - \frac{1}{K}\right)S = 3 \times 10^{-3}$

$\left(1 - \frac{1}{K}\right) = \frac{3 \times 10^{-3}}{4 \times 10^{-3} S}$

$1 - \frac{1}{K} = 0.75$

$\frac{1}{K} = 0.25$

$K = 4$

So, dielectric constant = 4

- 152.** Three capacitors of capacitance 2 pF, 3 pF and 4 pF are connected in parallel

- (i) What is the total capacitance of the combination ?
 (ii) Determine the charge on each capacitor if the combination is connected to (a) 100 V supply.

Ans :

OD 2014

Given, $C_1 = 2 \text{ pF}$, $C_2 = 3 \text{ pF}$, $C_3 = 4 \text{ pF}$

- (i) Total capacitance when connected in parallel,

$C_P = C_1 + C_2 + C_3$

$= 2 + 3 + 4 = 9 \text{ pF}$

- (ii) In parallel, the potential difference across each capacitor remains the same, i.e., $V = 100 \text{ Volt}$.

Charge on $C_1 = 2 \text{ pF}$ is

$q_1 = C_1 V = 2 \times 10^{-12} \times 100$

$= 2 \times 10^{-10} \text{ C}$

Charge on $C_2 = 3 \text{ pF}$ is

$q_2 = C_2 V = 3 \times 10^{-12} \times 100$

$= 3 \times 10^{-10} \text{ C}$

Charge on $C_3 = 4 \text{ pF}$ is

$q_3 = C_3 V = 4 \times 10^{-12} \times 100$

$= 4 \times 10^{-10} \text{ C}$

- 153.** Net capacitance of three identical capacitors in series is $1 \mu\text{F}$. What will be their net capacitance if connected in parallel ? Find the ratio of energy stored in the two configurations if they are both connected to the same source.

Ans :

OD 2015

Let each capacitor have a capacitance C . Then in series, we have

$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{3}{C} = 1 \mu\text{F}$

or

$C = 3 \mu\text{F}$

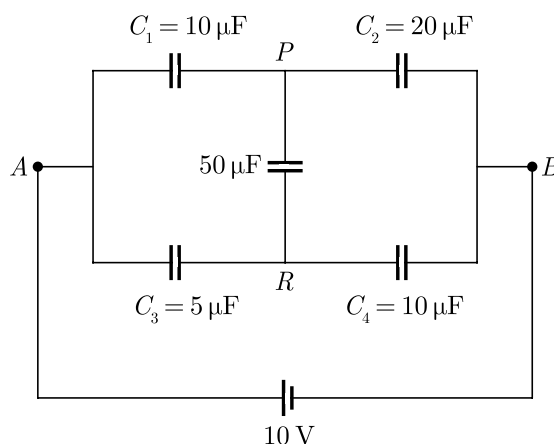
Hence, in parallel, we have

$C_P = C_1 + C_2 + C_3$

$= 3 + 3 + 3 = 9 \mu\text{F}$

$\frac{E_S}{E_P} = \frac{\frac{1}{2} C_S V^2}{\frac{1}{2} C_P V^2} = \frac{C_S}{C_P} = \frac{1}{9}$

- 154.** Calculate the equivalent capacitance between points A and B in the circuit below. If a battery of 10 V is connected across A and B, calculate the charge drawn from the battery by the circuit.



Alternately,

$$\begin{aligned} U_f : U_i &= \frac{1}{2} C_s \cdot V'^2 : \frac{1}{2} C V^2 \\ &= \frac{1}{2} \times \frac{C}{2} \times (2V)^2 : \frac{1}{2} C V^2 \\ &= C V^2 : \frac{C V^2}{2} = 2 : 1 \end{aligned}$$

- 148.** In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the separation between the plate is 3 mm.

- Calculate the capacitance of the capacitor.
- If this capacitor is connected to 100 V supply, what would be the change on each plate ?
- How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected ?

Ans :

Foreign 2008

Given,

area of each plate, $A = 6 \times 10^{-3} \text{ m}^2$

Distance between plates,

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

- Capacitance of parallel plate capacitor is given by

$$\begin{aligned} C &= \frac{\epsilon_0 A}{d} \\ C &= \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} \\ C &= 1.77 \times 10^{-11} \text{ F} \end{aligned}$$

- Charge on parallel plate capacitor is given by

$$Q = CV$$

Given, $V = 100 \text{ Volt}$

Now, $Q = 1.77 \times 10^{-12} \times 100$

$$Q = 1.77 \times 10^{-10} \text{ C} \quad \dots(1)$$

- Given, $K = 6$

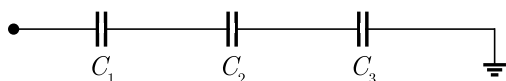
Now, $C' = KC$

$$Q' = KQ \quad [\text{From Equation (1)}]$$

$$Q' = 6 \times 1.77 \times 10^{-10}$$

$$Q' = 10.62 \times 10^{-10} \text{ C}$$

- 149.** Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, $C_1 = 20 \mu\text{F}$, $C_2 = 30 \mu\text{F}$, $C_3 = 15 \mu\text{F}$.



Ans :

SGP 2010

Capacitors C_1 , C_2 and C_3 are in series. So, its net capacitance is

$$\begin{aligned} \frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{20} + \frac{1}{30} + \frac{1}{15} \\ C_s &= \frac{20}{3} \mu\text{F} \end{aligned}$$

Net charge on the capacitors, C_1 , C_2 and C_3 remain same.

$$\begin{aligned} q &= C_s (V_A - V_E) \\ &= \frac{20}{3} \mu\text{F} \times (90 - 0) = 600 \mu\text{C} \end{aligned}$$

The potential difference across C_2 due to charge $600 \mu\text{C}$ is

$$V_2 = \frac{q}{C_2} = \frac{600}{30} = 20 \text{ V}$$

Energy stored in the capacitor C_2 ,

$$\begin{aligned} U_2 &= \frac{1}{2} \frac{q^2}{C_2} \quad \left(\text{or } \frac{1}{2} C_2 V_2^2 \right) \\ &= \frac{1}{2} \times 30 \mu\text{F} \times (20)^2 = 6000 \mu\text{J} \\ &= 6 \times 10^{-3} \text{ J} \end{aligned}$$

- 150.** Three capacitors each of capacitance 9 pF are connected in series :

- What is the total capacitance of the combination ?
- What is the potential difference across each capacitor if the combination is connected to 120 V supply ?

Ans :

OD 2015

- Given, $C_1 = C_2 = C_3 = 9 \text{ pF}$

When capacitors are connected in series, the equivalent capacitance C_s is given by

$$\begin{aligned} \frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3} \end{aligned}$$

- In series charge on each capacitor remains the same, so charge on each capacitor.

$$\begin{aligned} q &= C_s V \\ &= (3 \times 10^{-12} \text{ F}) \times (120 \text{ V}) \\ &= 3.6 \times 10^{-10} \text{ coulomb} \end{aligned}$$

Potential difference across each capacitor,

$$\begin{aligned} V &= \frac{q}{C_1} \\ &= \frac{3.6 \times 10^{-10}}{9 \times 10^{-12}} = 40 \text{ V} \end{aligned}$$

- (i) Energy stored in $12\mu\text{F}$ capacitor.
 (ii) Energy stored in $3\mu\text{F}$ capacitor.
 (iii) Total energy drawn from the battery.

Ans :

Comp 2017

Given, that energy stored in $6\mu\text{F}$ capacitor

- (i) Let V be the voltage across $6\mu\text{F}$ is E .

Also, $6\mu\text{F}$ and $12\mu\text{F}$ capacitors are in parallel.

Therefore, voltage across $12\mu\text{F}$

$$= \text{Voltage across } 6\mu\text{F capacitor}$$

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 6 \times V^2$$

$$V = \sqrt{\frac{E}{3}}$$

Energy stored in $12\mu\text{F}$

$$= \frac{1}{2} \times 12 \times \left(\sqrt{\frac{E}{3}}\right)^2 = 2E$$

- (ii) Since charge remains constant in series. Sum of charge on $6\mu\text{F}$ capacitor and $12\mu\text{F}$ capacitor is equal to charge on $3\mu\text{F}$ capacitor.

Using $Q = CV$,

Charge on $3\mu\text{F}$ capacitor,

$$Q = (6 + 12) \times V = 18 \times V$$

Energy stored in $3\mu\text{F}$ capacitor,

$$\begin{aligned} E &= \frac{Q^2}{2C} = \frac{(18)^2}{2 \times 3} \\ &= \frac{18 \times 18}{6} \left(\sqrt{\frac{E}{3}}\right)^2 \\ &= 18E \end{aligned}$$

- (iii) Total energy drawn from battery

$$= E + 2E + 18E = 21E$$

157. A parallel plate capacitor has its two plates kept 0.02 m apart. A dielectric slab of thickness 0.01 m is introduced between the plates with its faces parallel to them. The distance between the plates is readjusted to make the capacity of the capacitor $(2/3)^{\text{rd}}$ of its original value. Given that the dielectric constant of the slab equals 5, find the new distance between the plates.

Ans :

OD 2014, Comp 2011

Given, $d = 0.02\text{ m}$, $t = 0.01\text{ m}$, $K = 5$

$$C_1 = \frac{C}{5}$$

Using the relation, $C = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$... (1)

$$C = \frac{\epsilon_0 A}{\left(0.02 - 0.01 + \frac{0.01}{5}\right)}$$

$$C = \frac{\epsilon_0 A}{0.012} \quad \dots (2)$$

Again,

$$C_1 = \frac{\epsilon_0 A}{(d - 0.01 + 0.002)}$$

$$\frac{2C}{3} = \frac{\epsilon_0 A}{(d - 0.01 + 0.002)} \quad \dots (3)$$

Now dividing equation (3) by equation (2), we get

$$\frac{2\frac{C}{3}}{C} = \frac{\epsilon_0 A}{(d - 0.01 + 0.002)} \times \frac{0.012}{\epsilon_0 A}$$

$$2(d - 0.01 + 0.002) = 3 \times 0.012$$

$$2d - 0.02 + 0.004 = 0.036$$

$$2d = 0.036 + 0.02 + 0.004$$

$$2d = 0.052$$

$$d = 0.026\text{ m}$$

158. The capacitance of a parallel plate capacitor is $10\mu\text{F}$. When distance between its plates is 8 cm . If distance between the plates is reduced to 4 cm , find out the new value capacitance of capacitor ?

Ans :

OD 2011

Initial capacitance, $C_1 = 10\mu\text{F}$

Initial distance between plates,

$$d_1 = 8\text{ cm} = 0.08\text{ m}$$

and final distance between plates,

$$d_2 = 4\text{ cm} = 0.04\text{ m}$$

We know that Capacitance of a parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d} \propto \frac{1}{d}$$

Therefore,

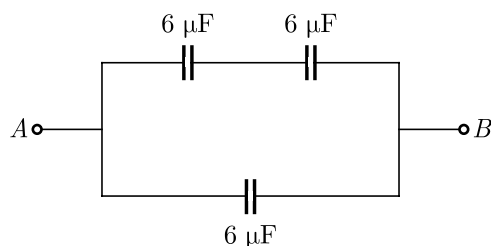
$$\frac{C_1}{C_2} = \frac{d_2}{d_1}$$

or

$$\begin{aligned} C_2 &= \frac{d_1}{d_2} \times C_1 \\ &= \frac{0.08}{0.04} \times 10 = 20\mu\text{F} \end{aligned}$$

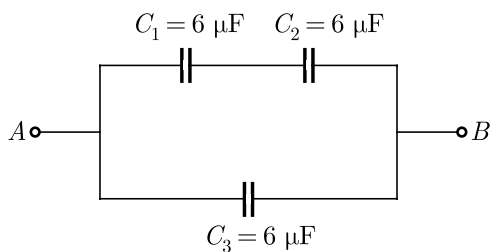
where C_2 is final capacitance of parallel plate capacitor

159. What is the equivalent capacity between A and B in the given figure ?



Ans :

Foreign 2009



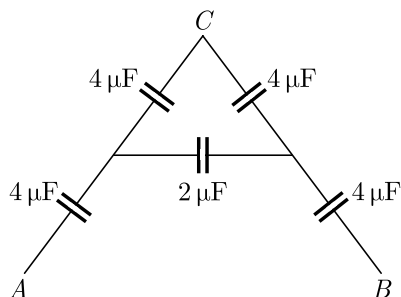
In the circuit C_1 and C_2 are arranged in series combination. Hence, Equivalent capacitance,

$$\begin{aligned} C' &= \frac{C_1 \times C_2}{C_1 + C_2} \\ &= \frac{6 \times 6}{6 + 6} \\ &= \frac{36}{12} = 3 \mu\text{F} \end{aligned}$$

Now C' and C_3 are arranged in parallel combination. Hence, equivalent capacitance between A and B ,

$$\begin{aligned} C_{eq} &= C' + C_3 \\ &= 3 + 6 = 9 \mu\text{F} \end{aligned}$$

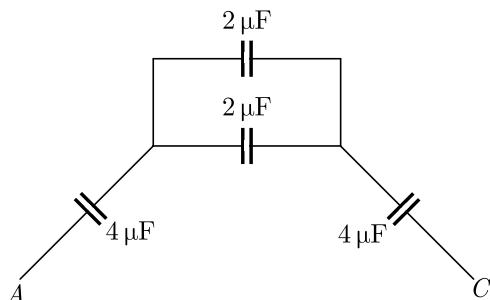
160. What is the equivalent capacity between points A and B ?



Ans :

OD 2012, Comp 2008

We can reduce the circuit as,



Equivalent capacitance across point C ,

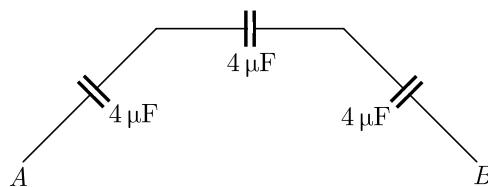
$$\frac{4 \times 4}{4 + 4} = 2 \mu\text{F}$$

Now $2 \mu\text{F}$ and $2 \mu\text{F}$ are in parallel.

So, equivalent capacitance,

$$2 + 2 = 4 \mu\text{F}$$

Further reduced diagram,



Now all there are in series so equivalent capacitance across A and B would be,

$$\begin{aligned} \frac{1}{C_{eq}} &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\ C_{eq} &= \frac{4}{3} \mu\text{F} \end{aligned}$$

161. Two capacitors $C_1 = 2 \mu\text{F}$ and $C_2 = 4 \mu\text{F}$ are connected in series and a potential difference (p.d.) of 1200 V is applied across it. What will be potential difference across $2 \mu\text{F}$?

Ans :

Delhi 2018

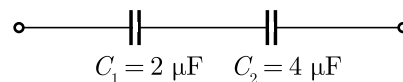
Given,

$$C_1 = 2 \mu\text{F} = 2 \times 10^{-6} \text{F}$$

$$C_2 = 4 \mu\text{F} = 4 \times 10^{-6} \text{F}$$

Potential difference, $V = 1200$ Volts

According to the question, capacitors are connected in series combination as shown in the figure,



Equivalent capacitance,

$$\begin{aligned} \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \\ &= \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} \\ C_{eq} &= \frac{4}{3} \mu\text{F} = \frac{4}{3} \times 10^{-6} \text{F} \end{aligned}$$

We know that,

$$\text{Capacitance, } C = \frac{Q}{V}$$

$$\text{where, } C = C_{eq} = \frac{4}{3} \times 10^{-6} \text{F}$$

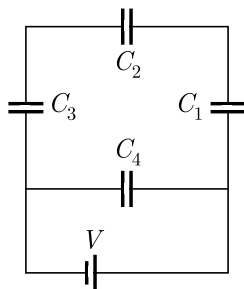
Hence,

$$\begin{aligned} Q &= C_{eq} V \\ &= \frac{4}{3} \times 10^{-6} \times 1200 \\ &= 1600 \times 10^{-6} \text{C} \end{aligned}$$

The Potential difference across $2 \mu\text{F}$ capacitor is given by,

$$\begin{aligned} V &= \frac{Q}{C} = \frac{1600 \times 10^{-6}}{2 \times 10^{-6}} \\ &= 800 \text{ Volt} \end{aligned}$$

- 162.** A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. What is the ratio of the charges on C_2 and C_4 ?



Ans :

SQP 2005

Capacitances, $C_1 = C$, $C_2 = 2C$
 $C_3 = 3C$, $C_4 = 4C$

And voltage of battery = V .

Since capacitors C_1 , C_2 and C_3 are in series combination, therefore their effective capacitance C_s is,

$$\begin{aligned}\frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} \\ &= \frac{6+3+2}{6C} = \frac{11}{6C}\end{aligned}$$

or $C_s = \frac{6C}{11}$

As C_1 , C_2 and C_3 are in series, therefore charge on each capacitor is same.

Therefore charge on C_2 capacitor,

$$q = C_s V = \frac{6CV}{11} \quad \dots(1)$$

Similarly, charge on C_4 capacitor,

$$q' = C_4 V = 4CV \quad \dots(2)$$

Dividing equation (1) by (2),

$$\frac{q}{q'} = \frac{6CV/11}{4CV} = \frac{6}{44} = \frac{3}{22}$$

- 163.** What is the work done in placing a charge of $8 \times 10^{-18} \text{ C}$ on a capacitor of capacity of capacity $100 \mu\text{F}$?

Ans :

Delhi 2014, OD 2009

Charge, $q = 8 \times 10^{-18} \text{ C}$

and capacity of capacitor,

$$C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$$

Potential developed due to the charge,

$$\begin{aligned}V &= \frac{q}{C} = \frac{8 \times 10^{-18}}{100 \times 10^{-6}} \\ &= 8 \times 10^{-14} \text{ V}\end{aligned}$$

Therefore, work done,

$$\begin{aligned}W &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times (100 \times 10^{-6}) \times (8 \times 10^{-14})^2 \\ &= 32 \times 10^{-32} \text{ J}\end{aligned}$$

- 164.** In a capacitor of capacitor $12 \mu\text{F}$, the distance between the plates is 3 mm . If a dielectric slab of width 2 mm and dielectric constant 2 is inserted between the plates, What will be the new capacitance of capacitor?

Ans :

SQP 2008

Initial capacitance of capacitor,

$$C_1 = 12 \mu\text{F} = 12 \times 10^{-6} \text{ F}$$

Distance between plates, $d = 3 \text{ mm}$

Width of dielectric slab, $t = 2 \text{ mm}$

Dielectric constant, $K = 2$

Initial capacitance of the capacitor without dielectric slab,

$$C_1 = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{3}$$

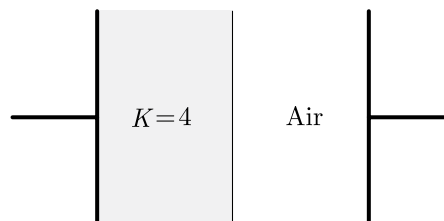
and final capacitance of the capacitor with dielectric slab,

$$\begin{aligned}C_2 &= \frac{\epsilon_0 A}{(d - t + \frac{t}{K})} \\ &= \frac{\epsilon_0 A}{3 - 2 + \frac{2}{2}} = \frac{\epsilon_0 A}{2}\end{aligned}$$

Therefore, $\frac{C_2}{C_1} = \frac{\frac{\epsilon_0 A}{2}}{\frac{\epsilon_0 A}{3}} = \frac{3}{2} = 1.5$

$$C_2 = 12 \times 1.5 = 18 \mu\text{F}$$

- 165.** Consider a parallel plate capacitor of capacity $10 \mu\text{F}$ with air filled in the gap between the plates. Now one-half of the space between the plates is filled with a dielectric of dielectric constant $K = 4$ as shown in the figure. How much capacitance of capacitor will be changed?



Ans :

OD 2013

Initial capacity of the capacitor when filled with air,

$$C = 10 \mu\text{F}$$

and dielectric constant dielectric,

$$K = 4$$

Capacity of the capacitor with half of the space filled with air,

$$C_1 = \frac{C}{2} = \frac{10}{2} = 5 \mu\text{F}$$

And capacity of the capacitor with half of the space filled with dielectric constant,

$$\begin{aligned} C_2 &= KC_1 \\ &= 4 \times 5 = 20 \mu\text{F} \end{aligned}$$

Therefore, total capacity of the capacitor,

$$\begin{aligned} C &= C_1 + C_2 \\ &= 5 + 20 = 25 \mu\text{F} \end{aligned}$$

- 166.** The capacitance of parallel plate capacitor becomes $\frac{4}{3}$ times its original value if a dielectric slab of thickness $t = \frac{d}{2}$ is inserted between the plates (where d is separation between the plates). What is the dielectric constant of the slab ?

Ans :

Foreign 2010, Comp 2001

Capacitance of the capacitor with dielectric slab,

$$C_2 = \frac{4}{3} C_1 w$$

where, C_1 = capacitance without dielectric slab.

and thickness of dielectric slab,

$$t = \frac{d}{2}$$

where, d = separation between the plates.

Initial capacitance of the capacitor without dielectric slab,

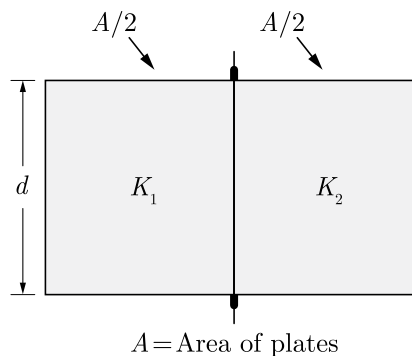
$$C_1 = \frac{\epsilon_0 A}{d}$$

and capacitance of the capacitor with dielectric slab of thickness $\frac{d}{2}$,

$$\begin{aligned} C_2 s a &= \frac{\epsilon_0 A}{(d - t + \frac{t}{K})} \\ \frac{4}{3} \frac{\epsilon_0 A}{d} &= \frac{\epsilon_0 A}{(d - \frac{d}{2} + \frac{d}{2K})} \\ \frac{4}{3d} &= \frac{1}{\frac{d}{2} + \frac{d}{2K}} \\ \frac{4}{3} &= \frac{2K}{K+1} \\ 4K+4 &= 6K \\ 2K &= 4 \\ K &= 2 \end{aligned}$$

where, K = Dielectric constant of the slab

- 167.** Two materials of dielectric constant K_1 and K_2 are filled between two parallel plates of a capacitor as shown in figure. Find out the net capacity of the capacitor?



Ans :

Delhi 2012

Dielectric constant of first material = K_1

Dielectric constant of second material = K_2

Capacity of the first capacitor,

$$\begin{aligned} C_1 &= \frac{K_1 \epsilon_0 (\frac{A}{2})}{d} \\ &= \frac{K_1 \epsilon_0 A}{2d} \end{aligned}$$

Similarly, capacity of the second capacitor,

$$C_2 = \frac{\epsilon_0 K_2 A}{2d}$$

Since, the two capacitors are connected in a parallel combination,

Therefore their net capacity of the capacitor,

$$\begin{aligned} C &= C_1 + C_2 \\ &= \frac{\epsilon_0 K_1 A}{2d} + \frac{\epsilon_0 K_2 A}{2d} \\ &= \frac{A \epsilon_0 (K_1 + K_2)}{2d} \end{aligned}$$

- 168.** Two charges $2 \mu\text{C}$ and $-2 \mu\text{C}$ are placed at points A and B , 5 cm apart. Depict an equipotential surface of the system.

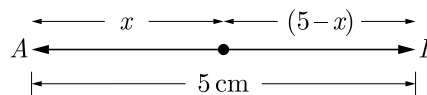
Ans :

Foreign 2013

Given,

$$q_A = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$q_B = -2 \mu\text{C} = -2 \times 10^{-6} \text{ C}$$



and

$$r = 5 \text{ cm}$$

$$\text{Potential, } V = \frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} + \frac{-2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}}$$

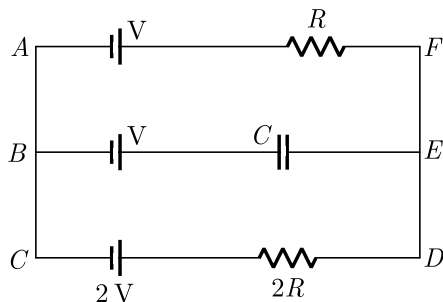
$$\frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} = \frac{2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}}$$

[since $V = 0$]

$$x = 5 - x$$

$$x = 2.5$$

169. In the given circuit in the steady state, obtain the expression for (a) the potential drop (b) the charge and (c) the energy stored in the capacitor, C .



Ans :

Delhi 2015, Foreign 2008

- (a) In steady state, current in the circuit $ACDFA$

$$I = \frac{2V - V}{2R + R} \\ = \frac{V}{3R}$$

Potential at point E , assuming $V_E = 0$

$$V_E = 2V - 2IR \\ = 2V - 2R \times \frac{V}{3R} = \frac{4V}{3}$$

Potential difference,

$$V_{BE} = V_E - V \\ = \frac{4V}{3} - V = \frac{V}{3}$$

Hence, potential drop across the capacitor is $\frac{V}{3}$.

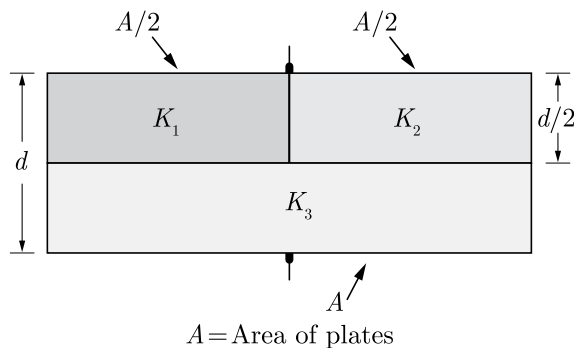
- (b) Charge on the capacitor q

$$= C\left(\frac{V}{3}\right) = \frac{1}{3} CV$$

- (c) Energy stored in the capacitor

$$U = \frac{1}{2} C\left(\frac{V}{3}\right)^2 = \frac{1}{18} CV^2$$

170. A parallel plate capacitor of area A , plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1 , K_2 and K_3 as shown in the figure. If a single dielectric material is to be used to have the same capacitance C in this capacitor, what is the value of dielectric constant K ?



Ans :

OD 2019

Area of parallel plate capacitor $= A$

Plate separation $= d$

Capacitance $= C$

and Different dielectric constants $= K_1, K_2$ and K_3

Capacitance of a capacitor,

$$C = \frac{\epsilon_0 AK}{d}$$

Therefore capacitance of first half portion,

$$C_1 = \frac{\epsilon_0 \left(\frac{A}{2}\right) K_1}{\left(\frac{d}{2}\right)} = \frac{\epsilon_0 AK_1}{d}$$

Similarly, capacitance of second half portion,

$$C_2 = \frac{\epsilon_0 \left(\frac{A}{2}\right) K_2}{\left(\frac{d}{2}\right)} = \frac{\epsilon_0 AK_2}{d}$$

and capacitance of third complete portion,

$$C_3 = \frac{\epsilon_0 AK_3}{\left(\frac{d}{2}\right)} = \frac{2\epsilon_0 AK_3}{d}$$

The capacitances C_1 and C_2 are in parallel combination.

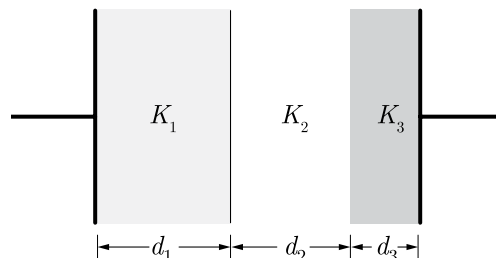
Therefore their equivalent capacitance,

$$C_{eq} = C_1 + C_2 \\ = \frac{\epsilon_0 AK_1}{d} + \frac{\epsilon_0 AK_2}{d} \\ = \frac{\epsilon_0 A}{d} (K_1 + K_2)$$

Now, the equivalent capacitance C_{eq} and C_3 are in a series combination. Therefore relation for their resultant capacitance C is,

$$\frac{1}{C} = \frac{1}{C_{eq}} + \frac{1}{C_3} \\ \frac{d}{\epsilon_0 AK} = \frac{d}{\epsilon_0 A (K_1 + K_2)} + \frac{d}{2\epsilon_0 AK_3} \\ \frac{d}{\epsilon_0 AK} = \frac{d}{\epsilon_0 A} \left[\frac{1}{(K_1 + K_2)} + \frac{1}{2K_3} \right] \\ \frac{1}{K} = \frac{1}{K_1 + K_2} + \frac{1}{2K_3}$$

171. What will be the capacity of a capacitor formed by a compound dielectric placed between the plates of a parallel plate capacitor as shown in figure ? (Area of plate $= A$)



Ans :

OD 2017

Given,

Area of plates = A Width of first dielectric = d_1 Width of second dielectric = d_2 Width of third dielectric = d_3 Dielectric constant of first medium = K_1 Dielectric constant second medium = K_2 Dielectric constant of third medium = K_3

Capacity of parallel plate capacitor with a compound dielectric,

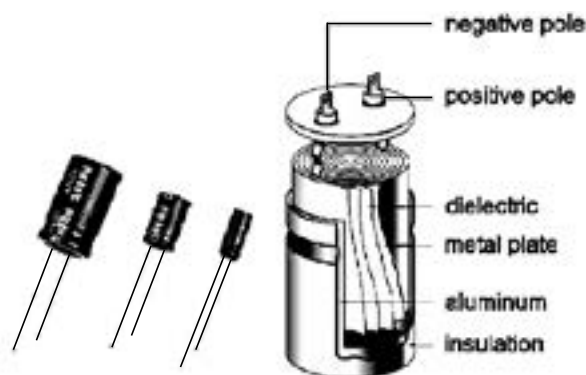
$$C = \frac{\epsilon_0 A}{\sum \left(\frac{d}{K} \right)}$$

$$= \frac{\epsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3} \right)}$$

CASE BASED QUESTIONS

172. Shikhaj was working on a project for science exhibition. He considered a capacitance of $2\mu\text{F}$ having a capacity of operating under 1 kV potential. When he reached to shop, he found that the shopkeeper is having a capacitors of $1\mu\text{F}$ of 400 V rating. Shikhaj calculated minimum number of capacitance of $1\mu\text{F}$ each so he could arrange to form a capacitor of $2\mu\text{F}$.

- What are the calculations done by Shikhaj ?
- Can there be a potential difference between two conductors of the same volume carrying equal positive charge?
- Is a dielectric a conductor?



Ans :

- Total potential difference across each row = 1000 Volt

Potential difference across each capacitor = 400 Volt

Number of capacitors in series

$$n = \frac{1000}{400} = 2.5 \simeq 3$$

Capacitance of capacitors in series,

$$C_s = \frac{1}{3} = 2$$

$$m = 6$$

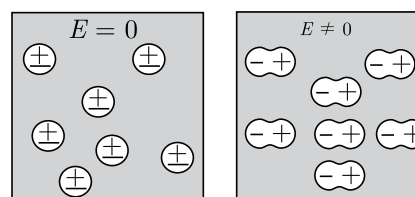
Hence total number of capacitance required

$$= m \times n = 6 \times 3 = 18$$

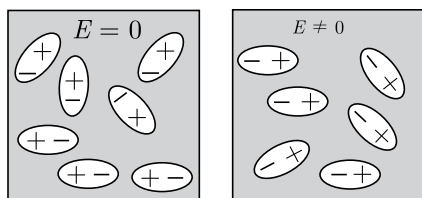
Total 6 row of capacitor in parallel with three capacitors in each row.

- Yes, Because two conductors of the same volume may have different shapes and hence difference capacitances.
- Dielectric materials are poor conductors of electricity because they do not have any loosely bound or free electrons that may drift through the material. Electrons are required to support the flow of an electric current.

173. Dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align with the field. When summed overall the molecules, there is then a net dipole moment in the direction of the external field, i.e., the dielectric is polarised. The extent of polarisation depends on the relative strength of two factors. The dipole potential energy in the external field tending to align the dipoles mutually opposite with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for non-polar molecules, but generally the alignment effect is more important for polar molecules. Thus in either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarization.



(a) Non-Polar molecules



(b) Polar molecules

- What do you mean by dielectric polarization?
- Calculate the polarisation vector of the material which has 100 dipoles per unit volume in a volume of 2 units?
- In which type of molecule positive and negative charges coincide with each other?

Ans :

- Dielectric polarization is the term given to describe the behaviour of a material when an external electric field is applied to it. It occurs when a dipole moment is formed in an insulating material because of an externally applied electric field.
 - 50
 - Non polar, A molecule in which the centre of mass of positive and negative charges coincide with each other is called a non-polar molecule. They normally have zero dipole moment. They have symmetrical shapes.
174. In Akash's classroom the fan above the teacher was running very slowly. Due to which his teacher was sweating and was restless and tired. All his classmates wanted to rectify this. They called for an electrician who came and changed the capacitor only after which the fan started running fast.



- What energy is stored in the capacitor and where?
- Does capacitor affect fan speed?
- How does a capacitor control fan speed?

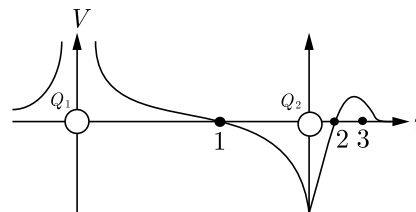
Ans :

- Electrical energy in the dielectric of the capacitor.
- When you increase the capacitance, the fan motor's voltage goes up, but the capacitor's goes down.

The fan's speed goes up. To increase the fan speed, you need to increase the capacitor value.

- They control the voltage across the fan is controlled by a capacitor, the voltage across the fan determines the fan speed. Capacitive regulators are energy efficient with linear speed control, quiet operation without humming noises and highly reliable than electronic type regulator.

175. The potential at any observation point P of a static electric field is defined as the work done by the external agent (or negative of work done by electrostatic field) in slowly bringing a unit positive point charge from infinity to the observation point. Figure shows the potential variation along the line of charges. Two-point charges Q_1 and Q_2 lie along a line at a distance from each other.

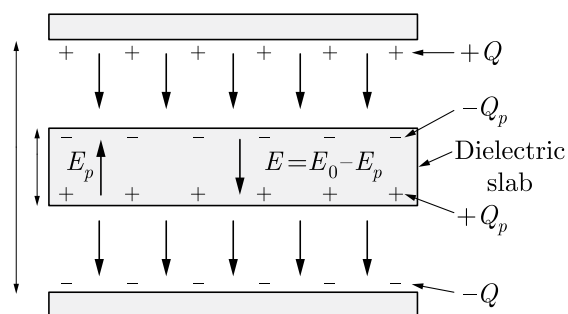


- At which of the points 1, 2 and 3 is the electric field is zero?
- What is the nature of charge Q_1 and Q_2 ?
- Which of the two charges Q_1 and Q_2 is greater in magnitude?

Ans :

- 3
- Positive and negative
- Q_1

176. A dielectric slab is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another. When a dielectric slab is placed between the plates, the field E_0 polarises the dielectric. This induces charge $-Q_p$ on the upper surface and $+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of E_0 as shown.



- (i) In a parallel plate capacitor, the capacitance increases from $4\mu\text{F}$ to $80\mu\text{F}$ on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?
- (ii) A parallel plate capacitor with air between the plates has a capacitance of 8 pF . The separation between the plates is now reduced half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in second case?
- (iii) A parallel plate capacitor of capacitance 1 pF has separation between the plates is d . When the distance of separation becomes $2d$ and wax of dielectric constant x is inserted in it the capacitance becomes 2 pF . What is the value of x ?

Ans :

- (i) 20, (ii) $80\mu\text{F}$, (iii) 4

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CHAPTER 3

Current Electricity

SUMMARY

1. ELECTRIC CURRENT

It is defined as the rate of flow of electric charge through any cross-section of a conductor, i.e.

$$I = \left(\frac{dq}{dt} \right)$$

The direct rate of flow of electric charge through any cross-section of a conductor is known as electric current.

$$I = \frac{q}{t} = \frac{ne}{t} \quad [\text{since } q = ne]$$

where, n = number of charged particles constitute the current.

2. CURRENT DENSITY

It is the ratio of the current at a point in conductor to the area of cross-section of the conductor at the point, i.e.

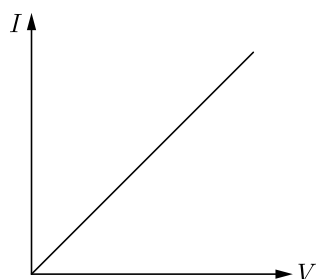
$$J = \frac{I}{A}$$

3. OHM'S LAW

At constant temperature, the potential difference V across the ends of a given metallic wire (conductor) in an circuit (electric) is directly proportional to the current flowing through it.

$$V \propto I$$

The variation of current w.r.t. applied potential difference is shown with the help of given graph.



$$V = IR$$

where, R = resistance of conductor

4. FLOW OF ELECTRIC CHARGES IN METALLIC CONDUCTORS

In case of solid conductor, large number of free electrons causes the strong current in them.

In the case of a liquid conductor, movement of positive and negative charged ions causes the electric current.

5. RESISTANCE OF A CONDUCTOR

Mathematically, it is the ratio of potential difference applied across the ends of conductor to the current flowing through it.

$$R = \frac{V}{I}$$

SI unit is ohm (Ω),

Resistance can also be written as,

$$R = \rho \frac{L}{A}$$

where, L = length of the conductor

A = area of cross-section

and ρ = constant, known as resistivity of the material

It depends upon nature of the material.

6. EFFECT OF TEMPERATURE ON RESISTANCE

For metals, resistance increases with rise in temperature. For insulators and semiconductors, resistance decreases with rise in temperature. For alloy, temperature coefficient of resistance is small.

Temperature coefficient of resistance is given by,

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

7. DRIFT VELOCITY

It is defined as the average velocity with which the free electrons move towards the positive end of a

conductor under the influence of an external electric field applied.

$$v_d = \frac{eE}{m}\tau$$

where, τ = relaxation time

E = electric field

m = mass

and e = charge on electron

8. RELATION BETWEEN DRIFT VELOCITY AND ELECTRIC FIELD

It is given by,

$$I = neAv_d$$

where, n = number density of free electrons

e = electronic charge

A = cross-sectional area

and v_d = drift velocity of an electron

The ratio of drift velocity of electron and the applied electric field is known as mobility.

$$\mu = \frac{v_d}{E} = \frac{q\tau}{m}$$

Hence, SI unit is $[\text{m}^2\text{s}^{-1}\text{V}^{-1}]$.

8.1 Relationship between Resistivity and Relaxation Time

$$\rho = \frac{m}{ne^2\tau}$$

where, τ = relaxation time

Specific resistance or resistivity ρ depends on the material of conductor, not on the length and cross-sectional area A , i.e. geometry of conductor.

8.2 Effect of Temperature on Resistivity

For metals, resistivity increases with increase in temperature.

For alloys, resistivity is very large but has a weak dependence on temperature.

9. CLASSIFICATION OF MATERIALS IN TERMS OF CONDUCTIVITY

For insulators, electrical conductivity is very small or nil. For conductors, electrical conductivity is very high. For semiconductors, electrical conductivity lies in between that of insulators and conductors.

10. CONDUCTANCE AND CONDUCTIVITY

Conductance is the reciprocal of resistance of conductor. Conductivity is the reciprocal of the

resistivity of conductor.

Electrical Conductivity is a property of the material it sell white electrical Conductivity is a property of a particulars electrical component the unit of conductance of semen and it is the reciprocal of the ohm. the unit of conduct unity are sometimes given as ohms/ meter.

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OBJECTIVE QUESTIONS

1. A current of 0.8 A flows in a conductor of $40\ \Omega$ for 1 minute. The heat produced in the conductor will be
 (a) 1445 J (b) 1536 J
 (c) 1569 J (d) 1640 J

Ans :

OD 2023

$$\text{Time} = 1\ \text{min} = 60\ \text{sec}$$

$$\text{Current, } I = 0.8\ \text{A}$$

$$\text{Resistance, } R = 40\ \Omega$$

$$\begin{aligned}\text{Heat produced } I^2 R t &= (0.8)^2 \times 40 \times 60 \\ &= 1536\ \text{J}\end{aligned}$$

Thus (b) is correct option.

2. A cell of emf E is connected across an external resistance R . When current I is drawn from the cell, the potential difference across the electrodes of the cell drops to V . The internal resistance ' r ' of the cell is
 (a) $\left(\frac{E-V}{E}\right)R$ (b) $\left(\frac{E-V}{R}\right)$
 (c) $\left(\frac{E-V}{I}\right)R$ (d) $\left(\frac{E-V}{V}\right)R$

Ans :

Delhi 2021

From ohm's law, let the current in the circuit,

$$i = \frac{V}{R}$$

PD. across the cell, $E = V + ir$

$$\begin{aligned}r &= \frac{E - V}{i} \\ &= \frac{E - V}{V/R} \\ &= \left(\frac{E - V}{V}\right)R\end{aligned}$$

Thus (d) is correct option.

3. Pieces of copper and silicon are initially at room temperature. Both are heated to temperature T . The conductivity of
 (a) both increases
 (b) both decreases
 (c) copper increases and silicon decreases
 (d) copper decreases and silicon increases

Ans :

SQP 2023

Copper is a conductor and hence its conductivity decreases with increase in temperature. Silicon is a semiconductor and hence its conductivity increases with increase in temperature.

Thus (d) is correct option.

4. Beams of electrons and protons move parallel to each other in the same direction. They
 (a) attract each other.
 (b) repel each other.
 (c) neither attract nor repel.
 (d) force of attraction or repulsion depends upon speed of beams.

Ans :

OD 2022

The flow of positive charge is taken as the direction of current. So, here the currents are in opposite direction, so they will repel each other.

Thus (b) is correct option.

5. Which element is used in electric heater?
 (a) Copper (b) Platinum
 (c) Tungsten (d) Nichrome

Ans :

Foreign 2017

Nichrome is used in electric heater because of the following reasons :

1. Its melting point is high.
2. Its resistivity is large.
3. It is not easily oxidised by the oxygen of the air when heated.

Thus (d) is correct option.

6. Kilowatt-hour (kWh) is the unit of.
 (a) energy (b) power
 (c) torque (d) force

Ans :

OD 2015, Comp 2010

The bigger unit of electric energy is kilowatt hour.

$$1\ \text{kW} = 1000\ \text{W}$$

$$1\ \text{kWh} = 3.6 \times 10^6\ \text{J}$$

Thus (a) is correct option.

7. To increase sensitivity of a potentiometer, its
 (a) area should be increased
 (b) current should be decreased
 (c) current should be increased
 (d) length should be increased

Ans :

SQP 2009

The sensitivity of potentiometer can be increased by reducing the potential gradient. This can be done in two way-

1. The sensitivity can be increased by increasing the length of the potentiometer wire.
2. For a potentiometer wire of fixed length, the potential gradient can be decreased by reducing the current in the circuit with the help of rheostat.

Thus (b) is correct option.

- (a) 4.5×10^{20} (b) 9.0×10^{20}
 (c) 4.5×10^{18} (d) 9.0×10^{18}

Ans :

SQP 2017

Current, $I = 300 \text{ mA} = 0.3 \text{ A}$ Time, $t = 4 \text{ min} = 240 \text{ s}$

And charge on an electron,

$$e = 1.6 \times 10^{-19} \text{ C}$$

Charge passing through a conductor,

$$q = I \times t \\ = 0.3 \times 240 = 72 \text{ C}$$

Therefore no. of electrons passed through the conductor,

$$n = \frac{q}{e} = \frac{72}{1.6 \times 10^{-19}} \\ = 4.5 \times 10^{20}$$

Thus (a) is correct option.

24. An energy source will supply a constant current into the load, if its internal resistance is
 (a) zero
 (b) equal to load resistance
 (c) very large than load resistance
 (d) non-zero but less than load resistance

Ans :

OD 2009

We know that current in the circuit,

$$I = \frac{E}{R + r}$$

Therefore an energy source will supply a constant current equal to $\frac{E}{R}$ into the load if its internal resistance r is zero.

Thus (a) is correct option.

25. The internal resistance of a cell is the actual resistance of
 (a) electrodes of the cell
 (b) vessel of the cell
 (c) material used in the cell
 (d) electrolyte used in the cell

Ans :

Delhi 2012, Comp 2010

Resistance offered by the electrolyte of a cell, when electric current flows from negative terminal to positive terminal through it is known as internal resistance of the cell. And it is denoted by r .

Thus (d) is correct option.

26. What is the resistance of a 40 W lamp which is lighted as full brilliance by a current of $\frac{1}{3} \text{ A}$?

- (a) 120Ω (b) 240Ω
 (c) 360Ω (d) 480Ω

Ans :

Foreign 2002

Power of lamp, $P = 40 \text{ W}$ Current, $I = \frac{1}{3} \text{ A}$

We know that resistance of lamp,

$$R = \frac{P}{I^2} = \frac{40}{\left(\frac{1}{3}\right)^2} \\ = 360 \Omega$$

Thus (c) is correct option.

27. Absorbed electrical energy is
 (a) Proportional to the potential difference
 (b) Inversely proportional to the potential difference
 (c) Proportional to the square of the potential difference
 (d) None of these

Ans :

OD 2017

Electrical power (P) in a circuit is the rate at which energy is absorbed or produced with in a circuit. Electrical power (P) is given by

$$P = \frac{V^2}{R}$$

Here, V = Potential difference R = ResistanceHence, $P \propto V^2$

Thus (c) is correct option.

28. If the length of a conductor is halved, then its conductance will be
 (a) halved (b) doubled
 (c) quadrupled (d) unchanged

Ans :

Foreign 2013

Initial length, $l_1 = l$ Final length, $l_2 = \frac{l}{2}$

We know that conductance of the conductor,

$$\sigma = \frac{A}{\rho l} \propto \frac{1}{l}$$

Therefore, $\frac{\sigma_1}{\sigma_2} = \frac{l_2}{l_1} = \frac{l/2}{l} = \frac{1}{2}$

$$\sigma_2 = 2\sigma_1$$

Thus conductance of conductor will be doubled.

Thus (b) is correct option.

29. The current flowing through a lamp, marked as 60 W and 240 V is

Ans :**Delhi 2016**

Increase in voltage = 2%

Power of lamp,

$$P = \frac{V^2}{R} \propto V^2$$

Therefore, if voltage increases by 2%, then increase in its power will be 4%.

Thus (d) is correct option.

17. Which of the following is a non-ohmic resistor?

- (a) copper (b) aluminium
(c) diode-valve (d) none of these

Ans :**Foreign 2008**

A diode-valve is a non-ohmic resistor, because the relation between voltage and resulting current is not linear.

Thus (c) is correct option.

18. The power of electric circuit is

- (a) VR (b) V^2R
(c) $\frac{V^2}{R}$ (d) V^2RI

Ans :**OD 2015**Electric power, $P = \frac{W}{t}$ Here, $W = VIt$

$$P = \frac{VIt}{t}$$

$$P = VI \quad \dots(1)$$

According to the ohm's law

$$V = IR$$

$$I = \frac{V}{R} \quad \dots(2)$$

From equation (1), we get

$$P = V \times \frac{V}{R} = \frac{V^2}{R}$$

Thus (c) is correct option.

19. The algebraic sum of all currents meeting at a point in an electrical circuit is

- (a) zero (b) infinite
(c) positive (d) negative

Ans :**OD 2016, 2015**

According to the Kirchhoff's first law "In an electric circuit, the algebraic sum of current at any junction is zero or the sum of currents entering a junction is equal to the sum of currents leaving that junction.

Thus (a) is correct option.

20. The drift velocity V_d and applied electric field E of a conductor are related as

- (a) $V_d \propto \sqrt{E}$ (b) $V_d \propto E$
(c) $V_d \propto E^2$ (d) $V_d = \text{constant}$

Ans :**SQP 2001**

The relation between drift velocity V_d and electric field ($\vec{E}$) is given by,

$$\vec{V} = \frac{e\vec{E}\tau}{m}$$

where, τ = relaxation time m = mass of electron e = charge on electronHence, $\vec{V} = \frac{e\vec{E}\tau}{m}$

$$\vec{V}_d \propto \vec{E}$$

Thus (b) is correct option.

21. A current of 10 ampere flows in a wire for 10 sec. If potential difference across the wire is 15 volt, the work done will be

- (a) 150 (b) 75 J
(c) 1500 J (d) 750 J

Ans :**OD 2012, SQP 2004**Current, $I = 10$ ampereTime, $t = 10$ secPotential difference, $V = 15$ volt

We know that,

$$V = \frac{W}{q}$$

Hence, $W = Vq = VIt$ [Since, $q = It$]
 $= 15 \times 10 \times 10 = 1500 \text{ J}$

Thus (c) is correct option.

22. If the resistivity of an alloy is ρ' and that of its constituent metal is ρ , then

- (a) $\rho' < \rho$ (b) $\rho' > \rho$
(c) $\rho' = \rho$ (d) none of these

Ans :**OD 2003**Resistivity of alloy = ρ' and Resistivity of its constituent metal = ρ

Resistivity of a metal increases, when it is converted into an alloy. Therefore,

$$\rho' > \rho$$

Thus (b) is correct option.

23. If a current of 300 mA is flowing in a conductor, then the number of electrons passed through the conductor in 4 min is (Charge on an electron = $1.6 \times 10^{-19} \text{ C}$)

55. A resistance R is connected across a cell of emf E and internal resistance r . A potentiometer now measures the potential difference between the terminals of the cell as V . Write the expression for r in terms of E , V and R .

Ans :

OD 2011

$$r = \left(\frac{E}{V} - 1 \right) R$$

56. Nichrome and Copper wires of same length and same radius are connected in series. Current/ is passed through them. Which wire gets heated up more? Justify your answer.

Ans :

Delhi 2017

Nichrome, since its resistance is high.

57. The emf of a cell is always greater than its terminal voltage. Why? Give reason.

Ans :

Comp 2018, SQP 2007

The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance.

58. Define electric current.

Ans :

Foreign 2015

The flow of electric charges through a conductor constitutes electric current. Quantitatively, electric current across an area held perpendicular to the direction of flow of charge is defined as the amount of charge across that area per unit time.

For a steady flow of charge,

$$I = \frac{Q}{t}$$

If the rate of flow of charge varies with time,

$$\text{Then, } I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

59. What is electromotive force?

Ans :

OD 2017

The emf of a source may be defined as the work done by the source in taking a unit positive charge from its lower potential terminal to the higher potential terminal. Or, it is the energy supplied by the source in taking a unit positive charge once round the complete circuit. It is equal to the terminal p.d. measured in open circuit.

$$\text{e.m.f.} = \frac{\text{Work done}}{\text{Charge}}$$

or

$$e = \frac{W}{q}$$

60. Alloys of metals have greater resistivity than their constituent metals. Why?

Ans :

Foreign 2006

In an alloy, like nichrome (Ni – Cr alloy), Ni^{2+} and

Cr^{3+} ions occupy random locations relative to each other. An electron, therefore, passes through a very random medium and is very frequently deflected. So there is a small relaxation time and hence large resistivity. In general, alloys have more resistivity than that of their constituent metals.

61. Define potential gradient. Give its units.

Ans :

OD 2015

The potential drop unit length of the potentiometer wire is known as potential gradient. It is given by

$$k = \frac{V}{l}$$

$$\text{SI unit of potential gradient} = \text{Vm}^{-1}$$

$$\text{Practical unit of potential gradient} = \text{V cm}^{-1}$$

62. Why is internal resistance of a secondary cell low?

Ans :

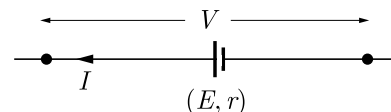
Delhi 2009

The secondary cell with provide higher value of maximum current so the value of internal resistance of secondary cell is lower than primary cell.

63. A cell of emf E and internal resistance r draws a current I . Write the relation between terminal voltage V in terms of E , I and r .

Ans :

SQP 2009



The terminal voltage $V < E$, so $V = E - Ir$

64. Define the current sensitivity of a galvanometer. Write its SI unit.

Ans :

Foreign 2017

Ratio of deflection produced in the galvanometer and the current flowing through it is called current sensitivity.

$$\text{Current sensitivity, } S_i = \frac{\theta}{I}$$

SI unit of current sensitivity S_i is division/ampere or radian/ampere.

65. Plot a graph showing variation of current versus voltage for the material GaAs.

Ans :

OD 2020

The variation of electric current with applied voltage for GaAs is as shown.

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- (a) 0.25 A (b) 1 A
(c) 2.5 A (d) 5 A

Ans :

SQP 2009

Power of lamp, $P = 60 \text{ W}$

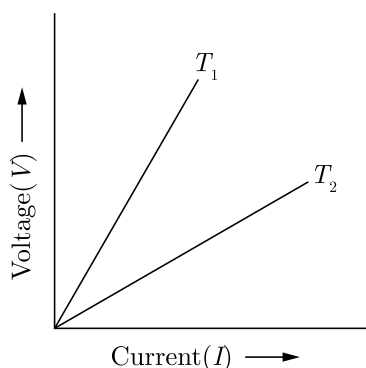
Voltage of lamp, $V = 240 \text{ Volt}$

We know that current flowing through the lamp,

$$I = \frac{P}{V} = \frac{60}{240} = 0.25 \text{ A}$$

Thus (a) is correct option.

30. The voltage V and current I graphs for a conductor at two different temperatures T_1 and T_2 are shown in the figure. The relation between T_1 and T_2 is



- (a) $T_1 > T_2$ (b) $T_1 \approx T_2$
(c) $T_1 = T_2$ (d) $T_1 < T_2$

Ans :

OD 2008

Ohm's law that resistance of a conductor,

$$R = \frac{V}{I}$$

From given figure we get that $\frac{V}{I}$ is the slope of the $V-I$ graph.

Since, $\left(\frac{V}{I}\right)_1 > \left(\frac{V}{I}\right)_2$

Therefore, $R_1 > R_2$... (1)

Resistance at a temperature T ,

$$R = R_0(1 + \alpha T)$$

or, $R \propto T$... (2)

From equation (1) and (2), we get

$$T_1 > T_2$$

Thus (a) is correct option.

31. The electric resistance of a certain wire of iron is R . If its length and radius are both doubled, then its
- (a) both resistance and specific resistance will remain unchanged.
(b) resistance will be doubled and specific resistance will be halved.

- (c) resistance will be halved and specific resistance will be doubled.
(d) resistance will be halved and specific resistance will remain unchanged.

Ans :

Delhi 2001, Foreign 2004

Initial resistance, $R_1 = R$

Final length of wire, $l_2 = 2l_1$

Final radius of wire, $r_2 = 2r_1$

Resistance of the wire of iron,

$$R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2} \propto \frac{l}{r^2}$$

Therefore, $\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \left(\frac{r_2}{r_1}\right)^2$

Here, l_1 = initial length of wire

R_2 = final resistance of the wire of iron

r_1 = initial radius of wire

$$= \frac{l_1}{2l_1} \times \left(\frac{2r_1}{r_1}\right)^2$$

$$= \frac{1}{2} \times 4 = 2$$

$$R_2 = \frac{R_1}{2} = \frac{R}{2}$$

Specific resistance of a wire depends upon nature of the material of wire and is independent of the dimensions of the wire. Therefore specific resistance of the wire remain unchanged.

Thus (d) is correct option.

32. A battery of 12 V and internal resistance 0.5Ω is connected across a variable resistance R . The value of R , for which the power delivered is maximum is equal to
- (a) 0.25Ω (b) 0.5Ω
(c) 1.2Ω (d) 2.4Ω

Ans :

Foreign 2006

Given,

Voltage of battery, $V = 12 \text{ Volt}$

Internal resistance, $r = 0.5 \Omega$

and variable resistance = R

For maximum power, the value of variable resistance is equal to the internal resistance of the battery,

i.e., $R = r = 0.5 \Omega$

Thus (b) is correct option.

33. Internal resistance of a cell of e.m.f. 12 V is 0.05Ω . It is connected to an unknown resistance. Voltage across the cell, when a current of 60 A is drawn from it, is
- (a) 15 V (b) 12 V
(c) 9 V (d) 6 V

8. The net charge on a current carrying conductor is
 (a) zero (b) constant
 (c) varying (d) negative

Ans :

OD 2003

Number of protons in a current carrying conductor at any instant is equal to the number of free electrons.

Therefore, the net charge on a current carrying conductor is zero.

Thus (a) is correct option.

9. A steady current flows in a metallic conductor of non-uniform cross-section. Which of the following quantity is constant along the conductor?
 (a) current (b) drift speed
 (c) current density (d) none of these

Ans :

Foreign 2015

Current remains constant through a conductor of non-uniform cross-section, because of the steady state.

In this case, the charge entering at one section is equal to the charge leaving at any other section.

Thus (a) is correct option.

10. The resistance of an incandescent lamp is
 (a) greater when switched ON
 (b) smaller when switched ON
 (c) greater when switched OFF
 (d) same whether it is switched OFF or ON

Ans :

SQP 2004

Resistance of an incandescent lamp depends upon the nature of the material used, its dimensions and physical conditions. Therefore it does not effect whether the lamp is switched OFF or ON. In other words, the resistance remains the same whether the bulb is switched off or on.

Thus (d) is correct option.

11. Wheatstone's bridge is used in measuring
 (a) High resistance
 (b) Low resistance
 (c) Both high and low resistance
 (d) Potential difference

Ans :

OD 2017

Wheatstone bridge can be used for the measurement of low resistances only.

Thus (b) is correct option.

12. The electrical resistance of a healthy man is
 (a) 50,000 Ω (b) 10,000 Ω
 (c) 1,000 Ω (d) 10 Ω

Ans :

OD 2011

The NIOSH states under dry conditions. The resistance offered by the human body may be as high as 10,000 ohms.

Thus (b) is correct option.

13. Electron-volt (eV) is the measure of
 (a) charge (b) potential difference
 (c) current (d) energy

Ans :

Foreign 2016, OD 2015

Electron-volt (eV) is a unit of energy equal to approximately 1.602×10^{-19} J. By definition it is the amount of energy gained by the charge of a single electron moved across an electric potential difference of one volt.

Thus (d) is correct option.

14. Dimension of electromotive force (e.m.f.) is.
 (a) ML^2T^{-2} (b) $ML^2T^{-2}I^{-1}$
 (c) MLT^{-2} (d) $ML^2T^{-3}I^{-1}$

Ans :

Delhi 2002

We know that,

$$\text{Electromotive force (emf)} = \frac{\text{Work done (W)}}{\text{Charge (Q)}} \dots (1)$$

Since, Work = force $\times$ displacement

$$\begin{aligned} \text{Hence, } Q &= [M^1L^1T^{-2}] \times [L] \\ &= [M^1L^2T^{-2}] \end{aligned}$$

$$\text{Charge, } Q = It = [IT]$$

From equation (1),

$$\text{e.m.f.} = \frac{[M^1L^2T^{-2}]}{[IT]} = [M^1L^2T^{-3}I^{-1}]$$

Thus (d) is correct option.

15. Potential difference between two electrodes of a galvanic cell in an open circuit, is known as
 (a) current
 (b) impedance
 (c) voltage
 (d) electromotive force

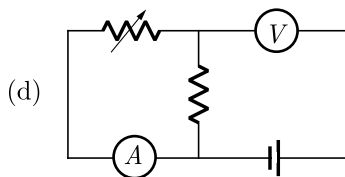
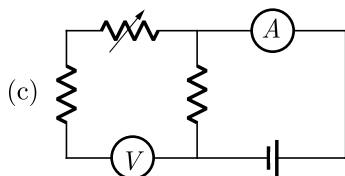
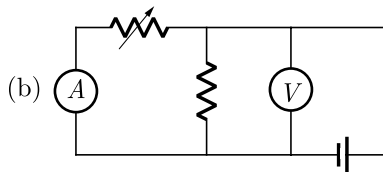
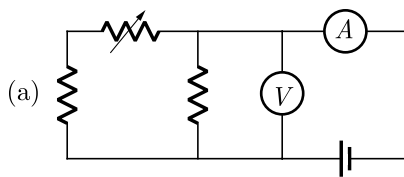
Ans :

OD 2003

In an open circuit, no current is drawn from the cell. And the potential difference between two electrodes of a galvanic cell, in an open circuit, is known as electromotive force of the cell.

Thus (d) is correct option.

16. If voltage across a lamp increases by 2%, then increase in its power will be
 (a) 1% (b) 2%
 (c) 3% (d) 4%



Ans :

Foreign 2013

We know that Ohm's law gives the resistance offered by a conductor by measuring steady current I flowing through the conductor and voltage drop across the ends of the conductor.

We also know that ammeter is always connected in series with the cell to measure current flowing through the conductor and voltmeter is always connected in parallel to the cell to measure voltage drop across the conductor. Therefore, option (a) is used to verify the Ohm's law.

Thus (a) is correct option.

ASSERTION AND REASON

38. **Assertion :** Voltmeter is connected in parallel with the circuit.

Reason : Resistance of a voltmeter is very large.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Voltmeter is a galvanometer with high resistance. It measures potential drop across any part of an electrical circuit. It is connected in parallel so that it does not draw any current itself (due to high resistance) and does not affect net resistance of the circuit.

Thus (b) is correct option.

39. **Assertion :** An electric bulb becomes dim, when the electric heater in parallel circuit is switched on.

Reason : Dimness decreases after sometime.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

The electric power of a heater is more than that of a bulb. As $P \propto \frac{1}{R}$, the resistance of heater is less than that of the electric bulb. When a heater connected in parallel to the bulb is switched on, it draws more current due to its lesser resistance, consequently, the current through the bulb decreases and so it becomes dim.

When the heater coil becomes sufficient hot, its resistance becomes more and hence it draws a little lesser current. Consequently the current through the electric bulb recovers.

Thus (b) is correct option.

40. **Assertion :** Ohm's law is applicable for all conducting elements.

Reason : Ohm's law is a fundamental law.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

A conducting device obeys ohm's law when the resistance of device is independent of the magnitude and polarity of the applied potential difference which happens in metallic conductors. Reason is false as ohm's law is not true for non-ohmic conductors such as junction diodes etc.

Thus (c) is correct option.

Ans :

SQP 2011

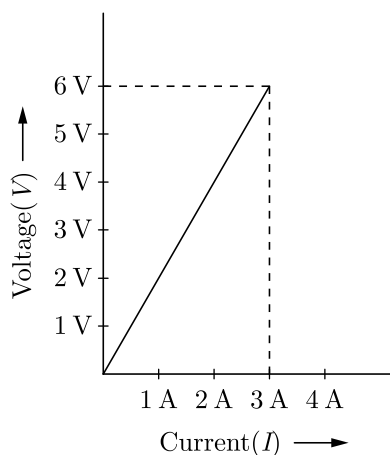
E.m.f. of cell, $E = 12$ VoltInternal resistance of cell, $r = 0.05 \Omega$ Current drawn from the cell, $I = 60$ A

Voltage across the cell,

$$\begin{aligned}
 V &= E - Ir \\
 &= 12 - (60 \times 0.05) \\
 &= 12 - 3 \\
 &= 9 \text{ Volt}
 \end{aligned}$$

Thus (c) is correct option.

34. The variation of voltage V and current I in a conductor is given below. The resistance of the conductor is



- (a) 1Ω (b) 2Ω
(c) 3Ω (d) 4Ω

Ans :

Foreign 2010

$V-I$ graph for the given conductor is a straight line. Thus it obeys Ohm's law. We find from the given graph that voltage is 6 V, when current in the conductor is 3 A.

Therefore, resistance of conductor,

$$R = \frac{V}{I} = \frac{6}{3} = 2 \Omega$$

Thus (b) is correct option.

35. A 100 W, 200 V bulb is connected to a 160 volts supply. The actual power consumption would be
(a) 64 W (b) 80 W
(c) 100 W (d) 125 W

Ans :

OD 2011

Power of bulb, $P = 100$ WVoltage of bulb, $V = 200$ VoltSupply voltage, $V_s = 160$ Volt

Resistance of bulb,

$$R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400 \Omega$$

Therefore, actual power consumption,

$$P = \frac{(V_s)^2}{R} = \frac{(160)^2}{400} = 64 \text{ W}$$

Thus (a) is correct option.

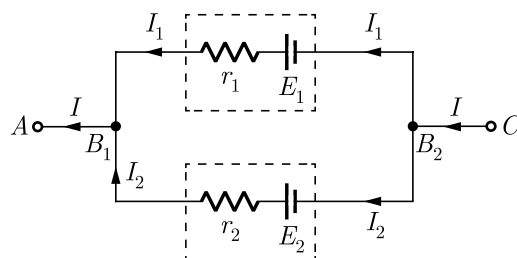
36. Two cells of emf ε_1 and ε_2 , internal resistance r_1 and r_2 , connected in parallel. The equivalent emf of the combination is

- (a) $\frac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{r_1 + r_2}$ (b) $\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$
(c) $\sqrt{\varepsilon_1 \times \varepsilon_2}$ (d) $\frac{\varepsilon_1 + \varepsilon_2}{2}$

Ans :

OD 2017, Delhi 2014

According to the question,



Potential difference between the terminal of first cell,

$$\begin{aligned}
 V &= V_{B_1} - V_{B_2} = \varepsilon_1 - I_1 r_1 \\
 I_1 &= \frac{\varepsilon_1 - V}{r_1} \quad \dots(1)
 \end{aligned}$$

Similarly,

$$I_2 = \frac{\varepsilon_2 - V}{r_2} \quad \dots(2)$$

$$\begin{aligned}
 I &= I_1 + I_2 \\
 &= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} \\
 &= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \\
 I &= \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \right) - V \left(\frac{r_1 + r_2}{r_1 r_2} \right)
 \end{aligned}$$

$$\begin{aligned}
 V \left(\frac{r_1 + r_2}{r_1 r_2} \right) &= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I \\
 V &= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2} \quad \dots(1)
 \end{aligned}$$

Now

$$V = E_{eq} - I r_{eq} \quad \dots(2)$$

After comparing equation. (1) and (2), we get

$$E_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Thus (b) is correct option.

37. Which of the following set up can be used to verify the Ohm's law?

free electrons in the magnetic field. On passing the current, the free electrons acquire drift velocity in a definite direction, hence magnetic force acts on them, unless the field has no perpendicular component.

Thus (a) is correct option.

47. **Assertion :** The electric bulbs glow immediately when switch is on.

Reason : The drift velocity of electrons in a metallic wire is very high.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

In a conductor there are large number of free electrons. When we close the circuit, the electric field is established instantly with the speed of electromagnetic wave which cause electron drift at every portion of the circuit. Due to which the current is set up in the entire circuit instantly. The current which is set up does not wait for the electrons flow from one end of the conductor to the another end. It is due to this reason, the electric bulb glows immediately when switch is on.

Thus (c) is correct option.

48. **Assertion :** The current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at the point.

Reason : A point charge when released from rest in a region having only electrostatic field always moves along electric lines of force.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

From relation $\vec{J} = \sigma \vec{E}$, the current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point. In space having non-uniform electric field, charges released from rest may not move along ELOF. Hence Assertion is correct while Reason is incorrect.

Thus (c) is correct option.

49. **Assertion :** Bending a wire does not effect electrical resistance.

Reason : Resistance of wire is proportional of resistivity of material.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Resistance wire $R = \frac{\rho}{A} l$, where ρ is resistivity of material which does not depend on the geometry of wire. Since when wire is bent resistivity, length and area of cross-section do not change, therefore resistance of wire also remain same.

Thus (a) is correct option.

VERY SHORT ANSWER QUESTIONS

50. The charging current for a capacitor is 0.25 A. What is the displacement current across its plates?

Ans :

Delhi 2019

The displacement current is equal to the charging current. So, displacement current is 0.25 A.

51. Two wires one of copper and other of manganin have same resistance and equal length. Which wire is thicker and why?

Ans :

OD 2014

Manganin wire is thicker because $R = \frac{\rho l}{A}$. As ρ increases, R also increases.

52. Define the term 'relaxation time' in a conductor

Ans :

Foreign 2016, Comp 2004

The average time between successive collisions of electrons in a conductor is known as relaxation time.

53. Write the expression for the drift velocity of charge carriers in a conductor of length L across which a potential difference V is applied.

Ans :

SQP 2017

Drift velocity, $v_d = \frac{eV}{mL} \tau$

54. How does the random motion of free electrons in a conductor get affected when a potential difference is applied across the ends?

Ans :

Foreign 2019

They start drifting in the same direction.

- 41. Assertion :** In a simple battery circuit, the point of the lowest potential is negative terminal of the battery.

Reason : The current flows towards the point of the higher potential, as it does in such a circuit from the negative to the positive terminal.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Positive terminal of a battery is point of highest potential and current flows from highest to lowest potential i.e. from positive to negative potential.

Thus (c) is correct option.

- 42. Assertion :** Long distance power transmission is done at high voltage.

Reason : At high voltage supply power losses are less.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

$$\text{Power loss} = i^2 R = \left(\frac{P}{V}\right)^2 R$$

[P = Transmitted power]

Thus (a) is correct option.

- 43. Assertion :** In a simple battery circuit, the point of the lowest potential is positive terminal of the battery.

Reason : The current flows towards the point of the higher potential, as it does in such a circuit from the negative to the positive terminal.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Positive terminal of the battery is point of highest potential and current flows from highest to lowest potential i.e. from positive to negative potential.

Thus (d) is correct option.

- 44. Assertion :** Kirchhoff's junction rule follows from conservation of charge.

Reason : Kirchhoff's loop rule follows from conservation of momentum.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Kirchhoff's loop rule follows from conservation of energy.

Thus (c) is correct option.

- 45. Assertion :** When current through a bulb decreases by 0.5%, the glow of bulb decreases by 1%.

Reason : Glow (Power) which is directly proportional to square of current.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

$$\text{Glow} = \text{Power } (P) = I^2 R$$

$$\begin{aligned} \text{Hence, } \frac{dP}{P} &= 2\left(\frac{dI}{I}\right) \\ &= 2 \times 0.5 = 1\% \end{aligned}$$

Thus (a) is correct option.

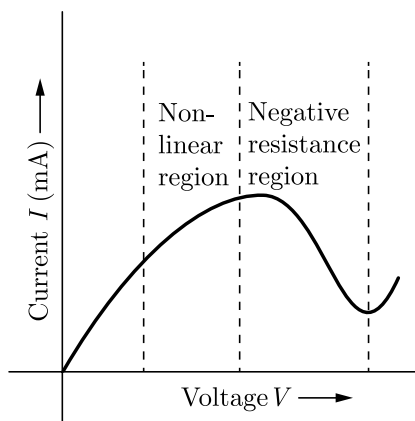
- 46. Assertion :** Free electrons always keep on moving in a conductor even then no magnetic force act on them in magnetic field unless a current is passed through it.

Reason : The average velocity of free electron is zero.

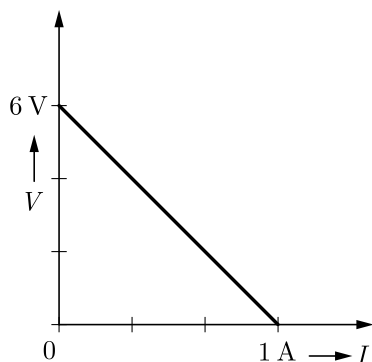
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

In the absence of the electric current, the free electrons in a conductor are in a state of random motion, like molecule in a gas. Their average velocity is zero. i.e. they do not have any net velocity in a direction. As a result, there is no net magnetic force on the



66. The plot at the variation of potential difference across a combination of three identical cells in series versus current is shown below. What is the emf and internal resistance of each cell?



Ans :

Foreign 2018

For a circuit

$$V = E_{eq} - I r_{eq} \quad \dots(1)$$

From graph, when $I = 0A$, when $V = 6V$ and when $I = 1A$, then $V = 0V$. Putting $V = 6V$ and $I = 0A$ in eq. (1)

$$6 = E_{eq} - 0 \cdot r_{eq}$$

$$E_{eq} = 6V$$

And when,

$$I = 1A$$

and

$$V = 0V$$

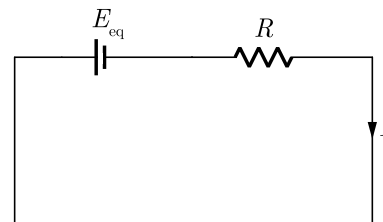
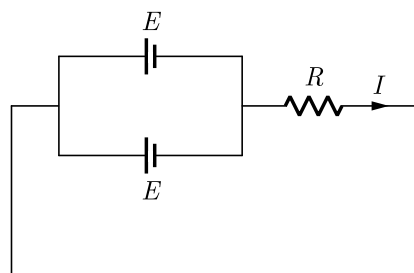
$$0 = 6 - I \cdot r_{eq}$$

$$r_{eq} = 6\Omega$$

67. Two identical cells, each of emf E , having negligible internal resistance, are connected in parallel with each other across an external resistance R . What is the current through this resistance?

Ans :

Comp 2021

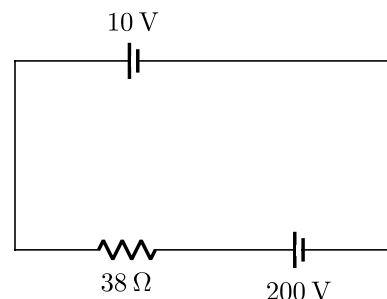


$$I = \frac{E_{eq}}{R + r_{eq}}$$

Given, internal resistance, $r = 0$

$$I = \frac{E_{eq}}{R}$$

68. A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of 38Ω as shown in the figure. Find the value of the current in the circuit.



Ans :

Delhi 2021, Foreign 2014

Since, the positive terminal of the batteries are connected together, so the equivalent emf of the batteries

$$E = 200 - 10 = 190 \text{ Volt}$$

Hence, the current in circuit,

$$I = \frac{E}{R} = \frac{190}{38} = 5A$$

69. Define conductivity of a material. Give its SI unit.

Ans :

SQP 2010

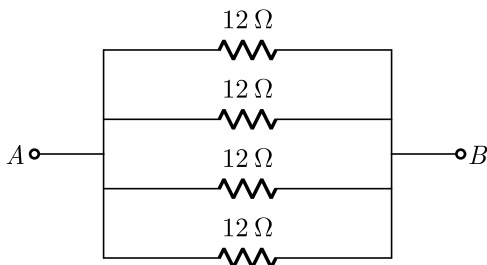
The reciprocal of the resistivity of a material is called its conductivity and is denoted by σ . Thus,

- 132.** Four $12\ \Omega$ resistance are connected in parallel. Three such combinations are connected in series. What will be the total resistance?

Ans :

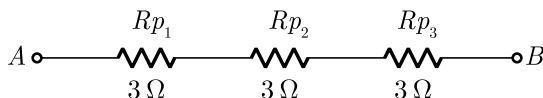
Comp 2018

Four $12\ \Omega$ resistance are connected in the parallel combination as shown in figure.



$$\begin{aligned}\text{Now, } \frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\ &= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \\ R_p &= \frac{12}{4} = 3\ \Omega\end{aligned}$$

Three such combination are connected in series as shown in the figure,



$$\begin{aligned}R_s &= R_{p1} + R_{p2} + R_{p3} \\ &= 3 + 3 + 3 \\ &= 9\ \Omega\end{aligned}$$

- 133.** An electron moves in a circular orbit of radius $5.3 \times 10^{-11}\text{ m}$ with a speed of $2.2 \times 10^6\text{ m-s}^{-1}$. What is the equivalent current in the orbit ?

Ans :

OD 2021

Radius of circular orbit, $r = 5.3 \times 10^{-11}\text{ m}$

and speed of electron, $v = 2.2 \times 10^6\text{ m-s}^{-1}$

Time-taken by an electron to complete one round along the circular orbit,

$$\begin{aligned}t &= \frac{\text{Circumference}}{\text{Speed}} \\ &= \frac{2\pi r}{v}\end{aligned}$$

Equivalent current in the orbit,

$$I = \frac{e}{t} = \frac{e}{\frac{2\pi r}{v}} = \frac{e \times v}{2\pi r}$$

where, e is charge on an electron equal to $1.6 \times 10^{-19}\text{ C}$

$$\begin{aligned}&= \frac{(1.6 \times 10^{-19}) \times (2.2 \times 10^6)}{2 \times 3.14 \times (5.3 \times 10^{-11})} \\ &= 1.1 \times 10^{-3}\text{ A}\end{aligned}$$

- 134.** An electric bulb is rated 100 W for 220 V A.C. supply of 50 Hz . Calculate

- The resistance of the bulb.
- The r.m.s. current through the bulb.

Ans :

Delhi 2020, SQP 2011

Power, $P = 100\text{ W}$

Voltage, $V = 220\text{ Volt}$

Frequency, $\nu = 50\text{ Hz}$

- We know that,

$$P = I^2 R$$

$$\text{Since, } I = \frac{V}{R}$$

$$\text{Hence, } P = \frac{V^2}{R} = \frac{220 \times 220}{100} = 484\ \Omega$$

- We know that,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$\begin{aligned}V_0 &= V_{\text{rms}} \times \sqrt{2} \\ &= 220\sqrt{2} = 311.12\end{aligned}$$

$$\begin{aligned}\text{Now, } I_{\text{rms}} &= \frac{I_0}{\sqrt{2}} = \frac{V_0}{R\sqrt{2}} \\ &= \frac{311.12}{484 \times \sqrt{2}} = 0.45\text{ A}\end{aligned}$$

- 135.** Two cells of emfs 1.5 V and 2.0 V having internal resistance $0.2\ \Omega$ and $0.3\ \Omega$ respectively are connected in parallel. Calculate the emf and internal resistance of the equivalent cell.

Ans :

SQP 2012

Given, $E_1 = 1.5\text{ Volt}$, $r_1 = 0.2\ \Omega$, $E_2 = 2.0\text{ Volt}$

$$r_2 = 0.3\ \Omega$$

emf of equivalent cell,

$$\begin{aligned}E &= \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} \\ &= \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \\ &= \left(\frac{1.5 \times 0.3 + 2 \times 0.2}{0.2 + 0.3} \right) \\ &= \frac{0.45 + 0.40}{0.5} = 1.7\text{ Volt}\end{aligned}$$

Internal resistance of equivalent cell,

$$\begin{aligned}\frac{1}{r} &= \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 r_2}{r_1 + r_2} \\ &= \left(\frac{0.2 \times 0.3}{0.2 + 0.3} \right) \Omega \\ &= \frac{0.06}{0.5} \Omega = 0.12\ \Omega\end{aligned}$$

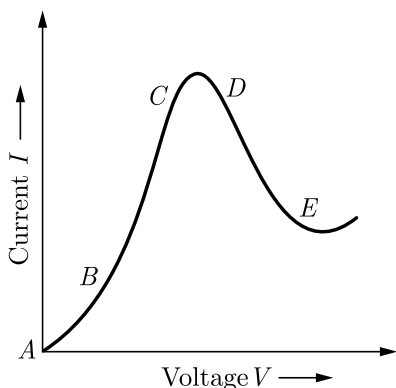
2. They have very small temperature coefficient. So their resistance does not change appreciably even for several degrees rise of temperature.
 3. They are least affected by atmospheric conditions like air, moisture, etc.
 4. Their contact potential with copper is small.
79. What is the advantage of using thick metallic strips to join wires in a potentiometer?

Ans :

OD 2019

The metal strips have low resistance and need not be counted in the potentiometer length l of the null point. One measures only their lengths along the straight segments (of length 1 metre each). This is easily done with the help of centimetre rulings or meter ruler and leads, to accurate measurements.

80. Graph showing the variation of current versus voltage to a material GaAs is shown in the figure. Identify the region of



- (i) negative resistance.
- (ii) where Ohm's law is obeyed ?

Ans :

Delhi 2020

- (i) In region DE , material GaAs (Gallium Arsenide) shows negative resistance, because slope $\frac{\Delta V}{\Delta I} < 0$.
- (ii) The region BC , approximately passes through the origin, (or current also increases with the increase of voltage). Hence, it follows Ohm's law and in this region $\frac{\Delta V}{\Delta I} > 0$.

SHORT ANSWER QUESTIONS

81. Define current density and relaxation time. Derive an expression for resistivity of a conductor in terms of number density of charge carriers in the conductor and relaxation time.

Ans :

OD 2023

Current density is the amount of current travelling per unit cross-section area is known as current density and expressed in ampere per square meter.

Relaxation time is the time gap between two successive electron collisions in a conductor. The relationship between the relaxation time (t) and drift velocity (v_d) is given below

$$v_d = -e \left(\frac{E_t}{m} \right)$$

Derivation : Using ohm's law, we have

$$V = IR$$

$$\frac{V}{I} = R$$

Drift velocity is given by,

$$v_d = \frac{eE}{m} t$$

But, electric field, $E = \frac{V}{l}$

$$v_d = \frac{eV}{ml} t$$

Also, we have

$$I = neAv_d$$

$$I = Ane \left(\frac{eV}{ml} t \right)$$

$$= \left(\frac{Ane^2 t}{ml} \right) V$$

i.e.

$$\frac{V}{I} = \frac{ml}{Ane^2 t} = R$$

Comparing this equation with

$$R = \rho \frac{l}{A}$$

We can deduce an expression for resistivity ρ which is given by

$$\rho = \frac{m_e}{\eta e^2 t}$$

where, η is the number density of charge carriers and t is the relaxation time.

82. Define the term resistance. Give physical explanation of the opposition by a conductor to the flow of current through it.

Ans :

SQP 2015, Comp 2002

Resistance of a conductor is the property by virtue of which it opposes the flow of current through it. Collisions are the basic cause of resistance. When a potential difference is applied across a conductor, its free electrons get accelerated. On their way, they frequently collide with the positive metal ions i.e., their motion is opposed and this opposition to the flow of electrons is called resistance. Larger the number of collisions per second, smaller is the relaxation time τ , and larger will be the resistivity ($\rho = m/ne^2\tau$).

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

or $\sigma = \frac{1}{\rho}$

The SI unit of conductivity is $\text{ohm}^{-1}\text{m}^{-1}$ or ohm m^{-1} or Sm^{-1} .

- 70.** Does the emf represent a force of potential energy or work done per unit charge or potential difference? Does emf have electrostatic origin?

Ans :

Delhi 2020

The term emf is a misnomer. Literally, emf means the force that pushes the electrons in a circuit. Since emf does not have simple electrostatic origin, so the concept of potential is not strictly applicable. It has the nature of work done per unit charge and that of force.

- 71.** Define the term conductivity of a conductor. On what factors does it depend?

Ans :

Delhi 2017

Conductivity of a conductor is defined as the current flowing per unit area per unit electric field applied. It denotes as σ . So, conductivity (σ) = $\frac{J}{E}$. It depends upon number density, i.e., nature of material and also relaxation time, i.e., temperature.

- 72.** Write the expression for the resistivity of the conductor and explain each term of the expression.

Ans :

Delhi 2011, OD 2009

The resistance of conductor is directly proportional to the length l of the conductor and inversely proportional to the area of the cross-section, A of the conductor i.e.,

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A}$$

$$\rho = \frac{RA}{l}$$

where, ρ = resistivity of conductor

R = resistance of conductor

- 73.** What are the advantages of a Wheatstone bridge method of measuring resistance over other methods?

Ans :

Comp 2010

1. It is a null method, hence the result is free from the effect of extra resistances (cell resistances) of the circuit.
2. Being null method, it is easier to detect a small change in deflection than to read a deflection directly.

- 74.** What do you mean by sensitivity of a Wheatstone bridge? On what factors does it depend?

Ans :

SQP 2017

A Wheatstone bridge is said to be sensitive if it shows a large deflection in the galvanometer for a small change of resistance in the resistance arm.

The sensitivity of the Wheatstone bridge depends on two factors:

1. Relative magnitudes of the resistances in the four arms of the bridge. The bridge is most sensitive when all the four resistances are of the same order.
2. Relative positions of battery and galvanometer.

- 75.** Define mobility of a charge carrier. What is its relation with relaxation time?

Ans :

Delhi 2011

It is defined as how fast electron moves from one place to another.

It is also defined as drift velocity per unit electric field. The SI unit of mobility is $\text{m}^2/\text{V} - \text{sec}$ and it is denoted as μ .

$$\begin{aligned}\mu &= \frac{|v_d|}{E} \\ &= \frac{eE\tau}{mE} = \frac{e\tau}{m} \\ \mu &\propto \tau\end{aligned}$$

- 76.** Define electrical conductivity of a conductor and give its SI unit.

Ans :

SQP 2007

The reciprocal of resistivity (ρ) of a material is called its electrical conductivity (σ), i.e.,

$$\sigma = \frac{1}{\rho}$$

- 77.** A wire of resistivity ρ is stretched to double its length. What will be its new resistivity?

Ans :

Delhi 2014

Resistivity of wire is given by,

$$\rho = \frac{RA}{l} \quad \dots(1)$$

Specific resistance or resistivity of the conductor depends upon the nature of the material and temperature of the conductor. So it remains same.

- 78.** Why alloys like constantan or manganin are used for making standard resistors?

Ans :

OD 2017

Use of alloys like constantan or manganin in making standard resistance coils. This is because of the following reasons :

1. These alloys have high value of resistivity.

For a hole, $\mu_h = \frac{e\tau_h}{m_h}$

SI unit of mobility = $\text{m}^2\text{V}^{-1}\text{S}^{-1}$ or $\text{ms}^{-1}\text{N}^{-1}\text{C}$

Practical unit of mobility = $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$

88. What is internal resistance of a cell? On what factors does it depend?

Ans :

OD 2016

The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called **internal resistance** of the cell.

Factors on which internal resistance of a cell depends :

1. Nature of the electrolyte.
2. It is directly proportional to the concentration of the electrolyte.
3. It is directly proportional to the distance between the two electrodes.
4. It varies inversely as the common area of the electrodes immersed in the electrolyte.
5. It increases with the decrease in temperature of the electrolyte.

89. Express Ohm's law in vector form.

Ans :

OD 2019

Vector form of Ohm's Law : If E is the magnitude of electric field in a conductor of length l , then the potential difference across its end is,

$$V = El$$

Also from Ohm's law, we can write

$$V = IR = \frac{I\rho l}{A}$$

Hence, $El = \frac{I}{A}\rho l$

$$E = \rho j$$

As the direction of current density $\vec{j}$ is same as that of electric field $\vec{E}$, we can write the above equation as,

$$\vec{E} = \rho \vec{j}$$

$$\vec{j} = \sigma \vec{E}$$

The above equation is the vector form of Ohm's law. It is equivalent to the scalar form,

$$V = RI$$

90. Define the terms electric energy and electric power. Give their units.

Ans :

Delhi 2010

Electric Power :

It is the rate at which an electric appliance converts energy into other forms of energy. Or, it is the rate at which work is done by a source of emf in maintaining

an electric current through a circuit.

$$\text{Electric power, } P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R}$$

SI unit of power is watt :

The power of an appliance is one watt if one ampere of current flows through it on applying a potential difference of 1 volt across it.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}}$$

$$= \frac{1 \text{ joule}}{1 \text{ coulomb}} \times \frac{1 \text{ coulomb}}{1 \text{ second}}$$

$$= 1 \text{ volt} \times 1 \text{ ampere}$$

$$1 \text{ W} = 1 \text{ Js}^{-1} = 1 \text{ VA}$$

$$1 \text{ kilowatt (kW)} = 1000 \text{ W}$$

Electric Energy :

It is the total work done in maintaining an electric current in an electric circuit for a given time.

$$\text{Electric energy} = \text{Electric power} \times \text{time}$$

$$W = Pt = VI t \text{ joule} = I^2 Rt \text{ joule}$$

Units of Electric Energy:

The commercial unit of electric energy is kilowatt-hour (kWh) or Board of Trade (B.O.T.) unit. It is the electric energy consumed by an appliance of power 1000 watt in one hour.

$$1 \text{ kWh} = 1000 \text{ Wh}$$

$$= 1000 \text{ W} \times 3600 \text{ s}$$

$$= 6 \times 10^6 \text{ J}$$

Another unit of electric energy is watt hour.

$$1 \text{ watt hour} = 1 \text{ W} \times 1 \text{ H}$$

$$= 3.6 \times 10^3 \text{ J}$$

91. Why is the use of a potentiometer preferred over that of a voltmeter for measurement of emf of a cell?

or

In what respect is the potentiometer better than a moving coil voltmeter in comparing the emfs of two cells.

Ans :

OD 2013, Comp 2002

Potentiometer is a null method device. No current is drawn from the cell at null point, thus there is no potential drop due to the internal resistance of the cell. So it measures the p.d. in an open circuit. But voltmeter requires a small current for its operation. So voltmeter measures the p.d. in a closed circuit which is less than the actual emf of the cell.

92. Three resistances R_1 , R_2 and R_3 are connected in series. Find their equivalent resistance.

(a) Charging current,

$$I = \frac{\varepsilon' - n\varepsilon}{R + nr} \quad \dots(1)$$

(b) Potential difference across the combination V is given by

$$-V - IR + \varepsilon' = 0$$

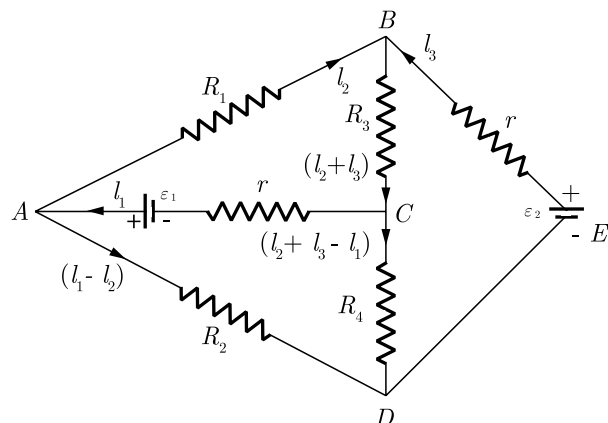
$$V = \varepsilon' - IR$$

$$V = \varepsilon' - \frac{(\varepsilon' - n\varepsilon)}{R + nr}$$

$$V = \frac{\varepsilon'(R + nr) - \varepsilon' + n\varepsilon}{R + nr}$$

$$V = \frac{\varepsilon'(R + nr - 1) + n\varepsilon}{R + nr}$$

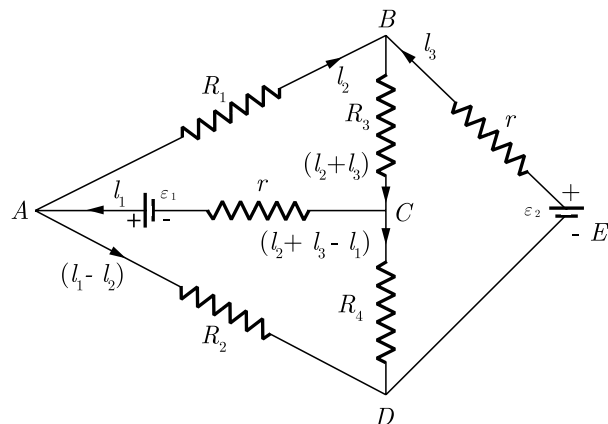
106. Use Kirchhoff's rules to write the three equations that may be used to obtain the values of three unknown currents in the branches (shown) of the given circuit.



Ans :

OD 2012

The distribution of current is shown in given figure.

Applying Kirchhoff's II law in loop $ABCA$

$$-I_2 R_1 - (I_2 + I_3) R_3 - I_1 r + \varepsilon_1 = 0$$

$$I_1 r + I_2 (R_1 + R_3) + I_3 R_3 = \varepsilon_1 \quad \dots(1)$$

Applying Kirchhoff's II law to loop $ACDA$,

$$-\varepsilon_1 + I_1 r - (I_2 + I_3 - I_1) R_4 + (I_3 - I_2) R_2 = 0$$

$$I_1 (r + R_2) - (R_2 + R_4) I_2 - I_3 R_4 = 0 \quad \dots(2)$$

In loop $BEDB$,

$$I_3 r - \varepsilon_2 + (I_2 + I_3 - I_1) R_4 + (I_2 + I_3) R_3 = 0$$

$$-I_1 R_4 + I_2 (R_3 + R_4) + I_3 (r + R_3 + R_4) = \varepsilon_2 \quad \dots(3)$$

Equations (1), (2) and (3) are required equations.

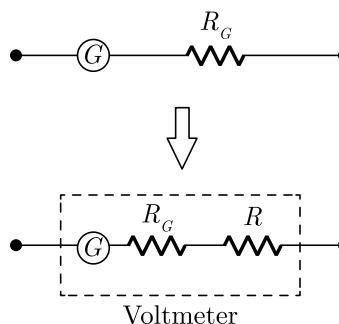
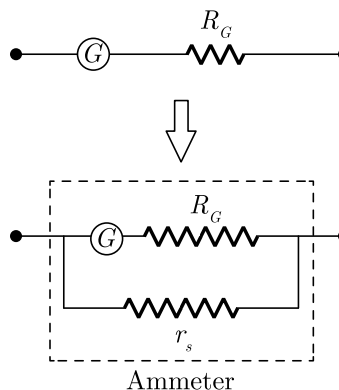
107. How is a galvanometer converted into a voltmeter and an ammeter? Draw the relevant diagrams and find the resistance of the arrangement in each case. Take resistance of galvanometer as G .

Ans :

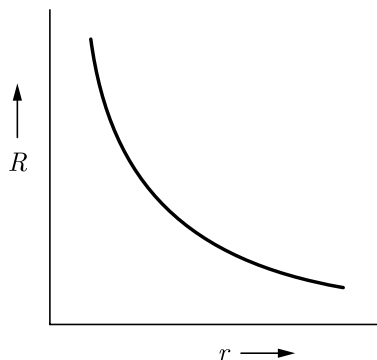
Delhi 2016, OD 2005

A galvanometer is converted into a voltmeter by connecting a high resistance R in series with it.

A galvanometer is converted into an ammeter by connecting a small resistance (called shunt) in parallel with it.

Resistance of voltmeter, $R_V = G + R$ Resistance of Ammeter, $R_A = \frac{Gr_s}{G + r_s}$ 

108. $V-I$ graph for two identical conductors of different materials A and B is shown in the figure. Which one of the two has higher resistivity?



SI unit of conductivity is $\text{mho} - \text{m}^{-1}$ (or siemen^{-1}).

97. Sketch a graph showing the variation of resistivity of carbon with temperature.

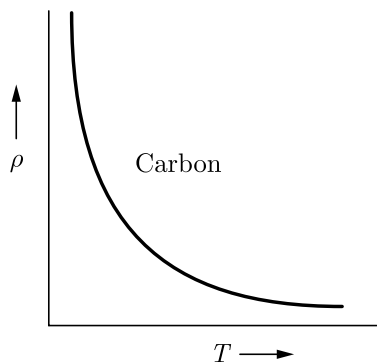
or

Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behaviour explained?

Ans :

Comp 2008

The resistivity of a typical semiconductor (carbon) decreases with increase a temperature. The graph is shown in figure.



In semiconductor the number density of free electrons n increases with increase in temperature T and consequently the relaxation period decreases. But the effect of increase in h has higher impact than decrease of τ . So, resistivity decreases with increase in temperature.

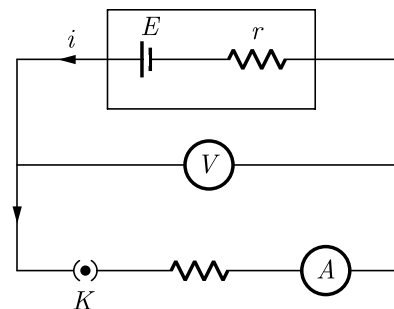
98. A cell of emf ' E ' and internal resistance ' r ' is connected across a variable resistor ' R '. Plot a graph showing the variation of terminal potential ' V ' with resistance R .

Predict from the graph the condition under which ' V ' becomes equal to ' E '.

Ans :

Comp 2009

From figure (a)

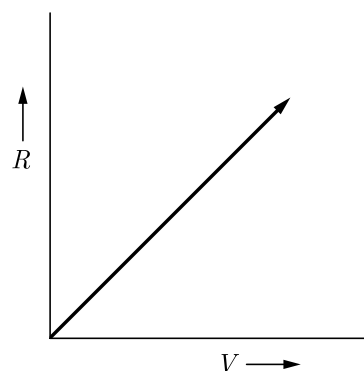


$$V = E - ir$$

and

$$V = iR$$

Required graph is as shown in figure (b)



When circuit is open (i.e., $i = 0$),

then

$$E = V$$

99. Explain the variation of conductivity with temperature for (i) a metallic conductor and (ii) ionic conductors.

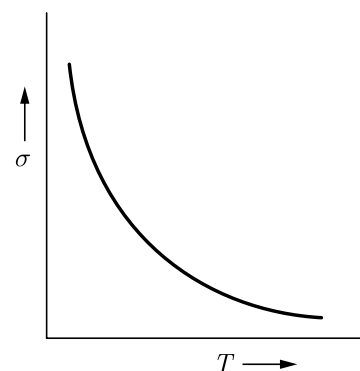
Ans :

OD 2019, SQP 2004

- (i) Conductivity of a metallic conductor

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

with raise of temperature, the collision of electrons with fixed lattice ions/atoms increases so that relaxation time (τ) decreases. Consequently, the conductivity of metals decreases with rise of temperature.



83. Give some important points of differences between electromotive force and potential difference.

Ans :

Comp 2021

Differences between electromotive force and potential difference are as follows:

	Electromotive Force	Potential Difference
1.	It is the work done by a source in taking a unit charge once round the complete circuit.	It is the amount of work done in taking a unit charge from one point of a circuit to another.
2.	It is equal to the maximum potential difference between the two terminals of a source when it is in an open circuit.	Potential difference may exist between any two points of a closed circuit.
3.	It exists even when the circuit is not closed.	It exists only when the circuit is closed.
4.	It is a cause. When emf is applied in a circuit, potential difference is caused.	It is an effect.
5.	It is equal to the sum of potential differences across all the components of a circuit including the p.d. required to send current through the cell itself.	Every circuit component has its own potential difference across its ends.
6.	It is larger than the p.d. across any circuit element.	It is always less than the emf.

84. How can we classify solids on the basis of their resistivity values?

Ans :

Delhi 2018

On the basis of their resistivity values, solids can be classified into following three categories :

- Conductors** : Metals have low resistivities in the range of $10^{-8} \Omega\text{m}$ to $10^{-6} \Omega\text{m}$. These are known as good conductors of electricity.
- Insulators** : These are the substances which have large resistivities, more than $10^4 \Omega\text{m}$. Insulators like glass and rubber have resistivities in the range 10^{14} to $10^{16} \Omega\text{m}$.
- Semiconductors** : These are the substances having resistivities between those of conductors and insulators i.e., between $10^{-6} \Omega\text{m}$ to $10^4 \Omega\text{m}$. Germanium and silicon are typical semiconductors.

85. Define current density. Is it a scalar or vector quantity? Give its SI unit.

Ans :

Delhi 2016

It is the amount of charge flowing per second per unit area normal to the flow of charge. It is a vector quantity having the same direction as that of the motion of the positive charge.

For normal flow of charge,

$$j = \frac{q/t}{A} = \frac{I}{A}$$

In general, $I = jA \cos \theta = \vec{j} \cdot \vec{A}$

SI unit of current density = Am^{-2} .

86. What are the factors on which resistance of a conductor depends? Give the corresponding relation.

Ans :

OD 2020

At a constant temperature, the resistance of a conductor depends on the following factors:

- Length** : The resistance R of a conductor is directly proportional to its length i.e., $R \propto l$.
- Area of cross-section** : The resistance R of a uniform conductor is inversely proportional to its area of cross-section A , i.e., $R \propto \frac{1}{A}$.
- Nature of the material** : The resistance of a conductor also depends on the nature of its material.

Combining the above factors, we get,

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

Where ρ is the constant of proportionality called resistivity or specific resistance of the material of the conductor. It depends on the nature of the material of the conductor and on the physical conditions like temperature and pressure but it is independent of its size or shape.

87. Define mobility of a charge carrier. Express it in terms of relaxation time. Give its SI and practical units.

Ans :

Foreign 2018, SQP 2010

The mobility of a charge carrier is the magnitude of the drift velocity acquired per unit electric field. It is given by

$$\mu = \frac{v_d}{E}$$

As drift velocity, $v_d = \frac{qE\tau}{m}$

Hence, $\mu = \frac{v_d}{E} = q \frac{\tau}{m}$

For an electron, $\mu_e = \frac{e\tau_e}{m_e}$

conductor, an electric field is produced and free electrons are acted upon an electric force (F_e). Due to this, electrons accelerate and keep colliding with each other and acquire a constant (average) velocity v_d called drift velocity.

Electric force on electron $F_e = -eE$

If m is the mass of electron, then its acceleration

$$a = \frac{F}{m} = \frac{-eE}{m}$$

Now,

$$v = u + at$$

Here, $u = 0$

$$t = \tau \text{ (relaxation time),}$$

$$\vec{v} = \vec{v}_d$$

$$\vec{v}_d = 0 - \frac{eE}{m} \tau$$

$$\vec{v}_d = -\frac{e\tau}{m} \vec{E}$$

(ii) When temperature increases, the collisions of electrons occur more frequently, so relaxation time decreases and hence, drift velocity decreases.

- 104.** Define the term 'resistivity' and write its S.I. unit. Derive the expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time.

or

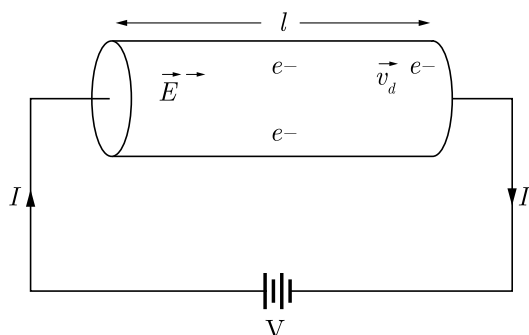
Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material.

Ans :

Comp 2012

The resistivity of the material of a conductor is defined as the resistance of the unit length and unit area of cross section of the conductor. Its S.I. unit is $\Omega \cdot m$.

Consider a conductor of length l and of uniform cross sectional area A . Let there be n free electrons per unit volume of the conductor. Let us apply a potential difference V across the ends of conductor along its length, then $E = \frac{V}{l}$



Though the free electrons in the conductor experience a force $\vec{F} = -e\vec{E}$ and thus, have an acceleration $\vec{a} = -e\frac{\vec{E}}{m}$, the drift with constant velocity $\vec{v}_d$ in the direction opposite to E .

Current I in the conductor can be written in terms of drift velocity of electrons as

$$I = ne^2 A v_d \quad \dots(1)$$

where

$$v_d = \frac{eE}{m} \cdot \tau \quad \dots(2)$$

Here, τ is called the relaxation time. It is the average time interval between two successive collisions of electrons with an atom or an ion.

From equations (1) and (2),

$$I = \frac{ne^2 AE}{m} \cdot \tau$$

$$I = \frac{ne^2 A V \tau}{m l}$$

$$\frac{V}{I} = \frac{m l}{ne^2 A \tau} = R \text{ (from Ohm's law)}$$

$$R = \frac{m}{ne^2 \tau} \frac{l}{A}$$

$$R = \rho \frac{l}{A}, \quad \text{where } \rho = \frac{m l}{ne^2 \tau}$$

Here, ρ is called the specific resistance or the resistivity of the material of conductor.

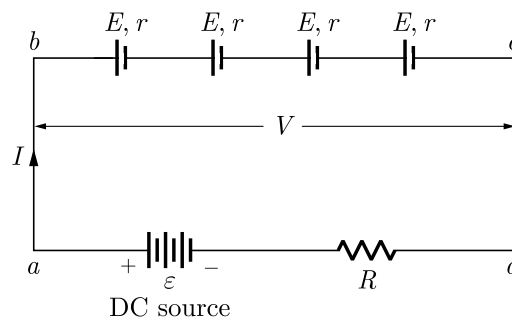
- 105.** n -identical cells, each of emf ε , internal resistance r connected in series are charged by a DC source of emf ε' , using a resistance R .

- Draw the circuit arrangement.
- Deduce expressions for (a) the charging current and (b) the potential difference across the combination of cells.

Ans :

SQP 2008, Comp 2016

- The circuit arrangement is shown in figure :



- Applying Kirchhoff's second law to the circuit $abcd a$

$$-ne - I(nr) - IR + \varepsilon' = 0$$

$$I = \frac{\varepsilon' - n\varepsilon}{R + nr}$$

Ans :

SQP 2013

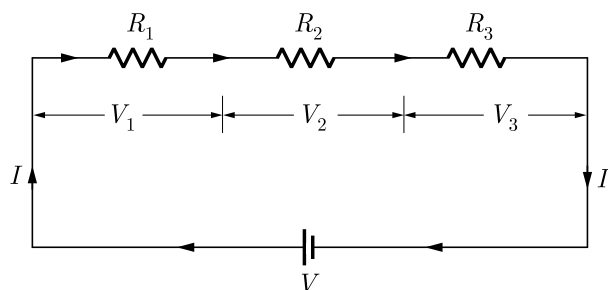


Figure shows three resistances R_1 , R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.

By Ohm's law, the potential drops across the three resistances are,

$$V_1 = IR_1$$

$$V_2 = IR_2$$

$$V_3 = IR_3$$

If R_s is the equivalent resistance of the series combination then we must have,

$$V = IR_s$$

But,

$$V = V_1 + V_2 + V_3$$

$$IR_s = IR_1 + IR_2 + IR_3$$

$$R_s = R_1 + R_2 + R_3$$

Thus when a number of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

93. What do you mean by the sensitivity of a potentiometer? How can we increase the sensitivity of a potentiometer?

Ans :

Foreign 2015

A potentiometer is sensitive if :

1. It is capable of measuring very small potential differences.
2. It shows a significant change in balancing length for a small change in the potential difference being measured.

The sensitivity of a potentiometer depends on the potential gradient along its wire. Smaller the potential gradient, greater will be the sensitivity of the potentiometer.

The sensitivity of a potentiometer can be increased by reducing the potential gradient. This can be done in two ways :

1. By increasing the length of the potentiometer wire.
2. By reducing the current in the circuit with the help of a rheostat.

94. What do you mean by heating effect of current? Explain its cause.

Ans :

Delhi 2019

The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current or joule heating.

Cause of heating effect of current : When a potential difference is applied across the ends of a conductor, its free electrons get accelerated in the opposite direction of the applied field. But the speed of the electrons does not increase beyond a constant drift speed. This is because during the course of their motion, the electrons collide frequently with the positive metal ions. The kinetic energy gained by the electrons during the intervals of free acceleration between collisions is transferred to the metal ions at the time of collision. The metal ions begin to vibrate about their mean positions more and more violently. The average kinetic energy of the ions increases. This increases the temperature of the conductor. Thus the conductor gets heated due to the flow of current.

95. Using theory of drift velocity, express Ohm's law.

Ans :

OD 2018, Foreign 2004

When a potential difference V is applied across a conductor of length l , the drift velocity in terms of V is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml} \quad \left(E = \frac{V}{l}\right)$$

If the area of cross-section of the conductor is A and the number of electrons per unit volume or the electron density of the conductor is n , then the current through the conductor will be

$$I = enAv_d = enA \cdot \frac{eV\tau}{ml}$$

or,

$$\frac{V}{I} = \frac{ml}{ne^2\tau A}$$

At a fixed temperature, the quantities m, l, n, e, τ and A , all have constant values for a given conductor. Therefore,

$$\frac{V}{I} = \text{a constant, } R$$

This proves Ohm's law for a conductor.

96. Plot a graph showing the variation of resistance of a conducting wire as a function of its radius, keeping the length of the wire and its temperature as constant.

Ans :

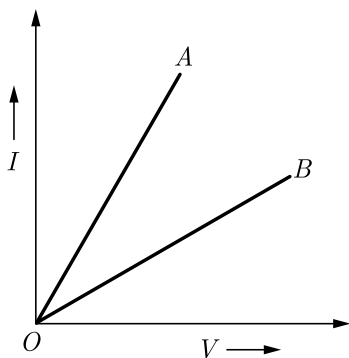
Foreign 2015

Resistance of a conductor of length l and radius r is given by

$$R = \rho \frac{l}{\pi r^2}$$

Hence,

$$R \propto \frac{1}{r^2}$$



Ans :

Delhi 2011

The resistivity of material B is higher.

Reason : If the same amount of the current flows through them, then $V_B > V_A$ and from Ohm's law $R_B > R_A$. Hence, the resistivity of the material B is higher.

LONG ANSWER QUESTIONS

109. What is Wheatstone bridge? Explain its use in detail.

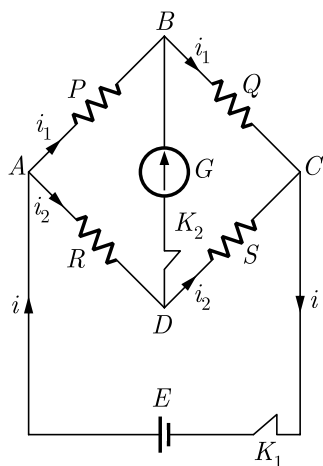
Ans :

Comp 2013

Wheatstone bridge is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances. A Wheatstone bridge consists of four resistances P , Q , R and S ; connected to form the arms of a quadrilateral $ABCD$. A battery of emf ϵ is connected between points A and C and a sensitive galvanometer between B and D , as shown in Figure.

Let S be the resistance to be measured. The resistance R is so adjusted that there is no deflection in the galvanometer. The bridge is said to be balanced when the potential difference across the galvanometer is zero so that there is no current through the galvanometer. In the balanced condition of the bridge,

$$\frac{P}{Q} = \frac{R}{S}$$



Unknown resistances, $S = \frac{Q}{P} \cdot R$

Knowing the ratio of resistances P and Q the resistance R , we can determine the unknown resistance S . That is why the arms containing the resistances P and Q are called ratio arms, the arm AD containing R standard arm and the arm CD containing S the unknown arm.

110. Define the terms drift velocity and relaxation time. Establish the relation between drift velocity of electrons and electric field applied to the conductor.

or

Derive an expression for the drift velocity of free electrons in a conductor in terms of relaxation time.

Ans :

OD 2019, Foreign 2011

In the absence of any electric field, the free electrons of a metal are in a state of continuous random motion. At room temperature, their random velocities correspond to 10^5 ms^{-1} . The average random velocity of free electrons is zero.

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} = 0$$

Thus, there is no net flow of charge in any direction.

In the presence of an external field $\vec{E}$, each electron experiences a force $-e\vec{E}$ in the opposite direction of $\vec{E}$ (since an electron has negative charge) and undergoes an acceleration $\vec{a}$ given by

$$\vec{a} = \frac{\text{Force}}{\text{Mass}} = -\frac{e\vec{E}}{m}$$

where m is the mass of an electron. As the electrons accelerate, they frequently collide with the positive metal ions or other electrons of the metal. Between two successive collisions, an electron gains a velocity component (in addition to its random velocity) in a direction opposite to $\vec{E}$. However, the gain in velocity lasts for a short time and is lost in the next collision.

The average time that elapses between two successive collisions of an electron is called relaxation time. It is given by

$$\tau = \frac{t_1 + t_2 + \dots + t_N}{N}$$

During the relaxation time τ , an electron gains an average velocity given by

$$\vec{v}_d = 0 + \vec{a} \tau \quad [\text{since } v = u + at]$$

$$\vec{v}_d = -\frac{e\vec{E} \tau}{m}$$

The parameter $\vec{v}_d$ is called drift velocity of electrons. It may be defined as the average velocity gained by the

- (ii) Conductivity of ionic conductor increases with increase of temperature because with increase of temperature, the ionic bonds break releasing positive and negative ions which are charge carriers in ionic conductors.

- 100.** Define the term current density of a metallic conductor. Deduce the relation connecting current density (J) and the conductivity σ of the conductor, when an electric field E , is applied to it.

Ans :

SQP 2010

Current density at a point in a conductor is defined as the amount of current flowing per unit area of the conductor around that point provided the area is held in a direction normal to the current,

$$J = \frac{I}{A}$$

Current density is a vector quantity. Its direction is the direction of motion of positive charge. The unit of current density is ampere/metre² or [Am⁻²].

Relation between J , σ and E

$$\begin{aligned} I &= nAev_d = nA \cdot e \left(\frac{eE}{m} \tau \right) \\ &= \frac{nAe^2 \tau E}{m} \\ \frac{I}{A} &= \frac{ne^2 \tau E}{m} \\ J &= \frac{I}{A} E \left[\text{since } J = \frac{I}{A} \text{ and } \rho = \frac{m}{ne^2 \tau} \right] \\ \text{Hence, } I &= \sigma E \left[\text{since } \sigma = \frac{1}{\rho} \right] \end{aligned}$$

- 101.** Define resistivity of a conductor. Plot a graph showing the variation of resistivity with temperature for a metallic conductor. How does one explain such a behaviour, using the mathematical expression of the resistivity of a material.

Ans :

SQP 2006

We know that, $R = \rho \frac{l}{A}$

If $l = 1, A = 1$

$$\rho = R$$

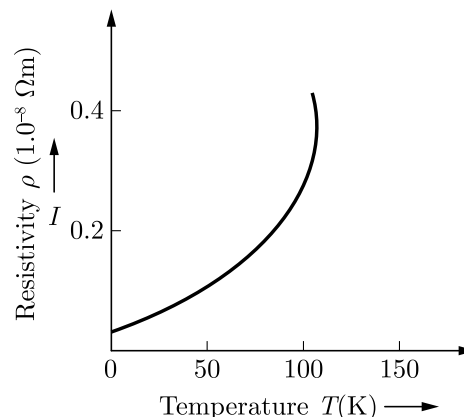
Thus, resistivity of a material is numerically equal to the resistance of the conductor having unit length and unit cross-sectional area.

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperature. Over a limited range of temperatures that is not too large, the resistivity of a metallic conductor is approximately given by,

$$\rho_T = \rho_0 [1 + \alpha(T - T_0)] \quad \dots(1)$$

Where ρ_T is the resistivity at a temperature T and ρ_0 is the same at a reference temperature T_0 . α is called the temperature co-efficient of resistivity.

Relation (1) implies that a graph of ρ_T plotted against T would be a straight line. At temperatures much lower than 0°C, the graph, however, deviates considerably from a straight line.



Resistivity ρ_1 of metallic conductor as a function of temperature.

- 102.** State Kirchhoff's rules of current distribution in an electrical network.

or

State Kirchhoff's rules. Explain briefly how these rules are justified.

Ans :

Delhi 2013, OD 2017

Junction rule in an electric circuit, the algebraic sum of currents in any junction is zero.

At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction.

$$\Sigma I = 0$$

Justification : This rule is based on the law of conservation of charge.

Loop rule : The algebraic sum of charges in potential around any closed loop involving resistors and cells in the loop must be zero.

$$\Sigma \Delta V = 0$$

Justification : This rule is based on the law of conservation of energy.

- 103.** (i) Derive an expression for drift velocity of free electrons.
(ii) How does drift velocity of electrons in a metallic conductor vary with increase in temperature? Explain.

Ans :

Foreign 2016

- (i) When a potential difference is applied across a

$$E_{eq} - Ir_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}$$

Hence, we have

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

and

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \dots(7)$$

- 113.** Define temperature coefficient of resistivity (α). Distinguish between metals, semiconductor and alloys on the basis of their α values.

Ans :

OD 2021

The temperature coefficient of resistivity may be defined as the increase in resistivity per unit resistivity per degree rise in temperature. It is given by

$$\alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)} = \frac{1}{\rho_0} \cdot \frac{d\rho}{dT}$$

Thus the resistivity ρ at any temperature T will be,

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad \dots(1)$$

where ρ_0 is the resistivity at a lower reference temperature T_0 (usually 20°C).

As, $R = \rho \frac{l}{A}$

i.e., $R \propto \rho$

Hence, equation (1) can be written in terms of resistance as

$$R_t = R_0(1 + \alpha t)$$

where, R_t = the resistance at $t^\circ\text{C}$

R_0 = the resistance at 0°C

and t = the rise in temperature

The unit of α is $^\circ\text{C}^{-1}$.

- For metals, α is positive i.e., resistance of metals increase with the increase in temperature.
 - For semiconductors (Ge, Si) and insulators, α is negative i.e., their resistance decreases with the increase in temperature.
 - For alloys like constantan and manganin, the temperature coefficient of resistance α is very small. So they are used for making standard resistors.
- 114.** What do you understand by the resistivity of a conductor? Discuss its temperature dependence for a
- Metallic conductor
 - Semiconductor
 - Ionic conductor
 - Electrolyte.

Ans :

Delhi 2020

Resistivity of a material is the resistance of a conductor of that material, having unit length and unit area of cross-section.

It is given by, $\rho = \frac{m}{ne^2\tau}$

Effect of Temperature on Resistivity

- Metallic Conductor :** In metals, the number of free electron is fixed. As the temperature increases, the amplitude of vibration of the atoms increases. The collisions of electrons with these atoms become more frequent. The relaxation time τ decreases. Hence the resistivity of a metallic conductor increases with the increase of temperature.
 - Semiconductor :** In case of semiconductors, the relaxation time τ does not change with temperature but the number density of free electrons increases exponentially with the increase in temperature. Consequently, the conductivity increases or resistivity decreases exponentially with the increase in temperature.
 - Ionic Conductor :** An ionic conductor has both positive and negative ions as the charge carriers. As the temperature increases, the electrostatic attraction between cations and anions decreases, the ions are more free to move and so the conductivity increases or resistivity decreases.
 - Electrolyte :** As the temperature increases, the inter-ionic attractions (solute-solute, solvent-solute and solvent-solvent type) decrease and also the viscous forces decrease, the ions move more freely. Hence conductivity increases or the resistivity decreases as the temperature of an electrolytic solution increases.
- 115.** Define terminal potential difference of a cell. When a battery of emf e and internal resistance r is connected to a resistance R , a current I flows through it. Derive the relation between e , I , r and R .

Ans :

SQP 2013, OD 2001

For definition of internal resistance, refer, answer to the above question.

Terminal Potential Difference

The potential drop across the terminals of a cell when a current is being drawn from it is called its terminal potential difference (V).

Relation between r , e and V

Consider a cell of emf e and internal resistance r connected to an external resistance R , as shown in figure. Suppose a constant current I flows through this circuit. By definition of emf,

$$e = \text{Work done by the cell in carrying a unit charge along the closed circuit}$$

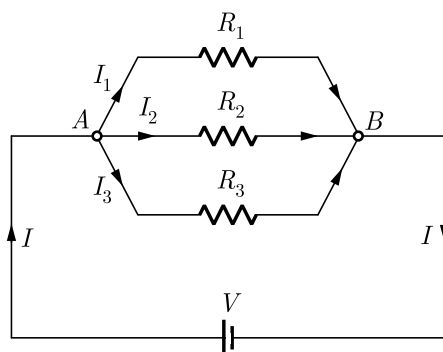
$$= \text{Work done in carrying a unit charge from A to B against external resistance } R + \text{Work done}$$

free electrons of a conductor in the opposite direction of the externally applied electric field.

111. Three resistances R_1 , R_2 and R_3 are connected in parallel. Find the equivalent resistance of the parallel combination.

Ans :

Foreign 2018



Resistances in Parallel

Figure shows three resistances R_1 , R_2 and R_3 connected in parallel between points A and B . Let V be the potential difference applied across the combination.

Let I_1 , I_2 and I_3 be the currents through the resistances R_1 , R_2 and R_3 respectively. Then the current in the main circuit must be,

$$I = I_1 + I_2 + I_3$$

Since all the resistances have been connected between the same two points A and B , therefore, potential drop V is same across each of them. By Ohm's law, the currents through the individual resistances will be,

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$

If R_p is the equivalent resistance of the parallel combination, then we must have,

$$I = \frac{V}{R_p}$$

But,

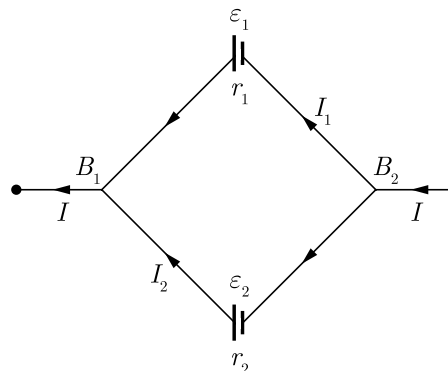
$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus when a number of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.

112. Two cells of emf ε_1 and ε_2 having internal resistances r_1 and r_2 respectively are connected in parallel as shown. Deduce the expressions for the equivalent emf and equivalent internal resistance of a cell which can replace the combination between the point B_1 and B_2 .



Ans :

OD 2009, Comp 2012

Consider a parallel combination of the cells I_1 and I_2 are the currents leaving the positive electrodes of the cells. At point B_1 , I_1 and I_2 flow in whereas current I flows out. Therefore, we have

$$I = I_1 + I_2 \quad \dots(1)$$

Let $V(B_1)$ and $V(B_2)$ be the potentials at B_1 and B_2 respectively.

Then, considering the first cell, the potential difference across its terminals is $V(B_1) - V(B_2)$. Hence, from equation $V = E - Ir$ we have

$$\begin{aligned} V &= V(B_1) - V(B_2) \\ &= \varepsilon_1 - I_1 r_1 \end{aligned} \quad \dots(2)$$

Points B_1 and B_2 are connected exactly similarly to the second cell. Hence, considering the second cell, we also have

$$\begin{aligned} V &= V(B_1) - V(B_2) \\ &= \varepsilon_2 - I_2 r_2 \end{aligned} \quad \dots(3)$$

Combining equations (1), (2) and (3), we have

$$\begin{aligned} I &= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} \\ &= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \end{aligned} \quad \dots(4)$$

Solving for (5), we have

$$V = \frac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2} \quad \dots(5)$$

If we want to replace the combination by a single cell, between B_1 and B_2 , of emf E_{eq} and internal resistance r_{eq} , we would have

$$V = E_{eq} - I r_{eq} \quad \dots(6)$$

from eqs, (5) and (6) we have

Total charge contained in length l of the conductor is $q = enAl$

All the electrons which enter the conductor at the right end will pass through the conductor at the left end in time,

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{l}{v_d}$$

Hence, Current, $I = \frac{q}{t} = \frac{enAl}{l/v_d}$

$$I = enAv_d$$

The equation relates the current I with the drift velocity v_d .

The current density j is given by,

$$j = \frac{I}{A} = env_d$$

2. **Deduction of Ohm's law :** If m is the mass of an electron and τ is the relaxation time, then drift velocity,

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml} \quad \left[\text{since } E = \frac{V}{l} \right]$$

Hence, Current, $I = enAv_d = enA \cdot \frac{eV\tau}{ml}$

$$\frac{V}{I} = \frac{ml}{ne^2\tau A}$$

At a fixed temperature, the quantities m , l , n , e , τ and A , all have constant value for a given conductor.

Therefore, $\frac{V}{I} = \text{a constant } R$

This proves Ohm's law for a conductor and here

$$R = \frac{ml}{ne^2\tau A}$$

is the resistance of the conductor.

3. **Resistivity in Terms of Electron Density and Relaxation Time :** The resistance R of a conductor of length l , area of cross-section A and resistivity ρ is given by,

$$R = \rho \frac{l}{A}$$

But, $R = \frac{ml}{ne^2\tau A}$

where, τ is the relaxation time. Comparing the above two equations, we get

$$\rho = \frac{m}{ne^2\tau}$$

Obviously, ρ is independent of the dimensions of the conductor but depends on its two parameters :

- Number of free electrons per unit volume or electron density of the conductor.
- The relaxation time τ , the average time between two successive collisions of an electron.

118. Two cells of emfs e_1 and e_2 , and internal resistances r_1 and r_2 are connected in parallel between the points A and B . Deduce the expressions for

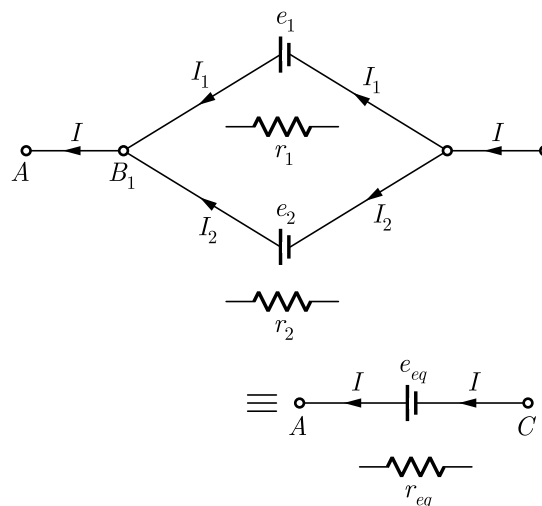
- the equivalent emf of the combination.
- the equivalent resistance of the combination.
- the potential difference between the points A and B .

Ans :

Delhi 2019, Comp 2005

As shown in figure, suppose two cells of emfs e_1 and e_2 and internal resistances r_1 and r_2 are connected in parallel between two points. Suppose the currents I_1 and I_2 from the positive terminals of the two cells flow towards the junction B_1 and current I flows out. Since as much charge flows in as flows out, we have

$$I = I_1 + I_2$$



A parallel Combination of two Cells is Equivalent to a Single Cell of emf e_{eq} and Internal Resistance r_{eq}

As the two cells are connected in parallel between the same two points B_1 and B_2 , the potential difference V across both cells must be same.

The potential difference between the terminals of first cell is,

$$V = V_{B_1} - V_{B_2} = e_1 - I_1 r_1$$

Hence, $I_1 = \frac{e_1 - V}{r_1}$

The potential difference between the terminals of e_2 is,

$$V = V_{B_1} - V_{B_2} = e_2 - I_2 r_2$$

Hence, $I_2 = \frac{e_2 - V}{r_2}$

Hence, $I = I_1 + I_2$

$$= \frac{e_1 - V}{r_1} + \frac{e_2 - V}{r_2}$$

in carrying a unit charge from B to A
against internal resistance r

$$e = V + V'$$

By Ohm's law,

$$V = IR$$

and

$$V' = Ir$$

$$e = IR + Ir = I(R + r)$$

Hence the current in the circuit is,

$$I = \frac{e}{R + r}$$

The terminal p.d. of the cell is given by,

$$V = IR = \frac{eR}{R + r}$$

Also,

$$V = e - V' = e - Ir$$

or terminal p.d. = emf - potential drop across the internal resistance.

Again, from the above equation, we get,

$$r = \frac{e - V}{I} = \frac{e - V}{V/R}$$

$$= \left(\frac{e - V}{V} \right)$$

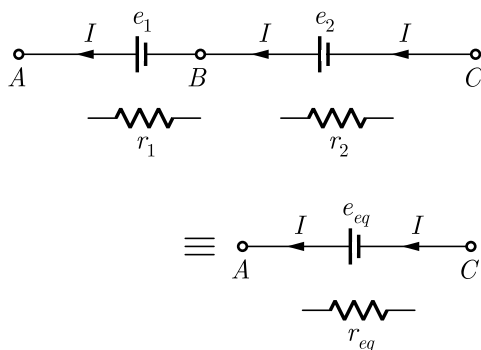
$$R = \left(\frac{e}{V} - 1 \right) R$$

- 116.** Two cells of different emfs and internal resistances are connected in series. Find expressions for the equivalent emf and equivalent internal resistance of the combination.

Ans :

OD 2018

As shown in figure, suppose two cells of emfs e_1 and e_2 and internal resistances r_1 and r_2 are connected in series between points A and C . Let I be the current flowing through the series combination.



A Series Combination of two Cells is Equivalent to a Single Cell of emf e_{eq} and internal resistance r_{eq}

Let V_A , V_B and V_C be the potential at points A , B and C respectively. The potential differences across the terminals of the two cells will be,

$$V_{AB} = V_A - V_B = e_1 - Ir_1$$

and

$$V_{BC} = V_B - V_C = e_2 - Ir_2$$

Thus the potential difference between the terminals A and C of the series combination is,

$$V_{AC} = V_A - V_C$$

$$= (V_A - V_B) + (V_B - V_C)$$

$$= (e_1 - Ir_1) + (e_2 - Ir_2)$$

$$V_{AC} = (e_1 + e_2) - I(r_1 + r_2)$$

If we wish to replace the series combination by a single cell of emf e_{eq} and internal resistance r_{eq} then,

$$V_{AC} = e_{eq} - Ir_{eq}$$

Comparing the last two equations, we get

$$e_{eq} = e_1 + e_2$$

and

$$r_{eq} = r_1 + r_2$$

For a series combination of n cells,

$$e_{eq} = e_1 + e_2 + e_3 + \dots + e_n$$

$$r_{eq} = r_1 + r_2 + r_3 + \dots + r_n$$

- 117.** 1. Derive an expression for the current density in terms of the drift speed of electrons.

or

Establish a relation between current and drift velocity.

2. Derive Ohm's law on the basis of the theory of electron drift.
3. Derive an expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time.

Ans :

OD 2017

1. **Relation between Electric Current and Drift Velocity :** Suppose a potential difference V is applied across a conductor of length l and of uniform cross-section A . The electric field E set up inside the conductor is given by

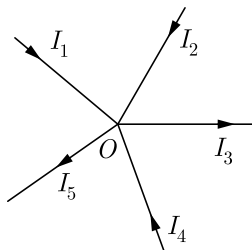
$$E = \frac{V}{l}$$

Under the influence of field $\vec{E}$, the free electrons begin to drift in the opposite direction $\vec{E}$ with an average drift velocity v_d .

Let the number of electrons per unit volume or electron density = n

Charge on an electron = e

Number of electrons in length l of the conductor = $n \times \text{volume of the conductor} = nAl$



Second Law (Kirchhoff's voltage rule)

This law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. It means that in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as loop rule.

Balanced Condition of Wheatstone Bridge

In figure, four resistances P , Q , R and S connected in the four arms of a parallelogram $ABCD$. Between B and D there is a sensitive galvanometer and cell is connected between A and C . K_1 and K_2 are two keys. By pressing the key K_1 , a current i is allowed to flow from the cell. At the point A , the current i is divided into two parts.

One part i_1 flows in the arm AB and the other part i_2 flows in the arm AD . The resistances P , Q , R and S are so adjusted that on pressing the key K_2 there is no deflection in the galvanometer G . That is, there is no current in the diagonal BD . Thus, the same current i_1 will flow in the arm BC as in the arm AB and the same i_2 will flow in the arm DC as in the arm AD .

Applying Kirchhoff's second law for the closed loop $BADB$, we have

$$\begin{aligned} -i_1 P + i_2 R &= 0 \\ P i_1 &= R i_2 \end{aligned} \quad \dots(1)$$

Similarly, for the closed loop $CBDC$, we have

$$\begin{aligned} i_1 Q + i_2 S &= 0 \\ Q i_1 &= S i_2 \end{aligned} \quad \dots(2)$$

Dividing Eq. (1) and by Eq. (2), we have

$$\begin{aligned} \frac{i_1 P}{i_1 Q} &= \frac{i_2 R}{i_2 S} \\ \text{or } \frac{P}{Q} &= \frac{R}{S} \end{aligned}$$

It is clear from this formula that if the ratio of the resistance P and Q and resistance R are known, then the unknown resistance S can be calculated. This is why, the arms AB and BC are called ratio arms, arm AD known arm and arm CD unknown arm.

121. 1. Obtain the formula for the power loss (i.e. power dissipated) in a conductor of resistance R carrying

a current.

2. Two heating elements of resistances R_1 and R_2 when operated at a constant supply of voltage V , consume powers P_1 and P_2 , respectively. Deduce the expressions for the power of their combination when they are in turn, connected in
- Series and
 - Parallel across their same voltage supply.

Ans :

OD 2017, SQP 2013

1. It is defined as the rate of electrical energy supplied per unit time to maintain flow of electric current through a conductor.

$$\text{Mathematically, } P = VI = I^2 R = \frac{V^2}{R}$$

Where, 1 watt = 1 volt $\times$ 1 ampere = 1 ampere-volt.

Power of an electric circuit is said to be one watt, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW).

Where 1 kW = 1000 W and MW = 10^6 W.

Commercial unit of electrical power is horse power (HP)

Where, 1 HP = 746 W.

2. To deduce the expression for the power of the combination, first find the equivalent resistance of the combination in the given conditions.

$$\text{Since, } P_1 = \frac{V^2}{R_1} \Rightarrow R_1 = \frac{V^2}{P_1}$$

$$\text{and } P_2 = \frac{V^2}{R_2} \Rightarrow R_2 = \frac{V^2}{P_2}$$

(a) In series combination,

$$R_s = R_1 + R_2 = \frac{V^2}{P_1} + \frac{V^2}{P_2}$$

$$R_s = V^2 \left(\frac{1}{P_1} + \frac{1}{P_2} \right) = V^2 \left(\frac{P_1 + P_2}{P_1 P_2} \right)$$

Now, let the power of heating element in series combination be P_s .

$$\begin{aligned} \text{Hence, } P_s &= \frac{V^2}{R_1 + R_2} \\ &= \frac{V^2}{V^2 \left(\frac{P_1 + P_2}{P_1 P_2} \right)} = \frac{P_1 P_2}{P_1 + P_2} \quad \dots(1) \end{aligned}$$

(b) In parallel combination

$$\begin{aligned} \frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{\frac{V^2}{P_1}} + \frac{1}{\frac{V^2}{P_2}} = \frac{P_1}{V^2} + \frac{P_2}{V^2} \end{aligned}$$

$$\frac{1}{R_p} = \frac{1}{V^2} (P_1 + P_2)$$

$$= \left(\frac{e_1}{r_1} + \frac{e_2}{r_2} \right) - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$V \left(\frac{r_1 + r_2}{r_1 r_2} \right) = \frac{e_1 r_2 + e_2 r_1}{r_1 r_2} - I$$

$$V = \frac{e_1 r_2 + e_2 r_1}{r_1 + r_2} - I \frac{r_1 r_2}{r_1 + r_2}$$

If we wish to replace the parallel combination by a single cell of emf e_{eq} and internal resistance r_{eq} , then

$$V = e_{eq} - I r_{eq}$$

Comparing the last two equations, we get

$$e_{eq} = \frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$$

and

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

We can express the above results in a simpler way as follows:

$$\frac{e_{eq}}{r_{eq}} = \frac{e_1}{r_1} + \frac{e_2}{r_2}$$

and

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For a parallel combination of n cells, we can write,

$$\frac{e_{eq}}{r_{eq}} = \frac{e_1}{r_1} + \frac{e_2}{r_2} + \dots + \frac{e_n}{r_n}$$

and

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}$$

- 119.** Obtain an expression for the heat developed in a resistor by the passage of an electric current through it. Hence state Joule's law of heating.

Ans :

Foreign 2016

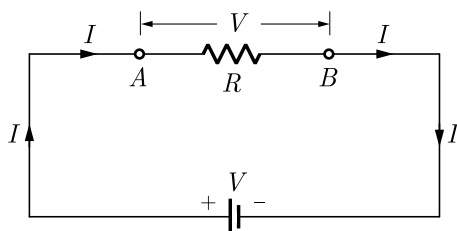
Consider a conductor AB of resistance R , shown in figure. A source of emf maintains a potential difference V between its ends A and B and sends a steady current I from A to B .

Clearly, $V_A > V_B$ and the potential difference across AB is,

$$V = V_A - V_B > 0$$

The amount of charge that flows from A to B in time t is,

$$q = It$$



Heat Produced in a Resistor

As the charge q moves through a decrease of potential of magnitude V , its potential energy decreases by the amount,

$$U = \text{Final P. E. at } B - \text{Initial P.E. at } A$$

$$= qV_B - qV_A = -q(V_A - V_B)$$

$$= -qV < 0$$

If the charges move through the conductor without suffering collisions, their kinetic energy would change. By conservation of energy, the change in kinetic energy must be,

$$K = -U = qV$$

$$= It \times V = VIt > 0$$

Thus, in case charges were moving freely through the conductor under the action of the electric field, their kinetic energy would increase as they move. But on the average, the electrons move with a steady drift velocity. This is because of the collisions of electrons with ions and atoms during the course of their motion. The kinetic energy gained by the electrons is shared with the metal ions. These ions vibrate more vigorously and the conductor gets heated up. The amount of energy dissipated as heat in conductor in time t is,

$$H = VIt \text{ joule}$$

$$= I^2 R t \text{ joule} = \frac{V^2 t}{R} \text{ joule}$$

$$= \frac{VIt}{1.18} \text{ cal} = \frac{I^2 R t}{4.18} \text{ cal}$$

$$= \frac{V^2 t}{4.18 R} \text{ cal}$$

The above equations are known as joule's law of heating. According to this law, the heat produced in a resistor is,

1. Directly proportional to the square of current for a given R .
2. Directly proportional to the resistance R for a given I .
3. Inversely proportional to the resistance R for a given, V .
4. Directly proportional to the time t for which the current flows through the resistor.

- 120.** Define Kirchhoff's laws of electrical circuit. Derive an expression for a balanced Wheatstone's bridge using Kirchhoff's law.

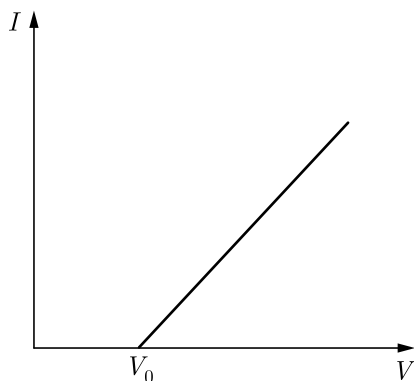
Ans :

Delhi 2019

Kirchoff's Laws of Electrical Circuit

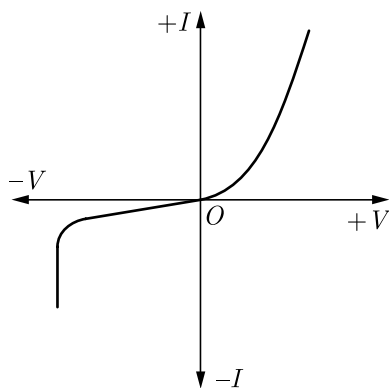
This law states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero. It is also known as junction rule.

2. The straight line $V-I$ graph does not pass through the origin. The $V-I$ graph for a water voltmeter is a straight line but not passing through the origin, as shown in Figure. So the electrolyte (water acidified with dil. H_2SO_4) is a non-ohmic conductor.



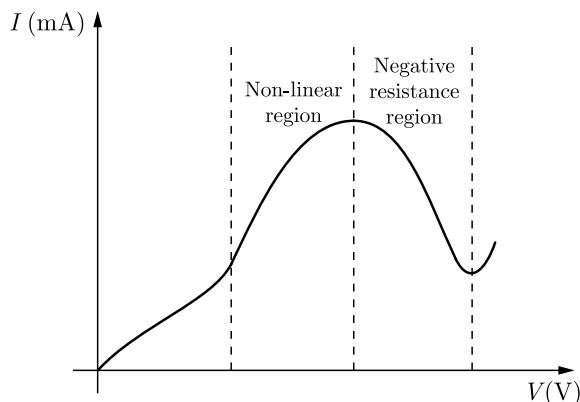
$V-I$ Graph for a Water Voltmeter

3. $V-I$ relationship depends on the sign of V for the same absolute value of V . I is the current for a certain V , then reversing the direction of V keeping its magnitude fixed, does not produce a current of same magnitude as I in the opposite direction, as shown in the $V-I$ graph for a junction diode in Figure.



$V-I$ Graph for a Junction Diode

4. $V-I$ relationship is non-unique. Figure shows the $V-I$ graph for the semiconductor GaAs. It exhibits non-linear behaviour i.e., there is more than one value of V for the same current I .



$V-I$ Graph for GaAs

124. (i) Define the term drift velocity.
 (ii) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend?
 (iii) Why alloys like constantan and manganin are used for making standard resistors?

Ans :

Delhi 2010, Comp 2007

- (i) The average velocity acquired by the free electrons of a conductor in a direction opposite to the externally applied electric field is called drift velocity.

Drift velocity, $v_d = \frac{-eE\tau}{m}$

where,

e = charge on electron

E = external electric field

τ = relaxation time

m = mass of electron

- (ii) The relation between current and drift velocity is,

$$I = -neAv_d \quad \dots(1)$$

Where, e is the charge on electron ($e = 1.6 \times 10^{-19} \text{ C}$)

$$v_d = \frac{-eE\tau}{m}$$

$$I = -neA\left(\frac{-e\tau}{m}E\right)$$

$$I = \frac{ne^2\tau}{m}AE \quad \dots(2)$$

Electric field at each point of wire,

$$E = \frac{V}{l} \quad \dots(3)$$

Now from (3),

$$I = \frac{ne^2\tau A}{m} \cdot \frac{V}{l}$$

$$\frac{V}{l} = \frac{m}{ne^2\tau} \cdot \frac{I}{A}$$

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$$1 \times 2 - I \times 3 = 3 - 5$$

$$2 - 3I = -2$$

$$3I = 4$$

$$I = \frac{4}{3} \text{ A}$$

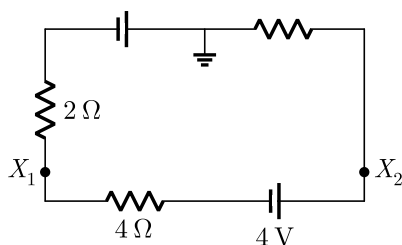
so, total current through R ,

$$I = 1 + \frac{4}{3} = \frac{7}{3} \text{ A}$$

Hence, Voltage drop across R ;

$$V = IR = \frac{7}{3} R \text{ Volt.}$$

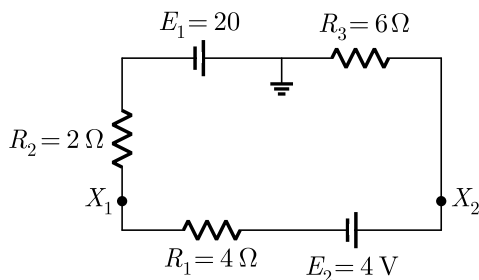
143. Determine the potentials at the points X_1 and X_2 in the circuit.



Ans :

SQP 2014, Comp 2011

The circuit diagram given in the question is,



Effective emf. of the given circuit,

$$E_{\text{eff}} = E_1 - E_2 = 20 - 4 = 16 \text{ Volt}$$

We know that,

Electric current in the circuit is given by,

$$I = \frac{E}{R_{\text{eq}}}$$

In the circuit, R_1 , R_2 and R_3 are connected in series combination,

$$\begin{aligned} \text{Hence, } R_{\text{eq}} &= R_1 + R_2 + R_3 \\ &= 6 + 2 + 4 = 12 \Omega \end{aligned}$$

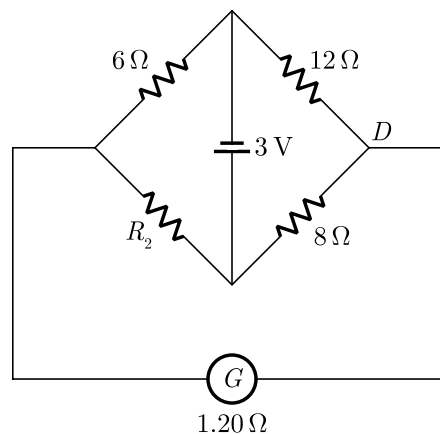
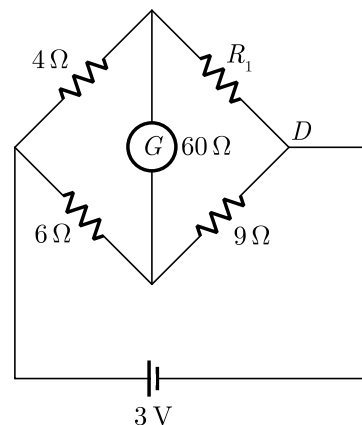
$$\text{Now, } I = \frac{16}{12} = \frac{4}{3} \text{ A}$$

Using the KVL,

$$\begin{aligned} \text{Potential at, } X_2 &= R_3 \times I \\ &= 6 \times \frac{4}{3} = 8 \text{ Volt} \end{aligned}$$

$$\begin{aligned} \text{Potential at, } X_1 &= -R_2 I - (-20) \\ &= -2 \times \frac{4}{3} + 20 \\ &= \frac{52}{3} = 17.3 \text{ Volt} \end{aligned}$$

144. Figure shows two circuits each having a galvanometer and a battery of 3 V. When the galvanometers in each arrangement do not show any deflection, obtain the ratio $\frac{R_1}{R_2}$.



Ans :

SQP 2009

For balanced Wheatstone bridge, if no current flows through the galvanometer.

$$\begin{aligned} \frac{4}{R_1} &= \frac{6}{9} \\ R_1 &= \frac{4 \times 9}{6} = 6 \Omega \end{aligned}$$

For another current,

$$\begin{aligned} \frac{6}{12} &= \frac{R_2}{8} \\ R_2 &= \frac{6 \times 8}{12} = 4 \Omega \end{aligned}$$

$$\text{Hence, } \frac{R_1}{R_2} = \frac{6}{4} = \frac{3}{2}$$

Substitute the value of R_2 in equation (1), we get

$$\begin{aligned} R_1 + \frac{48}{R_1} &= 16 \\ R_1^2 + 48 &= 16 R_1 \\ R_1^2 - 16 R_1 + 48 &= 0 \\ R_1^2 - 12 R_1 - 4 R_1 + 48 &= 0 \\ R_1(R_1 - 12) - 4(R_1 - 12) &= 0 \\ (R_1 - 4)(R_1 - 12) &= 0 \\ R_1 &= 4 \text{ or } 12 \Omega \end{aligned}$$

From equation (1),

$$R_2 = 12 \text{ or } 4 \Omega$$

- 129.** At room temperature, copper has free electron density of $8.4 \times 10^{28} \text{ m}^{-3}$. What will be drift velocity of electron in a copper conductor of cross-sectional area of 10^{-6} m^2 and carrying a current of 5.4 A ?

Ans :

Comp 2020

Electron density, $n = 8.4 \times 10^{28} \text{ m}^{-3}$

Cross-sectional area of conductor,

$$A = 10^{-6} \text{ m}^2$$

Current, $I = 5.4 \text{ A}$

Drift velocity of free electrons in a copper conductor,

$$V_d = \frac{I}{neA}$$

(where, e = Charge on an electron equal to $1.6 \times 10^{-19} \text{ C}$)

$$\begin{aligned} &= \frac{5.4}{(8.4 \times 10^{28}) \times (1.6 \times 10^{-19}) \times 10^{-6}} \\ &= \frac{5.4}{8.4 \times 1.6 \times 10^3} \\ &= 0.4 \times 10^{-3} \text{ m-s}^{-1} \\ &= 0.4 \text{ mm-s}^{-1} \end{aligned}$$

- 130.** A wire of resistance 16Ω is elongated to double its length. Find the new resistance.

Ans :

Delhi 2016

Resistance of wire, $R = 16 \Omega$

$$\text{Now } R = \frac{\rho l}{A} \quad \dots(1)$$

When wire is elongated to double its length. Its volume remains constant.

Volume of wire before elongation

$$= \text{Volume of wire after elongation}$$

$$Al = A' l'$$

Here, $l' = 2l$

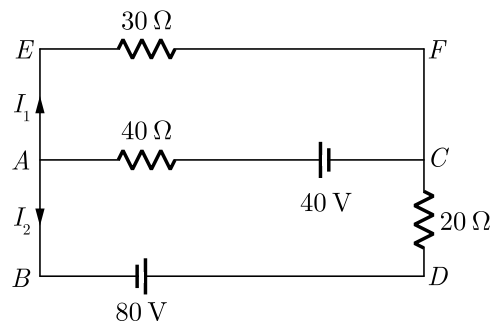
$$Al = A' (2l)$$

$$A' = \frac{A}{2}$$

Now,

$$\begin{aligned} R' &= \frac{\rho l'}{A'} = \frac{\rho (2l)}{\frac{A}{2}} = 4 \left(\frac{\rho l}{A} \right) \\ &= 4R \quad (\text{From Eq. 1}) \\ &= 4 \times 16 = 64 \Omega \end{aligned}$$

131.



- (a) Using Kirchhoff's rules, calculate the current in the arm AC of the given circuit.
 (b) On what principle does the meter bridge work? Why are the metal strips used in the bridge?

Ans :

Foreign 2013, Comp 2017

- (a) For the mesh $ERCAE$,

$$-30I_1 + 40 - 40(I_1 + I_2) = 0$$

$$\text{or } -7I_1 - 4I_2 = 4$$

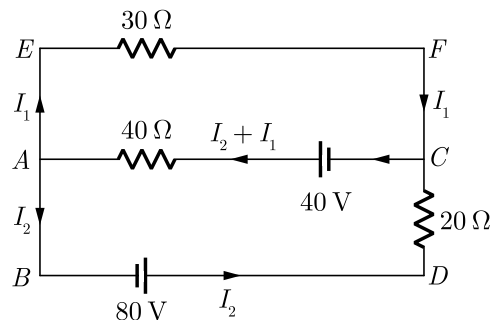
$$\text{or } 7I_1 + 4I_2 = 4 \quad \dots(1)$$

For the mesh $ACDBA$,

$$40(I_1 + I_2) - 40 + 20I_2 - 80 = 0$$

$$\text{or } 40I_1 + 60I_2 - 120 = 0$$

$$\text{or } 2I_1 + 3I_2 = 6 \quad \dots(2)$$



Solving eqs. (1) and (2), we get

$$I_1 = \frac{-12}{13} \text{ A}$$

$$I_2 = \frac{34}{13} \text{ A}$$

Current through arm,

$$AC = I_1 + I_2 = \frac{22}{13} \text{ A}$$

Now, power consumption in parallel combination.

$$P_p = \frac{V^2}{R_p} = V^2 \left(\frac{1}{R_p} \right)$$

$$P_p = V^2 \left[\frac{1}{V^2} (P_1 + P_2) \right]$$

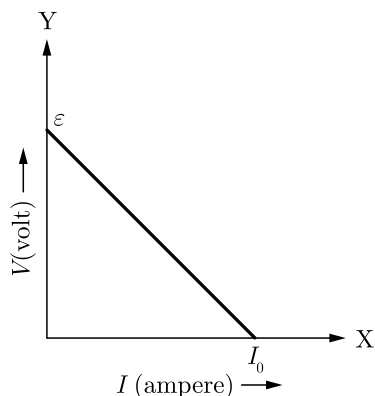
Hence, $P_p = P_1 + P_2$... (2)

122. (i) Plot a graph showing variation of voltage V_s the current drawn from the cell. How can one get information from this plot about the emf of the cell and its internal resistance?
- (ii) Two cells of emf is E_1 and E_2 internal resistance r_1 and r_2 are connected in parallel. Obtain the expression for the emf and internal resistance of a single equivalent cell that can replace this combination?

Ans :

SQP 2016, OD 2010

(i)

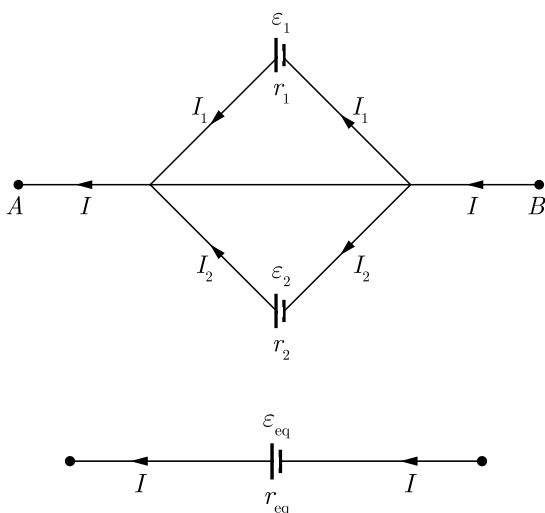


At, $I = 0, V = \varepsilon$

When, $V = 0, I = I_0, r = \frac{\varepsilon}{I_0}$

The intercept on y -axis gives the emf of the cell. The slope of graph gives the internal resistance.

(ii) $I = I_1 + I_2$... (1)



Let V = Potential difference between A and B .

Then, for cell ε_1 , $V = \varepsilon_1 - I_1 r_1$

$$I_1 = \frac{\varepsilon_1 - V}{r_1}$$

Similarly, for cell ε_2 ,

$$I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Substituting these values in equation (1)

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

or

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

Hence, V is given by,

$$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \right) - I \left(\frac{r_1 r_2}{r_1 + r_2} \right) \quad \dots (2)$$

Comparing the above equation with the equivalent circuit of emf ' ε_{eq} ' and internal resistance ' r_{eq} '

$$V = \varepsilon_{eq} - I r_{eq} \quad \dots (3)$$

(i) Therefore, $\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

(ii) $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

123. What are ohmic and non-ohmic conductors/resistors? State the conditions under which Ohm's law is not obeyed. Give one example of each type.

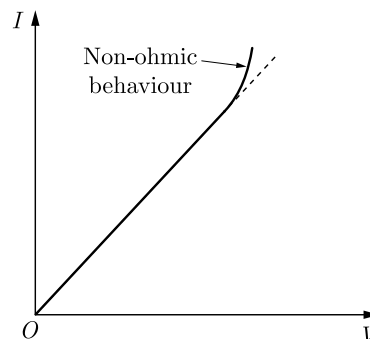
Ans :

Delhi 2019

Ohmic Conductors : The conductors which obey Ohm's law are called ohmic conductors. For these conductors, $V-I$ graph is a straight line passing through the origin. For example, a metallic conductor for small currents is an ohmic conductor, as shown in the $V-I$ graph of Figure.

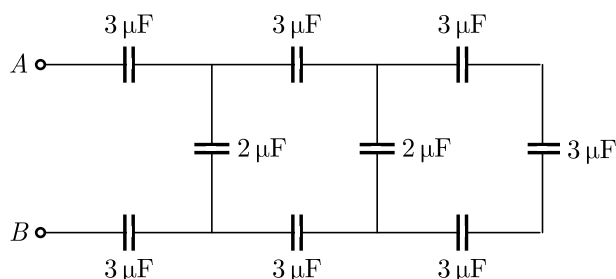
Non-ohmic Conductors : The conductors which do not obey Ohm's law are called non-ohmic conductors. The non-ohmic situations may be of the following types :

1. **$V-I$ relationship is non-linear.** When a large current flows through a metallic conductor, it gets heated up and its resistance increases. The $V-I$ graph becomes non-linear i.e., the conductor becomes non-ohmic at higher currents, as shown in Figure.



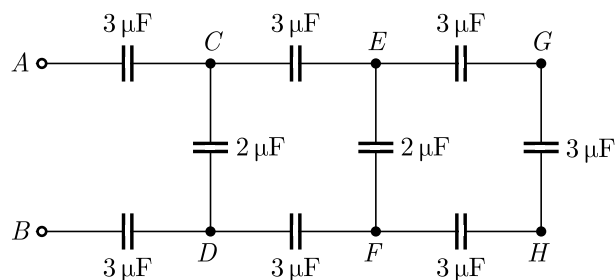
$V-I$ Graph for Metallic Conductor

136. Find equivalent capacity between A and B .



Ans :

Delhi 2012



In $EGHF$ three capacitor of $3\mu\text{F}$ are connected in series combination, Equivalent capacitance of this branch is given by,

$$\frac{1}{C'} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$C' = \frac{3}{3} = 1\mu\text{F}$$

C' and $2\mu\text{F}$ capacitor are connected in parallel combination.

Now, equivalent capacitance of $EGHF$,

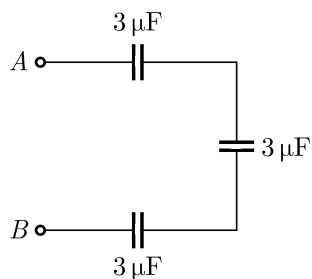
$$C_{EGHFE} = 2 + 1$$

$$= 3\mu\text{F}$$

Similarly equivalent capacitance of $CEFDC$,

$$C_{CEFDC} = 3\mu\text{F}$$

Now, circuit becomes $= 3\mu\text{F}$



$$\frac{1}{C_{AB}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$C_{AB} = 3/3 = 1\mu\text{F}$$

137. Use Kirchhoff's laws to determine the value of current I_1 in the given electrical circuit.

Ans :

OD 2020

From Kirchhoff's first law at junction C ,

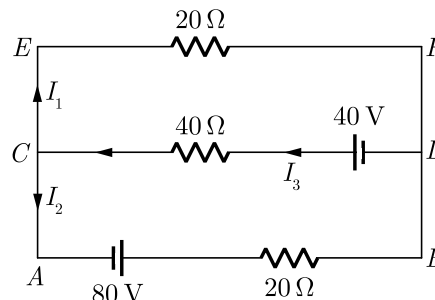
$$I_3 = I_1 + I_2 \quad \dots(1)$$

Applying Kirchhoff's second law in mesh $CDFEC$,

$$40I_3 - 40 + 20I_1 = 0$$

$$20(2I_3 + I_1) = 40$$

$$I_1 + 2I_3 = 2 \quad \dots(2)$$



Applying Kirchhoff's second law to mesh $ABFEA$,

$$80 - 20I_2 + 20I_1 = 0$$

$$20(I_1 - I_2) = -80$$

$$I_2 - I_1 = 4 \quad \dots(3)$$

Substituting value of I_3 from eq. (1) in eq. (2), we get

$$I_1 + 2(I_1 + I_2) = 2$$

$$3I_1 + 2I_2 = 2 \quad \dots(4)$$

Multiplying equation (3) by 2, we get

$$2I_2 - 2I_1 = 8 \quad \dots(5)$$

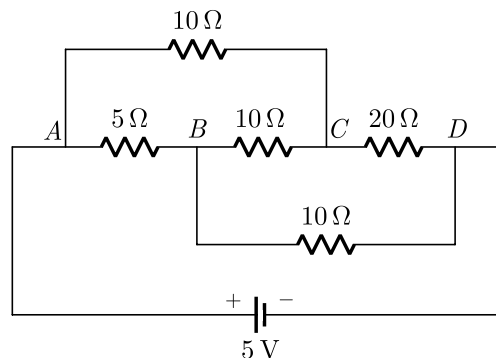
Subtracting eq. (4) from eq. (5), we get

$$5I_1 = -6$$

$$I_1 = -\frac{6}{5} \text{ A} = -1.2 \text{ A}$$

Note: Negative sign shows that current is in opposite direction.

138. Calculate the value of the current drawn from a 5 V battery in the circuit as shown:



$$R = \frac{m}{ne^2\tau} \cdot \frac{l}{A} \quad \dots(4)$$

We know,

$$R = \frac{\rho l}{A}$$

$$\rho = \frac{m}{ne^2\tau}$$

So, we can say that resistivity of a conductor is inversely proportional to number density of electrons and relaxation time.

- (iii) This is because constantan and manganin show very weak dependence of resistivity on temperature.

NUMERICAL QUESTIONS

- 125.** A current is passed through two coils connected in series. The potential difference across the first coil is 3 V and that of the second coil is 4.5 V. If the first coil has a resistance of $2\ \Omega$, what is the resistance of second coil?

Ans :

OD 2020

Potential difference across first coil,

$$V_1 = 3\text{ Volt}$$

Potential difference across second coil,

$$V_2 = 4.5\text{ Volt}$$

And resistance of first coil,

$$R_1 = 2\ \Omega$$

Current across the coils in series,

$$I = \frac{V_1}{R_1} = \frac{3}{2} = 1.5\text{ A}$$

Therefore resistance of second coil,

$$R_2 = \frac{V_2}{I} = \frac{4.5}{1.5} = 3\ \Omega$$

- 126.** When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electron is 2.5×10^{-4} m/s. If the electron density in the wire is $8 \times 10^{28}\text{ m}^{-3}$, calculate the resistivity of the material of wire.

Ans :

Foreign 2015

We know,

$$I = neAv_d$$

$$I = \frac{V}{R}$$

and

$$R = \rho \frac{l}{A}$$

So,

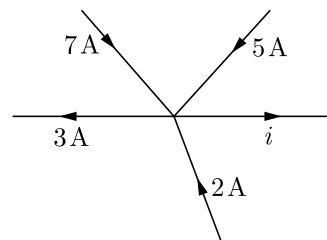
$$\frac{V}{R} = neAv_d$$

$$\frac{V}{nev_d l} = \frac{RA}{l}$$

$$\rho = \frac{V}{nev_d l}$$

$$\begin{aligned} \rho &= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1} \Omega\text{-m} \\ &= 1.56 \times 10^{-5} \Omega\text{-m} \\ &\approx 1.6 \times 10^{-5} \Omega\text{-m} \end{aligned}$$

- 127.** What is the value of i in the given circuit?



Ans :

OD 2017, Comp 2006

Apply the KCL at junction A, we get

$$\Sigma i = 0$$

$$7 + 5 + 2 - 3 - i = 0$$

$$14 - 3 - i = 0$$

$$i = 11\text{ A}$$

- 128.** When two resistors are connected in series and parallel, then their equivalent resistances are $16\ \Omega$ and $3\ \Omega$ respectively. Find out the resistance of each resistor.

Ans :

Comp 2019

Equivalent resistance in series,

$$R_s = 16\ \Omega$$

Equivalent resistance in parallel,

$$R_p = 3\ \Omega$$

Let two resistors are R_1 and R_2 respectively. According to the question, when resistor are connected in series combination then,

Equivalent resistance,

$$R_s = R_1 + R_2$$

$$16 = R_1 + R_2$$

$$R_1 + R_2 = 16$$

...(1)

Similarly in parallel combination,

$$R_p = \frac{R_1 \times R_2}{R_1 + R_2}$$

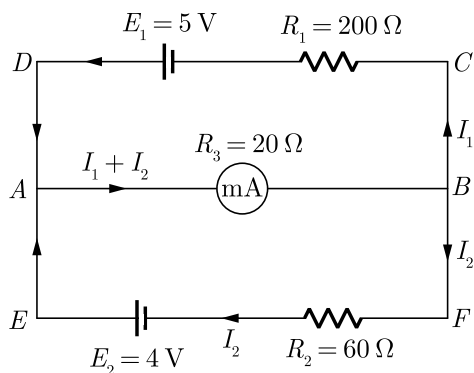
From equation (1),

$$3 = \frac{R_1 R_2}{16}$$

$$R_1 R_2 = 48$$

$$R_2 = \frac{48}{R_1}$$

...(2)



Using Kirchhoff's voltage law in closed loop $DABCD$

$$(I_1 + I_2)R_3 + I_1R_1 - E_1 = 0 \quad \dots(1)$$

In closed a loop $ABFEA$

$$(I_1 + I_2)R_3 + I_2R_2 - E_2 = 0 \quad \dots(2)$$

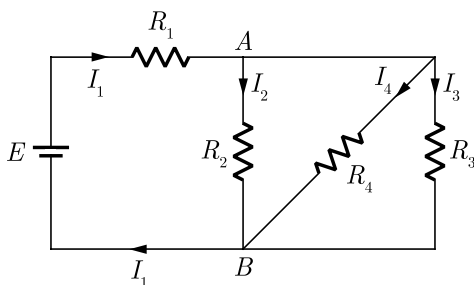
Multiplying eq. (1) by $(R_2 + R_3)$ and eq. (2) by R_3 and subtracting, we get

$$\begin{aligned} I_1 &= \frac{E_1R_2 + E_1R_3 - E_2R_3}{R_1R_2 + R_2R_3 + R_1R_3} \\ &= \frac{5 \times 60 + 5 \times 20 - 4 \times 20}{200 \times 60 + 60 \times 20 + 200 \times 20} \\ &= \frac{300 + 100 - 80}{12000 + 1200 + 4000} \\ &= \frac{320}{17200} \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } I_2 &= \frac{E_2R_1 + E_2R_3 - E_1R_3}{R_1R_2 + R_2R_3 + R_1R_3} \\ &= \frac{4 \times 200 + 4 \times 20 - 5 \times 20}{200 \times 60 + 60 \times 20 + 200 \times 20} \\ &= \frac{800 + 80 - 100}{17200} = \frac{780}{17200} \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Hence, total current in mA} &= I_1 + I_2 \\ &= \frac{320}{17200} + \frac{780}{17200} = \frac{1100}{17200} \\ &= \frac{11}{172} \text{ A} = 64 \text{ mA} \end{aligned}$$

- 141.** In the circuit shown, $R_1 = 4 \Omega$, $R_2 = R_3 = 5 \Omega$, $R_4 = 10 \Omega$ and $E = 6 \text{ V}$. Work out the equivalent resistance of the circuit and the current in each resistor.



Ans :

Here, R_2 , R_3 and R_4 are connected in parallel.

Delhi 2019

Their effective resistance R_p will be given by,

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_p} = \frac{1}{5} + \frac{1}{5} + \frac{1}{10}$$

or

$$\frac{1}{R_p} = \frac{2 + 2 + 1}{10}$$

$$= \frac{5}{10} = \frac{1}{2}$$

$$R_p = 2 \Omega$$

Total resistance of circuit,

$$R = R_1 + R_p = 4 + 2 = 6 \Omega$$

Current,

$$I = \frac{6}{6} = 1 \text{ A}$$

Potential drop across,

$$R_1 = I_1R_1 = 1 \times 4 = 4 \text{ Volt}$$

Potential drop across all other resistances,

$$= 6 - 4 = 2 \text{ Volt}$$

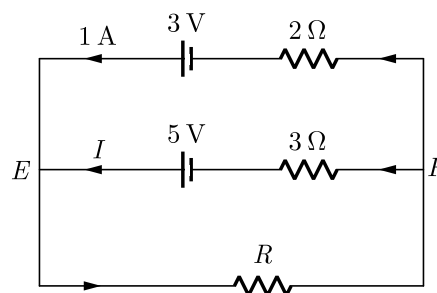
Current through R_2 and R_3 ,

$$I_2 = I_3 = \frac{2}{5} \text{ A}$$

Current through R_4 ,

$$I_4 = \frac{2}{10} = \frac{1}{5} \text{ A}$$

- 142.** Using Kirchhoff's rule in the given circuit, determine (i) the voltage drop across the unknown resistor R and (ii) the current I in the arm EF .

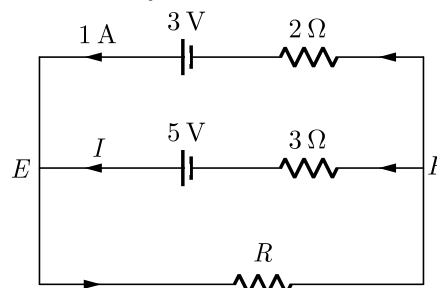


Ans :

OD 2014

Taking upper portion as mesh using Kirchhoff's second law

$$\sum IR = \sum E$$



Ans :

OD 2013

- (i) The effective resistance between B and E . as BC and CD are in series and BD , BCD and BED are in parallel.

$$\begin{aligned}\text{So, } \frac{1}{R_{BE}} &= \frac{1}{15} + \frac{1}{10} + \frac{1}{30} \\ &= \frac{2+3+1}{30}\end{aligned}$$

$$\text{or } R_{BE} = 5 \Omega$$

- (ii) Using Kirchhoff's law in loop.

$$0.2 \times 5 + 0.2R + 15 \times 0.2 = 8 - 3$$

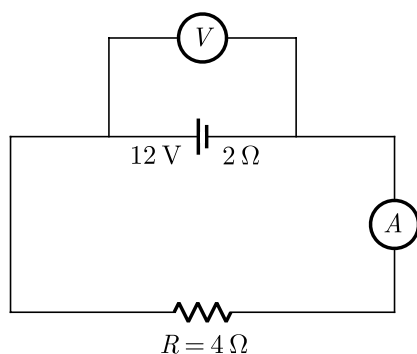
$$\text{or } 1 + 0.2R + 3 = 5$$

$$\text{or } 0.2R = 1$$

$$\text{or } R = 5 \Omega$$

$$\text{(iii) } V_{BE} = 5(0.2) = 1 \text{ Volt}$$

152. (a) The potential difference applied across a given resistor is altered so that heat produced per second increase by a factor of 9. By what factor does the applied potential difference change?
- (b) In the figure shown, an ammeter A and a resistor of 4Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2Ω . Calculate the voltmeter and ammeter readings.



Ans :

OD 2017, Comp 2014

- (a) Heat increase 9 factor, so current increased by 3 factor, so potential increased by 3 factor.

$$\text{also, } H = \frac{V^2}{R} t$$

$$H \propto V^2$$

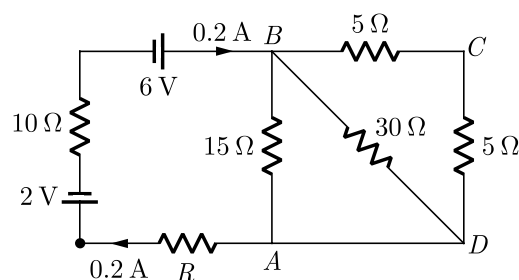
H increased by factor, so V increased by factor 3.

$$\text{(b) } I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A}$$

$$V = E - ir = 12 - 2 \times 2$$

$$= 12 - 4 = 8 \text{ Volt.}$$

153. Calculate the value of the resistance R in the circuit shown in the figure so that the current in the circuit is 0.2 A . What would be the potential difference between points A and B ?



Ans :

OD 2010

For BCD , equivalent resistance

$$R_1 = 5 + 5 = 10 \Omega$$

Across BA , equivalent resistance R_2

$$\begin{aligned}\frac{1}{R_2} &= \frac{1}{10} + \frac{1}{30} + \frac{1}{15} \\ &= \frac{3+1+2}{30} = \frac{6}{30} = \frac{1}{5}\end{aligned}$$

$$R_2 = 5 \Omega$$

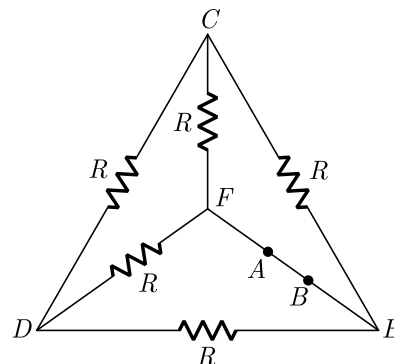
Potential difference

$$V_{BA} = I \times R_2 = 0.2 \times 5$$

$$V_{BA} = 1 \text{ Volt}$$

$$V_{AB} = -1 \text{ Volt}$$

154. (i) Calculate the equivalent resistance of the given electrical network between points A and B .
- (ii) Also find the current through CD and ACE , if a 10 V DC source is connected between A and B and the value of R is assumed as 2Ω .

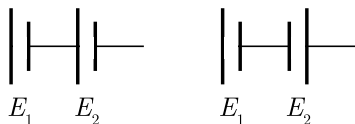


Ans :

Delhi 2014

- (i) We redraw the circuit as

145. Why is a potentiometer preferred over a voltmeter by determining the emf of a cell? Two cells of emf E and E_2 are connected together in two ways shown below :



The balance points in a given potentiometer experiment to these two combinations of cells are found to be at 351.0 cm and 70.2 cm respectively. Calculate the ratio of the emfs of the two cells.

Ans :

Delhi 2011, OD 2019

A potentiometer does not draw any current from the cell whose emf is to be determined, whereas a voltmeter always drawn some current. Therefore, emf measured by voltmeter is slightly less than actual value of emf of the cell.

From the given data,

For $\text{---} \text{---} \text{---}$ type of combination of cells
 $E_1 \ E_2$

$$E_1 + E_2 = K \cdot I_1 \quad \dots(1)$$

and for $\text{---} \text{---} \text{---}$ type of combination of cells
 $E_1 \ E_2$

$$E_1 + E_2 = K \cdot I_2 \quad \dots(2)$$

Here, $I_1 = 351 \text{ cm}$, $I_2 = 70.2 \text{ cm}$

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{I_1}{I_2}$$

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{351}{70.2}$$

$$\frac{E_1 + E_2}{E_1 - E_2} = 5$$

$$E_1 + E_2 = 5E_1 - 5E_2$$

$$4E_1 = 6E_2$$

$$\frac{E_1}{E_2} = \frac{3}{2}$$

146. Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell.

Ans :

Foreign 2016

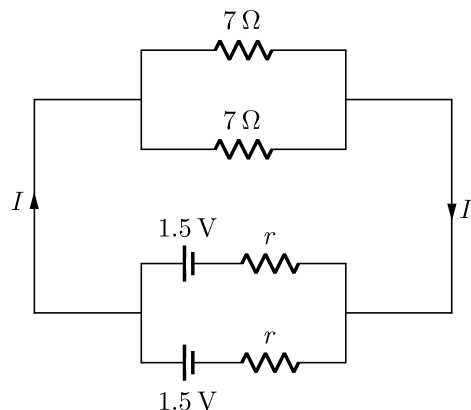
Given, $E = 1.5 \text{ Volt}$,

$V = 1.4 \text{ Volt}$.

Resistance of external circuit = Equivalent resistance of two resistances of 7Ω connected in parallel

or

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{7 \times 7}{7 + 7} = 3.5 \Omega$$



Let r' be the total internal resistance of the two cells, then

$$r' = \left(\frac{E - V}{V} \right) R$$

$$= \left(\frac{1.5 - 1.4}{1.4} \right) 3.5 = 0.25 \Omega$$

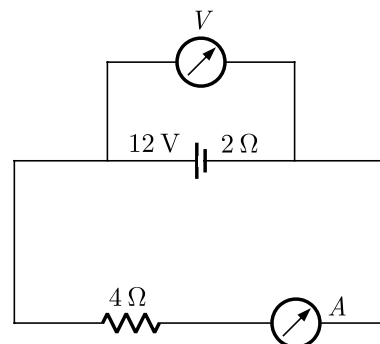
As the two cells of internal resistance r each have been connected in parallel, so

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r}$$

$$\frac{1}{0.25} = \frac{2}{r}$$

$$r = 0.25 \times 2 = 0.5 \Omega$$

147. A battery of emf 12 V and internal resistance 2Ω is connected to a 4Ω resistor as shown in the figure.



- Show that a voltmeter when placed across the cell and across the resistor in turn gives the same reading.
- To record the voltage and the current in the circuit why is voltmeter placed in parallel and ammeter in series in the circuit?

Ans :

Foreign 2016

- Case I :** When voltmeter is placed across the cell effective resistance of the circuit.

$$R_{eq} = (2 + 4) \Omega = 6 \Omega$$

Ans :

Foreign 2013

The equivalent Wheatstone bridge for the given combination is shown in figure.

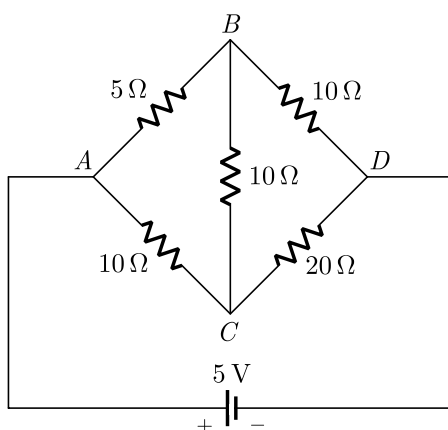
The resistance of arm ACD ,

$$R_{S_1} = 10 + 20 = 30 \Omega$$

Also, the resistance of arm ABD ,

$$R_{S_2} = 5 + 10 = 15 \Omega$$

Since, the condition $\frac{P}{Q} = \frac{R}{S}$ is satisfied, it is a balanced bridge.



No current flows along arm BC .

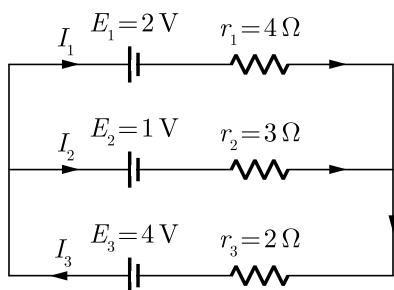
Equivalent resistance,

$$\begin{aligned} R_{eq.} &= \frac{R_{S_1} \times R_{S_2}}{R_{S_1} + R_{S_2}} = \frac{30 \times 15}{30 + 15} \\ &= \frac{30 \times 15}{45} = 10 \Omega \end{aligned}$$

Current draw from the source,

$$I = \frac{V}{R_{eq.}} = \frac{5}{10} = \frac{1}{2} \text{ A} = 0.5 \text{ A}$$

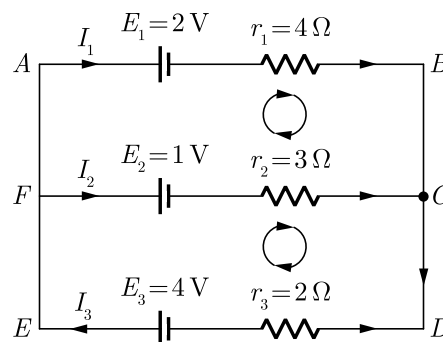
139. Using Kirchhoff's Law, write the expression for the currents I_1 , I_2 and I_3 in the circuit diagram shown.



Ans :

Comp 2015, OD 2003

For given circuit,



At F , applying junction rule,

$$I_3 = I_1 + I_2 \quad \dots(1)$$

In mesh $ABCFA$,

$$+2 + 4I_1 - 3I_2 - 1 = 0$$

$$4I_1 - 3I_2 = 1 \quad \dots(2)$$

In mesh $FCDEF$,

$$+1 + 3I_2 + 2I_3 - 4 = 0$$

$$3I_2 + 2I_3 = 3 \quad \dots(3)$$

On solving Eq. (1), (2), (3), we get

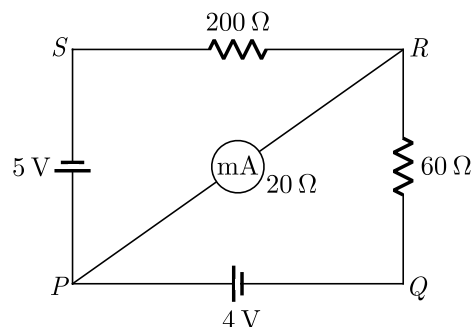
$$I_1 = \frac{2}{13} \text{ A}$$

$$I_2 = \frac{7}{13} \text{ A}$$

$$I_3 = \frac{9}{13} \text{ A}$$

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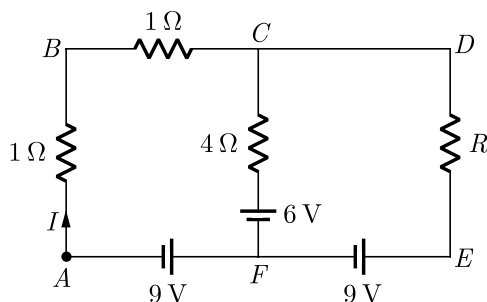
140. The network $PQRS$ shown in the circuit diagram, has the batteries of 4 V and 5 V and negligible internal resistance. A milliammeter of 20Ω resistance is connected between P and R . Calculate the reading in the milliammeter.



Ans :

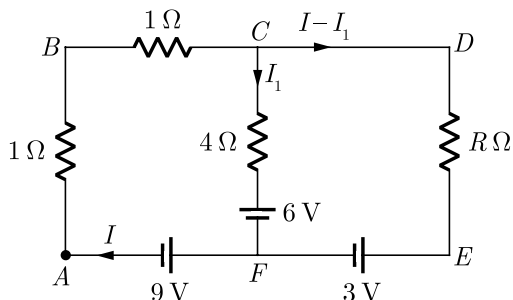
Delhi 2017, OD 2012

Let us redraw circuit as shown



Ans :

SQP 2008



Apply Kirchhoff's law in loop ABCFA :

$$I + I + 4I_1 = 9 - 6$$

$$2I + 4I_1 = 3 \quad \dots(1)$$

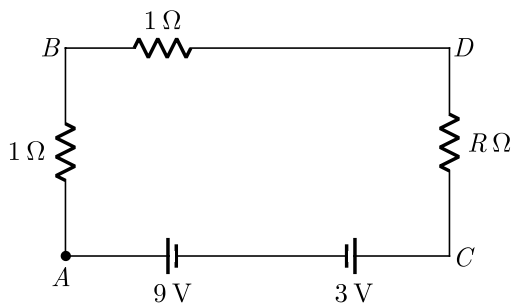
As there is no current flowing through the 4Ω resistance,

$$I_1 = 0$$

$$2I = 3$$

$$I = 1.5 \text{ A}$$

Thus, the current through resistance R is 1.5 A. As there is no current through branch CF , thus equivalent circuit will be



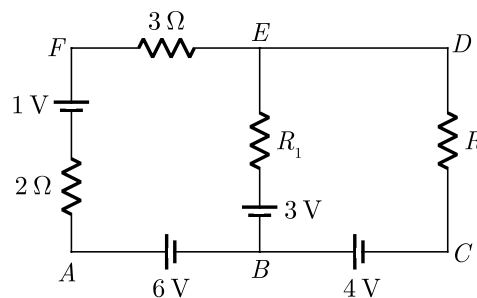
By applying Kirchhoff's loop law, we get

$$1.5 + 1.5 + R(1.5) - 9 = 3$$

$$R = 2\Omega$$

Potential difference between A and D = $9 - 3 = 6$ Volt.

150. Use Kirchhoff's rules to determine the potential difference between the points A and D when no current flows in the arm BE of the electric network shown in the figure.

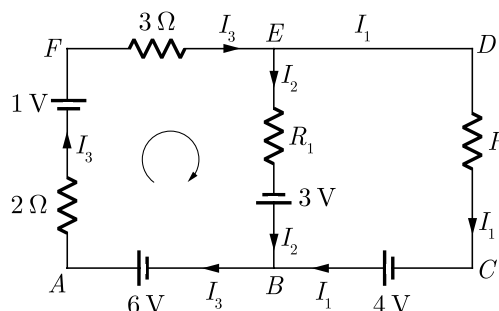


Ans :

OD 2015, Comp 2008

According to Kirchhoff's junction rule at E or B

$$I_3 = I_2 + I_1$$



Since, $I_2 = 0$ in the arm BE as given in the question.

$$I_3 = I_1$$

Using loop rule in loop AFEBA

$$+6V - 2I_3 + 1V - 3I_3 - I_2R_1 + 3V = 0$$

$$2I_3 + 3I_3 + I_2R_1 = 10 \text{ Volt}$$

Since,

$$I_2 = 0,$$

so

$$5I_3 = 10 \text{ Volt}$$

$$I_3 = 2 \text{ A}$$

The potential difference between A and D, along the branch AFCD of the closed circuit

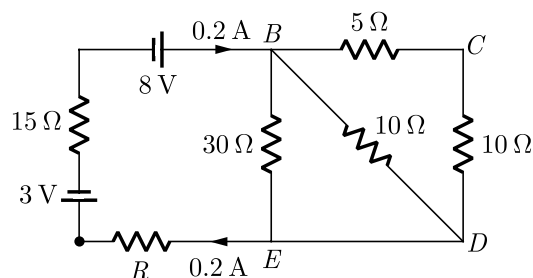
$$V_A - 2I_3 + 1V - 3I_3 - V_D = 0$$

$$V_A - V_D = 2I_3 - 1V + I_3$$

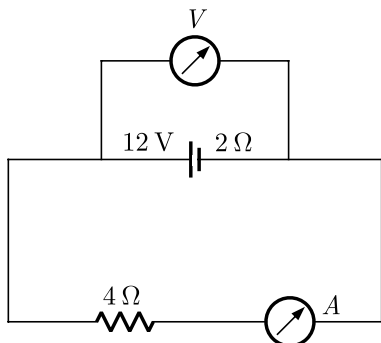
$$= 2 \times 2 - 1 + 3 \times 2$$

$$= 9 \text{ Volt}$$

151. Calculate the value of the resistance R in the circuit shown in the figure so that the current in the circuit is 0.2 A. What would be the potential difference between points B and E?



$$I = \frac{12}{6} \text{ A} = 2 \text{ A}$$



Terminal potential differences across the cell

$$V = E - Ir$$

$$V = 12 - 2(2) = 8 \text{ Volt}$$

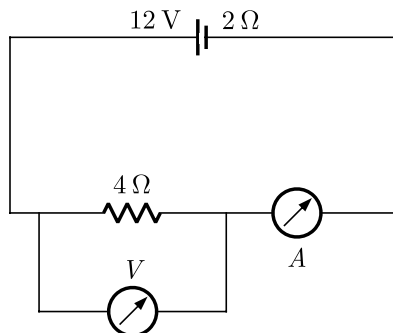
Case II : When voltmeter is placed parallel across the resistor

Voltage across resistance

$$= \text{current through resistance} \times \text{resistance}$$

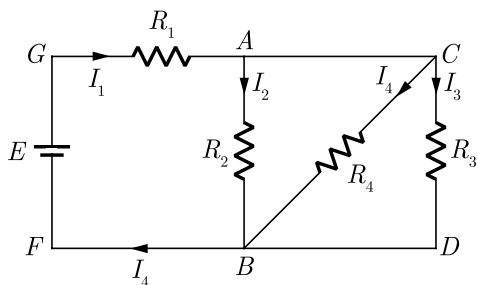
$$= 2 \times 4 = 8 \text{ Volt}$$

Hence, the voltmeter gives the same reading in the two cases.



(ii) In series, current in the circuit remains same. In parallel, potential in the circuit remains same.

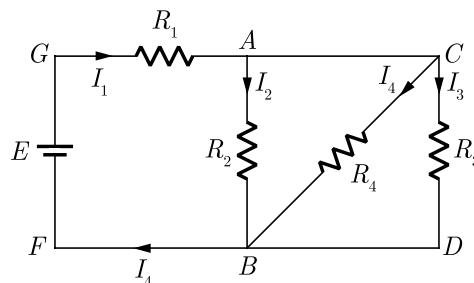
- 148.** In the circuit shown, $R_1 = 4 \Omega$, $R_2 = R_3 = 15 \Omega$, $R_4 = 30 \Omega$ and $E = 10 \text{ V}$. Calculate the equivalent resistance of the circuit and the current in each resistor.



Ans :

Delhi 2017, SQP 2013

Given network is as follows :



(i) As R_3 and R_4 are in parallel, so

$$\frac{1}{R_{34}} = \frac{1}{30} + \frac{1}{15} = \frac{1+2}{30} = \frac{3}{30} \Omega$$

or

$$R_{34} = 10 \Omega$$

Now R_2 and R_{34} are in parallel, so

$$\frac{1}{R_{234}} = \frac{1}{15} + \frac{1}{10} = \frac{2+3}{30} = \frac{5}{30}$$

$$R_{234} = 6 \Omega$$

Finally, R_1 and R_{234} are in series, so

$$R_{eq} = R_1 + R_{234} = 4 + 6 = 10 \Omega$$

(ii) Hence, current in circuit,

$$I_1 = \frac{E}{R_{eq}} = \frac{10\text{V}}{10\Omega} = 1 \text{ A}$$

For closed loop ACB , applying KVL,

$$15I_2 - 30I_4 = 0$$

$$I_2 = 2I_4 \quad \dots(1)$$

For closed loop $GABF$, applying KVL,

$$4I_1 + 15I_2 = 10$$

$$4 \times 1 + 15I_2 = 10$$

$$I_2 = \frac{6}{15} = 0.4 \text{ A} \quad \dots(2)$$

Using (i) and (ii), $I_4 = 0.2 \text{ A}$

Now at point A , applying KCL,

$$I_1 = I_2 + I_3 + I_4$$

$$I_3 = I_1 - I_2 - I_4$$

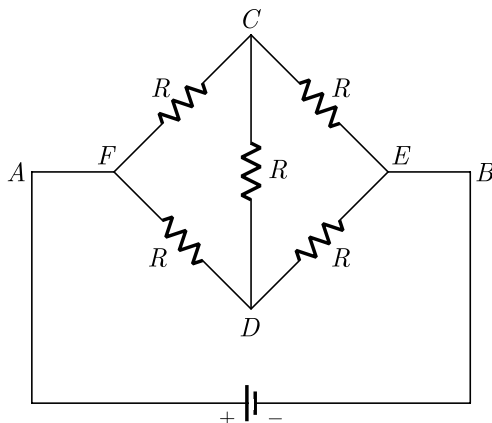
$$= 1 - 0.4 - 0.2 = 0.4 \text{ A}$$

so,

$$I_1 = 1 \text{ A}, I_2 = 0.4 \text{ A},$$

$$I_3 = 0.4 \text{ A}, I_4 = 0.2 \text{ A}$$

- 149.** Using Kirchhoff's rules determine the value of unknown resistance R in the circuit so that no current flows through 4Ω resistance. Also find the potential difference between A and D .



The network is a balanced Wheatstone's bridge and hence, resistance of branch CD is neglected.

Hence, Resistance of branch $RCE = R + R = 2R$ and Resistance of branch $FDE = R + R = 2R$.

Hence, Equivalent resistance between A and B

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$

(ii) If $R = 2\Omega$ then $R_{eq} = 2\Omega$

Hence, No current flows through CD in balanced condition of bridge.

$$\begin{aligned} \text{Current in arm, } ARCEB &= \frac{V}{2R} = \frac{10}{2 \times 2} \\ &= 2.5 \text{ A.} \end{aligned}$$

- 155.** A current flows in a wire of resistance 5Ω having potential difference 7 volt for 20 minutes. How much heat produced in the wire?

Ans :

Delhi 2020

Resistance of wire,

$$R = 5\Omega$$

Potential difference across the wire,

$$V = 7 \text{ volts}$$

Time, $t = 20 \text{ min} = 20 \times 60 \text{ sec}$

Heat produced in wire,

$$H = I^2 R t$$

Since, $V = IR$

Hence, $I = \frac{V}{R}$

$$\begin{aligned} &= \left(\frac{V}{R}\right)^2 R t = \frac{V^2}{R} \times t \\ &= \frac{(7)^2 \times 20 \times 60}{5} \\ &= \frac{7 \times 7 \times 20 \times 60}{5} \\ &= \frac{58800}{5} = 11760 \text{ J} \\ &= \frac{11760}{4.2} \text{ Cal} \quad (1\text{Cal} = 4.2 \text{ J}) \end{aligned}$$

$$= 2800 \text{ Cal}$$

- 156.** A cell of e.m.f. 1.5 V is connected with an ammeter of resistance 0.05Ω . If the current in the circuit is 2.0 A, what is the internal resistance of the cell?

Ans :

Delhi 2019, SQP 2006

E.m.f. of cell, $E = 1.5 \text{ Volt}$

Resistance of ammeter,

$$R = 0.05\Omega$$

Current in circuit, $I = 2.0 \text{ A}$

$$\text{Now } I = \frac{E}{R + r}$$

where, r = internal resistance of cell

$$\text{Hence, } I = \frac{E}{R + r}$$

$$2.0 = \frac{1.5}{0.05 + r}$$

$$2.0(0.05 + r) = 1.5$$

$$0.1 + 2r = 1.5$$

$$2r = 1.4$$

$$r = 0.7\Omega$$

- 157.** A wire 50 cm long and 1 mm^2 in cross-section carries a current of 4 A when connected to a 2 V battery. What is the specific resistivity of the wire?

Ans :

SQP 2013

Length of wire, $l = 50 \text{ cm} = 0.5 \text{ m}$

Cross-sectional area of wire

$$A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$$

Current in wire, $I = 4 \text{ A}$

Voltage of battery, $V = 2 \text{ Volt}$

Resistance of the wire

$$R = \frac{V}{I} = \frac{2}{4} = 0.5\Omega$$

Resistance of the wire,

$$R = \rho \times \frac{l}{A}$$

or,

$$\begin{aligned} \rho &= \frac{R \times A}{l} \\ &= \frac{0.5 \times 10^{-6}}{0.5} = 1 \times 10^{-6} \Omega\text{-m} \end{aligned}$$

where, ρ = Specific resistivity of the wire

- 158.** A uniform copper wire having mass of $2.4 \times 10^{-3} \text{ kg}$ carries a current of 1 A and has a potential difference of 1.8 V across its ends. If density and resistivity of copper are $9.6 \times 10^3 \text{ kg-m}^{-3}$ and $1.8 \times 10^{-8} \Omega\text{-m}$ respectively, What is the length of copper wire?

Now the resistances R_2 and 3Ω (between C and B) are in series combination. Therefore their equivalent resistance,

$$R_3 = R_2 + 3 = 3 + 3 = 6\Omega$$

Again the resistance R_3 and 3Ω (between A and B) are in parallel combination. Therefore equivalent resistance between A and B ,

$$R_{AB} = \frac{R_3 \times 3}{R_3 + 3} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

- 167.** Two copper wires are of the same length. One wire is twice as thick as the other. What is the resistances of the two wires are in the ratio ?

Ans :

OD 2010

Length of each wire $= l$

Thickness of first wire, $r_1 = r$

Thickness of second wire, $r_2 = 2r$

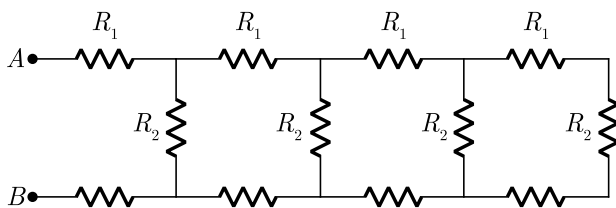
Resistance of the wire,

$$\begin{aligned} R &= \rho \times \frac{l}{A} \\ &= \rho \times \frac{l}{\pi r^2} \propto \frac{1}{r^2} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{R_1}{R_2} &= \left(\frac{r_2}{r_1}\right)^2 \\ &= \left(\frac{2r}{r}\right)^2 = \frac{4}{1} \end{aligned}$$

- 168.** A circuit with an infinite number of resistance is shown below. What is resultant resistance between A and B , when $R_1 = 1\Omega$ and $R_2 = 2\Omega$?



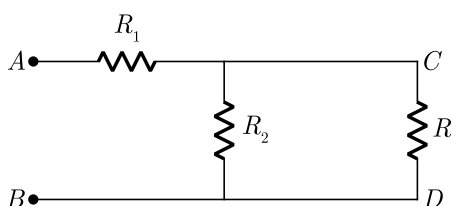
Ans :

Delhi 2013, SQP 2009

Resistances, $R_1 = 1\Omega$

and $R_2 = 2\Omega$

Since, the given circuit has infinite number of resistances, therefore the given circuit can be reduced as shown in the figure



For an infinite number of resistances, if the resistance between A and B is R , then resistance between C and D will also be equal to R .

Thus the resistances R and R_2 are in parallel combination.

Therefore their equivalent resistance,

$$R_{eq} = \frac{R \times R_2}{R + R_2} = \frac{R \times 2}{R + 2} = \frac{2R}{R + 2}$$

Now resistances R_{eq} and R_1 are in series combination.

Therefore their equivalent resistance,

$$\begin{aligned} R &= R_1 + R_{eq} \\ &= 1 + \frac{2R}{R + 2} \\ &= \frac{R + 2 + 2R}{R + 2} = \frac{3R + 2}{R + 2} \end{aligned}$$

$$R(R + 2) = 3R + 2$$

$$R^2 + 2R = 3R + 2$$

$$R^2 - R - 2 = 0$$

$$(R - 2)(R + 1) = 0$$

$$R = 2\Omega \text{ and } = -1\Omega$$

Hence,

$$R = 2\Omega$$

(Neglecting negative value)

- 169.** A bulb of 220 volt and 300 watt is connected across 110 volt circuit. What is the percentage reduction in power?

Ans :

Delhi 2019

Voltage of bulb, $V = 220$ Volt

Power of bulb, $P = 300$ W

Supply voltage, $V_s = 110$ Volt

Resistance of bulb,

$$R = \frac{V^2}{P} = \frac{(220)^2}{300} = \frac{484}{3}\Omega$$

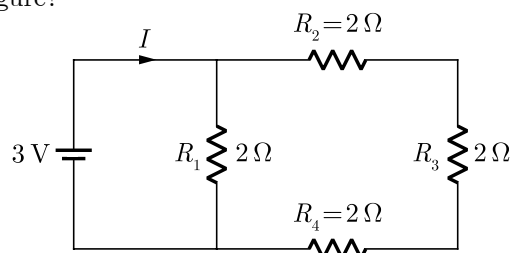
Power of the bulb,

$$P' = \frac{(V_s)^2}{R} = \frac{(110)^2}{484/3} = 75 \text{ W}$$

Therefore percentage reduction is power,

$$P'' = \frac{P - P'}{P} = \frac{300 - 75}{300} = 75\%$$

- 170.** What is the value of current I in the circuit shown in figure?



Equivalent resistance of two resistances when they are joined in series,

$$S = R_1 + R_2 = R + R = 2R$$

Similarly, equivalent resistance of two resistances when they are joined in parallel,

$$P = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{R \times R}{R + R} = \frac{R}{2}$$

Since, $S = nP$

Therefore, $2R = n \times \frac{R}{2}$

or $n = 4$

- 163.** A uniform wire of resistance R and length L is cut into four equal parts, each of length $\frac{L}{4}$, which are then connected in parallel combination. What will be effective resistance of the combination?

Ans :

SQP 2009

Resistance of single wire, $R_1 = R$

Length of single wire, $l_1 = L$

Length of each part, $l_2 = \frac{L}{4}$

Resistance of the wire,

$$R = \rho \times \frac{l}{A} \propto l$$

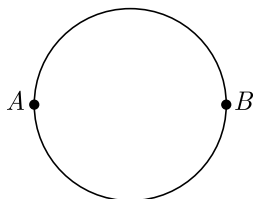
Therefore, $\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{L}{\frac{L}{4}} = 4$

or, $R_2 = \frac{R_1}{4} = \frac{R}{4}$

Relation for the effective resistance of the parallel combination is,

$$\begin{aligned} \frac{1}{R_{eff}} &= \left(\frac{1}{\frac{R}{4}} + \frac{1}{\frac{R}{4}} + \frac{1}{\frac{R}{4}} + \frac{1}{\frac{R}{4}} \right) \\ &= \left(\frac{4}{R} + \frac{4}{R} + \frac{4}{R} + \frac{4}{R} \right) = \frac{16}{R} \\ R_{eff} &= \frac{R}{16} \end{aligned}$$

- 164.** A wire of resistance $4R$ is bent in the form of a circle as shown in the figure. What is the effective resistance between the opposite ends A and B of the circle?



Ans :

Foreign 2017, OD 2012

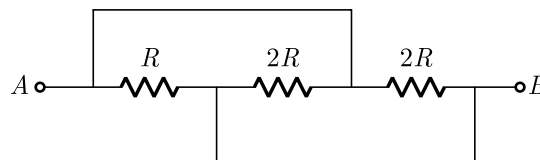
Resistance of wire $= 4R$

As the wire has a resistance of $4R$, therefore when

it is bent in the form of a circle, then it will behave as a combination of two wires each of resistance $2R$ connected in parallel combination between A and B . Therefore effective resistance between the ends A and B of the circle,

$$R_{AB} = \frac{2R \times 2R}{2R + 2R} = R$$

- 165.** What is the equivalent resistance between points A and B in the shown circuit?



Ans :

Delhi 2015

Resistances, $R_1 = R$

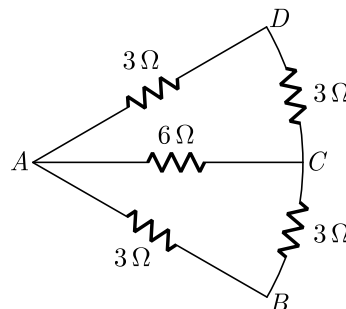
$R_2 = 2R$

and $R_3 = 2R$

In the given circuit that all the three resistances are in parallel combination. Therefore relation for equivalent resistance between A and B is,

$$\begin{aligned} \frac{1}{R_{AB}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\ &= \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} = \frac{2}{R} \\ R_{AB} &= \frac{R}{2} \end{aligned}$$

- 166.** What is the effective resistance between A and B in the circuit?



Ans :

Comp 2011

Resistance between A and D as well as D and C are in series combination.

Therefore their equivalent resistance,

$$R_1 = 3 + 3 = 6\Omega$$

Now the resistances R_1 and 6Ω (between A and C) are in parallel combination. Therefore their equivalent resistance,

$$R_2 = \frac{R_1 \times 6}{R_1 + 6} = \frac{6 \times 6}{6 + 6} = 3\Omega$$

Ans :

Foreign 2016, SQP2007

Mass of wire, $m = 2.4 \times 10^{-3} \text{ kg}$
 Current in wire, $I = 1 \text{ A}$
 Potential difference, $V = 1.8 \text{ Volt}$
 Density of copper, $d = 9.6 \times 10^3 \text{ kg-m}^{-3}$
 and its resistivity, $\rho = 1.8 \times 10^{-8} \Omega\text{-m}$
 Volume of wire,

$$\begin{aligned}
 Al &= \frac{m}{d} \\
 &= \frac{2.4 \times 10^{-3}}{9.6 \times 10^3} \\
 &= 0.25 \times 10^{-6} \quad \dots(1)
 \end{aligned}$$

Resistance of the wire,

$$\begin{aligned}
 R &= \rho \times \frac{l}{A} = \frac{V}{I} \\
 \frac{l}{A} &= \frac{V}{I\rho}
 \end{aligned}$$

where, l = Length of wire

$$= \frac{1.8}{1 \times (1.8 \times 10^{-8})} = 10^8 \quad \dots(2)$$

Multiplying equation (1) and (2),

$$\begin{aligned}
 l^2 &= 0.25 \times 10^2 = 25 \\
 l &= 5 \text{ m}
 \end{aligned}$$

- 159.** A platinum wire has a resistance of 10Ω at 0°C and 20Ω at 273°C . What is value of temperature coefficient of resistance of platinum?

Ans :

Comp 2018

Resistance of wire at 0°C , $R_0 = 10 \Omega$
 Initial temperature, $T_1 = 0^\circ \text{C}$
 Resistance of wire at 273°C , $R_{273} = 20 \Omega$
 Final temperature, $T_2 = 273^\circ \text{C}$
 Resistance of wire at 273°C ,

$$\begin{aligned}
 R_{273} &= R_0[1 + \alpha(T_2 + T_1)] \\
 20 &= 10[1 + \alpha(273 - 0)] \\
 &= 10 + 2730\alpha
 \end{aligned}$$

$$\begin{aligned}
 \text{or, } \alpha &= \frac{20 - 10}{2730} \\
 &= \frac{10}{2730} = 0.0037^\circ \text{C}^{-1}
 \end{aligned}$$

where, α = Temperature coefficient of resistance of platinum

- 160.** A series combination of two resistors 1Ω each is connected to a 12 V battery of internal resistance 0.4Ω . what is the value of current flowing through it?

Ans :

Delhi 2017

Resistance of each resistor, $R = 1 \Omega$
 Voltage of battery, $V = 12 \text{ Volt}$
 Internal resistance of battery, $r = 0.4 \Omega$

Since both the resistance are connected in a series combination, therefore total external resistance,

$$R' = 1 + 1 = 2 \Omega$$

Thus total resistance of the circuit,

$$\begin{aligned}
 R &= R' + r \\
 &= 2 + 0.4 = 2.4 \Omega
 \end{aligned}$$

And current flowing through the circuit,

$$I = \frac{V}{R} = \frac{12}{2.4} = 5 \text{ A}$$

- 161.** Two resistances 2Ω and 8Ω are connected in parallel. Similarly, two other resistances of 6Ω and 9Ω are connected in parallel. If both the combination are connected in series, what is the effective resistance of the system?

Ans :

Delhi 2012

First resistance, $R_1 = 2 \Omega$
 Second resistance, $R_2 = 8 \Omega$
 Third resistance, $R_3 = 6 \Omega$
 Fourth resistance, $R_4 = 9 \Omega$

Effective resistance of parallel combination of 2Ω and 8Ω resistances,

$$R = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = 1.6 \Omega$$

Similarly, effective resistance of parallel combination of 6Ω and 9Ω resistances,

$$R' = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{6 \times 9}{6 + 9} = \frac{54}{15} = 3.6 \Omega$$

Now effective resistance of the resistances R and R' are in series combination,

$$\begin{aligned}
 R_{\text{eff}} &= R + R' \\
 &= 1.6 + 3.6 = 5.2 \Omega
 \end{aligned}$$

- 162.** The resistance of series combination of two equal resistances is S . When they are joined in parallel, the total resistance is P . If $S = nP$, What is the possible value of n ?

Ans :

Foreign 2015

First resistance, $R_1 = R$
 Second resistance, $R_2 = R$

Equivalent resistance of two resistances, when they are joined in series = S and equivalent resistance of two resistances, when they are joined in parallel = P .

and $S = nP$

Ans :**OD 2018**Resistances, $R_1 = R_2 = R_3 = R_4 = 2\ \Omega$ Voltage, $V = 3\text{ Volt}$ We know that the resistances R_2 , R_3 and R_4 are in series combination.

Therefore their equivalent resistance,

$$R' = R_2 + R_3 + R_4 \\ = 2 + 2 + 2 = 6\ \Omega$$

Now, R' is in parallel combination with the resistance R_1 .

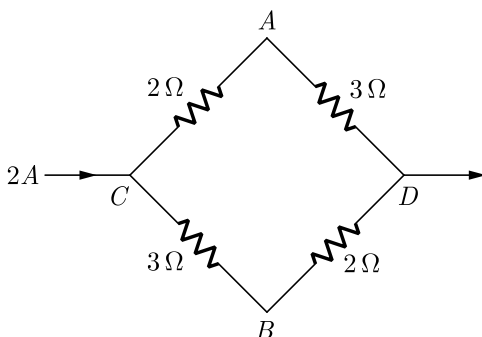
Therefore equivalent resistance of the circuit,

$$R = \frac{R_1 \times R'}{R_1 + R'} \\ = \frac{2 \times 6}{2 + 6} = \frac{12}{8} = 1.5\ \Omega$$

Thus current in the circuit,

$$I = \frac{V}{R} = \frac{3}{1.5} = 2\text{ A}$$

171. A current of 2 A flows in a system as shown in the figure. What is the potential difference between A and B ($V_A - V_B$) ?

**Ans :****SQP 2010, Foreign 2005**Resistances, $R_1 = 2\ \Omega$ $R_2 = 3\ \Omega$ $R_3 = 3\ \Omega$

and

 $R_4 = 2\ \Omega$ Here resistances $2\ \Omega$ and $3\ \Omega$ (in the upper) arm are in series combination.

Therefore their equivalent resistance,

$$R_U = 2 + 3 = 5\ \Omega$$

Similarly, equivalent resistance of the lower arm,

$$R_L = 3 + 2 = 5\ \Omega$$

Since the equivalent resistance of both arms are equal, therefore current flowing through the upper and lower arms will also be equal,

$$\text{i.e., } I_1 = I_2 = \frac{2}{2} = 1\text{ A}$$

Potential difference between C and A,

$$V_C - V_A = I_1 R_1 = 1 \times 2 = 2\text{ Volt} \quad \dots(1)$$

Similarly, potential difference between C and B,

$$V_C - V_B = I_2 R_2 = 1 \times 3 = 3\text{ Volt} \quad \dots(2)$$

Subtracting equation (1) from (2),

$$V_C - V_B - (V_C - V_A) = 3 - 2 \\ V_A - V_B = 1\text{ Volt}$$

172. In a Wheatstone's bridge, all the four arms have equal resistance R . If resistance of the galvanometer arm is also R , What is the equivalent resistance of the combination?

Ans :**Foreign 2013**

Resistances in four arms,

$$R_1 = R_2 = R_3 = R_4 = R$$

and resistance of galvanometer,

$$G = R$$

In a Wheatstone bridge, the resistance of galvanometer is ineffective.

Now resistances in upper arms are in a series combination. Therefore, their equivalent resistance,

$$R_U = R_1 + R_2 = R + R = 2R$$

Similarly, equivalent resistance of lower arms,

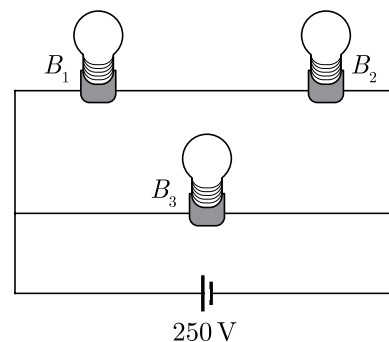
$$R_L = R_3 + R_4 = R + R = 2R$$

Now the equivalent resistance R_U and R_L are in parallel combination. Therefore, equivalent resistance of the combination,

$$R_{eq} = \frac{R_U \times R_L}{R_U + R_L} = \frac{2R \times 2R}{2R + 2R} = R$$

173. A 100 W bulb B_1 and two 60 W bulbs B_2 and B_3 , are connected to a 250 V source as shown in the figure. Now W_1 , W_2 and W_3 are the output powers of the bulbs B_1 , B_2 and B_3 respectively.

What is the relation between the output powers of bulbs?

**Ans :****Comp 2006**Power of bulb B_1 , $P_1 = 100\text{ W}$ Power of bulb B_2 , $P_2 = 60\text{ W}$

Power of bulb B_3 , $P_3 = 60 \text{ W}$
 Source voltage, $V = 250 \text{ Volt}$
 Output power of bulb, $B_1 = W_1$
 Output power of bulb, $B_2 = W_2$
 Output power of bulb, $B_3 = W_3$
 Resistance of bulb B_1 ,

$$R_1 = \frac{V^2}{P_1} = \frac{(250)^2}{100} = 625 \Omega$$

Similarly, resistance of bulb B_2 ,

$$R_2 = \frac{V^2}{P_2} = \frac{(250)^2}{60} = 1042 \Omega$$

and resistance of bulb B_3 ,

$$R_3 = \frac{V^2}{P_3} = \frac{(250)^2}{60} = 1042 \Omega$$

As the resistance R_1 and R_2 are connected in series.

Therefore, output power of bulb B_1 ,

$$\begin{aligned} W_1 &= \frac{V^2}{(R_1 + R_2)} \times R_1 \\ &= \frac{(250)^2}{(625 + 1042)^2} \times 625 \\ &= 14.1 \text{ W} \end{aligned}$$

Similarly, output power of bulb B_2 ,

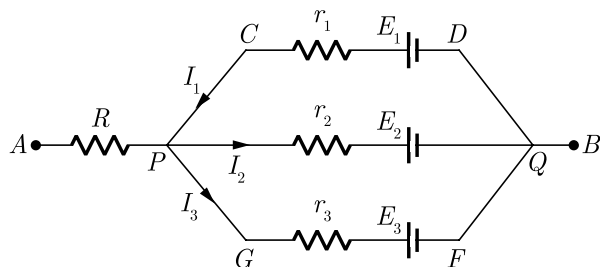
$$\begin{aligned} W_2 &= \frac{V^2}{(R_1 + R_2)^2} \times R_2 \\ &= \frac{(250)^2}{(625 + 1042)^2} \times 1042 \\ &= 23.4 \text{ W} \end{aligned}$$

and output power of bulb B_3 ,

$$\begin{aligned} W_3 &= \frac{V^2}{R_3^2} \times R_3 \\ &= \frac{(250)^2}{(1042)^2} \times 1042 = 60 \text{ W} \end{aligned}$$

Therefore, $W_1 < W_2 < W_3$

- 174.** In the circuit shown below, cell E_1 has e.m.f. of 3 V, cell E_2 has e.m.f. of 2 V, cell E_3 has e.m.f. of 1 V and $R = r_1 = r_2 = r_3 = 1 \Omega$. What is the potential difference between points A and B ?



Ans :

E.m.f. of first cell, $E_1 = 3 \text{ Volt}$

E.m.f. of second cell, $E_2 = 2 \text{ Volt}$
 E.m.f. of third cell, $E_3 = 1 \text{ Volt}$
 And resistances, $R = r_1 = r_2 = r_3 = 1 \Omega$
 Kirchhoff's first law, at junction P we get

$$I_1 = I_2 + I_3 \quad \dots(1)$$

From the Kirchhoff's second law that in the closed circuit PQDCP,

$$\begin{aligned} I_2 r_3 + E_2 - E_1 + I_1 r_1 &= 0 \\ (I_2 \times 1) + 2 - 3 + (I_1 \times 1) &= 0 \\ I_1 + I_2 &= 1 \\ (I_2 + I_3) + I_2 &= 1 \\ 2I_2 + I_3 &= 1 \quad \dots(2) \end{aligned}$$

Similarly, in the closed circuit PQFGP,

$$\begin{aligned} I_2 r_2 + E_2 - E_3 - I_3 r_3 &= 0 \\ (I_2 \times 1) + 2 - 1 - (I_3 \times 1) &= 0 \\ I_2 - I_3 &= -1 \quad \dots(3) \end{aligned}$$

Now, adding equations (2) and (3),

$$3I_2 = 0$$

$$I_2 = 0$$

Again from equation (3),

$$0 - I_3 = -1$$

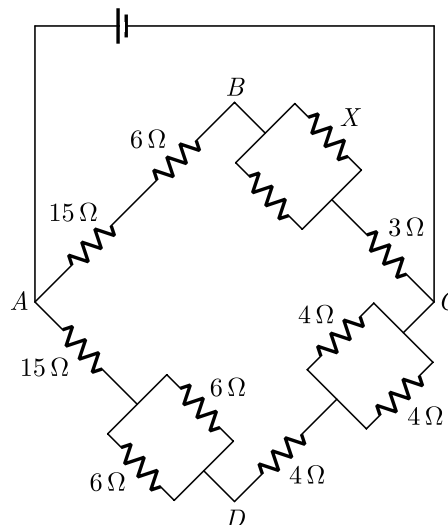
$$I_3 = 1 \text{ A}$$

and from equation (1), $I_1 = 0 + 1 = 1 \text{ A}$

Since, no current flows through r_2 and R , therefore potential difference between A and B is equal to the potential difference between P and Q (i.e., e.m.f. of cell E_2). Thus potential difference between A and B,

$$V_{AB} = E_2 = 2 \text{ Volt}$$

- 175.** In the given circuit, when the resistance between points B and D is zero. What is the value of the resistance X ?



Ans :

Delhi 2014

Resistance between B and D ,

$$R_{BD} = 0$$

Resistances between A and B are in series combination. Therefore, their equivalent resistance,

$$R_{AB} = 15 + 6 = 21 \Omega$$

Now, the resistances between A and D are 15Ω in series with 6Ω and 6Ω in parallel combination. Therefore, their equivalent resistance,

$$R_{AD} = 15 + \frac{6 \times 6}{6 + 6} = 15 + \frac{36}{12} = 18 \Omega$$

Similarly, equivalent resistance between D and C ,

$$R_{DC} = 4 + \frac{4 \times 4}{4 + 4} = 4 + \frac{16}{8} = 6 \Omega$$

and equivalent resistance between B and C ,

$$\begin{aligned} R_{BC} &= 3 + \frac{X \times 8}{X + 8} \\ &= \frac{11X + 24}{X + 8} \Omega \end{aligned}$$

As the resistance between BD is zero. Therefore, it is a balanced Wheatstone Bridge,

Hence,

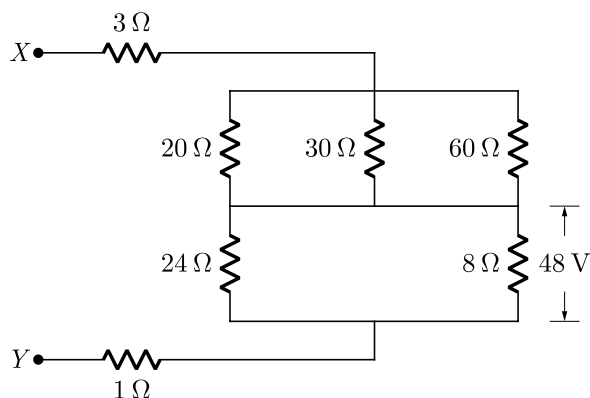
$$\begin{aligned} \frac{R_{AB}}{R_{AD}} &= \frac{R_{BC}}{R_{DC}} \\ \frac{21}{18} &= \frac{\frac{11X + 24}{X + 8}}{6} \end{aligned}$$

$$7X + 56 = 11X + 24$$

$$4X = 32$$

$$X = 8 \Omega$$

176. The potential difference across 8Ω resistance is 48 V as shown in the figure. What is the value of potential difference across X and Y terminal ?



Ans :

SQP 2008, Comp 2014

Potential difference across 8Ω resistor,

$$V = 48 \text{ Volt}$$

Resistances 24Ω and 8Ω are in parallel combination. Therefore their equivalent resistance,

$$R' = \frac{24 \times 8}{24 + 8} = 6 \Omega$$

Similarly, the resistances 20Ω , 30Ω and 60Ω are in parallel combination. Therefore relation for their equivalent resistance is,

$$\frac{1}{R''} = \frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{1}{10}$$

or,

$$R'' = 10 \Omega$$

Now the resistances 3Ω , R'' , R' and 1Ω are connected in series combination.Therefore effective resistance between X and Y ,

$$\begin{aligned} R_{XY} &= 3 + R' + R'' + 1 \\ &= 3 + 6 + 10 + 1 = 20 \Omega \end{aligned}$$

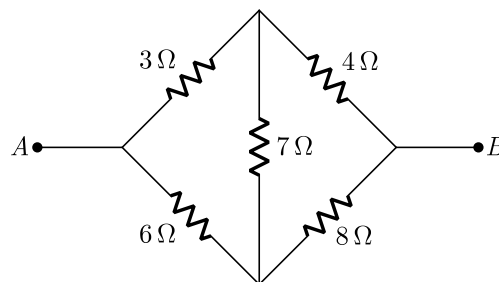
Current in the circuit,

$$I = \frac{V}{R'} = \frac{48}{6} = 8 \text{ A}$$

Therefore potential difference across X and Y ,

$$\begin{aligned} V_{XY} &= I \times R_{XY} \\ &= 8 \times 20 = 160 \text{ Volt} \end{aligned}$$

177. What is the equivalent resistance between A and B in the given figure?



Ans :

OD 2011, Delhi 2006

Resistances, $R_1 = 3 \Omega$, $R_2 = 4 \Omega$, $R_3 = 6 \Omega$ and $R_4 = 8 \Omega$, $R_5 = 7 \Omega$

In the given circuit,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} = \frac{1}{2}$$

Thus it is a case of balanced Wheatstone bridge, in which 7Ω resistance is ineffective.

Now resistances in the upper arms are in series combination. Therefore, their equivalent resistance,

$$R_U = R_1 + R_2 = 3 + 4 = 7 \Omega$$

Similarly, equivalent resistance of the lower arms,

$$R_L = R_3 + R_4 = 6 + 8 = 14 \Omega$$

Then,

$$\eta = \left(\frac{EI - I^2 r}{EI} \right) \times 100 \left(1 - \frac{Ir}{E} \right) \times 100$$

$$1 - \left(\frac{E}{R+r} \right) \left(\frac{r}{E} \right) \times 100$$

$$\eta = \left(\frac{R}{R+r} \right) \times 100$$

We know that output power of a source is maximum when the external resistance is equal to internal resistance, i.e., $R = r$.

- (i) Two fuse wires of same potential material are having length ratio 1 : 2 and ratio 4 : 1. What is the ratio of their current rating ?
- (ii) Efficiency of a battery when delivering maximum power is

Ans :

- (i) 8 : 1
- (ii) 40%

- 183.** Ram and Shyam purchased two electric tea kettles, *A* and *B* of same size, same thickness and same volume of 0.4 litre. They studied the specification of kettles as under

Kettle *A* :

Specific heat capacity = 1680 J/kgK

Mass = 200 g

Cost = ₹ 400

Kettle *B* :

Specific heat capacity = 2450 J/kgK

Mass = 400 g

Cost = ₹ 400

When kettle *A* is switched on with constant potential source, the tea begins to boil in 6 min. When kettle *B* is switched on with the same source separately, then tea begins to boil in 8 min. The efficiency of kettle is defined as

$$\frac{\text{Energy used for liquid heating}}{\text{Total energy supplied}}$$

They made discussion on specification and efficiency of kettles and subsequently prepared a list of questions to draw the conclusions. Some of them are as under (Assume specific heat of tea liquid as 4200 J/kgK and density 1000 kg/m³.)

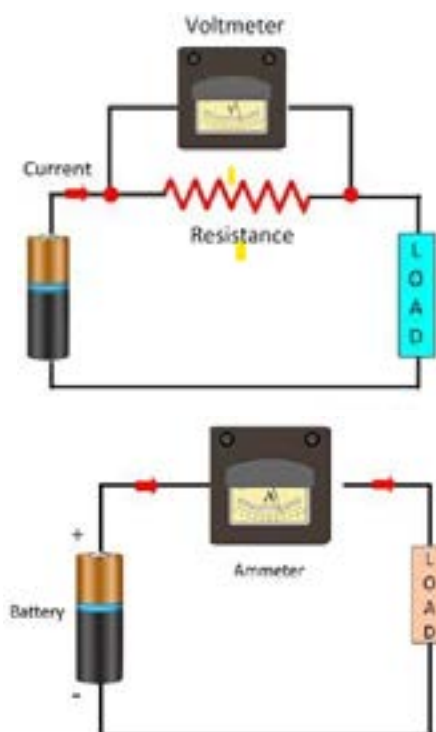


- (i) Calculate the efficiency of electric kettle *A* and *B* ?
- (ii) What is the ratio of efficiency consumed charges for one time boiling of tea in kettle *A* to that in kettle *B*?
- (ii) What is the relation between the resistance of electric kettle *A* and *B*?

Ans :

- (i) 83.34%, 62.5%
- (ii) 3 : 4
- (ii) $R_A = R_B$

- 184.** An ammeter and a voltmeter are connected in series to a battery with an emf of 10V. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decreases three times, whereas the reading of the ammeter increases two times.



- (i) Find the voltmeter reading after the connection of the resistance.
- (ii) If the resistance of the ammeter is 2 ohm. What is the resistance of the voltmeter?

Ans :

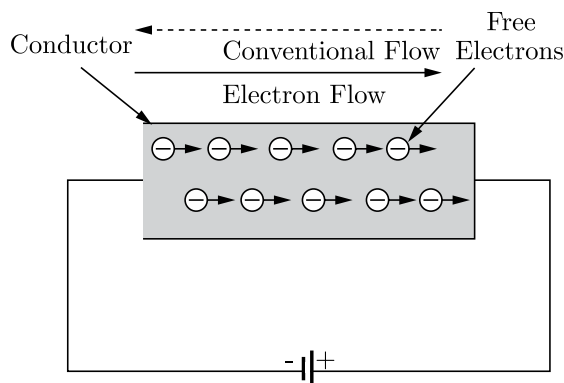
- (i) 2 Volt
- (ii) 3 ohm.

- 185.** In the circuit given in the figure, both batteries are ideal, e.m.f. E , of battery 1 has a fixed value, but e.m.f. E , of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through

Ans :

- (i) $8/3$ ohm.
- (ii) 2.25 A
- (ii) 9 W

- 181.** Electron move more easily through some conductors than others when potential difference is applied. The opposition of a conductor to current is called its resistance. Collisions are the basic cause of opposition. When potential difference is applied across the ends of a conductor, its free electrons get accelerated. On their way, they frequently collide with positive metal ions, i.e., their motion is opposed and this opposition to the flow of electron is called resistance. The number of collisions that the electrons make with atoms/ions depends on the arrangement of atoms or ions in the conductor. A long wire offers more resistance than short wire because there will be more collisions. A thick wire offers less resistance than a thin wire because in a thick wire more area of cross section is available for the flow of electrons. The resistance of metal increases when their temperature increases. Certain alloys such as constantan and manganin show very small changes of resistance with temperature and are used to make standard resistors. The resistance of semiconductor and insulator decreases as their temperature increases.



- (i) The dimensions of a block are $1\text{ cm} \times 1\text{ cm} \times 100\text{ cm}$. If the specific resistance of the material is 3.7×10^{-7} ohm. What is the resistance between two opposite rectangular base?
- (ii) Two wire of the same material have lengths l and $2l$ and areas of cross section $4A$ and A respectively. What is the ratio of their specific resistance?
- (ii) A wire of resistance R is stretched to twice of its original length. What is the new resistance of wire?

Ans :

- (i) As we know that,

$$\text{where, } R = \frac{\rho l}{A}$$

$$P = 3.7 \times 10^{-7} \text{ ohm}$$

$$\begin{aligned} l &= 10^{-2} \text{ m} \\ &= \frac{(3.7 \times 10^{-7} \times 10^{-2})}{1 \times 10^{-2}} \\ &= 3.7 \times 10^{-7} \Omega \end{aligned}$$

- (ii) The specific resistance does not depend upon l and A depends upon nature of material. Hence, the specific resistance of both wire remains same.

$$(ii) \quad \frac{R_2}{R_1} = \frac{12 \times A_1}{11 \times A_2}$$

when stretch volume remains constant hence

$$l_1 A_1 = l_2 \times A_2$$

$$\frac{A_1}{A_2} = \frac{l_2}{l_1}$$

$$\frac{A_1}{A_2} = \frac{2}{1}$$

Hence,

$$\begin{aligned} \frac{R_2}{R_1} &= \left(\frac{l_2}{l_1}\right) \times \frac{A_1}{A_2} \\ &= 2 \times 2 = 4 \end{aligned}$$

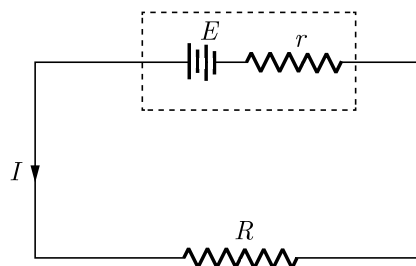
- 182.** Electric fuse is a protective device used in series with an electric circuit or an electric appliance to save it from damage due to overheating produced by strong current in the circuit or application. Fuse wire is generally made from an alloy of lead and tin which has high resistance and low melting point. It is connected in series in an electric installation. If a circuit gets accidentally short-circuited, a large current flows, then fuse wire melts every which causes a break in the circuit. The power through fuse (P') is equal to heat energy lost per unit area per unit time (h) (neglecting heat losses from ends of the wire).

$$P = I^2 R = h \times 2\pi r l \quad \left[R = \frac{\rho l}{\pi r^2} \right]$$

Where r and l are the length and radius of fuse wire, respectively.

A battery is described by its e.m.f. (E) and internal resistance (r). Efficiency of a battery (η) is defined as the ratio of the output power to the input power.

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$



But,

$$I = \frac{E}{R + r}, \text{ input power} = EI$$

$$\text{Output power} = EI - I^2 r$$

Now equivalent resistances R_U and R_L are in parallel combination.

Therefore equivalent resistance between A and B ,

$$R_{AB} = \frac{R_U \times R_L}{R_U + R_L} \\ = \frac{7 \times 14}{7 + 14} = \frac{14}{3} \Omega$$

- 178.** A 6V battery is connected to the terminals of a 3 m long wire of uniform thickness and of resistance 100Ω . What is the potential difference between two points on the wire, separated by a distance of 50 cm?

Ans :

Comp 2013

Voltage of battery, $V = 6 \text{ Volt}$

Length of wire, $L = 3 \text{ m}$

Resistance of wire, $R = 100 \Omega$

Length of small part of wire,

$$l = 50 \text{ cm} = 0.5 \text{ m}$$

Resistance of small part of wire,

$$R' = R \times \frac{l}{L} \\ = 100 \times \frac{0.5}{3} = \frac{50}{3} \Omega$$

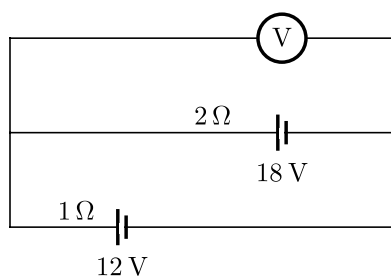
and current flowing through the wire,

$$I = \frac{V}{R} = \frac{6}{100} = \frac{3}{50} \text{ A}$$

Therefore, potential difference between two points on the wire,

$$V' = IR' = \frac{3}{50} \times \frac{50}{3} = 1 \text{ Volt}$$

- 179.** Two batteries, one of e.m.f. 18 V and internal resistance 2Ω and other of e.m.f. 12 volt and internal resistance 1Ω are connected as shown in figure. What is the reading of voltmeter?



Ans :

OD 2013

E.m.f. of first battery, $E_1 = 18 \text{ Volt}$

Internal resistance of first battery, $r_1 = 2 \Omega$

E.m.f. of second battery, $E_2 = 12 \text{ Volt}$

Internal resistance of second battery, $r_2 = 1 \Omega$

Reading of voltmeter = V

Kirchhoff's second law we get,

$$I r_1 - E_1 + I r_2 + E_2 = 0 \\ (I \times 2) - 18 + (I \times 1) + 12 = 0 \\ 3I = 6 \\ I = 2 \text{ A}$$

Therefore, voltage drop across r_1 ,

$$V_1 = I r_1 = 2 \times 2 = 4 \text{ Volt}$$

Thus reading of voltmeter,

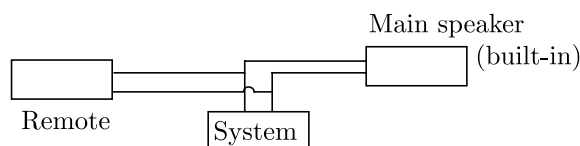
$$V = E_1 - V_1 \\ = 18 - 4 = 14 \text{ Volt}$$

CASE BASED QUESTIONS

- 180.** Most of the times we connect remote speakers to play music in another room along with the built-in speakers. These speakers are connected in parallel with the music system.

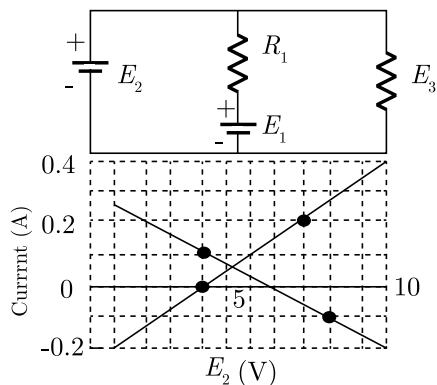


At the instant represented in the picture, the voltage across the speakers is 6.00 V. The resistance of the main speaker is 8Ω and the remote speaker has resistance 4Ω



- What is equivalent resistance of the speakers?
- What is the total current supplied by music system?
- What is the power dissipated in the 4 ohm resistance?

the two batteries as a function of E_2 , but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite to the direction of that battery's e.m.f. (direction of e.m.f. is from negative to positive).



- (i) What is the value of EMF of cell 1?
- (ii) What is the value of resistance R_1 ?
- (ii) Calculate the value of resistance R_2 ?

Ans :

- (i) 4 Volt
- (ii) 10 ohm
- (ii) 10 ohm

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CHAPTER 4

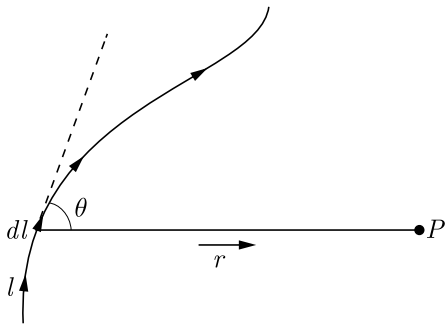
Moving Charge and Magnetism

SUMMARY

1. BIOT-SAVART LAW

It states that the magnetic field strength $\vec{dB}$ produced to a current element (of current I and length dl) at a point having position vector $\vec{r}$ relative to current element is,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$$



where, $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-meter
permeability of free space.

The magnitude of magnetic field is,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

where, θ is the angle between current element $I d\vec{l}$ and position vector $\vec{r}$ as shown in the figure. The direction of magnetic field $\vec{dB}$ is perpendicular to the plane containing $I d\vec{l}$ and $\vec{r}$.

Magnetic field at the centre of a circular current carrying conductor/coil.

$$B = \frac{\mu_0 I}{2r}$$

where, r is the radius of a circular loop.

For N turns of coil,

$$B = \frac{\mu_0 NI}{2r}$$

Magnetic field at the centre of semi-circular current carrying conductor.

$$B = \frac{\mu_0 I}{4r}$$

Magnetic field at the centre of an arc of circular current carrying conductor which subtends an angle θ at the centre.

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}$$

Magnetic field at any point lies on the axis of circular current carrying conductor.

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$

2. AMPERE'S CIRCUITAL LAW

It states that the line integral of magnetic field $\vec{B}$ along a closed path is equal to μ_0 times the current (I) passing through the closed path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

3. MAGNETIC FIELD DUE TO A CURRENT CARRYING SOLENOID

At the axis of a long solenoid, carrying a current I ,

$$B = \mu_0 n I$$

where, n = number of turns per unit length

Magnetic field at one end of solenoid,

$$B_{\text{end}} = \frac{\mu_0 n I}{2}$$

The polarity of any end is determined by using Ampere's right hand rule.

4. MAGNETIC FIELD DUE TO A TOROID (CIRCULAR SOLENOID)

Magnetic field within the turns of toroid,

$$B = \frac{\mu_0 NI}{2\pi r} = \mu_0 n I$$

where, $n = \frac{N}{2\pi r}$ and r is average radius

Magnetic field outside the toroid is zero.

5. FORCE ON A MOVING CHARGED PARTICLE IN MAGNETIC FIELD

The force on a charged particle moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ is given by,

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

This is known as Lorentz force.

The direction of this force is determined by using Fleming's left hand rule.

The direction of this force is perpendicular to both $\vec{v}$ and $\vec{B}$. When $\vec{v}$ is parallel to $\vec{B}$, then $\vec{F}_m = 0$.

When $\vec{v}$ is perpendicular to $\vec{B}$, then $\vec{F}_m$ is maximum, i.e., $\vec{F}_m = qv\vec{B}$.

SI Unit of Magnetic Field

Since, $F = qvB\sin\theta$

$$B = \frac{F}{qv\sin\theta}$$

If, $F = 1 \text{ N}$

$$q = 1 \text{ C}$$

$$v = 1 \text{ m/s}$$

$$\sin\theta = 1$$

$$\theta = 90^\circ$$

Hence, SI unit of $B = \frac{1\text{N}}{(1\text{C})(1\text{m/s})}$
 $= 1 \text{ NA}^{-1}\text{m}^{-1}$
 $= 1 \text{ T} = 1 \text{ tesla (T)}$

6. RESONANCE CONDITION FOR THE ACCELERATION OF CHARGED PARTICLES

The frequency of revolution of charged particle must be equal to the frequency of radio frequency voltage source.

The frequency of revolution of the particle is,

$$f = \frac{qB}{2\pi m}$$

where, B is the magnetic field inside the dees, q is the charge on the particle and m is its mass. This frequency is called cyclotron frequency.

Energy gained per revolution $= 2qV$

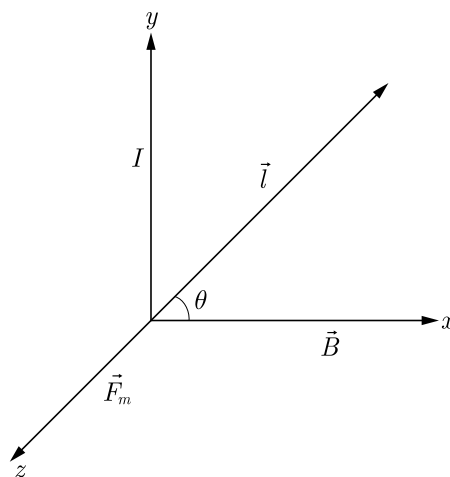
Energy gained in n - revolutions,

$$E = 2nqV$$

$$= \frac{B^2 q^2 R^2}{2m}$$

where, R is radius of circular track of charged particle.

7. MAGNETIC FORCE ON A CURRENT CARRYING CONDUCTOR



Magnetic force on a current carrying conductor of length (l) is given by

$$\vec{F}_m = I(\vec{l} \times \vec{B})$$

Magnitude of force is,

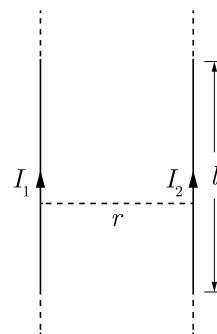
$$F_m = IlB\sin\theta$$

Direction of force $\vec{F}$ is normal to $\vec{l}$ and $\vec{B}$ given by Fleming's Left Hand Rule. If $\theta = 0$ (i.e., $\vec{l}$ is parallel to $\vec{B}$), then magnetic force is zero.

8. MAGNETIC FORCE BETWEEN TWO STRAIGHT PARALLEL CURRENT CARRYING CONDUCTORS IS GIVEN BY

Magnetic force between two straight parallel current carrying conductors is given by

$$\frac{F}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$



Torque Experienced by a Current Loop (of area $\vec{A}$)

Carrying current I in a uniform magnetic field $\vec{B}$ is given by,

$$\vec{\tau} = NI(\vec{A} \times \vec{B}) = \vec{M} \times \vec{B}$$

where, $\vec{M} = NI \vec{A}$ is magnetic moment of loop

The unit of magnetic moment in SI system is ampere $\times$ metre² (Am^2).

Potential Energy of a Current Loop in a Magnetic Field

When a current loop of magnetic moment M is placed in a magnetic field, then potential energy of magnetic dipole is,

$$U = -\vec{M} \cdot \vec{B} = -MB \cos \theta$$

1. When $\theta = 0$, $U = -MB$ (minimum or stable equilibrium position).
2. When $\theta = \pi$, $U = +MB$ (maximum or unstable equilibrium position).
3. When $\theta = \frac{\pi}{2}$, potential energy is zero.

9. MOVING COIL GALVANOMETER

Is a device used to detect the current in electrical circuit. It is based on the principle that a current carrying loop placed in a uniform magnetic field experiences torque. The current sensitivity and voltage sensitivity of galvanometer depends on number of turns of coil magnetic field B , area A of the coil and torsion constant k of the spring or suspension wire.

Current sensitivity,
$$I_s = \frac{\theta}{I} = \frac{NBA}{k}$$

Its SI unit is rad/A or div/A.

Voltage sensitivity,
$$V_s = \frac{\theta}{V} = \frac{\theta}{IR}$$

$$= \frac{I_s}{R} = \frac{NBA}{kR}$$

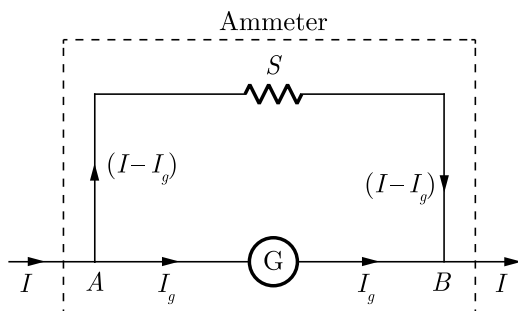
Its SI unit is rad/V or div/V.

Torque or moment of galvanometer,

$$G = \frac{\theta}{I} = \frac{NBA}{k}$$

A galvanometer can be converted into an ammeter by connecting a very low resistance (shunt S) in parallel with galvanometer whose value is given by,

$$S = \frac{I_g G}{I - I_g}$$



where, G = resistance of galvanometer

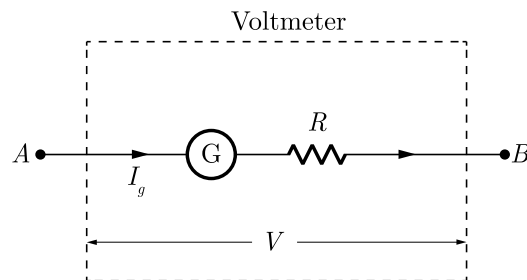
I_g = current through galvanometer

I = total current in circuit

S = resistance of the shunt (low resistance)

A galvanometer can be converted into voltmeter by connecting a very high resistance R in series with galvanometer which is given by

$$R = \frac{V}{I_g} - G$$



where,

I_g = current through the galvanometer

G = resistance of galvanometer and

V = potential difference across the terminal A and B

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OBJECTIVE QUESTIONS

1. A long straight wire of radius 'a' carries a steady current I. The current is uniformly distributed across its area of cross-section. The ratio of magnitude of magnetic field $\vec{B}$ at $\frac{a}{2}$ and $\vec{B}_2$ at distance $2a$ is
- (a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 4

Ans :

OD 2023

Consider two American loops of radius $a/2$ and $2a$.

Applying ampere's circuital law for these loops, we get

$$\oint B \cdot dL = \mu_0 I_{\text{enclosed}}$$

For the smaller loop,

$$B_1 \times 2\pi \frac{a}{2} = \mu_0 \times \frac{1}{\pi a^2} \times \left(\frac{a}{2}\right)^2$$

$$\mu_0 I \times \frac{1}{4} = \frac{\mu_0 I}{4}$$

$$B_1 = \frac{\mu_0 I}{4\pi a}$$

and for bigger. loop,

$$B_2 = \frac{\mu_0 I}{2\pi(2a)} \frac{\mu_0 I}{4\pi a}$$

$$\frac{B_1}{B_2} = 1$$

Thus (b) is correct option.

2. The coil of a moving coil galvanometer is wound over a metal frame in order to
- (a) reduce hysteresis
(b) increase sensitivity
(c) increase moment of inertia
(d) provide electromagnetic damping

Ans :

SQP 2009

We know that the coil of a moving coil galvanometer is wound over a metal frame, in order to provide electromagnetic damping, so that the galvanometer becomes dead beat i.e., on passing current, galvanometer shows a steady deflection without any oscillation.

Thus (d) is correct option.

3. The direction of magnetic field produced by a current-carrying small element of any shape is given by
- (a) Lenz law
(b) Newton's law
(c) Right-hand thumb rule
(d) Fleming left-hand rule

Ans :

Comp 2010

We know that magnetic field produced by a current-carrying small element of any shape,

$$(d\vec{B}) = \frac{\mu_0}{4\pi} i \left(\frac{d\vec{l} \times \vec{r}}{r^3} \right)$$

and the direction of magnetic field ($d\vec{B}$) is given by Right-hand thumb rule.

Thus (c) is correct option.

4. A charge moving with uniform velocity produces
- (a) only an electric field (b) only a magnetic field
(c) electromagnetic field (d) none of these

Ans :

OD 2017

A moving charge produces electric and magnetic field. Hence, an electromagnetic field is present around a moving charge.

Thus (c) is correct option.

5. A charge q moves in a region where electric field $\vec{E}$ and magnetic field $\vec{B}$ both exist, then force on it is
- (a) $q(\vec{v} \times \vec{E})$ (b) $q\vec{E}$
(c) $q\{\vec{E} + (\vec{v} \times \vec{B})\}$ (d) $q\{\vec{B} + (\vec{v} \times \vec{E})\}$

Ans :

SQP 2016, Comp 2009

A charge q in an electric field $\vec{E}$ experiences the electric force,

$$\vec{F}_e = q\vec{E} \quad \dots(1)$$

The magnetic force experienced by the charge q moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$ is given by,

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \quad \dots(2)$$

$$\begin{aligned} \text{Total force, } F &= \vec{F}_e + \vec{F}_m \\ &= q\vec{E} + q(\vec{v} \times \vec{B}) \\ &= q\{\vec{E} + (\vec{v} \times \vec{B})\} \end{aligned}$$

Thus (c) is correct option.

6. A proton is moving in a space with constant velocity in an electric field E and magnetic field B . The angle between electric field and magnetic field should be
- (a) 0° (b) 30°
(c) 45° (d) 90°

Ans :

Delhi 2015

When a charged particle (i.e., proton) is moving in space with a constant velocity in electric field E and magnetic field B , then angle between electric field and magnetic field should be 90° .

Thus (d) is correct option.

7. A circular coil of radius r carries a current I . The magnetic field at its center is B . At what distance from the centre, on the axis of the coil, the magnetic field will be $B/8$

- (a) $\sqrt{2} R$ (b) $2R$
(c) $\sqrt{3} R$ (d) $3R$

Ans :

SQP 2009

Magnetic field at distance x from the centre,

$$B_{\text{axis}} = \frac{\text{magnetic field at center } (B)}{8}$$

$$B_{\text{axis}} = \frac{B}{8}$$

$$\frac{\mu_0 Ni R^2}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 Ni}{8 \times 2R}$$

$$8R^3 = (R^2 + x^2)^{3/2}$$

This is possible only when $x = \pm \sqrt{3} R$.

Hence, $\sqrt{3} R$ distance from the centre magnetic field is equal to magnetic field at centre.

Thus (c) is correct option.

8. When a charged particle moves perpendicular to a magnetic field, then

- (a) speed of the particle is changed
(b) speed of the particle remains unchanged
(c) direction of the particle remains unchanged
(d) acceleration of the particle remains unchanged

Ans :

OD 2011

When a charged particle moves perpendicular to a magnetic field, then the particle moves in a circular path with a uniform speed.

As a result of this, the particle experiences a centripetal force, but the speed of the particle remains unchanged.

Thus (b) is correct option.

9. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields, which are parallel to each other. The particle will move in a

- (a) circle (b) helix
(c) cycloid (d) straight line

Ans :

Delhi 2017

When a charged particle is released from rest in a region of steady and uniform electric and magnetic fields, which are parallel to each other, then charged particle will move along the electric field in a straight line, because magnetic field has no effect on it.

Thus (d) is correct option.

10. The dimensional formula for $\frac{1}{2}\epsilon_0 E^2$ is identical to that of

- (a) $\frac{B^2}{2\mu_0}$ (b) $\frac{1}{2}B^2\mu_0$
(c) $\frac{\mu_0^2}{2B}$ (d) $\frac{1}{2}B\mu_0^2$

Ans :

Foreign 2011

Dimension of $\frac{1}{2}\epsilon_0 E^2$ is given by,

$$\begin{aligned} \frac{1}{2}\epsilon_0 E^2 &= \frac{\text{Energy}}{\text{Volume}} \\ &= \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = [\text{ML}^{-1}\text{T}^{-2}] \end{aligned}$$

Dimension of $\frac{B^2}{2\mu_0}$ is given by,

$$\begin{aligned} &= \frac{[\text{MT}^{-2}\text{A}^{-1}]^2}{[\text{MLT}^{-2}\text{A}^{-2}]} \\ &= [\text{M}^1\text{L}^{-1}\text{T}^{-2}] \end{aligned}$$

[Since, $B = \frac{F}{qV\sin\theta}$, $\mu_0 = \frac{4\pi rF}{I_1 I_2}$]

Thus (a) is correct option.

11. Dimension of magnetic field is

- (a) $\text{I}^{-1}\text{ML}^0\text{T}^{-2}$ (b) $\text{I}^0\text{MLT}^{-2}$
(c) IMLT^{-1} (d) $\text{IM}^{-1}\text{L}^{-1}\text{T}^{-2}$

Ans :

OD 2008, Delhi 2015

We have

$$F = qvB\sin\theta$$

$$B = \frac{F}{qv\sin\theta}$$

$\sin\theta$ is a dimensionless quantity

$$\begin{aligned} &= \frac{[\text{M}^1\text{L}^1\text{T}^{-2}]}{[\text{IT}][\text{LT}^{-1}]} \quad [\text{Since, } q = \text{IT}] \\ &= [\text{ML}^0\text{T}^{-2}\text{I}^{-1}] \end{aligned}$$

Thus (a) is correct option.

12. If a current i ampere flows in a long straight thin walled tube, then magnetic induction at any point inside the tube is

- (a) zero (b) infinite
(c) $\frac{2i}{r}$ Tesla (d) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}$ Tesla

Ans :

SQP 2002

Current in the long straight thin walled tube = i .

Current flows only on the surface of a long straight thin walled tube. Therefore, magnetic induction at any point inside the tube is zero.

Thus (a) is correct option.

13. Magnetic field due to a long straight conductor of length l , carrying current I , at a point, distance d from it is given by

will move in a circular path and radius of this circular path,

$$r = b - a = \frac{mv}{qB}$$

Therefore, particle can not enter the region $x > b$.

And if the particle is to enter the region $x > b$,

Then, $r > (b - a)$

$$\frac{mv}{Bq} > (b - a)$$

$$v > \frac{q(b - a)B}{m}$$

where, v = minimum velocity of particle

Thus (c) is correct option.

21. Two circular coils of diameters 10 cm and 20 cm have the same no. of turns and currents. The ratio of magnetic field inductions produced at the centre of the coils, when connected in series is

- (a) 1:2 (b) 2:1
(c) 3:2 (d) 2:3

Ans :

OD 2014

Diameter of first coil, $d_1 = 10$ cm

Radius of first coil, $r_1 = 5$ cm

Diameter of second coil, $d_2 = 20$ cm

Radius of second coil, $r_2 = 10$ cm

Magnetic field induction produced at the centre of a current-carrying circular coil,

$$B = \frac{\mu_0 n I}{2r} \propto \frac{1}{r}$$

Therefore, $\frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{10}{5}$

$$B_1 : B_2 = 2 : 1$$

Thus (b) is correct option.

22. The magnetic field induction at one end on the axis of a long solenoid having n turns per unit length and carrying I ampere current is

- (a) $\mu_0 n I$ (b) $\frac{\mu_0 n I}{2}$
(c) $\frac{\mu_0 n I}{4}$ (d) zero

Ans :

Delhi 2003, SQP 2017

Number of turns per unit length = n

and Current in solenoid = I

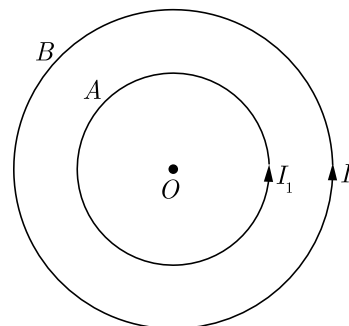
Magnetic field induction at one end of a long solenoid,

$$B = \frac{\mu_0 n I}{2}$$

where, μ_0 = Absolute permeability of free space

Thus (b) is correct option.

23. A and B are two concentric circular conductors of centre O and carrying currents I_1 and I_2 as shown in the adjacent figure. If ratio of their radii is 1:2 and ratio of the magnetic flux densities at O due to A and B is 1:3, then the value of I_1/I_2 is



(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Ans :

Foreign 2002

Radius of conductor A , $r_1 = r$

Radius of conductor B , $r_2 = 2r$

Magnetic flux density due to conductor A ,

$$B_1 = B$$

and magnetic flux density due to conductor B ,

$$B_2 = 3B$$

Magnitude of magnetic flux density at the centre of a current-carrying circular conductor,

$$B = \frac{\mu_0 I}{2r} \propto \frac{I}{r}$$

Therefore, $\frac{B_1}{B_2} = \frac{I_1}{I_2} \times \frac{r_2}{r_1}$

$$\frac{1}{3} = \frac{I_1}{I_2} \times \frac{2}{1}$$

$$\frac{I_1}{I_2} = \frac{1}{6}$$

Thus (a) is correct option.

24. The magnetic field at the centre of a circular loop of area A is B . The magnetic moment of the loop will be

- (a) $\frac{BA^2}{\mu_0 \pi}$ (b) $\frac{BA^{3/2}}{\mu_0}$
(c) $\frac{BA^{3/2}}{\mu_0 \pi}$ (d) $\frac{2BA^{3/2}}{\mu_0 \sqrt{\pi}}$

Ans :

OD 2012

Area of circular loop = A

Magnetic field at the centre of loop = B

Magnetic moment, $M = i \cdot A$... (1)

Magnetic field due to current-carrying long straight wire at the point,

$$B = \frac{\mu_0}{2\pi} \times \frac{I}{r} \propto \frac{1}{r}$$

Therefore, $\frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{20}{5} = 4$

$$B_2 = \frac{B_1}{4} = \frac{B}{4}$$

where, B_2 = magnetic field at second point

Thus (b) is correct option.

- 18.** A current passing through a circular coil of two turns produces a magnetic field of 8 T at its centre. The coil is then rewound, so as to have four turns and current is passed through it is doubled. Now magnetic field at the centre of the coil will be

- (a) 64 T (b) 32 T
(c) 16 T (d) 8 T

Ans :

Delhi 2015

Initial number of turns, $n_1 = 2$

Initial magnetic field at centre, $B_1 = 8$ T

Final no. of turns in coil, $n_2 = 4$

Final current in the coil, $I_2 = 2I_1$

(where, I_1 is the initial current)

Since, circumference of two coil is equal to circumference of four turn coil,

Therefore, $2 \times 2\pi r_1 = 4 \times 2\pi r_2$

$$r_1 = 2r_2$$

Magnetic field at the centre of a current-carrying circular coil,

$$B = \frac{\mu_0 n I}{2r} \propto \frac{nI}{r}$$

Therefore, $\frac{B_1}{B_2} = \frac{n_1}{n_2} \times \frac{I_1}{I_2} \times \frac{r_2}{r_1}$

$$= \frac{2}{4} \times \frac{I_1}{2I_1} \times \frac{r_2}{2r_2} = \frac{1}{8}$$

$$B_2 = 8B_1 = 8 \times 8 = 64 \text{ T}$$

Where, B_2 = final magnetic field at centre of coil

Thus (a) is correct option.

- 19.** Two circular coils of radii 15 cm and 30 cm are made of similar wires and have same number of turns. If they are connected in parallel, then ratio of their magnetic inductions at the centre of the coils is

- (a) 1:2 (b) 2:1
(c) 1:4 (d) 4:1

Ans :

OD 2004

Radius of first coil, $r_1 = 15 \text{ cm} = 0.15 \text{ m}$

Radius of second coil, $r_2 = 30 \text{ cm} = 0.3 \text{ m}$

and no. of turns in each coil = n

Resistance of coil,

$$R = 2\pi r n \times \rho$$

where, ρ = resistance per unit length

Therefore, current in coil,

$$I = \frac{V}{R} = \frac{V}{2\pi r n \rho}$$

Magnetic induction at the centre of a current-carrying circular coil,

$$\begin{aligned} B &= \frac{\mu_0 n I}{2r} \\ &= \frac{\mu_0 n}{2r} \times \frac{V}{2\pi r n \rho} \\ &= \frac{\mu_0}{4\pi r^2} \times \frac{V}{\rho} \end{aligned}$$

Since, both the coils are connected in parallel, therefore potential difference V across both the coils is same and resistance per unit length of the wires will also be same, because they are made of similar wires.

Thus, $B \propto \frac{1}{r^2}$

Therefore, $\frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.3}{0.15}\right)^2 = \frac{4}{1}$

$$B_1 : B_2 = 4 : 1$$

Thus (d) is correct option.

- 20.** A particle of mass m and charge q moves with a constant velocity v along the positive x -direction. It enters a region containing a uniform magnetic field B directed along the negative z -direction, extending from $x = a$ to $x = b$. The minimum value of v required, so that the particle can just enter the region of $x > b$ is

- (a) $\frac{qbB}{m}$ (b) $\frac{qaB}{m}$
(c) $\frac{q(b-a)B}{m}$ (d) $\frac{q(b+a)B}{2m}$

Ans :

SQP 2002, Foreign 2011

Mass of particle = m

Charge on particle = q

Velocity of particle = v

...(directed along positive x -direction)

Uniform magnetic field = B

...(directed along negative z -direction,

extending from $x = a$ to $x = b$)

When a charged particle enters in a uniform magnetic field, directed vertically upwards, then the particle

- (a) $B = \frac{\mu_0}{4\pi} \times \frac{2I}{d}$ (b) $B = \frac{\mu_0}{4\pi} \times \frac{I}{d}$
 (c) $B = \frac{\mu_0}{4\pi} \times \frac{2I}{d^2}$ (d) $B = \frac{\mu_0}{\pi} \times \frac{2I}{d}$

Ans :

SQP 2012

Length of the straight conductor = l Current in the conductor = I and distance of the point from it = d

The magnetic field at a point due to a long current-carrying conductor,

$$B = \frac{\mu_0}{4\pi} \times \frac{2I}{d}$$

where, μ_0 = absolute permeability of free space.

Thus (a) is correct option.

14. At what distance of a point, from a long straight wire carrying a current of 12 A will the magnetic field be equal to 3×10^{-5} T ?

- (a) 8 cm (b) 16 cm
 (c) 32 cm (d) 64 cm

Ans :

Delhi 2011

Current in wire, $I = 12$ Aand Magnetic field, $B = 3 \times 10^{-5}$ T

Distance of the point from the wire,

$$\begin{aligned} r &= \frac{\mu_0}{2\pi} \times \frac{I}{B} \\ &= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{12}{3 \times 10^{-5}} \\ &= 8 \times 10^{-2} \text{ m} = 8 \text{ cm} \end{aligned}$$

where μ_0 = absolute permeability of free spaceequal to $4\pi \times 10^{-7} \text{ WbA}^{-1}\text{m}^{-1}$

Thus (a) is correct option.

15. The magnitude of magnetic field at a point due to a current-carrying small element does not depend upon
 (a) current in the element
 (b) length of the element
 (c) diameter of the element
 (d) distance of the point from the element

Ans :

OD 2001

Magnitude of magnetic field at a point due to a current-carrying small element,

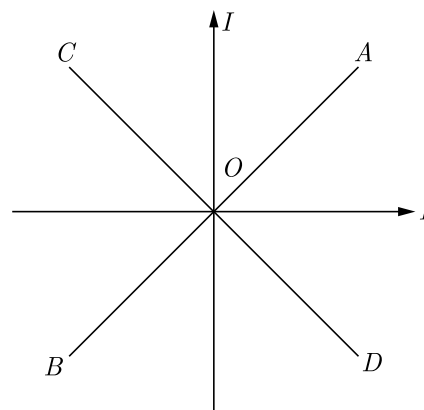
$$(d\vec{B}) = \frac{\mu_0 i dl \sin \theta}{4\pi r^2}$$

where i = Current in the element, l = Length of the element and r = Distance of the point from the element.

Therefore, magnitude of magnetic field at a point due to a current-carrying small element does not depend upon the diameter of the element.

Thus (c) is correct option.

16. Two equal electric currents are flowing perpendicular to each other as shown in the figure. AB and CD are perpendicular to each other and symmetrically placed with respect to the currents. Where do we expect the resultant magnetic field to be zero ?



- (a) on AB
 (b) on CD
 (c) on both AB and CD
 (d) on both OD and BO

Ans :

Foreign 2006, OD 2012

Direction of the magnetic field, due to current, is given by the Right-hand thumb rule. Since, the components of magnetic field due to two electric currents on the axis AB cancel each other, therefore, the resultant magnetic field on AB will be zero.

Thus (a) is correct option.

17. An electric current passes through a long straight copper wire. At a distance 5 cm from the straight wire, the magnetic field is B . The magnetic field at 20 cm from the straight wire would be

- (a) $\frac{B}{6}$ (b) $\frac{B}{4}$
 (c) $\frac{B}{3}$ (d) $\frac{B}{2}$

Ans :

SQP 2016

Distance of first point from the wire,

$$r_1 = 5 \text{ cm}$$

Magnetic field at first point,

$$B_1 = B$$

and distance of second point from the wire,

$$r_2 = 20 \text{ cm}$$

Since, $B = \frac{\mu_0 i}{2R}$

$$i = \frac{2BR}{\mu_0} \quad \dots(2)$$

$$A = \pi R^2 \text{ (for circular loop)} \quad \dots(3)$$

Substituting the value of Eq. (3) and (2) in Eq. (1), we get,

$$\begin{aligned} M &= \frac{2BR}{\mu_0} \times A \\ &= \frac{2BR \times A}{\mu_0} = \frac{2BRA}{\mu_0} \times \frac{\sqrt{A}}{\sqrt{A}} \\ &= \frac{2BRA^{3/2}}{\mu_0 \sqrt{\pi R^2}} \quad [\text{From Eq. (3)}] \\ &= \frac{2BA^{3/2}}{\mu_0 \sqrt{\pi}} \end{aligned}$$

Thus (d) is correct option.

- 25.** A proton moving with a velocity $2.5 \times 10^7 \text{ ms}^{-1}$, enters a magnetic field of intensity 2.5 T at an angle 30° with the magnetic field. The force on the proton is

- (a) $3 \times 10^{-12} \text{ N}$ (b) $5 \times 10^{-12} \text{ N}$
(c) $6 \times 10^{-12} \text{ N}$ (d) $9 \times 10^{-12} \text{ N}$

Ans :

OD 2011

Velocity of proton, $v = 2.5 \times 10^7 \text{ ms}^{-1}$

Intensity of magnetic field, $B = 2.5 \text{ T}$

and angle between magnetic field and direction of motion of proton,

$$\theta = 30^\circ$$

Force acting on the charged particle in magnetic field,

$$F = qvB \sin \theta$$

where, q = charge of proton equal to $1.6 \times 10^{-19} \text{ C}$
 $= (1.6 \times 10^{-19}) \times (2.5 \times 10^7)$
 $\times 2.5 \times \sin 30^\circ$

$$\begin{aligned} &= (10 \times 10^{-12}) \times 0.5 \\ &= 5 \times 10^{-12} \text{ N} \end{aligned}$$

Thus (b) is correct option.

- 26.** A current-carrying coil is placed in a uniform magnetic field of induction B . The current in the coil is I , it has n turns and A is the face area of coil and normal to the surface makes an angle θ with B . The torque experienced by the coil is

- (a) $nIAB\theta$ (b) $nIAB \cos \theta$
(c) $nIAB \sin \theta$ (d) $nIAB \tan \theta$

Ans :

SQP 2014

Magnetic field induction $= B$

Current in coil $= I$

Numbers of turns in coil $= n$

Area of coil $= A$

and angle between normal to the surface of coil and magnetic field $= \theta$.

Torque experienced by the coil,

$$\tau = nIAB \sin \theta$$

Thus (c) is correct option.

- 27.** The radius of a circular path in which an electron will move, when subjected to a perpendicular uniform magnetic field (B), is

- (a) $\frac{me}{B}$ (b) $\frac{mB}{e}$
(c) $\frac{Be}{mv}$ (d) $\frac{mv}{Be}$

Ans :

Delhi 2009

Centrifugal force on the electron,

$$\begin{aligned} F_1 &= m\omega^2 r \\ &= m\left(\frac{v}{r}\right)^2 r = \frac{mv^2}{r} \end{aligned}$$

And magnetic force on the moving charge in the magnetic field,

$$F_2 = Bev$$

Equating these two forces,

$$\begin{aligned} \frac{mv^2}{r} &= Bev \\ r &= \frac{mv}{Be} \end{aligned}$$

where, e = charge on electron

and r = Radius of circular path

Thus (d) is correct option.

- 28.** A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to the magnetic field B . The time-taken by the particle to complete one revolution is

- (a) $\frac{2\pi m}{qB}$ (b) $\frac{2\pi qB}{m}$
(c) $\frac{2\pi mq}{B}$ (d) $\frac{2\pi q^2 B}{m}$

Ans :

OD 2007, Comp 2002

Mass of particle $= m$

Charge on particle $= q$

Radius of circular path $= r$

and, Magnetic field $= B$

Magnetic force on the charged particle in magnetic field,

$$F_1 = Bqv$$

and centrifugal force on the charged particle moving in circular path,

- 38. Assertion :** In electric circuits, wires carrying currents in opposite directions are often twisted together.

Reason : If the wires are not twisted together, the combination of the wires forms a current loop, the magnetic field generated by the loop might affect adjacent circuits or components.

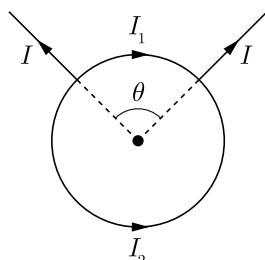
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

If the wires are twisted together, they can be formed as a single wire carrying currents in opposite directions. In this pattern, in wires no magnetic field is induced which does not affect adjacent circuits.

Thus (a) is correct option.

- 39. Assertion :** The magnetic field at the centre of the circular coil in the following figure due to the currents I_1 and I_2 is zero.



Reason : $I_1 = I_2$ implies that the fields due to the current I_1 and I_2 will be balanced.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

$$\frac{I_1}{I_2} = \frac{2\pi - \theta}{\theta}$$

$$I_1 \theta = I_2 (2\pi - \theta) \quad \dots(1)$$

$$B_1 = \frac{\theta}{2\pi} \cdot \frac{\mu_0 I_1}{2R}$$

and

$$B_2 = \frac{2\pi - \theta}{2\pi} \cdot \frac{\mu_0 I_2}{2R}$$

Using (1), we get

$$B_1 = B_2$$

Thus (d) is correct option.

- 40. Assertion :** The magnetic field produced by a current carrying solenoid is independent of its length and cross-sectional area.

Reason : The magnetic field inside the solenoid is uniform.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Magnetic field inside the solenoid at point P is given by,

$$B = \frac{\mu_0}{4\pi} (2\pi n i) [\sin \alpha + \sin \beta]$$

where n = no. of turns per unit length = N/ℓ

Thus it is clear that magnetic field is independent of length and cross sectional area.

Also the magnetic field within the solenoid is uniform and parallel to the axis of the solenoid.

Thus (b) is correct option.

- 41. Assertion :** Free electrons always keep on moving in a conductor even then no magnetic force act on them in magnetic field unless a current is passed through it.

Reason : The average velocity of free electron is zero.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

In the absence of the electric current, the free electrons in a conductor are in a state of random motion, like molecular in a gas. Their average velocity is zero. i.e. they do not have any net velocity in a direction. As a result, there is no net magnetic force on the free electrons in the magnetic field. On passing the current, the free electrons acquire drift velocity in a definite direction, hence magnetic force acts on them, unless the field has no perpendicular component.

Thus (b) is correct option.

- 42. Assertion :** If the current in a solenoid is reversed in direction while keeping the same magnitude, the magnetic field energy stored in the solenoid decreases.

Reason : Magnetic field energy density is proportional to square of current.

Since, when current in both the conductors flows in the same direction, therefore force between them will be attractive.

Thus (a) is correct option.

- 33.** A current-carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon
- area of loop
 - shape of loop
 - no. of turns in loop
 - strength of current and magnetic field

Ans :

Foreign 2017

Torque acting on a current-carrying loop in a uniform magnetic field,

$$\tau = nIAB \sin \theta$$

Therefore, torque does not depend on the shape of loop.

(where I = current, A = area of loop, B = magnetic field, n = no. of turns in loop and θ = Angle between magnetic field and normal to the surface of coil)

Thus (b) is correct option.

- 34.** On connecting a battery to the two corners of a diagonal of a square conductor frame of side a , the magnitude of magnetic field at the centre will be
- zero
 - $\frac{\mu_0}{\pi a}$
 - $\frac{\mu_0}{2\pi a}$
 - $\frac{2\mu_0}{\pi a}$

Ans :

OD 2005

When a battery is connected two corners of a diagonal of a square conductor frame, then the conductor frame can be taken as two sets of parallel wires carrying currents in the same direction. Magnetic field at the centre due to two parallel wires carrying currents in the same direction will be equal in magnitude, but opposite in directions.

Therefore, magnitude of the magnetic field at the centre will be zero.

Thus (a) is correct option.

- 35.** A long solenoid has 800 turns per meter length of solenoid. A current of 1.6 A flows through it. The magnetic field induction at the end of the solenoid on its axis is
- 4×10^{-4} T
 - 8×10^{-4} T
 - 16×10^{-4} T
 - 32×10^{-4} T

Ans :

Delhi 2013

Number of turns per metre length,

$$n = 800$$

Current in solenoid, $I = 1.6$ A

Magnetic field induction at the end of the solenoid on its axis,

$$\begin{aligned} B &= \frac{\mu_0 n I}{2} \\ &= \frac{(4\pi \times 10^{-7}) \times 800 \times 1.6}{2} \\ &= 8.038 \times 10^{-4} \approx 8 \times 10^{-4} \text{ T} \end{aligned}$$

(where μ_0 = Absolute permeability of free space equal to $4\pi \times 10^{-7} \text{ Wb} - \text{A}^{-1} - \text{m}^{-1}$)

Thus (b) is correct option.

- 36.** A circular loop of area 0.01 m^2 carrying a current of 10 A, is held perpendicular to a magnetic field of intensity 0.1 T. The torque acting on the loop is
- zero
 - 0.01 N-m
 - 0.1 N-m
 - 0.8 N-m

Ans :

OD 2010, SQP 2005

Area of circular loop, $A = 0.01 \text{ m}^2$

Current in the loop, $I = 10$ A

Angle between the magnetic field and normal to the surface of coil,

$$\theta = 0^\circ$$

Magnetic field, $B = 0.1$ T

Torque acting on the loop,

$$\begin{aligned} \tau &= IAB \sin \theta \\ &= 10 \times 0.01 \times 0.1 \times \sin 0^\circ \\ &= 0 \end{aligned}$$

Thus (a) is correct option.

ASSERTION AND REASON

- 37. Assertion :** To convert a galvanometer into an ammeter a small resistance is connected in parallel with it.
- Reason :** The small resistance increases the combined resistance of the combination.
- Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 - Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 - Assertion is correct but Reason is incorrect.
 - Assertion is incorrect but Reason is correct.

Ans :

An ammeter should have a low resistance which we get when we connect low resistance in parallel with galvanometer.

Thus (c) is correct option.

$$F_2 = \frac{mv^2}{r}$$

Since charged particle is moving in circular path,

Therefore,

$$F_1 = F_2$$

$$Bqv = \frac{mv^2}{r}$$

$$v = \frac{Bqr}{m}$$

Time-taken by the particle to complete one revolution

$$= \frac{\text{Circumference}}{\text{Velocity}}$$

$$= \frac{2\pi r}{v} = \frac{2\pi r}{\frac{Bqr}{m}}$$

$$= \frac{2\pi m}{Bq}$$

Thus (a) is correct option.

- 29.** A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then

- (a) its velocity will increase
(b) its velocity will decrease.
(c) it will turn towards left.
(d) it will turn towards right.

Ans :

Foreign 2010

When an electron is projected in a region of uniform electric and magnetic fields acting along the same direction, then electric field will apply force on the electron in the opposite direction to the motion, whereas magnetic field will not apply any force on the electron. Thus the electron experiences retardation.

As a result of this, velocity of the electron will decrease. Thus (b) is correct option.

- 30.** Two thin, long, parallel wires, separated by a distance d carry a current of (i) A in the same direction. They will

- (a) attract each other with a force of $\mu_0 i^2 / (2\pi d)$
(b) repel each other with a force of $\mu_0 i^2 / (2\pi d)$
(c) attract each other with a force of $\mu_0 i^2 / (2\pi d^2)$
(d) repel each other with a force of $\mu_0 i^2 / (2\pi d^2)$

Ans :

SGP 2003

Distance between wires = d

and current in each wire = i A

We know that force on each wire due to current,

$$F = \frac{\mu_0 i^2}{2\pi d}$$

Since, current flows in both the parallel wires in the

same direction, therefore they attract each other with a force of $\mu_0 i^2 / 2\pi d$.

Thus (a) is correct option.

- 31.** Two solenoids of the same length having number of turns in the ratio of 2:3 are connected in series. The ratio of magnetic fields at their centres is

- (a) 2:1 (b) 3:1
(c) 2:3 (d) 3:2

Ans :

Delhi 2014

No. of turns in first solenoid, $N_1 = 2N$

No. of turns in second solenoid, $N_2 = 3N$

We know that number of turns per unit length,

$$n = \frac{N}{L}$$

where, L = length of solenoid

Magnetic field at the centre of solenoid,

$$B = \mu_0 n I$$

$$= \mu_0 \left(\frac{N}{L} \right) I \propto N$$

Therefore,
$$\frac{B_1}{B_2} = \frac{N_1}{N_2} = \frac{2N}{3N} = \frac{2}{3}$$

$$B_1 : B_2 = 2 : 3$$

Thus (c) is correct option.

- 32.** Two long straight parallel conductors separated by a distance of 0.5 m carry currents of 5 A and 8 A in the same direction. The force per unit length experienced by each other is

- (a) 1.6×10^{-5} N (attractive)
(b) 1.6×10^{-5} N (repulsive)
(c) 16×10^{-5} N (attractive)
(d) 16×10^{-5} N (repulsive)

Ans :

OD 2000, Delhi 2017

Distance between conductors,

$$d = 0.5 \text{ m}$$

Current in first conductor,

$$I_1 = 5 \text{ A}$$

and current in second conductor,

$$I_2 = 8 \text{ A}$$

Force experienced by each conductor due to currents,

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$= \frac{(4\pi \times 10^{-7}) \times 5 \times 8 \times 1}{2\pi \times 0.5}$$

$$= 1.6 \times 10^{-5} \text{ N}$$

(where, μ_0 = Absolute permeability of free space equal to $4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1}$)

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Reversing the direction of the current reverses the direction of the magnetic field. However, it has no effect on the magnetic field energy density, which is proportional to the square of the magnitude of the magnetic field.

Thus (d) is correct option.

- 43. Assertion :** A charge, whether stationary or in motion produces a magnetic field around it.

Reason : Moving charges produce only electric field in the surrounding space.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

A charge, whether stationary or in motion, produces an electric field around it. If it is in motion, then in addition to the electric field, it also produces a magnetic field, because moving charges produce magnetic field in the surrounding space.

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

- 44.** Using the concept of force between two infinitely long parallel current carrying conductors define one ampere of current.

Ans :

Delhi 2020

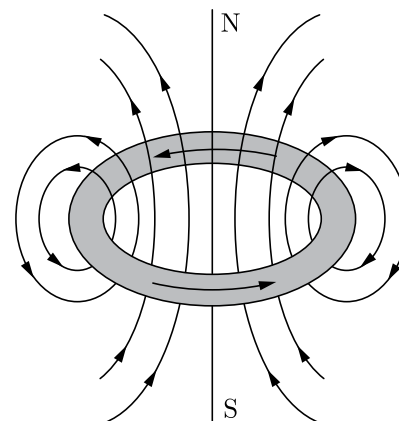
One ampere is that value of current which flows through two straight, parallel infinitely long current carrying conductors placed in air or vacuum at a distance of 1 m and they experience a force of attractive or repulsive nature of magnitude 2×10^{-7} N/m on their unit length.

- 45.** Draw the magnetic field lines due to a current carrying loop.

Ans :

Comp 2021

Magnetic field lines due to a current carrying loop are given by



- 46.** Is the steady electric current the only source of magnetic field? Justify your answer.

Ans :

OD 2020

Yes, the net magnetic force acting on a wire carrying a steady (constant) electric current I in an external magnetic field B and is given by

$$F = IdlB$$

- 47.** State Ampere's circuit law.

Ans :

Delhi 2016

It states that the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_0 times the total current I passing through this closed circuit.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

- 48.** Write the expression, in a vector form, for the Lorentz magnetic force $\vec{F}$ due to a charge moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. What is the direction of the magnetic force ?

Ans :

Delhi 2021, OD 2012

Force, $\vec{F} = q(\vec{v} \times \vec{B})$

Obviously, the force on charged particle is perpendicular to both velocity $\vec{v}$ and magnetic field $\vec{B}$.

- 49.** Must every magnetic field configuration have a north pole and a south pole? Explain with example.

Ans :

Foreign 2011

Ans :

No, because the concept of magnetic poles is fictitious; the fundamental quantity in magnetism is magnetic moment. Therefore, a toroid (endless solenoid) and a straight conductor do not have north and south poles.

- 50.** What will be the path of a charged particle moving along the direction of a uniform magnetic field ?

Ans :

OD 2019

The path of particle will remain unchanged (since magnetic force $F_m = qvB\sin\theta = 0$).

When $\sin 2\theta$ would be maximum than the tangent galvanometer has maximum,

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1} 1$$

$$2\theta = \sin^{-1} \sin 90^\circ \text{ [Since, } \sin 90 = 1 \text{]}$$

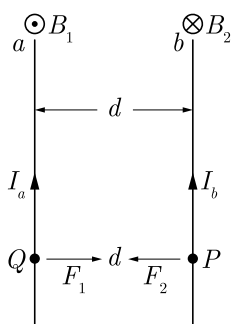
$$\theta = \frac{90}{2} = 45^\circ$$

70. Two long straight parallel conductors a and b carrying steady currents I_a and I_b respectively are separated by a distance d . Write the magnitude and direction, what is the nature and magnitude of the force between the two conductors?

Ans :

Delhi 2011

Let, a and b be two long straight parallel conductors I_a and I_b are the current flowing through them and separated by a distance d . Magnetic field induction at a point P on a conductor b due to current I_a passing through a is



$$B_1 = \frac{\mu_0 2I_a}{4\pi d}$$

Now, unit length of b will experience a force as

$$F_2 = B_1 I_b \times 1 = B_1 I_b$$

$$F_2 = \frac{\mu_0}{4\pi} \frac{2I_a I_b}{d}$$

Conductor a also experiences the same amount of force directed towards b . Hence, a and b attract each other.

71. Define one tesla using the expression for the magnetic force acting on a particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$.

Ans :

OD 2013

One tesla is the magnetic field in which a charge of 1C moving with a velocity of 1ms^{-1} , normal to the magnetic field, experiences a force of 1 N.

$$B = \frac{F}{qv \sin \theta}$$

If $F = 1 \text{ N}$

$$q = 1 \text{ C}$$

$$v = 1 \text{ ms}^{-1} \text{ and } \theta = 90^\circ$$

$$\begin{aligned} \text{Then, SI unit of } B &= \frac{1 \text{ N}}{1 \text{ C} \cdot 1 \text{ ms}^{-1} \cdot \sin 90^\circ} \\ &= 1 \text{ NA}^{-1} \text{ m}^{-1} = 1 \text{ tesla} \end{aligned}$$

72. (i) State Biot-Savart law in vector form expressing the magnetic field due to an element $\vec{dl}$ carrying current I at a distance $\vec{r}$ from the element.
(ii) Write the expression for the magnitude of the magnetic field at the centre of a circular loop of radius r carrying a steady current I . Draw the field lines due to the current loop.

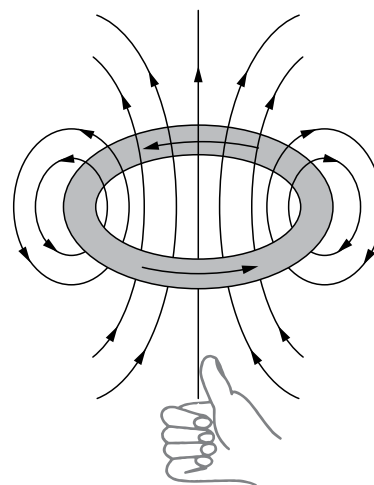
Ans :

Delhi 2014, OD 2009

- (i) It states that for a small current element dl the magnetic field at a distance r is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \hat{r})}{r^2}$$

- (ii) Magnetic field at the centre of a circular loop is given by the field lines are as shown :



73. Show how the equation for Ampere's circuital law, viz.

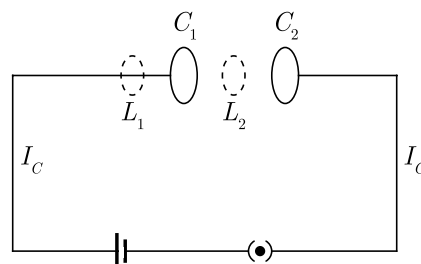
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

is modified in the presence of displacement current.

Ans :

OD 2019

During charging of a capacitor, let at any instant transient current (I_C) flows through the wire.



to its coil. So, the galvanometer gives full scale deflection.

2. In converting a galvanometer into an ammeter, a very small suitable resistance is connected in parallel to its coil. The remaining part of the current i.e., $(I - I_g)$ flows through the resistance.

Here, I = Circuit current

and I_g = Current through galvanometer

59. Write any two important points of similarities between Coulomb's law for the electrostatic field and Biot-Savart's law for the electrostatic field and Biot-Savart's law for the magnetic field.

Ans :

Delhi 2020

Similarities of electrostatic field and magnetic field:

1. Follows the principle of superposition.
 2. Depends inversely on the square of distance from source to the point of interest.
60. Two particles A and B of masses m and $2m$ have charges q and $2q$ respectively. They are moving with velocities v_1 and v_2 respectively. In the same direction, enters the same magnetic field B acting normally to their direction of motion. If the two forces F_A and F_B acting on them are in the ratio of 1 : 2, find the ratio of their velocities.

Ans :

Comp 2019, OD 2011

Ratio of forces acting on the two particles,

$$\frac{F_A}{F_B} = \frac{qv_1 B \sin 90^\circ}{(2q)v_2 B \sin 90^\circ}$$

$$= \frac{1}{2} = \frac{v_1}{2v_2}$$

$$\frac{v_1}{v_2} = 1$$

$$v_1 : v_2 = 1 : 1$$

61. A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the direction of initial velocity of the particle?

Ans :

SQP 2013

Magnetic force on the particle,

$$F_m = qvB \sin \theta$$

The particle will remain undeflected if,

$$\sin \theta = 0$$

$$\theta = 0^\circ \text{ or } 180^\circ$$

Therefore, the initial velocity of the particle should be parallel or anti-parallel to the magnetic field (i.e., either east to west or west to east).

SHORT ANSWER QUESTIONS

62. Write an expression for the maximum kinetic energy acquired by charged particles accelerated by a cyclotron.

Ans :

OD 2018

The ions will attain maximum velocity near the periphery of the dees. If v_0 is the maximum velocity acquired by the ions and r_0 is the radius of the dees, then

$$\frac{mv_0^2}{r_0} = qv_0 B$$

$$v_0 = \frac{qBr_0}{m}$$

The maximum kinetic energy of the ions will be,

$$K_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{qBr_0}{m}\right)^2$$

$$K_0 = \frac{q^2 B^2 r_0^2}{2m}$$

63. Give some points of similarities and differences between Biot-Savart law for the magnetic field and Coulomb's law for the electrostatic field.

Ans :

Delhi 2021

According to Coulomb's law, the electric field produced by a charged element is,

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

According to Biot-Savart law, the magnetic field produced by a current element $I dl$ is,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \theta}{r^2}$$

On comparing the above two equations, we can note the following points:

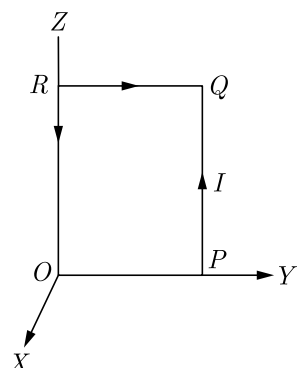
Points of Similarity

1. Both fields depend inversely on the square of the distance from the source.
2. Both are long range fields.
3. The principle of superposition is applicable to both fields.

Points of Difference

1. The magnetic field is produced by a vector source: the current element $I \vec{dl}$. The electrostatic field is produced by a scalar source: the electric charge dg .
2. The direction of the electrostatic field is along the displacement vector joining the source and the field point. The direction of the magnetic field is perpendicular to the plane containing the displacement vector $\vec{r}$ and the current element $I \vec{dl}$.
3. In Bio-Savart law, the magnitude of the magnetic field is proportional to the sine of the angle between

51. A square coil $OPQR$ of side a carrying a current I , is placed in the $Y-Z$ plane as shown here. Find the magnetic moment associated with this coil.



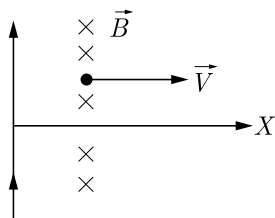
Ans :

Foreign 2015

The magnetic moment associated with the coil, is

$$\vec{\mu}_m = Ia^2 \hat{i}$$

52. A long straight wire carries a steady current I along the positive Y -axis in a coordinate system. A particle of charge $+Q$ is moving with a velocity $\vec{v}$ along the X -axis. In which direction will the particle experience a force ?



Ans :

OD 2013

From relation, $\vec{F} = qvB[\hat{i} \times (-\hat{k})] = +qvB(\hat{j})$

Magnetic force $\vec{F}$ along $+Y$ axis.

or

From Fleming's left hand rule, thumb points along $+Y$ direction, so the direction of magnetic force will be along $+Y$ axis (or in the direction of flow of current).

53. Under what condition is the force acting on a charge moving through a uniform magnetic field is minimum?

Ans :

SQP 2013

$F_m = qvB \sin \theta$; for minimum force $\sin \theta = 0$, i.e., force is minimum when charged particle move parallel or anti-parallel to the field.

54. What is the nature of magnetic field in a moving coil galvanometer?

Ans :

Delhi 2017

The nature of magnetic field in a moving coil galvanometer is radial.

55. An electron moving through a field remains deflected. Is it possible that there is no magnetic field. Explain.

Ans :

Delhi 2016

Yes, it is possible. When a charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$ and $\vec{v}$ makes an angle θ with $\vec{B}$ then force on charge is given by,

$$\vec{F} = q\vec{v}B \sin \theta$$

If,

$$\theta = 0^\circ \text{ or } 180^\circ$$

Then,

$$F = 0 \text{ (Since, } \sin 0^\circ \text{ or } \sin 180^\circ = 0)$$

Hence, a charged particle moving parallel or anti-parallel to a magnetic field does not experience any force in the magnetic field.

56. A circular coil of closely wound N turns and radius r carries a current I . Write the expressions for the following:

1. The magnetic field at its centre.
2. The magnetic moment of this coil.

Ans :

Foreign 2013, Comp 2011

1. Magnetic field at centre due to circular current carrying coil,

$$B = \frac{\mu_0 NI}{2r}$$

2. Magnetic moment,

$$M = NIA = NI(\pi r^2) \\ = \pi NI r^2$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns.

57. When a charged particle moving with velocity $\vec{v}$ is subjected to magnetic field $\vec{B}$, the force acting on it is non-zero. Would the particle gain any energy?

Ans :

OD 2016

This is because the charge particle moves on a circular path.

$$\vec{F} = q(\vec{v} \times \vec{B})$$

and power dissipated

$$P = \vec{F} \cdot \vec{v}$$

$$= q(\vec{v} \times \vec{B}) \cdot \vec{v}$$

$$= q(\vec{v} \times \vec{v}) \cdot \vec{B}$$

$$\text{(since } \vec{v} \times \vec{v} = 0)$$

$$= 0$$

The particle does not gain any energy.

58. Explain, giving reasons, the basic difference in converting a galvanometer into (1) a voltmeter and (2) an ammeter.

Ans :

Foreign 2010

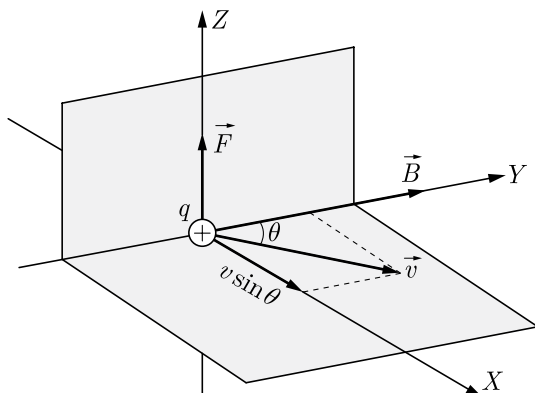
1. In converting a galvanometer into a voltmeter, a very high suitable resistance is connected in series

67. State the factors on which the force acting on a charge moving in a magnetic field depends. Write the expression for this force. When is this force minimum and maximum?

Ans :

OD 2019

As shown in figure, suppose a charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$ at angle θ .



Magnetic Lorentz Force

From experiments, it is found that the charge q experiences a force $\vec{F}$ such that,

1. The force is proportional to the magnitude of the magnetic field, i.e., $F \propto B$
2. The force is proportional to the charge q , i.e., $F \propto q$
3. The force is proportional to the component of the velocity v in the perpendicular direction of the field B , i.e., $F \propto v \sin \theta$

Combining the above factors, we get

$$F \propto Bqv \sin \theta$$

$$F = kqvB \sin \theta$$

The unit of magnetic field is so defined that the proportionality constant k becomes unity in the above equation. Then

$$F = qvB \sin \theta$$

As the direction of $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$, so we can express $\vec{F}$ as

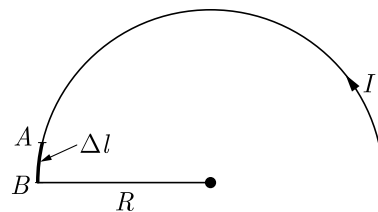
$$\vec{F} = q(\vec{v} \times \vec{B})$$

68. Applying Biot-Savart's law deduce the expression for the magnetic field at the centre of a semicircular loop of radius R carrying current I .

Ans :

Delhi 2020

Consider a semicircular coil of radius R carrying current I in anti-clock wise direction as shown in the figure according to Biot Savart's law the magnetic field due to current element AB at centre O is,



$$\Delta B = \frac{\mu_0}{4\pi} \frac{I \Delta l \sin \theta}{R^2}$$

where θ is angle between current element AB and the line joining the element to centre O .

Here, $\theta = 90^\circ$

$$\text{Hence, } \Delta B = \frac{\mu_0}{4\pi} \frac{I dl}{R^2}$$

Resultant magnetic field,

$$B = \int \Delta B$$

$$= \int \frac{\mu_0 I dl}{4\pi R^2} = \frac{\mu_0 I}{4\pi R^2} \int dl$$

For Semicircle, $\int dl = \pi R$

$$= \frac{\mu_0 I}{4\pi R^2} \times \pi R = \frac{\mu_0 \cdot I}{4R}$$

69. Show that a tangent galvanometer measures that current with maximum accuracy which produces 45° deflection.

Ans :

Foreign 2013, SQP 2008

For a tangent galvanometer,

$$i = k \tan \theta \quad \dots (1)$$

$$\frac{di}{d\theta} = k \sec^2 \theta$$

$$\left[\text{Since, } \frac{d(\tan \theta)}{d\theta} = \sec^2 \theta \right]$$

$$\frac{di}{i} = \frac{k \sec^2 \theta d\theta}{i}$$

From Eq. (1), we get

$$\frac{di}{i} = \frac{k \sec^2 \theta d\theta}{k \tan \theta}$$

$$\frac{di}{i} = \frac{1}{\cos^2 \theta} \times \frac{\cos \theta d\theta}{\sin \theta}$$

$$\left[\text{Since, } \sec \theta = \frac{1}{\cos \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\frac{di}{i} = \frac{d\theta}{\cos \theta \sin \theta}$$

$$\frac{di}{i} = \frac{2 d\theta}{2 \cos \theta \sin \theta}$$

$$\frac{di}{i} = \frac{2 d\theta}{\sin 2\theta} \quad [\text{since } 2\theta = 2 \sin \theta \cos \theta]$$

$$d\theta = \frac{\sin 2\theta di}{2i}$$

the current element $I \vec{dl}$ and displacement vector $\vec{r}$ while there is no such angle dependence in the Coulomb's law for the electrostatic field.

64. Define magnetic field in terms of the force on a moving charge. Hence define one tesla.

Ans :

Comp 2020

Definition of Magnetic Field

$$\text{Magnetic Field } B = \frac{F}{qv \sin \theta}$$

$$\text{If } q = 1, v = 1, \theta = 90^\circ, \sin 90^\circ = 1$$

$$\text{Then, } B = F$$

Thus the magnetic field at a point may be defined as the force acting on a unit charge moving with a unit velocity at right angles to the direction of the field.

SI Unit of Magnetic Field

$$\text{Again, } B = \frac{F}{qv \sin \theta}$$

$$\text{If } F = 1 \text{ N}, q = 1 \text{ C}, v = 1 \text{ ms}^{-1} \text{ and } \theta = 90^\circ, \text{ then}$$

$$\begin{aligned} \text{Hence, SI unit of } B &= \frac{1 \text{ N}}{1 \text{ C} \cdot 1 \text{ ms}^{-1} \cdot \sin 90^\circ} \\ &= \frac{1 \text{ N}}{1 \text{ A} \cdot 1 \text{ m}} \quad [\text{since } \text{Cs}^{-1} = \text{A}] \\ &= 1 \text{ NA}^{-1} \text{m}^{-1} = 1 \text{ tesla} \end{aligned}$$

Thus the SI unit of magnetic field is tesla (T).

One tesla is that magnetic field in which a charge of 1 C moving with a velocity of 1 ms^{-1} at right angles to the field experiences a force of one newton.

65. State the Bio-Savart law for the magnetic field due to a current carrying element. Use this law to obtain a formula for magnetic field at the centre of a circular loop of radius R carrying a steady current I . Indicate the direction of the magnetic field.

Ans :

Comp 2021, Foreign 2005

According to Bio-Savart's law, the magnetic field due to a current element $\vec{dl}$ carrying current I at a point P at distance r from it is given by,

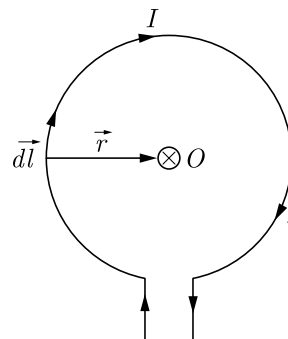
$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin \theta}{r^2}$$

where, μ_0 is permeability of free space and θ , is the angle between $\vec{dl}$ and $\vec{r}$.

Magnetic field at the centre of a circular current carrying coil: Consider a circular loop of wire of radius r carrying a current I . Consider a current element $\vec{dl}$ of the loop.

The direction of $\vec{dl}$ is along the tangent, so $\vec{dl} \perp \vec{r}$. From Bio-Savart law, magnetic field at the centre O due to this current element is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin 90^\circ}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2}$$



Magnetic Field at the Centre of a Circular Current Loop

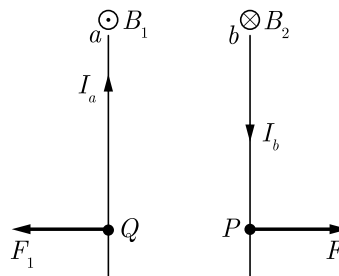
The magnetic field due to all such current elements will point into the plane of paper at centre O as shown by encircled cross $\otimes$. Hence the total magnetic field at the centre O is,

$$\begin{aligned} B &= \int dB = \int \frac{\mu_0 I}{4\pi} \cdot \frac{dl}{r^2} \\ &= \frac{\mu_0 I}{4\pi r^2} \int dl \\ &= \frac{\mu_0 I}{4\pi r^2} \cdot l = \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r \\ &= \frac{\mu_0 I}{2r} \end{aligned}$$

66. Show with the help of a diagram, how the force between the two conductors would change when the currents in them flow in the opposite directions.

Ans :

OD 2015



Now, let the direction of current in conductor b be reversed. The magnetic field B_2 at point P due to current I_a flowing through a will be downwards.

Similarly, the magnetic field B_1 at point Q due to current I_b passing through b will also be downward as shown. The force on a will be, therefore towards the left. Also, the force on b will be towards the right. Hence, the two conductors will repel each other as shown.

Applying Ampere's circuital law for loops L_1 and L_2 , we get

$$\oint_{L_1} \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

$$\oint_{L_2} \vec{B} \cdot d\vec{l} = \mu_0 \times 0 = 0$$

This violates the concept of continuity of electric current. Maxwell introduced the concept of displacement current flowing in space due to varying electric field such that

$$I_C = I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

This maintained continuity of current.

Hence, Modified Ampere's circuital law is given by,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_C + I_D)$$

74. Considering the case of a parallel plate capacitor being charged, show how one is required to generalise Ampere's circuital law to include the term due to displacement current.

Ans :

OD 2017, SQP 2010

Ampere's circuital law conduction current during charging of a capacitor was found inconsistent. Therefore, Maxwell modified Ampere's circuital law by introducing displacement current.

Ampere's circuit law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ was modified to

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_C + I_D)$$

It is called modified Ampere's circuital law or Ampere-Maxwell's circuital law.

The displacement current arising due to time varying electric field is given by $I_D = \varepsilon_0 \frac{d\phi_E}{dt}$

Therefore, modified Ampere's circuital law may be expressed as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

The following inferences can be drawn from the above discussion

- The conduction and displacement currents are individually discontinuous, but the currents together possess the property of continuity through any closed electric circuit.
- The displacement current is precisely equal to the conduction current when the two present in different parts of the circuit.
- The displacement current arises due to the rate of change of electric flux (or electric field) between the two plates of the capacitor.
- Just as the conduction current is the displacement current is also the source of magnetic field.

75. Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Show that no work is done by this force on the charged particle.

Ans :

Delhi 2019

Magnetic Lorentz force,

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

$$\vec{F} \perp \vec{v}$$

Force is perpendicular to displacement made by charge particle.

Hence, $W = Fd \cos 90^\circ = 0$

[since, F and displacement are perpendicular to each other]

$$W = 0$$

No work is done by magnetic Lorentz force on the charge particle.

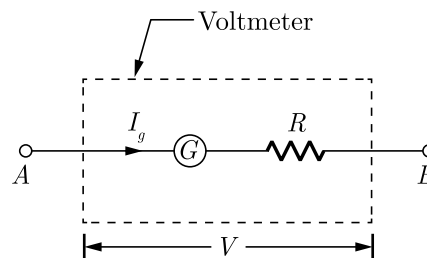
76. How is a moving coil galvanometer converted into a voltmeter? Explain giving the necessary circuit diagram and the required mathematical relation used.

Ans :

SQP 2009

A galvanometer can be converted into a voltmeter by connecting a very high resistance R in series with it.

Let R is so chosen that current I_g gives full deflection in the galvanometer where I_g is the range of galvanometer.



Let galvanometer G of resistance R , range I_g is to be converted into voltmeter of range V (volt). Now,

$$V = I_g (G + R)$$

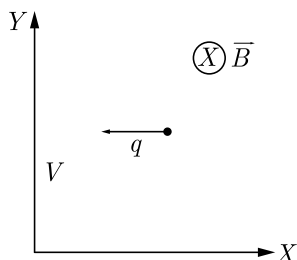
$$R + G = \frac{V}{I_g}$$

$$R = \frac{V}{I_g} - G$$

The appropriate scale need to be graduated to measure potential difference.

77. Write the expression for the force, $\vec{F}$ acting on a charged particle of charge q , moving with a velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields.

95. (i) A point charge q moving with speed V enters a uniform magnetic field B that is acting into the plane of the paper as shown. What is the path followed by the charge q and in which plane does it move ?
- (ii) How does the path followed by the charge get affected if its velocity has a component parallel to $\vec{B}$?
- (iii) If an electric field $\vec{E}$ is also applied such that the particle continues moving along the original straight line path, what should be the magnitude and direction of the electric field $\vec{E}$?



Ans :

SQP 2011

- (i) The force experienced by the charge particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

When $\vec{v}$ is perpendicular to $\vec{B}$, the force on the charge particle acts as the centripetal force and makes it move along a circular path. Path followed by charge is anticlockwise in $X-Y$ plane. The point charge moves in the plane perpendicular to both $\vec{v}$ and $\vec{B}$.

- (ii) A component of velocity of charge particle is parallel to the direction of the force of the electric field, the force experienced due to that component will be zero. This is because $F = qvB \sin 0^\circ = 0$. Thus, particle will move in straight line.

Also, the force experienced by the component perpendicular to $\vec{B}$ moves the particle in a circular path. The combined effect of both the components will move the particle in a helical path.

- (iii) Magnetic force on the charge, q

$$\begin{aligned}\vec{F}_B &= q(\vec{v} \times \vec{B}) \\ &= q[v(-\hat{i}) \times B(-\hat{k})]\end{aligned}$$

Hence, for moving charge, q in its original path

$$\vec{F}_E + \vec{F}_B = 0$$

$$\vec{F}_B = qvB(-\hat{j})$$

$$\text{Hence, } \vec{E} = vB(-\hat{j})$$

Taking magnitude both side,

$$|\vec{E}| = q \frac{vB}{q} = vB$$

Direction of Lorentz magnetic force is negative along Y -axis.

Therefore, direction of $\vec{E}$ is positive along Y -axis.

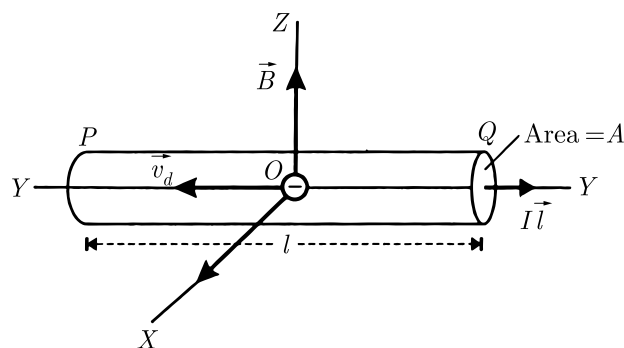
96. Derive a mathematical expression for the force acting on a current carrying straight conductor kept in a magnetic field. State the rule used to determine the direction of this force. Under what conditions if this force (1) zero and (2) maximum?

Ans :

OD 2019, SQP 2013

As shown in figure, consider a conductor PQ of length l , area of cross-section A , carrying current I along +ve Y -direction. The field $\vec{B}$ acts along +ve Z -direction. The electrons drift towards left with velocity $\vec{v}_d$. Each electron experiences a magnetic Lorentz force along +ve X -axis, which is given by,

$$\vec{f} = -e(\vec{v}_d \times \vec{B})$$



Force on a Current in a Magnetic Field

In n is the number of free electrons per unit volume, then total number of electrons in the conductor is,

$$N = n \times \text{volume} = nAl$$

Total force on the conductor is,

$$\begin{aligned}\vec{F} &= N\vec{f} = nAl[-e(\vec{v}_d \times \vec{B})] \\ &= enA[-l\vec{v}_d \times \vec{B}]\end{aligned}$$

If $I\vec{l}$ represents a current element vector in the direction of current, then vectors $\vec{l}$ and $\vec{v}_d$ will have opposite directions and we can take,

$$-l\vec{v}_d = v_d\vec{l}$$

$$\text{Hence, } \vec{F} = enAv_d(\vec{l} \times \vec{B})$$

$$\vec{F} = I(\vec{l} \times \vec{B}) \quad [\text{since } enAv_d = I]$$

Magnitude of Force

The magnitude of the force on the current carrying conductor is given by

$$F = IlB \sin \theta$$

where θ is the angle between the direction of the magnetic field and the direction of flow of current.

80. A steady current I_1 flows through a long straight wire. Another wire carrying steady current I_2 in the same direction is kept close and parallel to the first wire. Show with the help of a diagram, how the magnetic field due to the current I_1 exert a magnetic force on the second wire. Deduce the expression for this force.

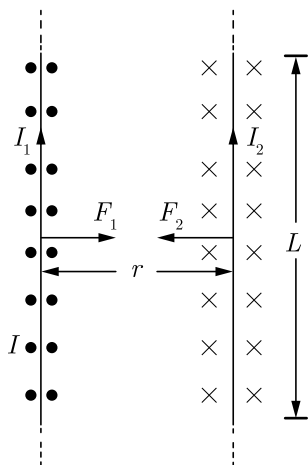
Ans :

OD 2011

Let two infinitely long straight current carrying conductor carry currents I_1 and I_2 in the same direction. Magnetic field B_1 due to first wire on seconds, i.e.,

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} \quad \dots(1)$$

The magnetic field is perpendicular to the plane of paper and directed inwards i.e., (X) type.



Now, magnetic force on L length of second wire is given by

$$F_2 = I_2 B_1 L \sin 90^\circ$$

$$F_2 = I_2 \left(\frac{\mu_0}{4\pi} \cdot \frac{2I_1}{r} \right) L$$

$$\frac{F_2}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \dots(2)$$

By Fleming's left hand rule, the direction of force F_2 is perpendicular to the second wire in the plane of paper towards the first wire.

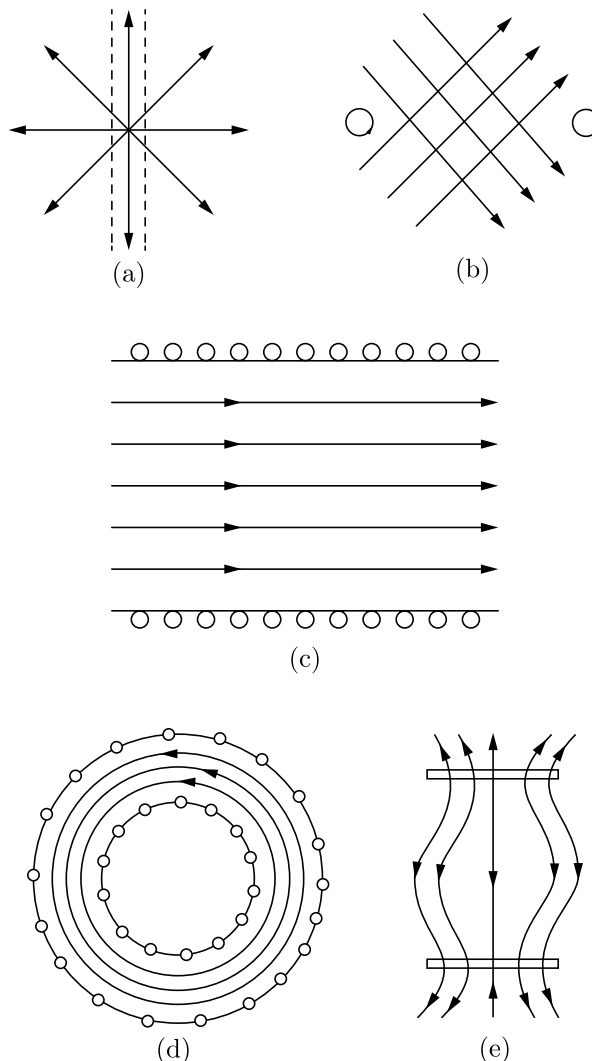
Similarly, magnetic force on 1st wire is given by

$$\frac{F_1}{L} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \quad \dots(3)$$

The force F_1 is directed towards the second wire.

Thus, two straight parallel current carrying conductor have the same direction of flow of currents attracting each other.

81. Following figures represent magnetic field lines drawn by some students. Point out which figures represent wrong field lines and why ?



Ans :

Foreign 2013

- (a) Wrong, because magnetic field lines never emanate from a point.
 (b) Wrong, because magnetic field lines never intersect.
 (c) Wrong, because magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined. The lines should curve out at both ends and meet eventually to form closed loops.
 (d) Right, because magnetic field is completely confined within a toroid.
 (e) Wrong, because the magnetic field lines are emerging from the upper plate and diverging at the lower plate; the net flux through a closed surface bounding the upper plate is not zero; this violates Ampere's circuital law. Actually these represent electric field lines when the upper plate is positively charged and the lower plate is negatively charged.

Ans :

SQP 2014

Force $\vec{F}$ acting on a charge q moving with velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$ is

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

Consider a region in which magnetic field, electric field and velocity of charge particle are perpendicular to each other.

To move charge particle undeflected the net force acting on the particle must be zero i.e., the electric force must be equal and opposite to the magnetic force.

$$qE = qvB$$

$$v = \frac{E}{B}$$

The direction of electric and magnetic forces are in opposite direction. Their magnitudes are in such a way that they cancel out each other to give net force zero so that the charge particle does not deflect.

78. Define current sensitivity and voltage sensitivity of a galvanometer. Increasing the current sensitivity may not necessarily increase the voltage sensitivity of a galvanometer. Justify.

Ans :

Delhi 2012

Current Sensitivity : The current sensitivity of a galvanometer is defined as the deflection product in the galvanometer when a unit charge flows through it. If a current i produces a deflection ϕ in the galvanometer, then current sensitivity is

$$\frac{\phi}{i} = \frac{NAB}{C} \quad \dots(1)$$

Voltage Sensitivity : The voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across its coil. If a voltage V produces a deflection ϕ in the galvanometer, then voltage sensitivity is

$$\frac{\phi}{V} = \frac{\phi}{iR} \quad \dots(2)$$

To explain that increasing the current sensitivity may not necessarily increase the voltage sensitivity let us assume that we double the number of turns i.e., $N \rightarrow 2N$, then

$$\text{Current sensitivity, } \frac{\phi}{i} \rightarrow 2 \frac{\phi}{i} \quad \dots(3)$$

So, current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. So,

$$N \rightarrow 2N \text{ and } R \rightarrow 2R$$

$$\text{So, voltage sensitivity, } \frac{\phi}{V} \rightarrow \frac{\phi}{V}$$

[As $\phi = \left(\frac{NAB}{C}\right) \frac{1}{R}$ remains unchanged.]

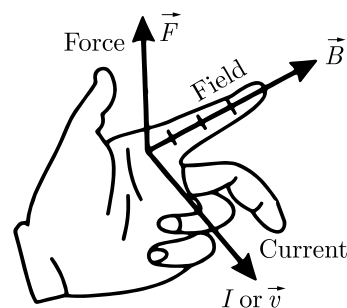
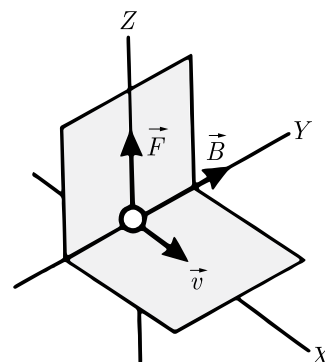
79. State the rules to find the direction of force on a charge moving in a perpendicular magnetic field.

Ans :

Delhi 2017, OD 2003

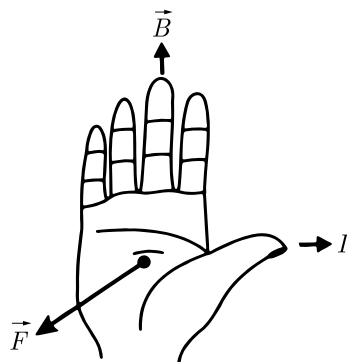
Rules for finding the direction of force on a charged particle moving perpendicular to a magnetic field :

1. **Fleming's Left Hand Rule :** Stretch the thumb and the first two fingers of the left hand mutually perpendicular to each other. If the forefinger points in the direction of the magnetic field, central finger in the direction of current, then the thumb gives the direction of the force on the charged particle.



Fleming's Left Hand Rule

2. **Right Hand (palm) Rule :** Open the right hand and place it so that tips of the fingers point in the direction of the field $\vec{B}$ and thumb in the direction of velocity $\vec{v}$ of the positive charge, then the palm faces towards the force $\vec{F}$.

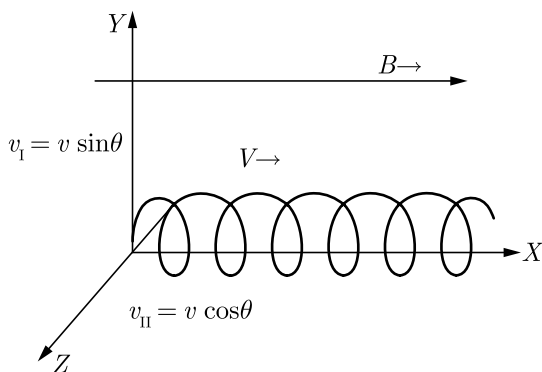


Right Hand Palm Rule

85. A uniform magnetic field $\vec{B}$ is set up along the positive X -axis. A particle of charge q and mass m moving with a velocity v enters the field at the origin in XY -plane such that it has velocity components both along and perpendicular to the magnetic field $\vec{B}$. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation.

Ans :

Delhi 2015



The path of the charged particle will be helix. As, the charge moves linearly in the direction of the magnetic field with velocity $v \cos \theta$ and also describe the circular path due to velocity $v \sin \theta$.

Time taken by the charge to complete one circular rotation,

$$T = \frac{2\pi r}{v_1} \quad \dots(1)$$

$$f = qv_1 B$$

and $\frac{mv_1^2}{r} = qv_1 B \quad \dots(2)$

From equation (1) and (2), we get

$$\frac{v_1^2 m}{r} = qv_1 B$$

$$\frac{v_1 m}{qB} = r$$

$$T = \frac{2\pi v_1 m}{qB \cdot v_1}$$

$$T = \frac{2\pi m}{Bq}$$

Distance moved by the particle along the magnetic field in one rotation (pitch of the helix path)

$$= v_{\parallel} \times T \quad (\text{since } v_{\parallel} = v_{\text{parallel}})$$

$$= v \cos \theta \times \frac{2\pi m}{Bq}$$

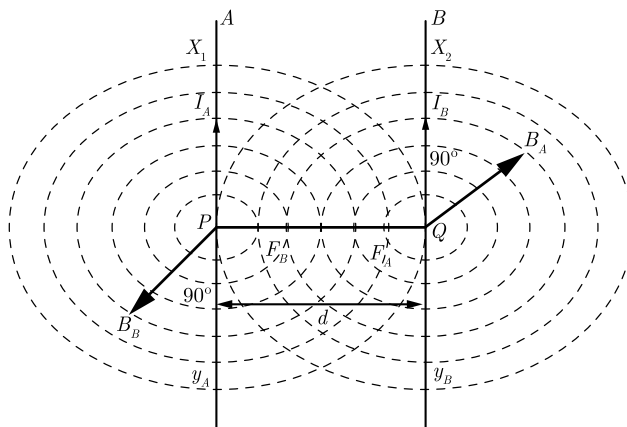
$$P = \frac{2\pi m v \cos \theta}{qB}$$

86. Two long parallel conductors A and B separated by a distance (d) carry steady currents I_A and I_B respectively. Show, by using Biot-Savart law and the Lorentz force that the magnitude of the force on a segment L of conductor B due to A is given by $F_{BA} = \frac{\mu_0 I_A I_B L}{2\pi d}$

Ans :

Foreign 2012, Comp 2001

Consider two parallel conductors A and B of length l long carrying currents I_A and I_B in same direction placed in vacuum at a distance d apart.



Magnetic field due to current I_A flowing through conductor A is given by

$$B_A = \frac{\mu_0 I_A}{2\pi d} \quad \dots(1)$$

Now, we can say that conductor $x_B y_B$ carrying current I_B is placed in magnetic field B , so force per unit length acting on it be F_A (since $F = ILB \sin \theta$), where $\theta = 90^\circ$

$$F_A = I_B B_A = \frac{\mu_0}{4\pi} \cdot \frac{2I_A I_B}{d} = \frac{\mu_0 I_A I_B}{2\pi d} \quad \dots(2)$$

Similarly, magnetic field due to I_B flowing through conductor B is given by,

$$B_B = \frac{\mu_0 I_B}{2\pi d}$$

and force F_B on A will be

$$F_B = I_A B_B = I_A \frac{\mu_0 I_B}{2\pi d} = \frac{\mu_0 \cdot I_A I_B}{2\pi d} \quad \dots(3)$$

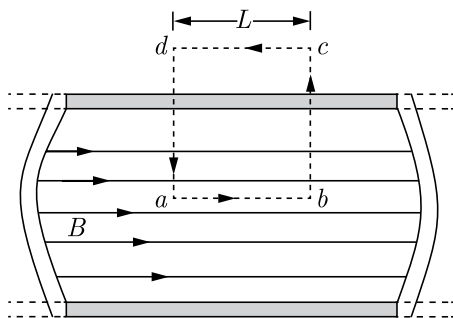
It is clear from equations (2) and (3), forces F_A and F_B are equal and opposite in direction (B_1 and B_2 are parallel but opposite in direction).

82. A long solenoid of length L having N turns carries a current I . Deduce the expression for the magnetic field in the interior of the solenoid.

Ans :

Comp 2018, SQP 2010

Consider a rectangular path $abcd$ of length L as shown in figure below. Let us apply Ampere's circuital law to this rectangular path, so that, we have



$$\begin{aligned}\oint \vec{B} \cdot d\vec{L} &= \oint_a^b \vec{B} \cdot d\vec{L} + \oint_b^c \vec{B} \cdot d\vec{L} + \oint_c^d \vec{B} \cdot d\vec{L} + \oint_d^a \vec{B} \cdot d\vec{L} \\ &= \oint_a^b \vec{B} \cdot d\vec{L} + 0 + 0 + 0 = BL \quad \dots(1)\end{aligned}$$

Since, $\oint_b^c \vec{B} \cdot d\vec{L} = \oint_d^a \vec{B} \cdot d\vec{L} = 0$ [since, $\phi = 90^\circ$]

$$\oint_c^d \vec{B} \cdot d\vec{L} = 0 \quad [\text{since, } B = 0]$$

But by Ampere's circuital law, we have

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 NI = \mu_0 (nL) I \quad \dots(2)$$

From equations (1) and (2), we have

$$BL = \mu_0 nLI$$

or $B = \mu_0 nI$

But $n = N/L$, therefore, we have

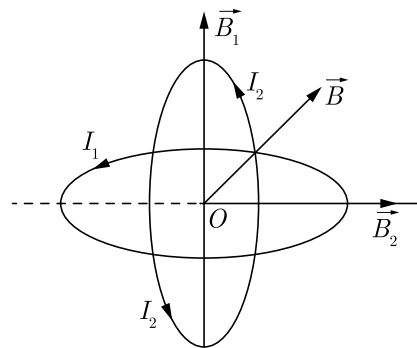
$$B = \mu_0 \frac{NI}{L}$$

This gives the value of magnetic field inside a solenoid.

83. Two identical coils, each of radius R and number of turns N are lying in perpendicular planes such that their centres coincide. Find the magnitude and direction of the resultant magnetic field at the centre of the coils, if they are carrying currents I and $\sqrt{3}I$ respectively.

Ans :

Comp 2008



Here,

$$I_1 = I, I_2 = \sqrt{3}I, N_1 = N_2 = N$$

$$R_1 = R_2 = R$$

The magnetic field induction at the centre O of two concentric coils inclined at 90° is

$$\begin{aligned}B &= \sqrt{B_1^2 + B_2^2} \\ &= \sqrt{\left(\frac{\mu_0 N_1 I_1}{2R_1}\right)^2 + \left(\frac{\mu_0 N_2 I_2}{2R_2}\right)^2} \\ &= \sqrt{\frac{\mu_0^2 N^2 I^2}{4R^2} + \frac{\mu_0 N (\sqrt{3}I)^2}{4R^2}} \\ &= \frac{\mu_0 NI}{2R} \sqrt{1+3} = \frac{\mu_0 NI}{R}\end{aligned}$$

84. Write the expression for the generalised form of Ampere's circuital law. Discuss its significance and describe briefly how the concept of displacement current is explained through charging/discharging of a capacitor in an electric circuit.

Ans :

OD 2015

The generalised form of Ampere's circuital law, modified by Maxwell states that

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 (I_C + I_D) \\ &= \mu_0 \left(1 + \epsilon_0 \frac{d\phi_E}{dt}\right)\end{aligned}$$

While dealing with the charging of a parallel plate capacitor with varying current. It was found that Ampere's circuital law is not logically consistent, because $\oint \vec{B} \cdot d\vec{l}$ has not the same value on the two sides of a plate of charged capacitor. The inconsistency of Ampere's circuital law was removed by Maxwell by predicting the presence of displacement current in the region between the plates of capacitor, when the charge on the capacitor is changing with time. So, displacement current is the missing term (which is related with the changing electric field which passes through the surface between the plates of capacitor).

In this way, Maxwell pointed out that for consistency of Ampere's circuital law, there must be displacement current I_D along with the conduction current in the closed loop as $(I_C + I_D)$ has the property of continuity.

LONG ANSWER QUESTIONS

90. Explain Biot-Savart law. Derive expression for the magnetic field at the centre of a current carrying circular loop.

Ans :

Comp 2019

According to Biot-Savart's Law

The magnitude of magnetic field induction (dB) at a point P due to a current element depends on the following factors.

1. $dB \propto I$ (i.e. magnetic field is directly proportional to the current flowing through the conductor).
2. $dB \propto dl$ (i.e. magnetic field is directly proportional to the length of the element).
3. $dB \propto \sin \theta$ (i.e. magnetic field is directly proportional to the sine of angle between the length of element and line joining the element to point (P)).
4. $dB \propto \frac{1}{r^2}$ (i.e. magnetic field is inversely proportional to the square of distance between the element and point P). Combining all the above relations,

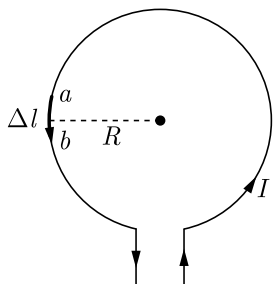
$$dB \propto \frac{Idl \sin \theta}{r^2}$$

The relation is called **Biot-Savart's law**.

Magnetic Field at the Centre of Circular Loop

Consider a circular coil of radius R carrying current I in anticlockwise direction. Say, O is the centre of coil, at which magnetic field is to be computed. The coil may be supposed to be formed of a large number of current elements. Consider a small current element ab of length Δl . According to Biot Savart law the magnetic field due to current element ab at centre O is,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{R^2}$$



where, θ is angle between current element ab and the line joining the element to the centre O . Here $\theta = 90^\circ$, because current element at each point of circular path is perpendicular to the radius.

Therefore magnetic field produced at O , due to current element ab is,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{R^2}$$

According to Maxwell's right hand rule, the direction of magnetic field at O is upward, perpendicular to the plane of coil. The direction of magnetic field due to all current elements is the same. Therefore the resultant magnetic field at the centre will be the sum of magnetic field due to all current elements.

Thus,

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{Idl}{R^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \int dl$$

But,

$$\int dl = \text{total length of circuit coil}$$

$$= 2\pi R \text{ (for one-turn)}$$

Hence,

$$B = \frac{\mu_0}{4\pi} \frac{I}{R^2} \cdot 2\pi R$$

or

$$B = \frac{\mu_0 I}{2R}$$

If the coil contains N -turns, then

$$B = \frac{\mu_0 NI}{2R}$$

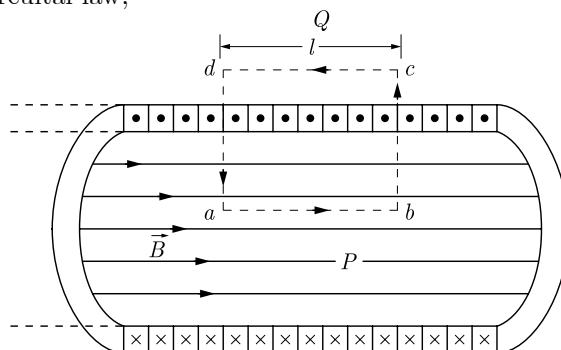
Here, current in the coil is anticlockwise and the direction of magnetic field is perpendicular to the plane of coil upward; but if the current in the coil is clockwise, then the direction of magnetic field will be perpendicular to the plane of coil downward.

91. A long solenoid with closely wound turns has n turns, per unit of its length. A steady current I flows through this solenoid. Use Ampere's circuital law to obtain an expression, for the magnetic field, at a point on its axis and close to its midpoint.

Ans :

Comp 2020, OD 2014

The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside it shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked $\odot$ and enters the plane of paper at points marked $\otimes$. To determine the magnetic field $\vec{B}$ at any inside point, consider a rectangular closed path $abcd$ as the Amperian loop. According to Ampere's circuital law,



The Magnetic Field of a very Long Solenoid

Hence, force between two parallel current carrying conductors on a segment L of conductor B due to A is given by

$$F = \frac{\mu_0 I_A I_B L}{2\pi d}$$

87. A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto $V/2$ volts. Find the resistance, in terms of R_1 and R_2 , required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2 .

Ans :

Delhi 2015

Let I_g be the current through galvanometer at full deflection to measure V volts,

$$V = I_g(G + R_1) \quad \dots(1)$$

According to question

$$\frac{V}{2} = I_g(G + R_2) \quad \dots(2)$$

$$2V = I_g(G + R_3) \quad \dots(3)$$

To measure for conversion of range dividing equation (1) by equation (2),

$$2 = \frac{G + R_1}{G + R_2}$$

$$G = R_1 - 2R_2$$

Putting the value of G in equation (1), we have

$$I_g = \frac{V}{R_1 - 2R_2 + R_1}$$

$$I_g = \frac{V}{2R_1 - 2R_2}$$

Substituting the value of G and I_g in equation (3), we have

$$2V = \frac{V}{2R_1 - 2R_2}(R_1 - 2R_2 + R_3)$$

$$4R_1 - 4R_2 = R_1 - 2R_2 + R_3$$

$$R_3 = 3R_1 - 2R_2$$

88. How is a galvanometer converted into a voltmeter and an ammeter ? Draw the relevant diagrams and find the resistance of the arrangement in each case. Take resistance of galvanometer as G .

or

Briefly explain why and how a galvanometer is converted into an ammeter.

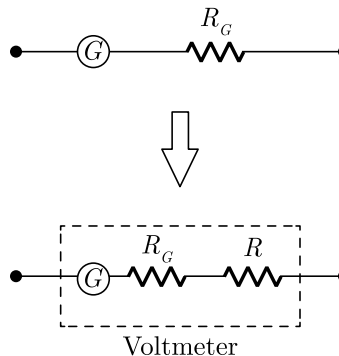
Ans :

SQP 2023, OD 2016

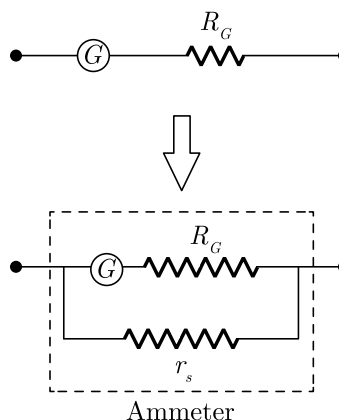
A galvanometer is converted into a voltmeter by connecting a high resistance R in series with it.

A galvanometer is converted into an ammeter by connecting a small resistance (called shunt) in parallel with it.

Resistance of voltmeter, $R_V = G + R$



Resistance of Ammeter, $R_A = \frac{Gr_s}{G + r_s}$



89. In a certain region of space, electric field $\vec{E}$ and magnetic field $\vec{B}$ are perpendicular to each other. An electron enters in the region perpendicular to the directions of both $\vec{B}$ and $\vec{E}$ and moves undeflected. Find the velocity of the electron.

Ans :

SQP 2010

Net force on electron moving in the combined electric field $\vec{E}$ and magnetic field $\vec{B}$ is

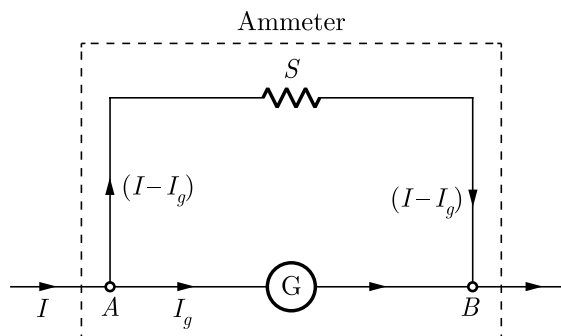
$$\vec{F} = -e[\vec{E} + \vec{v} \times \vec{B}]$$

Since electron moves undeflected then $\vec{F} = 0$

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

$$|\vec{E}| = |\vec{v}| \times |\vec{B}|$$

$$|\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|}$$



Conversion of a Galvanometer into an Ammeter

Let, G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - I$ = the required current range of the ammeter

S = shunt resistance

$I - I_g$ = current through the shunt.

As galvanometer and shunt are connected in parallel. So,

P.D. across the galvanometer = P.D. across the shunt

$$I_g G = (I - I_g) S$$

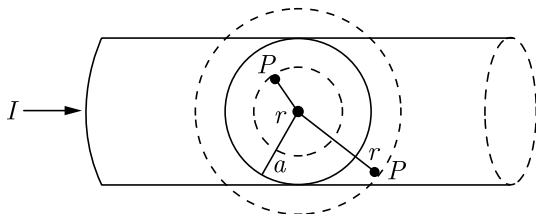
or
$$S = \frac{I_g}{I - I_g} \times G$$

So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Hence an ammeter is a shunted or low resistance galvanometer.

Its effective resistance is,

$$R_A = \frac{GS}{G+S} < S$$

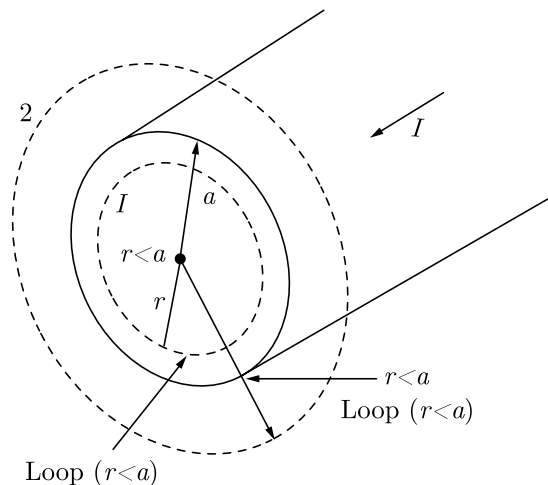
94. Figure shows a long straight wire of a circular cross-section of radius a and carrying steady current I . The current I is uniformly distributed across this cross-section. Derive the expressions for the magnetic field in the region $r < a$ and $r > a$.



Ans :

OD 2019

The current is distributed uniformly across the cross-section of radius a .



Current passes per unit in the cross-section = $\frac{I}{\pi a^2}$

Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2 \right) = \frac{I r^2}{a^2} \quad \dots (1)$$

- (i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire. Applying Ampere's circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I'$$

$$\oint B dl \cos 0^\circ = \mu_0 \left(\frac{I r^2}{a^2} \right) \quad [\text{From equation (1)}]$$

$$B \oint dl = \mu_0 \frac{I r^2}{a^2}$$

$$B \times 2\pi r = \mu_0 \frac{I r^2}{a^2}$$

$$B = \frac{\mu_0 I r^2}{2\pi a^2}$$

$$B \propto r$$

- (ii) Considering a loop of radius r whose centre lies at the axis of wire and ($r > a$) as shown in outer dotted line. Current I threads the loops. Applying Ampere's circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint B dl \cos 0^\circ = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B \propto \frac{1}{r}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{Total current through the loop } abcd$$

Now,

$$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

$$\text{But, } \int_b^c \vec{B} \cdot d\vec{l} = \int_b^c B dl \cos 90^\circ = 0$$

$$\int_d^a \vec{B} \cdot d\vec{l} = \int_d^a B dl \cos 90^\circ = 0$$

$$\int_c^d \vec{B} \cdot d\vec{l} = 0$$

As, $B = 0$ for points outside the solenoid.

$$\begin{aligned} \text{Hence, } \oint \vec{B} \cdot d\vec{l} &= \int_a^b \vec{B} \cdot d\vec{l} \\ &= \int_a^b B dl \cos 0^\circ = B \int_a^b dl = Bl \end{aligned}$$

where, l is length of the side ab of the rectangular loop $abcd$.

Let, number of turns per unit length of the solenoid = n . Then number of turns in length l of the solenoid = nl .

Hence, total current threading the loop $abcd = nIl$.

$$\text{Hence, } Bl = \mu_0 nIl$$

$$\text{or } B = \mu_0 nI$$

It can be easily shown that the magnetic field at the end of the solenoid is just one half of that at its middle. Thus,

$$B_{\text{end}} = \frac{1}{2} \mu_0 nI$$

- 92.** A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current I passing through it.

Ans :

Foreign 2011

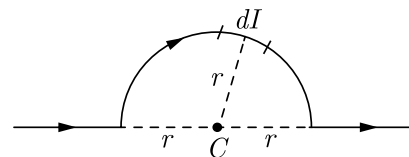
When a straight wire is bent into semi-circular loop, then there are two parts which can produce the magnetic field at the centre one is circular part and other is straight part due to which field is zero.

Length L is bent into semi-circular loop.

Length of wire = Circumference of semi-circular wire

$$L = \pi r$$

$$r = \frac{L}{\pi} \quad \dots(1)$$



Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C .

Using Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{r^2}$$

[since $Idl \perp r$, hence $\theta = 90^\circ$]

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

Net magnetic field at C due to semi-circular loop,

$$\begin{aligned} B &= \int_{\text{semi-circle}} \frac{\mu_0 Idl}{4\pi r^2} \\ &= \frac{\mu_0 I}{4\pi r^2} \int_{\text{semi-circle}} dl \\ &= \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L \end{aligned}$$

But,

$$r = \frac{L}{\pi}$$

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} \\ &= \frac{\mu_0}{4\pi} \times \frac{IL}{L^2} \times \pi^2 \\ &= \frac{\mu_0 I \pi}{4L} \end{aligned}$$

This is the required expression.

- 93.** How will you convert a galvanometer into an ammeter of range $0 - I$ amperes? What is the effective resistance of an ammeter?

Ans :

Foreign 2015, SQP 2010

An ordinary galvanometer cannot as such be used as an ammeter to measure current in a circuit. This is because of two reasons: (1) Galvanometer is a very sensitive device, it gives a full-scale deflection with a small current of a few μA . (2) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will decrease the value of current in the circuit. To overcome these difficulties, a small resistance, called shunt resistance, is connected in parallel with the galvanometer coil, so that most of the current passes through the shunt.

1. If,

$$\theta = 0^\circ \text{ or } 180^\circ$$

Then, $F = I\ell B(0) = 0$

Thus a current carrying conductor placed parallel to the direction of the magnetic field does not experience any force.

2. If,

$$\theta = 90^\circ$$

Then, $F = I\ell B \sin 90^\circ = I\ell B$

$$F_{\max} = I\ell B$$

Thus a current carrying conductor placed perpendicular to the direction of a magnetic field experience a maximum force.

Direction of Force

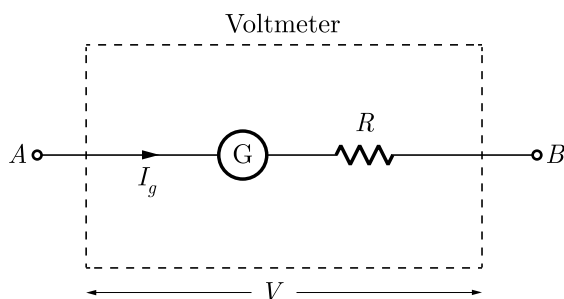
The direction of force on a current carrying conductor placed in a perpendicular magnetic field is given by Fleming's left hand rule. Stretch the thumb and the first two fingers of the left hand in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, central finger in the direction of current, then the thumb gives the direction of force on the conductor. In Fig. 4.24, the field $\vec{B}$ is along $+Z$ -direction, the current I along $+Y$ -direction and so the force $\vec{F}$ acts along $+X$ -direction.

97. How can a galvanometer of resistance G be converted into a voltmeter to read a maximum potential difference of V volts? Support your answer with related mathematical expression.

Ans :

Comp 2012

A galvanometer can be used as a voltmeter to measure voltage across any circuit element. For this it must be connected in parallel with that element. Further, it must draw a very small current, otherwise the voltage required to be measured will decrease substantially. To ensure this, a large resistance R is connected in series with the galvanometer.



Conversion of a Galvanometer into a Voltmeter

Let, G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - V$ = required range of the voltmeter and

R = the high series resistance which

restricts the current to safe limit I_g

Hence, total resistance in the circuit = $R + G$

By Ohm's law,

$$I_g = \frac{\text{Potential difference}}{\text{Total resistance}} = \frac{V}{R + G}$$

$$R + G = \frac{V}{I_g}$$

$$R = \frac{V}{I_g} - G$$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Hence a voltmeter is a high resistance galvanometer. Its effective resistance is,

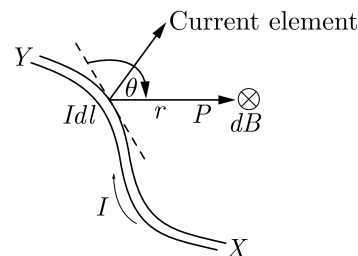
$$R_V = R + G \gg G$$

98. (a) State Biot-Savart law and express this law in the vector form.
 (b) Two identical circular coils, P and Q each of radius R , carrying currents 1 A and $\sqrt{3}$ A, respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ planes. Find the magnitude and direction of the net magnetic field at the centre of the coils.

Ans :

Delhi 2014, OD 2004

- (a) According to Biot-Savart's law the magnitude of the magnetic field $\vec{dB}$ due to a small element of length dl of a current carrying wire at a point P , is proportional to the current I , the element length dl and is inversely proportional to the square of the distance r . It is also proportional to $\sin\theta$, where θ is the angle between $\vec{dl}$ and $\vec{r}$.



Its direction is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ in vector form

$$\vec{dB} \propto \frac{I\vec{dl} \times \vec{r}}{r^3}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl} \cdot \vec{r}}{r^3}$$

The distance moved along the magnetic field in one rotation is called pitch of the helical path.

$$\begin{aligned}\text{Pitch} &= v_{||} \times T = v \cos \theta \times \frac{2\pi m}{qB} \\ &= \frac{2\pi m v \cos \theta}{qB}\end{aligned}$$

- 101.** Explain Biot-Savart law. With its help derive an expression for the magnetic field at any point on the axis of a current carrying circular loop.

Ans :

OD 2017

According to Biot-Savart's law, the magnitude of magnetic field induction (dB) at a point P due to a current element depends on the following factors:

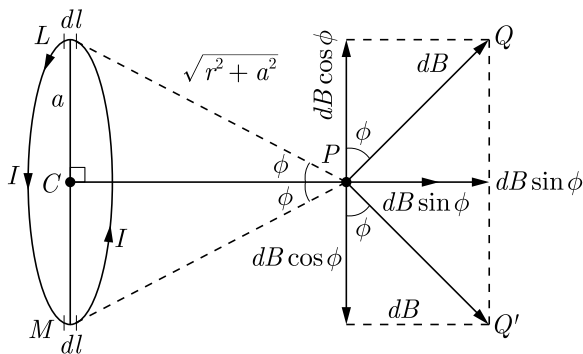
1. $dB \propto I$ (i.e., magnetic field is directly proportional to the current flowing through the conductor).
2. $dB \propto dl$ (i.e. magnetic field is directly proportional to the length of the element.)
3. $dB \propto \sin \theta$ (i.e., magnetic field is directly proportional to the sine of angle between the length of element and line joining the element to point, P).

Combining all the above relation $dB \propto \frac{Idl \sin \theta}{r^2}$.

This relation is called Biot-Savart's law.

Magnetic Field on the Axis of a Circular Current Carrying Loop

Let us consider a circular loop of radius a with centre C . Let the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the direction shown. Suppose P is any point on the axis of a coil at a distance r from the centre C .



Now, consider a current element Idl on top L , where current comes out of paper normally, whereas at bottom M , current enters into the plane paper normally.

Since, $LP \perp dl$

Also, $MP \perp dl$

Hence, $LP = MP = \sqrt{r^2 + a^2}$

The magnetic field at a point P due to the current element Idl , according to Biot-Savart's law is given by,

$$\begin{aligned}dB &= \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{(r^2 + a^2)} \\ &= \frac{\mu_0}{4\pi} \cdot \frac{Idl}{(r^2 + a^2)}\end{aligned}$$

where, a = radius of circular loop

and r = distance of point P from the centre

C along the axis.

According to right hand screw rule, the direction of dB is perpendicular to LP and along PQ , where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ' , where $PQ' \perp MP$.

Now, resolving dB due to current element at L and M . So, $dB \cos \phi$ components balance each other and net magnetic field is given by,

$$\begin{aligned}B &= \oint dB \sin \phi \\ &= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}} \\ &\left[\text{Since, In } \triangle PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]\end{aligned}$$

$$= \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$\oint dl = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a)$$

or

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$

For N turns, the net magnetic field is given by,

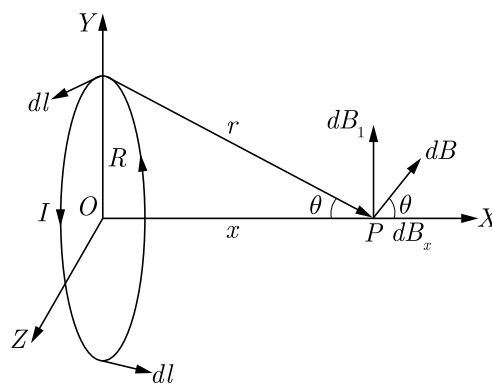
$$B = \frac{\mu_0 N I a^2}{2(r^2 + a^2)^{3/2}}$$

- 102.** Using Biot-Savart's law, derive the expression for the magnetic field in the vector form at a point on the axis of a circular current carrying loop.

Ans :

Comp 2013

Magnetic field on the axis of a circular current loop.



$I \rightarrow$ Current in the loop,

$R \rightarrow$ Radius of the loop,

100. Discuss the motion of a charged particle in a uniform magnetic field with initial velocity (1) parallel to the field, (2) perpendicular to the magnetic field and (3) at an arbitrary angle with the field direction.

Ans :

Comp 2021, OD 2013

When a charged particle having charge q and velocity $\vec{v}$ enters a magnetic field $\vec{B}$, it experiences a force,

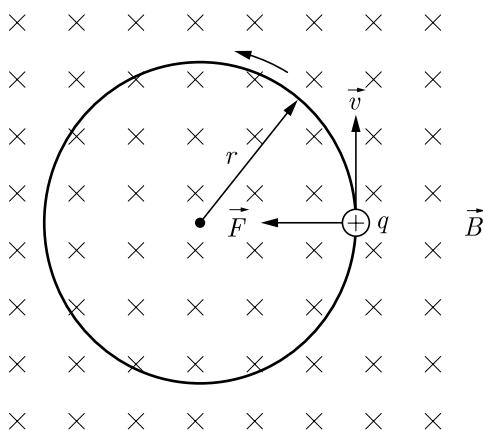
$$\vec{F} = q(\vec{v} \times \vec{B})$$

The direction of this force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnitude of this force is,

$$F = qvB \sin \theta$$

Following three cases are possible:

1. **When the Initial Velocity is Parallel to the Magnetic Field :** Here $\theta = 0^\circ$, so $F = qvB \sin 0^\circ = 0$. Thus the parallel magnetic field does not exert any force on the moving charged particle. The charged particle will continue to move along the line of force.
2. **When the Initial Velocity is Perpendicular to the Magnetic Field :** Figure shows a magnetic field $\vec{B}$ directed normally into the plane of paper, so shown by small crosses. A charge $+q$ is projected with a speed v in the plane of the paper. The velocity is perpendicular to the magnetic field. A force $F = qvB$ acts on the particle perpendicular to both $\vec{v}$ and $\vec{B}$. This force continuously deflects the particle sideways without changing its speed and the particle will move along a circle perpendicular to the field. Thus the magnetic force provides the centripetal force. Let r be the radius of the circular path.



Centripetal force,

$$\frac{mv^2}{r} = \text{Magnetic force, } qvB$$

$$r = \frac{mv}{qB}$$

Period of revolution,

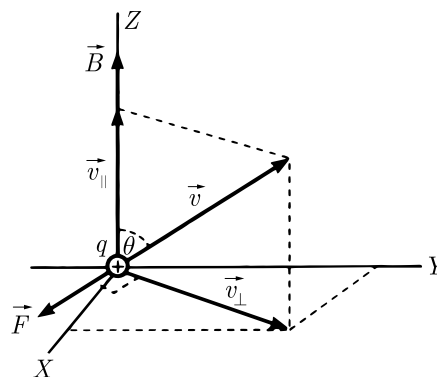
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}$$

The frequency of revolution is,

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m}$$

This frequency is independent of v and r and is called cyclotron frequency.

3. **When the Initial Velocity Makes an Arbitrary Angle with the Field Direction :** Consider a charged particle q entering a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ inclined at an angle θ with the direction of $\vec{B}$.



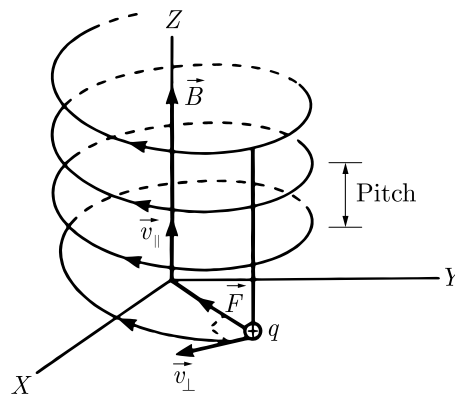
The Perpendicular Component : $v_\perp = v \sin \theta$ of the initial velocity makes the charge move along a circular path of radius,

$$r = \frac{mv_\perp}{qB} = \frac{mv \sin \theta}{qB}$$

The period of revolution is,

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi}{v \sin \theta} \cdot \frac{mv \sin \theta}{qB} = \frac{2\pi m}{qB}$$

The Parallel Component : $v_\parallel = v \cos \theta$ of the initial velocity makes it move along the direction of the magnetic field. Hence the resultant path of the charged particle will be a helix, with its axis along the direction of $\vec{B}$.



Helical Motion of a Charged Particle in a Magnetic Field

($\vec{dl}$ is directed along the length of the wire in the direction of current and $\vec{r}$ is the vector joining the centre of current element to the point P)

(b) Field due to current in coil P is

$$\vec{B}_2 = \frac{\mu_0 I_1}{2R} \cdot \hat{k}$$

(Assuming current to be anticlockwise as seen from +ve Z -axis)

and that due to current in coil Q is

$$\vec{B}_2 = \frac{\mu_0 I_2}{2R} \hat{j}$$

(Assuming current to be anticlockwise as seen from positive X -axis)

Hence, net field, $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$\vec{B} = \left(\frac{\mu_0 I_1}{2R}\right) \hat{k} + \left(\frac{\mu_0 I_2}{2R}\right) \hat{j}$$

Since, $I_1 = 2 \text{ A}$ and $I_2 = \sqrt{3} \text{ A}$

$$= \left(\frac{\mu_0}{2R}\right) \hat{k} + \left(\frac{\sqrt{3}\mu_0}{2R}\right) \hat{j}$$

$$\begin{aligned} \vec{B} &= \sqrt{\left(\frac{\mu_0}{2R}\right)^2 + \left(\frac{\sqrt{3}\mu_0}{2R}\right)^2} \\ &= \frac{\mu_0}{2R} \sqrt{1+3} = \frac{\mu_0}{2R} \times 2 \end{aligned}$$

Hence, $\vec{B} = \frac{\mu_0}{R}$

The field is directed in XZ plane.

Let θ be the angle of $\vec{B}$ with positive X -axis.

$$\text{Then, } \tan \theta = \left(\frac{\mu_0}{2R}\right) / \left(\frac{\sqrt{3}\mu_0}{2R}\right)$$

$$\begin{aligned} \text{or } \tan \theta &= \frac{1}{\sqrt{3}} \\ \theta &= 30^\circ \end{aligned}$$

Thus, $\vec{B}$ is directed in XZ plane making an angle of 30° with X -axis.

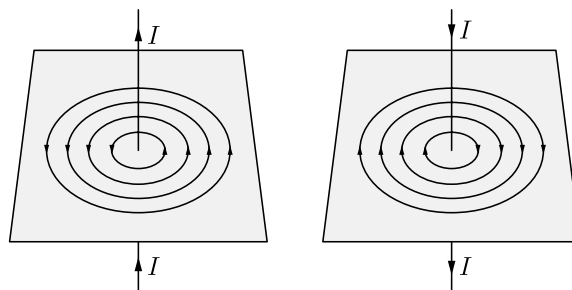
99. Sketch the magnetic lines of force of straight current carrying conductor. State the rules used to find the direction of this magnetic field.

Ans :

OD 2021

Magnetic Field Pattern of a Straight Current Carrying Conductor

The magnetic lines of force of a straight current carrying conductor are concentric circles with the wire at the centre and in a plane perpendicular to the wire. If the current flows upwards, the lines of force have anticlockwise sense [Figure (a)] and if the current flows downwards, then the lines of force have clockwise sense [Figure (b)].

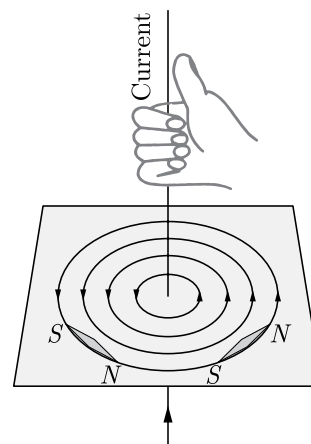


Magnetic Lines of Force of a Straight Current Carrying Conductor

Rules for Finding the Direction of Magnetic Field Due to Straight Current Carrying Conductor

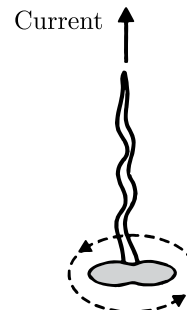
Either of the following two rules can be used for this purpose.

1. **Right Hand Thumb Rule :** If we hold the straight conductor in the grip of our right hand in such a way that the extended thumb points in the direction of current, then the direction of the curl of the fingers will give the direction of the magnetic field.



Right Hand Rule for Field Due to a Straight Conductor

2. **Maxwell's Cork Screw Rule :** If a right-handed screw be rotated along the wire so that it advances in the direction of current, then the direction in which the thumb rotates gives the direction of the magnetic field.



Cork Screw Rule for Field Due to a Straight Conductor

X -axis $\rightarrow$ Axis of the loop,

$x \rightarrow$ Distance OP

$dl \rightarrow$ Conducting element of the loop

According to Biot-Savart's law, the magnetic field at P is

$$dB = \frac{\mu_0 I |dl \times r|}{4\pi r^3}$$

$$r^2 = x^2 + R^2$$

$$dl \times r = r dl$$

[since, they are perpendicular]

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{(x^2 + R^2)}$$

dB has two components $-dB_x$ and dB_1 , dB_1 is cancelled out and only the x -component remains.

$$dB_x = dB \cos \theta$$

$$\cos \theta = \frac{R}{(x^2 + R^2)^{1/2}}$$

Hence,
$$dB_x = \frac{\mu_0 Idl}{4\pi} \cdot \frac{R}{(x^2 + R^2)^{3/2}}$$

Summation of dl over the loop is given by $2\pi R$.

$$B = B_x \hat{i} = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} \hat{i}$$

103. 1. Explain principle and working of a moving coil galvanometer. Derive an expression for current sensitivity.
2. Compare between moving coil galvanometer and moving magnet galvanometer.

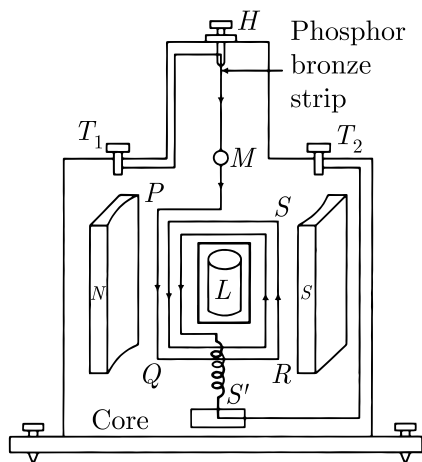
Ans :

SQP 2006, OD 2017

1. Moving Coil Galvanometer

Principle

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



Construction

The moving coil galvanometer consists of a coil with many turns free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it.

Working

Suppose the coil $PQRS$ is suspended freely in the magnetic field. Let l be length PQ or RS of the coil, b be breadth QR or SP of the coil, N be number of turns in the coil and area of each turn of the coil, $A = l \times b$. Let B be strength of the magnetic field in which coil is suspended and I is current passing through the coil in the direction $PQRS$.

Let at any instant, α be the angle, which normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular coil carrying current when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin \alpha$. Due to deflecting torque, the coil rotates and suspension wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze stripe. The,

Total restoring torque produced $= k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$NIBA = k\theta$$

$$I = \frac{k}{NBA} \theta = G\theta$$

where, $\frac{k}{NBA} = G$

It is known as galvanometer constant.

i.e. $\theta \propto I$. It means that the deflection produced is proportional to the current flowing through the galvanometer.

Current sensitivity

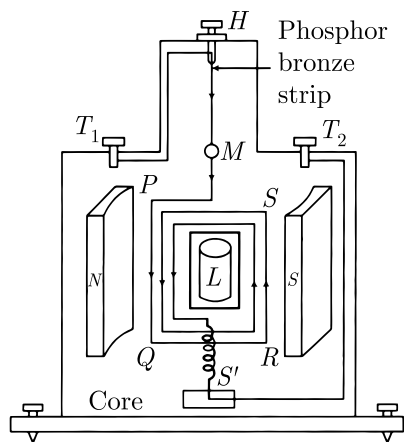
Current sensitivity of the galvanometer is the deflection per unit current flowing through it.

It is given by,
$$I_s = \frac{\theta}{I} = \frac{NAB}{k}$$

Its unit is rad/A or div/A.

2. Compare between Moving Coil Galvanometer and Moving Magnet Galvanometer

The difference between moving coil galvanometer and moving magnet galvanometer



Working : Suppose, the coil $PQRS$ is suspended freely in the magnetic field.

Let l = length PQ or RS of the coil

b = breadth QR or SP of the coil

n = number of turns in the coil

Area of each turns of the coil, $A = l \times b$

Let B = strength of the magnetic field in which coil is suspended.

I = current passing through the coil in the direction of $PQRS$.

Let at any instant of time, α be the angle which normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular current carrying coil when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin \alpha$.

Due to this deflecting torque, the coil rotates and suspended wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip.

Then, total restoring torque produced = $k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$NIBA = k\theta$$

$$\text{or } I = \frac{k}{NBA} \theta = G\theta$$

$$\text{where, } \frac{k}{NBA} = G \quad [\text{constant for a galvanometer}]$$

It is known as galvanometer constant.

Current sensitivity of the galvanometer is the deflection per unit current.

$$\frac{\phi}{I} = \frac{NAB}{k}$$

Voltage sensitivity is the deflection per unit voltage.

$$\begin{aligned} \text{Hence, } \frac{\phi}{V} &= \frac{NAB}{k} \left(\frac{I}{V} \right) \\ &= \frac{NAB}{k} \frac{1}{R} \quad [\text{since, } V = IR] \end{aligned}$$

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field, i.e., the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil.

107. (i) A straight thick long wire of uniform circular cross-section of radius a is carrying a steady current I . The current is uniformly distributed across the cross-section. Use Ampere's circuital law to obtain a relation showing the variation of the magnetic field (B_r) inside and outside the wire with distance, ($r \leq a$) and ($r > a$) of the field point from the center of its cross-section. What is the magnetic field at the surface of this wire? Plot a graph showing the nature of this variation.
- (ii) Calculate the ratio of magnetic field at a point $a/2$ above the surface of the wire to that at a point $a/2$ below its surface. What is the maximum value of the field of this wire?

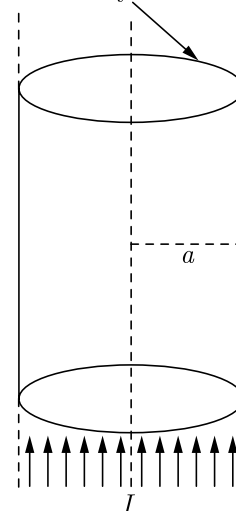
Ans :

Foreign 2015, OD 2012

- (i) **Magnetic field due to a straight thick wire of uniform cross-section :** Consider an infinitely long cylindrical wire of radius a , carrying current I . Suppose that the current is uniformly distributed over whole cross-section of the wire. The cross-section of wire is circular. Current per unit cross-sectional area.

$$i = \frac{I}{\pi a^2} \quad \dots(1)$$

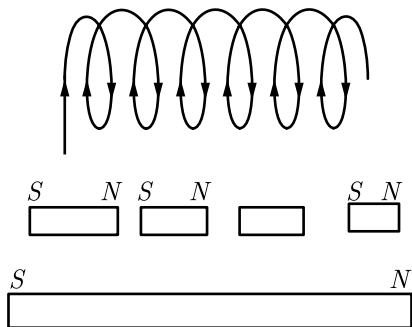
Circular cross-section of cylinder



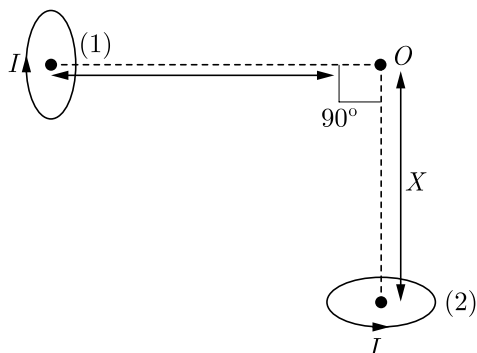
The solenoid can be regarded as a combination of circular loops placed side by side, each behaving like a magnet of magnetic moment IA , where I is the current and A area of the loop.

These magnets neutralise each other except at the ends where south and north poles appear.

Magnetic moment of bar magnet = NIA



- 105.** Two very small identical circular loop (1) and (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axis perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O .



Ans :

Delhi 2014

The magnetic field at a point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ia^2}{(a^2 + r^2)^{3/2}}$$

where, I = current through the loop

a = radius of the loop

r = distance of O from the centre of the

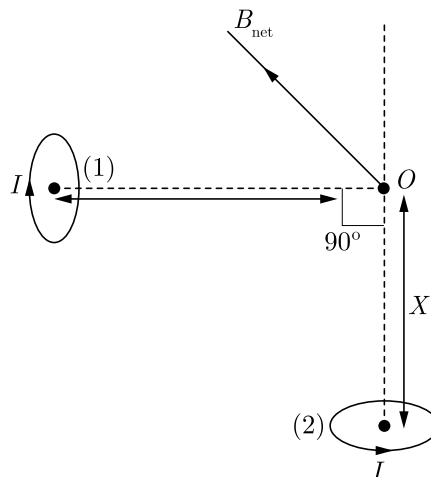
loop

Since, I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ia^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown.

The direction of the net magnetic field will be as shown below



The magnitude of the net magnetic field is given by

$$B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$$

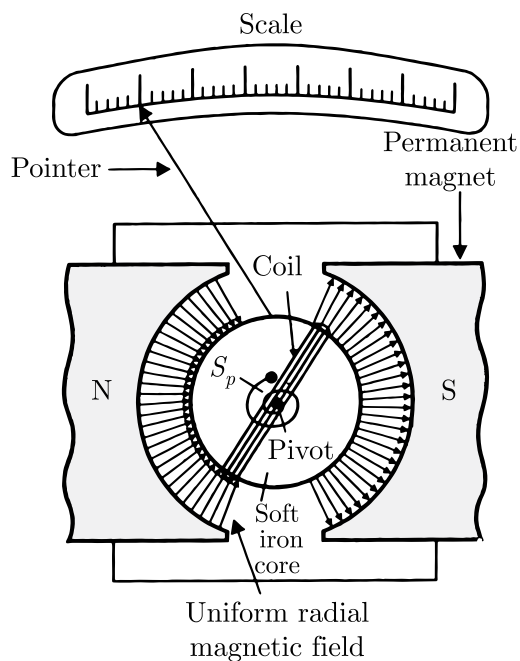
$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\pi Ia^2}{(a^2 + x^2)^{3/2}}$$

- 106.** Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil?

Ans :

Delhi 2019, OD 2007

Moving Coil Galvanometer Principle : Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a net torque.

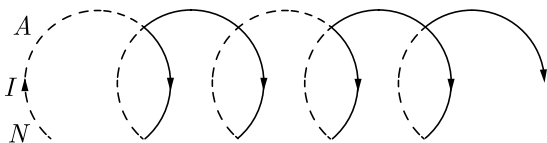


	Moving Coil-Galvanometer	Moving Magnet Galvanometer
1.	The deflection in moving coil galvanometer is proportional to the current flowing through it.	It is also known as tangent galvanometer. The current flowing through the tangent galvanometer is directly proportional to the deflection in the coil
2.	Its sensitivity is high.	Its sensitivity is low.
3.	A moving coil galvanometer is based on the principle of torque experienced by a current carrying coil placed in strong magnetic field.	Moving coil galvanometer is based on the tangent law.

104. Using Ampere's circuital law, derive an expression for the magnetic field along the axis of a toroidal solenoid.

or

- (i) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius r , having n turns per unit length and carrying a steady current I .
- (ii) An observer to the left of a solenoid of N turns each of cross-section area A observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$.



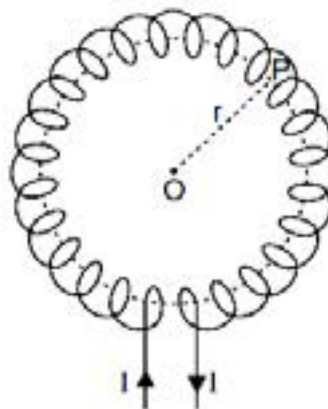
Ans :

SQP 2013, OD 2015

Magnetic field due to a toroidal solenoid : A long solenoid shaped in the form of closed ring is called a toroidal solenoid (or endless solenoid).

Let n be the number of turns per unit length of toroid and I the current flowing through it. The current causes the magnetic field inside the turns of the solenoid. The magnetic field inside the turns of the solenoid. The magnetic lines of force inside the toroid are in the form of concentric circles. By symmetry the magnetic field has the same magnitude at each point of circle and is along the tangent at every point on the circle.

- (i) **For points inside the core of toroid :** Consider a circle of radius r in the region enclosed by turns of toroid. Now we apply Ampere's circuital law to this circular path, i.e.,



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \dots(1)$$

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos 0 \\ = B \cdot 2\pi r$$

$$\text{Length of toroid} = 2\pi r$$

$$N = \text{Number of turns in toroid} = n(2\pi r)$$

and current in one-turn = I

$$\text{Current enclosed by circular path} = (n2\pi r) \cdot I$$

Equation (1) gives

$$B2\pi r = \mu_0(n2\pi rI)$$

$$B = \mu_0 nI$$

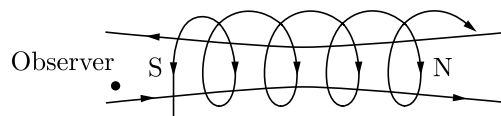
- (ii) **For points in the open space inside the toroid :** No current flows through the Amperian loop, so $I = 0$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I = 0 \\ |B|_{\text{inside}} = 0$$

- (iii) **For points in the open space exterior to the toroid :** The net current entering the plane of the toroid is exactly cancelled by the net current leaving the plane of the toroid.

$$\oint \vec{B} \cdot d\vec{l} = 0 \\ |B|_{\text{exterior}} = 0$$

For observer, current is flowing in clockwise direction hence we will see magnetic field lines going towards south pole.



Magnetic field at external points ($r > a$) : We consider a circular path of radius $r(> a)$ passing through external point P , concentric with circular cross-section of wire. By symmetry the strength of magnetic field at every point of circular path is same and the direction of magnetic field is tangential to path at every point. So line integral of magnetic field $\vec{B}$ around the circular path

$$\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos 0^\circ = B2\pi r$$

Current enclosed by path = Total current on circular cross-section of cylinder = I

By Ampere's circuital law

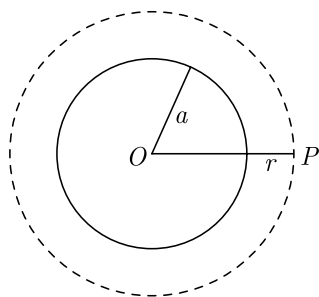
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{current enclosed by path}$$

$$B2\pi r = \mu_0 \times I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

This expression is same as the magnetic field due to a long current carrying straight wire.

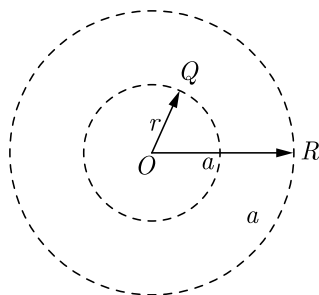
This shows that for external points the current flowing in wire may be supposed to be concerned at the axis of cylinder.



Magnetic Field at Internal Points ($r < a$) : Consider a circular path of radius $r(< a)$, passing through internal point Q , concentric with circular cross-section of the wire. In this case the assumed circular path encloses only a path of current carrying circular cross-section of the wire.

Current enclosed by path = current per unit cross-section $\times$ cross section of assumed circular path

$$\begin{aligned} &= i \times \pi r^2 = \left(\frac{I}{\pi a^2} \right) \times \pi r^2 \\ &= \frac{Ir^2}{a^2} \quad [\text{using (1)}] \end{aligned}$$



By Ampere's circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times \text{current closed by path}$$

$$B \cdot 2\pi r = \mu_0 \times \frac{Ir^2}{a^2}$$

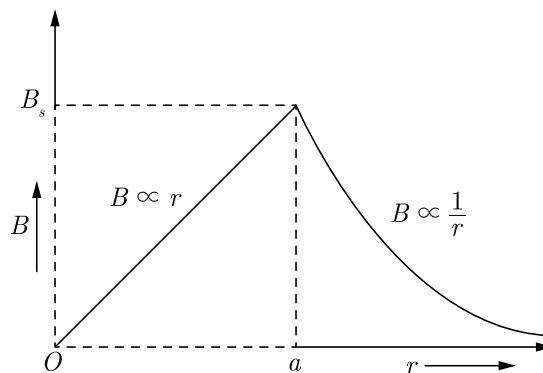
$$B = \frac{\mu_0 Ir}{2\pi a^2}$$

Clearly, magnetic field strength inside the current carrying wire is directly proportional to distance of the point from the axis of wire.

At surface of cylinder $r = a$, so magnetic field at surface of wire

$$B_s = \frac{\mu_0 I}{2\pi a} \quad (\text{maximum value})$$

The variation of magnetic field strength (B) with distance (r) from the axis of wire for internal and external points is shown in figure.



$$(ii) \quad B_{\text{outside}} = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi \left(a + \frac{a}{2} \right)} = \frac{\mu_0 I}{3\pi a}$$

$$B_{\text{inside}} = \frac{\mu_0 Ir}{2\pi a^2} = \frac{\mu_0 I(a/2)}{2\pi a^2} = \frac{\mu_0 I}{4\pi a}$$

$$\frac{B_{\text{outside}}}{B_{\text{inside}}} = \frac{4}{3}$$

Maximum value of magnetic field is at the surface

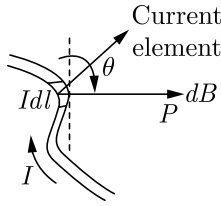
$$\text{given by } B_{\text{outside}} = \frac{\mu_0 I}{2\pi a}.$$

108. Write using Biot-Savart law, the expression for the magnetic field $\vec{B}$ due to an element $d\vec{l}$ carrying current I at a derive the expression for the magnetic field due to a current carrying loop of radius R at a point P distance x from its centre along the axis to the loop.

Ans :

OD 2018

According to Biot-Savart's law, conductor is placed in air or vacuum,

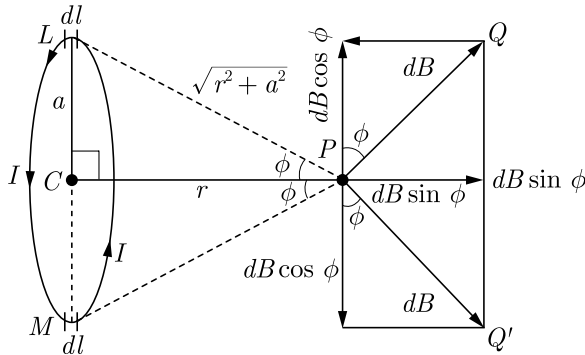


then,
$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

In vector form, Biot-Savart's law can be written as,

$$dB \propto \frac{Idl \times r}{r^3} = \frac{\mu_0}{4\pi} \frac{Idl \times r}{r^3}$$

Let us consider a circular loop of radius a with centre C . Let, the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the direction shown in the figure. Suppose P is any point on the axis at a distance r from the centre.



Now, consider a current element $I dl$ on top L , where current comes out of paper normally whereas at bottom (M) enters into the plane paper normally.

Hence, $LP \perp dl$

Also, $MP \perp dl$

Hence, $MP = LP = \sqrt{r^2 + a^2}$

The magnetic field at P due to current element $I dl$.

According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{(r^2 + a^2)^{3/2}}$$

where, a = radius of circular loop

r = Distance of point P from centre along the axis.

The direction of dB is perpendicular to LP and along PQ , where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element $I dl$ at bottom and direction is along PQ' , where $PQ' \perp MP$.

Now, resolving dB due to current element at L and M $dB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{I dl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

[since, In $\triangle PCL$, $\sin \phi = \frac{a}{\sqrt{r^2 + a^2}}$]

$$= \frac{\mu_0}{4\pi} \cdot \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a)$$

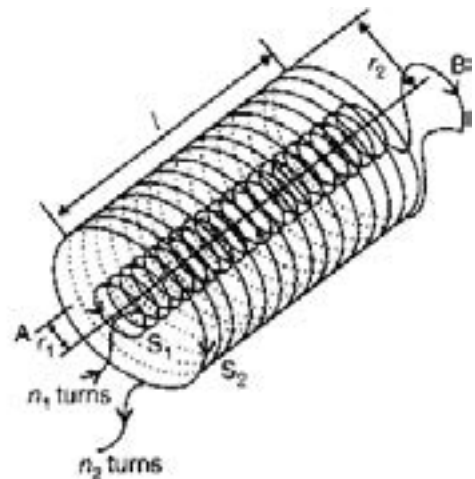
or

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$

For N turns,
$$B = \frac{\mu_0 N I a^2}{2(r^2 + a^2)^{3/2}}$$

The direction is along the axis and away from the loop.

109. (i) State Ampere's circuital law expressing it in the integral form.
- (ii) Two long co-axial insulated solenoids S_1 and S_2 of equal length are wound one over the other as shown in the figure. A steady current I flows through the inner solenoid S_1 to the other end B which is connected to the outer solenoid S_2 through which the same current I flows in the opposite direction so, as to come out at end A . If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point
- inside on the axis and
 - outside the combined system.



Ans :

Delhi 2012

- (i) Ampere's circuital law states that the line integral of magnetic field (B) around any closed path in vacuum is μ_0 times the net current (I) threading the area enclosed by the curve.

Mathematically,
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Ampere's law is applicable only for an Amperian loop as the Gauss's law is used for Gaussian surface in electrostatics.

- (ii) According to Ampere's circuital law, the net magnetic field is given by $B = \mu_0 n \hat{i}$.

(a) The net magnetic field is given by

$$\begin{aligned} B_{\text{net}} &= B_2 - B_1 \\ &= \mu_0 n_2 I_2 - \mu_0 n_1 I_1 \\ &\quad [\text{since, } I_2 = I_1 = I] \\ &= \mu_0 I (n_2 - n_1) \end{aligned}$$

The direction is from B to A .

- (b) As the magnetic field due to S_1 is confined solely inside S_1 as the solenoids are assumed to be very long. So, there is no magnetic field outside S_1 due to current in S_1 , similarly there is no field outside S_2 .

Hence, $B_{\text{net}} = 0$

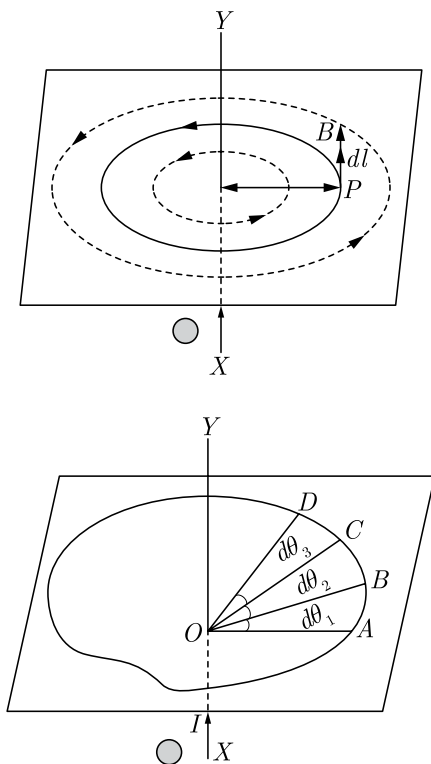
110. Explain how Biot-Savart law enables one to express the Ampere's circuital law in the integral form, viz.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where, I is the total current passing through the surface.

Ans :

When current in the coil is in the anti-clockwise direction.



Consider any arbitrary closed path perpendicular to the plane of paper around a long straight conductor

XY carrying current from X to Y , lying in the plane in the plane of paper.

Let, the closed path be made of large number of small elements, where

$$AB = dl_1, BC = dl_2, CD = dl_3$$

Let $d\theta_1, d\theta_2, d\theta_3$ be the angles subtended by the various elements at point O through which conductor is passing. Then,

$$d\theta_1 + d\theta_2 + d\theta_3 + \dots = 2\pi$$

Suppose these small elements $AB, BC, CD, \dots$ are small circular arcs of radii $r_1, r_2, r_3, \dots$ respectively.

Then, $d\theta_1 = \frac{dl_1}{r_1}$

$$d\theta_2 = \frac{dl_2}{r_2}, d\theta_3 = \frac{dl_3}{r_3}$$

If $\vec{B}_1, \vec{B}_2, \vec{B}_3$ are the magnetic field induction at a point along the small elements $\vec{dl}_1, \vec{dl}_2, \vec{dl}_3, \dots$ then from Biot-Savart's law we know that for the conductor of infinite length, magnetic field is given by

$$B_1 = \frac{\mu_0 2I}{4\pi r_1}$$

$$B_2 = \frac{\mu_0 2I}{4\pi r_2}$$

$$B_3 = \frac{\mu_0 2I}{4\pi r_3} \dots$$

In case of each elements, the magnetic field induction $\vec{B}$ and current element vector $\vec{dl}$ are in the same direction. Line integral of B around closed path is

$$\begin{aligned} \oint \vec{B} \cdot d\vec{l} &= \vec{B}_1 \cdot \vec{dl}_1 + \vec{B}_2 \cdot \vec{dl}_2 + \vec{B}_3 \cdot \vec{dl}_3 + \dots \\ &= B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + \dots \\ &= \frac{\mu_0 2I}{4\pi r_1} dl_1 + \frac{\mu_0 2I}{4\pi r_2} dl_2 + \frac{\mu_0 2I}{4\pi r_3} dl_3 + \dots \\ &= \frac{\mu_0 2I}{4\pi} \left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + \dots \right] \\ &= \frac{\mu_0 2I}{4\pi} [d\theta_1 + d\theta_2 + d\theta_3 + \dots] \\ &= \frac{\mu_0}{4\pi} 2I \times 2\pi = \mu_0 I \end{aligned}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Which is an expression of Ampere's circuital law.

NUMERICAL QUESTIONS

111. An electron of energy 2000 eV describes a circular path in magnetic field of flux density 0.2 T. What is the radius of path? Take, $e = 1.6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg.

$$r = 3 \times 10^{-2} \text{ m}$$

$$\theta = \frac{3\pi}{2} \text{ rad}$$

Magnetic field induction at O due to current through circular path ACB is,

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi r} \theta \\ &= 10^{-7} \times \frac{10}{(3 \times 10^{-2})} \times \frac{3\pi}{2} \\ &= 1.57 \times 10^{-4} \text{ T} \end{aligned}$$

118. A student records the following data for the magnitudes (B) of the magnetic field at axial points at different distance x from the centre of a circular coil of radius a carrying a current. Verify (for any two) that these observations are in good agreement with the expected theoretical variation of B with x .

x	$x = 0$	$x = a$	$x = 2a$	$x = 3a$
B	B_0	$0.25\sqrt{2}B_0$	$0.039\sqrt{5}B_0$	$0.010\sqrt{10}B_0$



Ans :

OD 2019, SQP 2007

We have,

Magnetic field at axial point,

$$B = \frac{\mu_0 Ina^2}{2(a^2 + x^2)^{3/2}}$$

1. At,

$$x = 0$$

$$B_0 = \frac{\mu_0 In}{2a}$$

2. At,

$$x = a$$

$$\begin{aligned} B &= \frac{\mu_0 Ina^2}{2(a^2 + a^2)^{3/2}} = \frac{\mu_0 Ina^2}{2(2a^2)^{3/2}} \\ &= \frac{\mu_0 nI}{2^{5/2}a} = \frac{B_0}{2^{5/2}a} \\ &= \frac{B_0}{2^2 \cdot 2^{1/2}} = 0.25\sqrt{2} B_0 \end{aligned}$$

3. At,

$$x = 2a$$

$$\begin{aligned} B &= \frac{\mu_0 Ina^2}{2(a^2 + 4a^2)^{3/2}} \\ &= \frac{\mu_0 Ina^2}{2(5a^2)^{3/2}} = \frac{\mu_0 In}{2.5 \cdot \sqrt{5}a} \\ &= \frac{1}{5\sqrt{5}} B_0 = \frac{0.2\sqrt{5}}{\sqrt{5} \times \sqrt{5}} B_0 \end{aligned}$$

$$= 0.040B_0 \approx 0.039B_0$$

4. At,

$$x = 3a$$

$$\begin{aligned} B &= \frac{\mu_0 Ina^2}{2(a^2 + 9a^2)^{3/2}} \\ &= \frac{\mu_0 Ina^2}{2(10a^2)^{3/2}} = \frac{\mu_0 In}{2.10 \cdot \sqrt{10}a} \\ &= \frac{B_0}{10\sqrt{10}} = \frac{\sqrt{10}B_0}{100} \\ &= 0.010\sqrt{10}B_0 \end{aligned}$$

Hence, data are in good agreement for all values of x .

119. A current of 5 A is flowing from South to North in a straight wire. Find the magnetic field due to a 1cm piece of wire at a point 1 m North-East from the piece of wire.

Ans :

SQP 2013

Given, $I = 5$

$$dl = 1 \text{ cm} = 0.01 \text{ m}$$

$$r = 1 \text{ m}$$

$$\theta = 45^\circ \quad [\text{Since, direction is North-East}]$$

$$\begin{aligned} \text{Hence, } dB &= \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin \theta}{r^2} \\ &= 10^{-7} \times \frac{5 \times 0.01 \times \sin 45^\circ}{(1)^2} \\ &= 3.54 \times 10^{-9} \text{ T} \end{aligned}$$

Its direction is vertically downwards.

120. An element $dl = dx \hat{i}$ (where, $dx = 1 \text{ cm}$) is placed at the origin and carries a large current $I = 10 \text{ A}$. What is the magnetic field on the Y -axis at a distance of 0.5 m ?

Ans :

Foreign 2011

Hence, $dl = dx = 1 \text{ cm} = 10^{-2} \text{ m}$

$$I = 10 \text{ A}$$

$$r = 0.5 \text{ m}$$

Using Bio-Savart's law,

$$\begin{aligned} dB &= \frac{\mu_0}{4\pi} \cdot \frac{Idl \times r}{r^3} \\ &= \frac{\mu_0}{4\pi} \cdot \frac{Idx}{r^2} (\hat{i} \times \hat{j}) \\ &= \frac{\mu_0}{4\pi} \cdot \frac{Idx}{r^2} \hat{k} \quad [\text{since } \hat{i} \times \hat{j} = \hat{k}] \\ &= \frac{10^{-7} \times 10 \times 10^{-2}}{(0.5)^2} \hat{k} \\ &= 4 \times 10^{-8} \hat{k} \text{ T} \end{aligned}$$

114. An Infinitely long straight current carrying wire, produces a magnetic field B , at a point distant a from it. What must be the radius of a circular loop, so that, for the same current through, it, the magnetic field at:

1. Its centre is equal to $B/2$?
2. An axial point, distance equal to the radius of the loop is equal to B ?

Ans :

Comp 2018

$$B = \frac{10^{-7} \times 2I}{a} = \frac{\mu_0 I}{2\pi a}$$

$$\begin{aligned} 1. \quad B_c &= \frac{B}{2} \\ &= \frac{10^{-7} \times 2I}{a \times 2} \\ &= \frac{10^{-7} \times 2\pi l}{R} \end{aligned}$$

$$R = 2\pi a$$

$$\begin{aligned} 2. \quad B_{\text{axis}} &= \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} \\ B &= \frac{\mu_0 I}{2\pi a} \end{aligned}$$

As, $x = R$

Hence, $\frac{R^2}{(R^2 + R^2)^{3/2}} = \frac{1}{\pi a}$

$$\frac{R^3}{2^{3/2} R^{2 \times 3/2}} = \frac{1}{\pi a}$$

$$\frac{R^2}{\sqrt{8} R^3} = \frac{1}{\pi a}$$

$$\frac{1}{\sqrt{8}} = \frac{R}{\pi a}$$

$$R = \frac{\pi a}{\sqrt{8}}$$

115. An electron having velocity $\vec{v} = 2\hat{i} + 3\hat{j}$ is projected into a uniform magnetic field $\vec{B} = 4\hat{k}$. Calculate the time period (T) of motion of the electron.

Ans :

OD 2020, Comp 2002

Velocity, $\vec{v} = 2\hat{i} + 3\hat{j}$

Magnetic field, $\vec{B} = 4\hat{k}$

Time period, $T = \frac{2\pi r}{v} \quad \dots(1)$

Since, $v = \frac{eBr}{m_e} \quad \dots(2)$

After substituting the value of Eq. (2) in Eq. (1), we get

Now, $T = \frac{2\pi m_e}{eB}$

Here, mass of electron,

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

Hence,

$$\begin{aligned} T &= \frac{2\pi m}{eB} \\ &= \frac{2 \times 3.14 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 4} \\ &= 8.9 \times 10^{-12} \text{ sec} \end{aligned}$$

116. A toroid has a core of inner radius 20 cm and outer radius 22 cm around which 4200 turns of a wire are wound. If current in the wire is 10 A, find the then magnetic field inside the core of toroid.

Ans :

OD 2009

Inner radius, $r_1 = 20 \text{ cm} = 0.2 \text{ m}$

Outer radius, $r_2 = 22 \text{ cm} = 0.22 \text{ m}$

No. of turns, $N = 4200$

Current in toroid, $r = \frac{r_1 + r_2}{2}$
 $= \frac{0.2 + 0.22}{2} = 0.21 \text{ m}$

Therefore, total length of toroid,

$$L = \text{circumference of toroid}$$

$$= 2\pi r = 2\pi \times 0.21 = 0.42\pi$$

and number of turns per unit length of toroid,

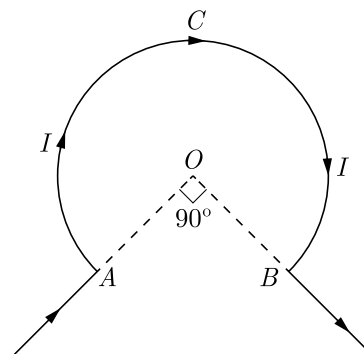
$$n = \frac{N}{L} = \frac{4200}{0.42\pi} = \frac{10000}{\pi}$$

Magnetic field inside the core of toroid,

$$\begin{aligned} B &= \mu_0 nI \\ &= (4\pi \times 10^{-7}) \times \frac{10000}{\pi} \times 10 \\ &= 0.04 \text{ T} \end{aligned}$$

(where μ_0 = Absolute permeability of free space equal to $4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1}$)

117. The wire shown in the figure carries a current of 10 A. Determine the magnitude of magnetic field induction at the centre O . (Given the radius of the bent coil is 3 cm.)



Ans :

Delhi 2019

Given, $I = 10 \text{ A}$

$$r = 3 \text{ cm}$$

Ans :

OD 2017

Here, energy of electron,

$$E' = 2000 \text{ eV}$$

$$E' = 2000 \times 1.6 \times 10^{-19}$$

$$J = 3.2 \times 10^{-16} \text{ Joule}$$

Now,

$$B = 0.2 \text{ T}$$

$$r = ?$$

$$E' = \frac{1}{2}mv^2$$

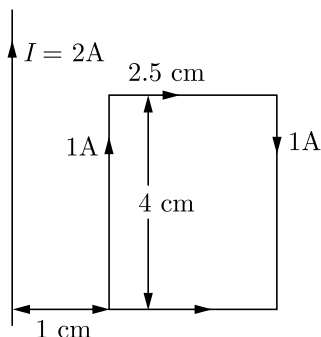
$$v = \sqrt{\frac{2E'}{m}}$$

$$Bev = \frac{mv^2}{r}$$

$$\text{Hence, } r = \frac{mv}{Be}$$

$$\begin{aligned} &= \frac{m}{Be} \sqrt{\frac{2E'}{m}} = \frac{\sqrt{2E'm}}{Be} \\ &= \frac{\sqrt{2 \times 3.2 \times 10^{-16} \times 9 \times 10^{-31}}}{0.2 \times 1.6 \times 10^{-19}} \\ &= 7.5 \times 10^{-4} \text{ m} \end{aligned}$$

112. A rectangular loop of wire of size $2.5 \text{ cm} \times 4 \text{ cm}$ carries steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of the force on the loop due to the current carrying wire.



Ans :

Comp 2021, SQP 2014

There will be force of attraction between the straight wire and 4 cm long arm of loop nearer to the straight conductor.

$$\begin{aligned} F_1 &= \frac{\mu_0}{4\pi} \frac{2 \times 2 \times 1}{(2 \times 10^{-2})} \times (4 \times 10^{-2}) \\ &\quad [\text{towards straight conductor}] \\ F_1 &= 8 \times 10^{-7} \text{ N} \end{aligned} \quad \dots(1)$$

Similarly, force on other 4 cm arm of loop, away from the straight conductor.

$$F_2 = \frac{\mu_0}{4\pi} \times \frac{2 \times 2 \times 1}{(4.5 \times 10^{-2})} \times (4 \times 10^{-2})$$

$$F_2 = 3.55 \times 10^{-7} \text{ N} \quad \dots(2)$$

[away from conductor]

- (i) Since, F_1 and F_2 are of different magnitudes, therefore, do not form couple and hence

$$\text{Torque, } \tau = 0$$

- (ii) Net force on loop,

$$F = F_1 - F_2$$

[towards straight conductor]

$$F = 8 \times 10^{-7} - 3.55 \times 10^{-7}$$

$$F = 4.45 \times 10^{-7} \text{ N}$$

The forces on two branches of loop are equal in magnitude and opposite in the directions, hence they balance each other.

113. An ammeter of resistance 0.80Ω can measure current upto 1.0 A.

- What must be the value of shunt resistance to enable the ammeter to measure current upto 5.0 A?
- What is the combined resistance of the ammeter and the shunt?

Ans :

Delhi 2020

Resistance of ammeter,

$$R_A = 0.80 \Omega$$

Maximum current across ammeter,

$$I_A = 1.0 \text{ A}$$

So, voltage across ammeter,

$$V = IR = 1.0 \times 0.80 = 0.8 \text{ V}$$

Let the value of shunt is x .

1. Resistance of ammeter with shunt,

$$R = \frac{R_A x}{R_A + x} = \frac{0.8x}{0.8 + x}$$

Current through ammeter,

$$I = 5 \text{ A}$$

$$\left(\frac{0.8x}{0.8 + x} \right) \times 5 = 0.8$$

$$0.8x \times 5 = 0.8(0.8 + x)$$

$$4x = 0.64 + 0.8x$$

$$x = \frac{0.64}{3.2} = 0.2$$

So, the shunt resistance is 0.2 ohm.

2. Combined resistance of the ammeter and the shunt,

$$\begin{aligned} R &= \frac{0.8x}{0.8 + x} = \frac{0.8 \times 0.2}{0.8 + 0.2} \\ &= \frac{0.16}{1} = 0.16 \Omega \end{aligned}$$

- 121.** Two wires A and B have the same length equal to 44 cm and carry a current of 10 A each. Wire A is bent into a circle and wire B is bent into a square.

- Obtain the magnitudes of the fields at the centres of the two wires.
- Which wire produces a greater magnetic field at its centre?

Ans :

Delhi 2016

Current $I = 10$ A

Length of each wire = 44 cm

- Let r be the radius of wire A when it is bent into a circle.

$$2\pi r = 44$$

$$r = \frac{7}{100} \text{ m}$$

Magnetic field at the center of the circular coil carrying current is given by,

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \\ &= 10^{-7} \times 2 \times \frac{22}{7} \times 10 \times \frac{100}{7} \\ &= 9 \times 10^{-5} \text{ T} \end{aligned}$$

When another wire is bent into a square of each side L , then

$$4L = 44$$

$$L = 11 \text{ cm} = 0.11 \text{ m}$$

Since, magnetic field induction at a point, at perpendicular distance a from the linear conductor carrying current is given by,

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi a} (\sin \theta_1 + \sin \theta_2) \\ B &= 4 \times \frac{\mu_0 I}{4\pi a} (\sin 45^\circ + \sin 45^\circ) \\ &= 4 \times 10^{-7} \times \frac{10}{(11/100)} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \\ &= 10.3 \times 10^{-5} \text{ T} \end{aligned}$$

- The magnetic field due to a square will be more than that due to a circle of same perimeter.

- 122.** A solenoid of length 50 cm having 100 turns carries a current of 2.5 A. Find the magnetic field (1) in the interior of the solenoid, (2) at one end of the solenoid.

Ans :

OD 2018, Comp 2006

Here, $I = 2.5$ A

$$n = \frac{100}{0.50} = 200$$

- $$B = \mu_0 n I$$

$$= 4\pi \times 10^{-7} \times 200 \times 2.5$$

$$B = 6.28 \times 10^{-4} \text{ T}$$

2.

$$\begin{aligned} B &= \frac{\mu_0 n I}{2} \\ &= \frac{4\pi \times 10^{-7} \times 200 \times 2.5}{2} \\ &= 3.14 \times 10^{-4} \text{ T} \end{aligned}$$

- 123.** A solenoid of length 1.0 m and 3.0 cm diameter has 5 layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of solenoid? Also, calculate the magnetic flux from a cross-section of the magnetic flux solenoid at the centre of solenoid.

Ans :

SQP 2013

Number of turns, $N = 850 \times 5$

$$l = 1 \text{ m}$$

$$I = 5 \text{ A}$$

Area of cross-section,

$$A = \pi r^2 = \frac{22}{7} \left(\frac{3}{2} \times 10^{-2} \right)^2 \text{ m}^2$$

Magnetic field at the center of solenoid,

$$\begin{aligned} B &= \mu_0 N I / l \\ &= 4\pi \times 10^{-7} \times (850 \times 5) \times 5 / 1 \\ &= 2.671 \times 10^{-2} \text{ T} \end{aligned}$$

Hence, Magnetic flux = BA

$$\begin{aligned} &= 2.671 \times 10^{-2} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2} \right)^2 \\ &= 1.89 \times 10^{-5} \text{ Wb} \end{aligned}$$

- 124.** A magnetic field of 100 (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2 . The maximum current carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1} . Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic.

Ans :

Foreign 2010

Magnetic field, $B = 100$

$$G = 100 \times 10^{-4} \text{ T}$$

To design the solenoid, let us find the product of current and number of turns in the solenoid.

The magnitude of magnetic field,

$$B = \mu_0 n I$$

$$\text{or } n I = \frac{10^{-2}}{4} \times 3.14 \times 10^{-7}$$

$$n I = 7961 \approx 8000$$

Here, the product of nI is 8000.

Answer the following questions :

- Explain the reason of jumping of the ring when the switch is dosed in the circuit.
- What will happen if the terminals of the battery are reversed and the switch is closed ? Explain.
- Explain the two laws that help us to understand this phenomenon.
- Briefly explain various ways to increase the strength of magnetic field produced by a given solenoid.

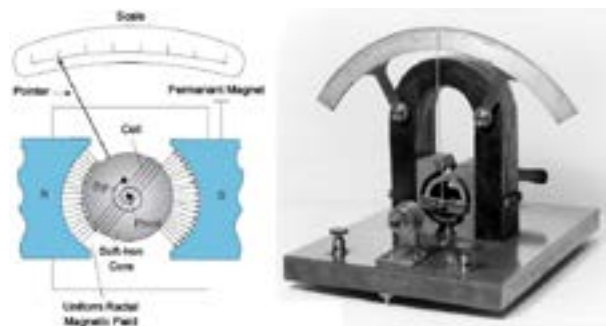
Ans :

OD 2023

- By Lenz's law, the magnetic field produced by the induced eddy current in the ring opposes the changing magnetic field of the solenoid. Consequently, the two magnetic fields repel each other making the ring jump.
- If the terminals of the battery are reversed, then current will be cut off. The cut keeps the ring from turning into a magnet, so ring will not jump.
- The jumping ring is a demonstration of Faraday's law of electromagnetic induction and also of Lenz's law.
Lenz's law states that an induced current in a closed conducting. Loop will appear in such a direction that it oppose the change that produces it.
Faraday's first law of electromagnets induction describes the induction of emf in a conductor and second law quantifies the emf produced in the conductor.
- The strength of the magnetic field produced by the solenoid can be increased.
 - By placing soft iron inside the solenoid.
 - By increasing the number of turns of the coil.
 - By increasing the current flowing through the solenoid.

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130. Ajay and Sandeep were asked by their teacher to perform an experiment using a galvanometer. Before doing the experiment they were very keen to know the different parts of the galvanometer which was given to them in the form of a small box. They approached the teacher and asked for the permission. The teacher thought it would be a good idea if the galvanometer be opened before the whole class and explained its construction and working to all of them.



- State briefly the working principle of the galvanometer.
- What is the shape of the magnets used and why is it so designed ?

Ans :

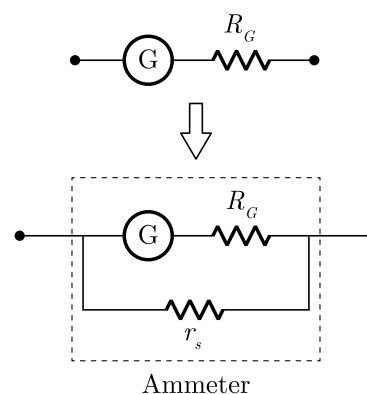
OD 2015

- When current flows through a coil kept in uniform radial magnetic field, it experiences a torque which tends to rotate the coil and produces an angular deflection. The deflection of the coil is directly proportional to the current.

$$\theta = \frac{NAB}{K} \cdot I$$

- The shape of the magnets used is concave. It is designed to produce a radial magnetic field.

131. The galvanometer cannot be used as an ammeter to measure the value of the current directly as it is a very sensitive device. It gives a full-scale deflection for current of the order of μA . For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of current in the circuit.



be read by the cash dispenser machine. If you will mishandle such magnetic strip of it, then its stored data may become corrupted.

- (i) How do magnetic strips work?
- (ii) A long wire carries a steady current. It is first bent into a circular loop and magnetic field at its centre is found to be B_0 . Then the same wire is bent into a circular coil of n turns. Find the magnetic field at the centre point now.



Ans :

- (i) This standard tape strip contains three magnetic tracks that are used to store the card's code data. The card is usually presented to the reader by swiping or inserting it into the reader, which obtains the card's code using a magnetic head that detects the magnetic field generated by its strip.
- (ii) Let radius of circular loop be R , so the magnetic field at the centre,

$$B_0 = \frac{\mu_0 I}{2R} \quad \dots(1)$$

In the second case, radius of the coil be r having n turns, then

$$2\pi r \cdot n = 2\pi R$$

$$r = \frac{R}{n}$$

Then, magnetic field at the centre of the coil,

$$B = \frac{n\mu_0 I}{2r} = \frac{n\mu_0 I}{2R/n} = \frac{n^2\mu_0 I}{2R} = n^2 B_0 \quad [\text{From equation (1)}]$$

Thus,

$$B = n^2 B_0$$

- 128.** Deepak needed an ammeter of range 0 to 6 A for his project work. He went to physics laboratory and market for it. But it was not available to him. But a galvanometer of resistance of 15Ω and its range of 0 to 4 mA was available in the laboratory, then he decided to convert such galvanometer into ammeter of given range.

- (i) How does an ammeter work?
- (ii) How did he convert such galvanometer into ammeter of desired range? Explain showing by calculations.

Ans :

- (i) It is not mechanical, or digital. It uses an analog to digital converter (ADC) to measure the voltage across the shunt resistor. The ADC is read by a microcomputer that performs the calculations to display the current through the resistor.

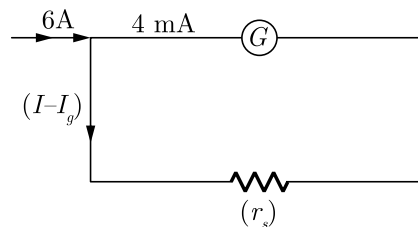
- (ii) Given, $R_G = 15 \Omega$

$$I_g = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}$$

$$I = 6 \text{ A}$$

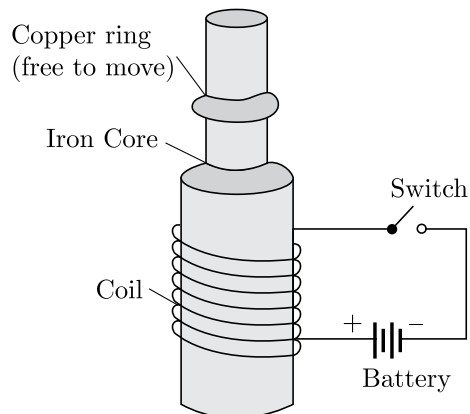
For converting galvanometer into ammeter of given range, We join a small shunt resistance r_s in parallel to the galvanometer.

$$I_g \times R_G = (I - I_g) r_s$$



$$\begin{aligned} r_s &= \frac{I_g}{I - I_g} \times R_G \\ &= \frac{4 \times 10^{-3}}{6 - (4 \times 10^{-3})} \times 15 \\ &\approx 10^{-2} \Omega = 10 \text{ m}\Omega \end{aligned}$$

- 129.** Consider the experimental set up shown in the figure. This jumping ring experiment is an outstanding demonstration of some simple laws of Physics. A conducting non-magnetic ring is placed over the vertical core of a solenoid. When current is passed through the solenoid, the ring is thrown off.



Current, $I = 8 \text{ A}$

Number of turns, $n = 1000$

The other design is,

$$I = 10 \text{ A}$$

and $n = 800/\text{m}$

This is the most appropriate design as per the requirement.

- 125.** Two concentric coils each of radius equal to $2\pi \text{ cm}$ are placed at right angles to each other. If 3 A and 4 A are the currents flowing in each coil respectively, find the resultant magnetic field at the centre of the coils.

Ans :

OD 2004, Delhi 2015

$$\begin{aligned} \text{Radius of each circular loop, } r &= 2\pi \text{ cm} \\ &= 2\pi \times 10^{-2} \text{ m} \end{aligned}$$

$$\text{Current in first loop, } i_1 = 3 \text{ A}$$

$$\text{Current in second loop, } i_2 = 4 \text{ A}$$

Magnetic field at the centre of first circular coil due to its current,

$$\begin{aligned} B_1 &= \frac{\mu_0 i_1}{2r} = \frac{(4\pi \times 10^{-7}) \times 3}{2 \times (2\pi \times 10^{-2})} \\ &= 3 \times 10^{-5} \text{ T} \end{aligned}$$

Similarly, magnetic field at the centre of second circular coil due to its current,

$$\begin{aligned} B_2 &= \frac{\mu_0 i_2}{2r} \\ &= \frac{(4\pi \times 10^{-7}) \times 4}{2 \times (2\pi \times 10^{-2})} \\ &= 4 \times 10^{-5} \text{ T} \end{aligned}$$

Since, both the coils are perpendicular to each other, therefore directions of their magnetic fields will also be perpendicular to each other. Thus, resultant magnetic field at the centre of the coils,

$$\begin{aligned} B &= \sqrt{(B_1)^2 + (B_2)^2} \\ &= \sqrt{(3 \times 10^{-5})^2 + (4 \times 10^{-5})^2} \\ &= 5 \times 10^{-5} \text{ Wb} \cdot \text{m}^{-2} \end{aligned}$$

CASE BASED QUESTIONS

- 126.** Rahul's uncle was advised by his doctor to undergo an MRI scan test of his chest who also gave him an estimate of the cost. Not knowing much about the significance of this test and finding it to be too expensive he first hesitated. When Rahul learnt about this, he decided to take help of his family, friends and neighbours and arranged for the money. He convinced

his uncle to undergo this test so as to enable the doctor to diagnose the disease. He got the test done and the resulting information greatly helped the doctor to give him proper treatment.

- Assuming that the MRI scan test involved a magnetic field of 0.1 T , find the maximum and minimum values of the force that this field could exert on a proton moving with a speed of 10^4 ms^{-1} . State the condition under which the force can be minimum.
- What is the principal mechanism for producing an MRI signal?



Ans :

- A strong magnetic field is required by using an expensive setup.

Magnetic force on moving charge particle in uniform magnetic field $\vec{B}$ can be given as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\text{or } |F| = qvB \sin \theta$$

- Maximum force at $\theta = 90^\circ$

$$\begin{aligned} F &= qvB = 1.6 \times 10^{-19} \times 10^4 \times 0.1 \\ &= 1.6 \times 10^{-16} \text{ N} \end{aligned}$$

- Minimum force at $\theta = 0^\circ$ and 180°

$$F = 0$$

i.e., charge particles either move parallel or anti-parallel to the magnetic field lines.

- Magnetic resonance imaging (MRI) uses the movement of protons within a magnetic field to generate an image. Within the constant magnetic field of an MRI scanner, tissues that contain free hydrogen nuclei (protons) generate varying signals when pulses of radio frequency (RF) energy are applied to them.

- 127.** A credit card of an old villager was not working properly. Rakesh observed the card and found its stored data was corrupted due to mishandling of magnetic strip. He suggested him that brown strip on such card is a magnet. It carries a code which can

- (i) How is a moving coil galvanometer converted into an ammeter of desired range?
- (ii) A moving coil galvanometer of resistance G gives a full-scale deflection for a current I_g . It is converted into an ammeter of range 0- I ampere. What should be the value of shunt resistance to convert it into an ammeter of desired range?
- (iii) Which one will have the greatest resistance – a micro-ammeter, a milli-ammeter, an ammeter?
- (iv) What is the resistance of ammeter?

Ans :

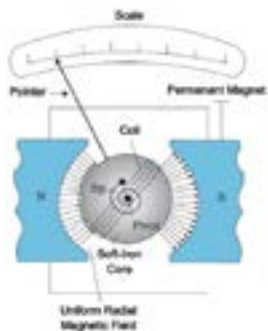
(i) Connecting a shunt resistance in parallel.

(ii). $S = \frac{IG}{I - I_g}$

(iii) Micro-ammeter

(iv) $\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S}$

- 132.** The galvanometer is a device used to detect the current flowing in a circuit or a small potential difference applied to it consists of a coil with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field formed by using concave pole pieces of a magnet. When a current flow through the coil, a torque acts on it.



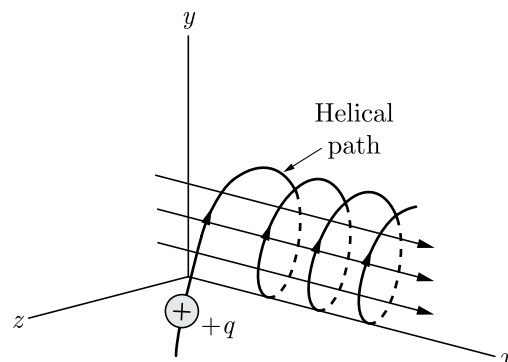
- (i) What is the principle of moving coil galvanometer?
- (ii) If the field is radial, what is the angle between magnetic moment of galvanometer coil and the magnetic field?
- (iii) Why pole pieces are made concave in the moving coil galvanometer?

Ans :

- (i) A current-carrying coil when placed in an external magnetic field experiences magnetic torque. The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.
- (ii) 30°
- (iii) The pole pieces of the galvanometer are made concave so that the current-carrying coil and the magnetic field are always perpendicular to each other. Concave magnetic pole always produces a

radial magnetic field. The radial field will always be perpendicular to the conductor rotating about the axis.

- 133.** If velocity has a component along B , this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. The motion in a plane perpendicular to magnetic field is a circular one, thereby producing a helical motion.



- (i) What is the expression of radius of charged particle in the uniform magnetic field ?
- (ii) An electron, proton, He^+ and Li^{++} are projected with the same velocity perpendicular to a uniform magnetic field. Which one will experience maximum magnetic force?
- (iii) What is the work done by the magnetic field on the charge particle moving perpendicular to a uniform magnetic field?
- (iv) How much distance moved by a charged particle along the magnetic field in one rotation, when velocity has a component parallel to magnetic field?

Ans :

(i) $R = \frac{mv}{qB}$

(ii) Li^{++}

(iii) Zero

(iv) $\frac{2\pi mvcos\theta}{qB}$

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CHAPTER 5

Magnetism and Matter

SUMMARY

1. MAGNETIC FIELD LINES

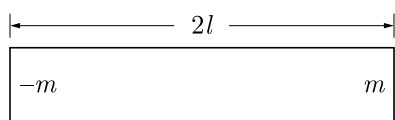
These are imaginary lines which give pictorial representation for the magnetic field inside and around the magnet.

Properties of electric field lines are given as below:

1. These lines form continuous closed loops.
2. The tangent to the field line gives direction of the field at that point.
3. Larger the density of the lines, stronger will be the magnetic field.
4. These line do not intersect each other.

2. MAGNETIC DIPOLE MOMENT OF A CURRENT LOOP AND REVOLVING ELECTRON

Magnetic dipole moment of a magnet, $M = m \times 2l$ A-m².



Where m is a pole strength, $2l$ is separation between poles.

Magnetic dipole moment of a current loop,

$$M = NIA \text{ A-m}^2$$

The direction of m is perpendicular of the plane of loop and given by right hand thumb rule.

Magnetic dipole moment of a revolving electron = $IA = e\nu\pi r^2$

where ν is frequency, r is radius of orbit.

$$M = \frac{e}{2m_e} L \text{ A-m}^2$$

where $L = m_e v r$ is angular momentum of revolving electron.

3. BAR MAGNET AS AN EQUIVALENT SOLENOID

The expression of magnetic field at distance r from the centre is given by,

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}}{r^3}$$

This expression is equivalent to that of bar magnet.

4. TORQUE OF BAR MAGNET

Torque on a bar magnet in a uniform magnetic field is,

$$\tau = MB \sin \theta = M \times B$$

where, θ is the angle between M and B . Its S.I. units is joule per tesla (JT⁻¹).

5. POTENTIAL ENERGY OF A MAGNETIC DIPOLE

Potential energy of a magnetic dipole in a magnetic field is given by,

$$U = -MB \sin \theta = -M \times B$$

where, θ is the angle between M and B .

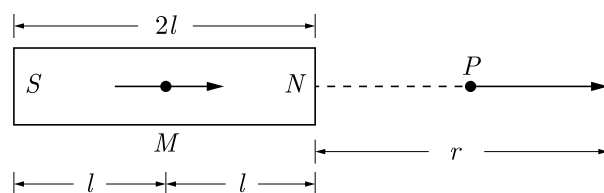
6. WORK DONE IN ROTATING THE DIPOLE

Work done in rotating the dipole in a uniform magnetic field from θ_1 to θ_2 is given by

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

7. MAGNETIC FIELD DUE TO A BAR MAGNET

1. At an axial point (end-on-position).



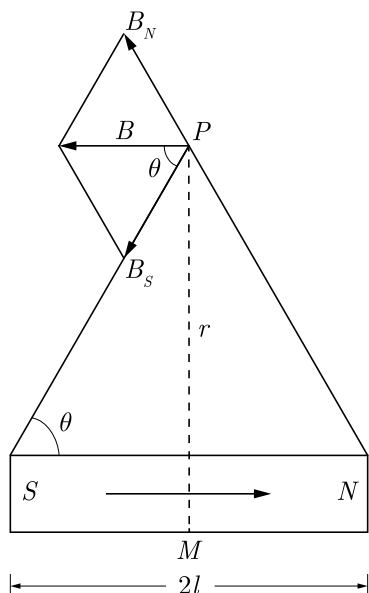
$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}}{r^3} \quad (\text{Since } r \gg l)$$

The direction of magnetic field is along the direction of magnetic dipole moment M .

2. At on equatorial point (broadside-on-position)

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{\vec{M}}{r^3}$$

The direction of magnetic field is parallel to the magnetic dipole and opposite to the direction of dipole moment.



8. MAGNETISM AND GAUSS' LAW

The net magnetic flux (φ_B) through any closed surface is always zero.

$$\text{i.e., } \varphi_B = \Sigma B \cdot \Delta S = 0$$

9. TERMS RELATED TO MAGNETIC PROPERTIES

Terms related to magnetic properties of material are given as below:

1. **Intensity of Magnetisation, I** : The magnetic dipole moment produced per unit volume field of specimen of magnetic material in magnetising field is known as intensity of magnetisation.

$$I = \frac{M}{V} = \frac{m}{A}$$

where, M = magnetic dipole moment,

m = pole strength,

v = volume of specimen

and A = cross-section area

2. **Magnetic Permeability** : It is equal to the ratio of magnetic induction and magnetising field.

$$\mu = \frac{B}{H}$$

3. **Magnetic Susceptibility** : It is equal to the ratio of intensity of magnetisation and magnetising field.

$$\chi_m = \frac{I}{H}$$

It has no unit.

10. MAGNETIC INDUCTION

It is defined as the number of magnetic lines of induction crossing per unit area through the magnetic substance.

$$B = \mu_0(H + I)$$

where, μ_0 = permeability of free space

H = magnetising field

I = intensity of magnetisation

11. CLASSIFICATION OF MAGNETIC MATERIALS

Magnetic materials may be classified into three categories

1. **Diamagnetic Substances** : These are the substances in which feeble magnetism is produced in a direction opposite to the applied magnetic field. These substances are repelled by a strong magnet. These substances have small negative values of susceptibility χ and positive low value of relative permeability μ_r , i.e.,

$$-1 \leq \chi < 0 \text{ and } 0 \leq \mu_r < 1$$

The examples of diamagnetic substances are bismuth, antimony, copper, lead, water, nitrogen (at STP) and sodium chloride.

2. **Paramagnetic Substances** : These are the substances in which feeble magnetism is induced in the same direction as the applied magnetic field. These are feebly attracted by a strong magnet. These substances have small positive values of M and χ and relative permeability μ_r greater than 1, i.e.,

$$0 < \chi < \varepsilon \text{ and } 1 < \mu_r < 1 + \varepsilon$$

where ε is a small positive number. The examples of paramagnetic substances are platinum, aluminium, calcium, manganese, oxygen (at STP) and copper chloride.

3. **Ferromagnetic Substances** : These are the substances in which a strong magnetism is produced in the same direction as the applied magnetic field. These are strongly attracted by a magnet. These substances are characterised by large positive values of M and χ and values of μ_r much greater than 1, i.e.,

$$\chi \gg 1, \mu_r \gg \chi$$

OBJECTIVE QUESTIONS

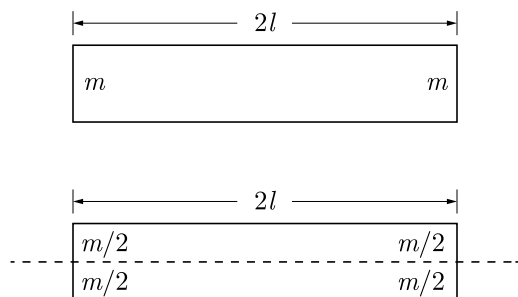
1. A bar magnet of magnetic moment M is cut into two parts of equal length. The magnetic moment of either part is

(a) M (b) $M/2$
(c) $2M$ (d) Zero

Ans :

OD 2018

If a bar magnet of magnetic moment M is cut into two parts as shown in figure,



Dipole moment, $M = m \times 2l$... (1)

When bar magnet is cut into two parts of equal length. Then,

Dipole moment, $M' = m/2 \times 2l$
 $= \frac{1}{2} \times (m \times 2l)$

From Eq. (1), we get

$$M' = \frac{M}{2}$$

Thus (b) is correct option.

2. Magnetic dipole moment is a vector quantity directed from

(a) South to North Pole
(b) North to South Pole
(c) East to West direction
(d) West to East direction

Ans :

OD 2016

Magnetic dipole moment is a vector quantity directed from South pole to North pole.

Thus (a) is correct option.

3. A wire of magnetic dipole moment M and L is bent into shape of a semicircle of radius r . What will be its new dipole moments?

(a) M (b) $\frac{M}{2\pi}$
(c) $\frac{M}{\pi}$ (d) $\frac{2M}{\pi}$

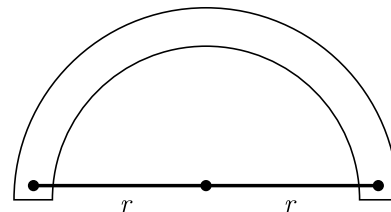
Ans :

SQP 2017

According to the question,

Magnetic dipole moment of wire is given by,

$$M = m \times L \quad \dots (1)$$



Now, Circumference of semicircle

= length of the wire

$$\pi r = L$$

$$r = \frac{L}{\pi} \quad \dots (2)$$

Magnetic dipole moment of semicircle is given by,

$$M' = m \times 2r$$

$$M' = m \times 2 \times \frac{L}{\pi} \quad \text{From Eq. (2)}$$

$$= \frac{m \times 2L}{\pi}$$

$$M' = \frac{2M}{\pi} \quad \text{From Eq. (1)}$$

Thus (d) is correct option.

4. Permeability μ of a ferromagnetic substance

(a) $\mu \gg 1$ (b) $\mu = 1$
(c) $\mu < 1$ (d) $\mu = 0$

Ans :

Foreign 2013, OD 2014

The magnetic permeability μ of a magnetic substance gives the ratio of the magnetic flux density in the material B to the intensity of external magnetising field H in which it is placed.

Thus, We have

$$\mu = \frac{B}{H}$$

A ferromagnetic substance is strongly magnetised in the direction of the external magnetising field. Thus B is very larger than H . Therefore, its permeability μ is enormously greater than one i.e. $\mu \gg 1$.

Thus (a) is correct option.

5. S.I. unit of pole strength is

(a) N (b) $N/A - m$
(c) $A - m$ (d) T

Ans :

OD 2017, 2010

Magnetic dipole moment = Pole strength $\times$ Length

20. Most of the substance show which of the following magnetic property?

(a) diamagnetism (b) paramagnetism
(c) ferromagnetism (d) both (b) and (c)

Ans :

Foreign 2016

All substances show diamagnetism, as it is universal. Since both the paramagnetism and ferromagnetism in substances have diamagnetism in weak, therefore they are hard to detect.

Thus (a) is correct option.

21. A diamagnetic material in a magnetic field moves

(a) from weaker to stronger parts
(b) perpendicular to the field
(c) from stronger to weaker parts
(d) in none of the above directions

Ans :

SCP 2015

Diamagnetic materials are magnetised in the opposite direction of the magnetising field. Therefore diamagnetic material moves from stronger to weaker parts of the magnetic field.

Thus (c) is correct option.

22. If a diamagnetic substance is brought near north or south pole of a bar magnet, then it is

(a) attracted by the poles
(b) repelled by the poles
(c) attracted by the north pole and repelled by the south pole
(d) repelled by the north pole and attracted by the south pole

Ans :

OD 2008

When diamagnetic substances are magnetised in the opposite direction of the magnetising field. That is why, when a diamagnetic substance is brought near the north or south pole of a bar magnet, it is repelled by the poles.

Thus (b) is correct option. A super conductor exhibits perfect

(a) ferrimagnetism (b) ferromagnetism
(c) paramagnetism (d) diamagnetism

Ans :

Delhi 2015

A substance with permeability less than unity is called diamagnetic. Since the superconductor has a zero permeability, therefore it exhibits perfect diamagnetism.

Thus (d) is correct option.

23. The magnetic susceptibility does not depend upon the temperature in

(a) ferrite substances
(b) ferromagnetic substances
(c) diamagnetic substances
(d) paramagnetic substances

Ans :

SCP 2002

Susceptibility of diamagnetic substances is constant and has low negative value (-10^{-6} to -10^{-7}). It does not depend upon the temperature.

Thus (c) is correct option.

24. A frog can be levitated in magnetic field produced by a current in a vertical solenoid placed below the frog. This is possible because the body of the frog behaves as

(a) paramagnetic (b) diamagnetic
(c) ferromagnetic (d) anti-ferromagnetic

Ans :

Delhi 2017

When diamagnetised materials are placed in a magnetic field, they are feebly magnetised in the opposite direction of the magnetising field or repelled. Since the frog levitated in magnetic field produced by a current in vertical solenoid placed below the frog due to repulsion, therefore body of the frog behaves as diamagnetic.

Thus (b) is correct option.

25. The domain formation is a necessary feature of

(a) diamagnetism (b) paramagnetism
(c) ferromagnetism (d) all of these

Ans :

OD 2016, Foreign 2010

A ferromagnetic substance is composed of tiny crystals-like regions. And within each crystal there are one or more domains. And in each domain, there is a perfect alignment of the elementary moment represented by an arrow. That is why domain, there is a perfect alignment of the elementary moment represented by an arrow. That is why, domain formation is a necessary feature of ferromagnetism.

Thus (c) is correct option.

26. Substances which are strongly attracted by magnetic field i.e., move from weaker to stronger side of the field are termed as

(a) diamagnetic substances
(b) paramagnetic substances
(c) ferromagnetic substances
(d) non-magnetic substances

12. The magnetic lines of force inside a bar magnet
 (a) do not exist
 (b) are from north-pole to south-pole of the magnet
 (c) are from south-pole to north-pole of the magnet
 (d) depend upon the area of cross-section of the bar magnet

Ans :

SQP 2011

Magnetic lines of force, due to a bar magnet, form a closed loop and outside the bar magnet, these lines go from north-pole to south-pole, whereas inside the bar magnet, these lines travel from south-pole to north-pole of the magnet.

Thus (c) is correct option.

13. The magnetic field at a distance r from a short bar magnet is directly proportional to
 (a) r^2 (b) r^{-3}
 (c) $r^{2/3}$ (d) r^4

Ans :

OD 2017

Magnetic field due to a short bar magnet at a distance r from itself,

$$B = \frac{\mu_0 M}{4\pi r^3} \propto r^{-3}$$

where, M = Magnetic moment of magnet

Thus (b) is correct option.

14. At the magnetic poles of the earth, a compass needle will be
 (a) vertical
 (b) horizontal
 (c) inclined at 10° with the vertical
 (d) inclined at 45° with the horizontal

Ans :

Delhi 2004

Horizontal component of earth's magnetic field at the magnetic poles of the earth is zero.

Therefore, a compass needle at this place will be vertical.

Thus (a) is correct option.

15. Which of the following is an example for diamagnetic substances?
 (a) copper (b) nickel
 (c) aluminium (d) iron

Ans :

OD 2014

Copper is an example of diamagnetic substance. And nickel, aluminium, iron are substances.

Thus (a) is correct option.

16. If a bar magnet is dropped down in an infinitely long vertical copper tube, then the magnet will move continuously
 (a) increasing velocity and acceleration.
 (b) increasing velocity but constant acceleration.
 (c) decreasing velocity and ultimately comes to rest.
 (d) increasing velocity and ultimately acquires a constant terminal velocity.

Ans :

Delhi 2001

When a bar magnet is dropped down in an infinitely long vertical copper tube, its velocity continuously increases due to the gravitational attraction. As a result of this, the velocity of bar magnet continuously goes on increasing but having constant acceleration due to free fall under gravity.

Thus (b) is correct option.

17. At which place, the earth's magnetism becomes horizontal?
 (a) magnetic pole (b) geographical pole
 (c) magnetic equator (d) magnetic meridian

Ans :

Foreign 2010

Magnetic field of the earth becomes horizontal at a place where angle of magnetic dip is zero.

Since, the angle of magnetic dip at magnetic meridian is zero, therefore earth's magnetism becomes horizontal at that place.

Thus (d) is correct option.

18. Metals getting magnetised by orientation of atomic magnetic moments in external magnetic field are called
 (a) diamagnetics (b) paramagnetics
 (c) ferromagnetics (d) anti-magnetics

Ans :

OD 2017, Comp 2011

Those metals which are magnetised by orientation of atomic magnetic moments in external magnetic field are called paramagnetics.

Thus (b) is correct option.

19. Liquid oxygen remains suspended between two poles of a magnet, because it is
 (a) diamagnetic (b) paramagnetic
 (c) ferromagnetic (d) anti-ferromagnetic

Ans :

Delhi 2012

Liquid oxygen (O_2) is paramagnetic, because each oxygen molecule has two unpaired electrons. This is why, it remains suspended between two poles of the magnet.

Thus (b) is correct option.

$$\begin{aligned}\text{Pole strength} &= \frac{\text{Magnetic dipole moment}}{\text{length}} \\ &= \frac{A - m^2}{m} = A - m\end{aligned}$$

Thus (c) is correct option.

6. S.I. unit of magnetic moment is
 (a) JT^{-2} (b) Am^2
 (c) JT (d) Am^{-1}

Ans :

OD 2015

The magnetic moment of a magnetic dipole is defined as the product of its pole strength and magnetic length, $2l$ i.e.,

$$M = m \times 2l$$

SI unit of magnetic moment is ampere-meter² (Am^2).

Thus (b) is correct option.

7. When the intensity of magnetic field is increased four times, the time period of suspended magnetic needle becomes
 (a) double (b) half
 (c) four times (d) one-fourth less

Ans :

OD 2002

The time period of suspended magnetic needed is given by,

$$T = 2\pi \sqrt{\frac{I}{mB}}$$

i.e.,

$$T \propto \frac{1}{\sqrt{B}}$$

Hence,

$$\frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{4B_1}{B_1}}$$

$$T_2 = \frac{T_1}{2}$$

The time period of suspended magnetic needed becomes half.

Thus (b) is correct option.

8. The direction of null points are on the equatorial line of a bar magnet, when the north pole of the magnet is pointing towards
 (a) north (b) south
 (c) east (d) west

Ans :

SPQ 2016

When the north pole of a bar magnet is pointing towards north direction, then magnetic lines of force pass through the magnet.

Thus (a) is correct option.

9. In tan A and tan B positions of a magnet the magnet fields at a distance d are represented by B_1 and B_2 respectively, then

$$(a) B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2}, B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{3/2}}$$

$$(b) B_1 = B_2; d \gg 1$$

$$(c) B_1 = 2B_2; d \gg 1$$

$$(d) \text{Both a and c}$$

Ans :

OD 2009

According to the deflection magnetometer, the magnetic field in tan A and tan B position is given by,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2} \quad (\text{for tan } A \text{ position})$$

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{3/2}} \quad (\text{for tan } B \text{ position})$$

If $d \gg l$ then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{d^3}$$

and

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

$$\frac{B_1}{B_2} = 2$$

$$B_1 = 2B_2$$

$$d \gg 1$$

Thus (d) is correct option.

10. Magnetic lines of force
 (a) always intersect.
 (b) are always closed.
 (c) do not pass through vacuum.
 (d) tend to crowd far way from the poles of a magnet.

Ans :

Foreign 2012, Comp 2004

Outside the magnet, magnetic lines of force travel from north to south-pole, whereas inside the magnet, these lines, travel from south to north-pole.

That is why, the magnet lines of force are always closed.

Thus (b) is correct option.

11. If a material, placed in a magnetic field is thrown out of it, then the material is
 (a) diamagnetic (b) paramagnetic
 (c) ferromagnetic (d) non-magnetic

Ans :

Delhi 2006

Diamagnetic materials are repelled when they are placed in a magnetic field. That is why, diamagnetic material is thrown out if it.

Thus (a) is correct option.

Ans :

SQP 2003

Ferromagnetic substances are strongly magnetised in the direction of magnetising field. Therefore ferromagnetic substances move from weaker to stronger side of the magnetic field.

Thus (c) is correct option.

27. If χ_{m_1} and χ_{m_2} are the susceptibilities of a diamagnetic substance at temperatures T_1 and T_2 respectively, then
- (a) $\chi_{m_1} = \chi_{m_2}$ (b) $\chi_{m_1} T_1 = \chi_{m_2} T_2$
- (c) $\chi_{m_1} T_2 = \chi_{m_2} T_1$ (d) $\chi_{m_1} \sqrt{T_1} = \chi_{m_2} \sqrt{T_2}$

Ans :

Delhi 2005

Given:

Initial susceptibility, $T_1 = \chi_{m_1}$

Final susceptibility, $T_2 = \chi_{m_2}$

Susceptibility of a diamagnetic substance is independent of temperature. And it does not obey Curie's law.

Therefore, $\chi_{m_1} = \chi_{m_2}$

Thus (a) is correct option.

28. Susceptibility of a magnetic substance is found to depend on temperature and the strength of the magnetising field. The material is a
- (a) diamagnet (b) ferromagnet
- (c) paramagnet (d) superconductor

Ans :

OD 2011

Ferromagnetism of a substance decreases with the rise in temperature. Moreover, a ferromagnetic substance gets strongly magnetised along the direction of the magnetic field. Therefore the material is ferromagnet. Thus (b) is correct option.

29. The vertical component of the earth's magnetic field is zero at a place where the angle is dip
- (a) 0° (b) 45°
- (c) 60° (d) 90°

Ans :

Foreign 2009

Given:

Vertical component of earth's magnetic field,

$$B_V = 0$$

Vertical component of the earth's magnetic field at a place,

$$B_V = B \sin \delta$$

Therefore, If $B_V = 0$, Then

$$\sin \delta = 0 \quad (\text{where, } \delta \text{ is Angle of dip})$$

$$\delta = 0$$

Thus (a) is correct option.

ASSERTION AND REASON

30. **Assertion** : Diamagnetic substances exhibit magnetism.

Reason : Diamagnetic materials do not have permanent magnetic dipole moment.

- (a) Both Assertion and Reason are true and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion and Reason are true and Reason is NOT the correct explanation of Assertion.
- (c) Assertion is true and Reason is false.
- (d) Assertion is false and Reason is also false.

Ans :

Diamagnetic material exhibits magnetism in reverse direction and due to the absence of unpaired electron in diamagnetic material it does not exhibit permanent magnetic dipole moment.

Thus (b) is correct option.

31. **Assertion** : We cannot think of a magnetic field configuration with three poles.

Reason : A bar magnet does exert a torque on itself due to its own field.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Magnet field may be formed with the help of three poles. A bar magnet does not exert a torque on itself due to its own field.

Thus (d) is correct option.

32. **Assertion** : Magnetic Resonance Imaging (MRI) is a useful diagnostic tool for producing images of various parts of human body.

Reason : Protons of various tissues of the human body play a role in MRI.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

MRI is useful diagnostic tool for producing images of various parts of human body because it makes use

of magnetic property of spinning proton inside the nucleus.

Thus (a) is correct option.

- 33. Assertion :** Diamagnetic materials can exhibit magnetism.

Reason : Diamagnetic materials have permanent magnetic dipole moment.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Diamagnetic material exhibits magnetism in reverse direction. R is a wrong statement. Because due to absence of unpaired electron in diamagnetic material it does not exhibit permanent magnet dipole moment. Thus (c) is correct option.

- 34. Assertion :** Ferro-magnetic substances become paramagnetic above Curie temperature.

Reason : Domains are destroyed at high temperature.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Susceptibility of ferro magnets decreases with increase of temperature. At a transition temperature is increased, dipoles acquire kinetic energy and are disoriented, hence domain internal interaction called exchange coupling disappears.

Thus (a) is correct option.

- 35. Assertion :** The ferromagnetic substance do not obey Curie's law.

Reason : At Curie point a ferromagnetic substance start behaving as a paramagnetic substance.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

The susceptibility of ferromagnetic substance decreases with the rise of temperature in a complicated manner. After Curies point in the susceptibility of ferromagnetic substance varies inversely with its absolute temperature. Ferromagnetic substance obeys Curie's law only above its Curie point.

Thus (c) is correct option.

- 36. Assertion :** The ferromagnetic substance do not obey Curie's law.

Reason : At Curie point a ferromagnetic substance start behaving as a paramagnetic substance.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

The susceptibility of ferromagnetic substance decreases with the rise of temperature in a complicated manner. After Curie point in the susceptibility of ferromagnetic substance varies inversely with its absolute temperature. Ferromagnetic substance obeys Curie's law only above its Curies point.

Thus (b) is correct option.

- 37. Assertion :** A paramagnetic sample display greater magnetisation (for the same magnetic field) when cooled.

Reason : The magnetisation does not depend on temperature.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

A paramagnetic sample display greater magnetisation when cooled, this is because at lower temperature, the tendency to disrupt the alignment of dipoles (due to magnetising field) decreases on account of reduced random thermal motion.

Thus (d) is correct option.

- 38. Assertion :** The poles of magnet can not be separated by breaking into two pieces.

Reason : The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

When a magnet is cut into pieces, each piece becomes new magnet.

$$M' = \frac{ml}{2} = \frac{M}{2}$$

Thus (b) is correct option.

VERY SHORT ANSWER QUESTIONS

- 39.** The horizontal component of the earth's magnetic field at a place is B and angle of dip is 60° what is the value of vertical component of earth's magnetic field at equator?

Ans :

OD 2021

On the equator, the values of both angles of dip (δ) and vertical component of earth's magnetic field is zero. So, in this case, $B_V = 0$.

- 40.** A magnetic needle, free to rotate in a vertical plane, orients itself vertically at a certain place on the Earth. What are the values of (i) horizontal component of Earth's magnetic field and (ii) angle of dip at this

Ans :

Comp 2020

- (i) 0° ,
 (ii) 90°

- 41.** A small magnet is pivoted to move freely is the magnetic meridian. At what place on earth's surface will the magnet be vertical?

Ans :

Delhi 2020, SQP 2013

Magnet will be vertical at magnetic pole of the earth.

- 42.** Relative permeability of a material $\mu_r = 0.5$. identify the nature of the magnetic material and write its relation of magnetic susceptibility.

Ans :

Foreign 2014

The nature of magnetic material is a diamagnetic.

$$\mu_r = 1 + \chi_m$$

- 43.** Where on the earth's surface is the value of angle of dip maximum?

or

Where on the surface of earth is the angle of dip 90° ?

Ans :

OD 2020

Angle of dip (90°) is maximum at magnetic poles.

- 44.** Where on the earth's surface is the value of vertical component of earth's magnetic field zero?

Ans :

OD 2018

Vertical component of earth's magnetic field is zero at magnetic equator.

- 45.** Which of the following substances are diamagnetic? Bi, Al, Cu, Ca, and Ni

Ans :

Delhi 2019

Diamagnetic substances are

- (i) Bi,
 (ii) Cu.

- 46.** The Susceptibility of a magnetic material is -4.2×10^{-6} . Name the type of magnetic material it represents.

Ans :

OD 2019

Negative susceptibility represents diamagnetic substance.

- 47.** The Susceptibility of a magnetic material is 1.9×10^{-5} . Name the type of magnetic material it represents.

Ans :

Foreign 2011

The small and positive Susceptibility of 1.9×10^{-5} represents paramagnetic substance.

- 48.** The permeability of a magnetic material is 0.9983. Name the type of magnetic materials it represents.

Ans :

Comp 2018, OD 2011

$\mu_r < 1$, so magnetic material is diamagnetic

- 49.** What are permanent magnets ?

Ans :

SQP 2013

Substances that retain their attractive property for a long period of time at room temperature are called permanent magnets.

- 50.** Why is the core of an electromagnet made of ferromagnetic materials?

Ans :

OD 2010

Ferromagnetic material has a high retentive. So on passing current thorough windings it gains suffice magnetism immediately.

- 51.** What is the importance of radial magnetic field in a moving coil galvanometer?

Ans :

Delhi 2014

- The plane of the coil remains parallel to the direction of magnetic field. So, the galvanometer is linear.
- It is a stronger magnetic field as compared to the magnetic field produced by the flat pieces of a field magnet.

The effect of the torque $\vec{\tau}$ is to make the magnet align itself parallel to the field $\vec{B}$.

Definition of magnetic dipole moment. If in Eq. (1),

$$B = 1$$

Then, $\theta = 90^\circ$

Hence, the magnetic dipole moment may be defined as the torque acting on a magnetic dipole placed perpendicular to a uniform magnetic field of unit strength.

(ii) SI unit of magnetic moment.

SI unit of magnetic moment. As,

$$m = \frac{\tau}{B \sin \theta}$$

$$\text{SI units of } m = \frac{1 \text{ Nm}}{1 \text{ T} \cdot 1}$$

$$= \text{NmT}^{-1} \text{ or } \text{JT}^{-1} \text{ or } \text{Am}^2$$

(iii) Torque on a magnet

When the magnet lies along the direction of the magnetic field,

$$\theta = 0^\circ$$

$$\sin \theta = 0$$

$$\tau = 0$$

Thus the torque is minimum.

When the magnet lies perpendicular to the direction of the field,

$$\theta = 90^\circ$$

$$\sin \theta = 1$$

$$\tau = mB$$

Thus the torque is maximum and $\tau_{\max} = mB$.

NUMERICAL QUESTIONS

94. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times its vertical component. What is the angle of dip at that place?

Ans :

OD 2015

Horizontal components of earth's magnetic field,

$$B_H = \sqrt{3} \times B_V$$

where B_V = Vertical component of earth's magnetic field

Relation for the angle of dip δ at a place is,

$$\tan \delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3} \times B_V} = \frac{1}{\sqrt{3}}$$

$$\delta = 30^\circ$$

95. A bar magnet of magnetic moment 5.0 Am^2 has poles 20 cm apart. Calculate the pole strength.

Ans :

Comp 2016

$$\text{Given, } m = 5.0 \text{ Am}^2$$

$$2l = 20 \text{ cm} = 0.20 \text{ m}$$

$$\text{As, } m = q_m \times 2l$$

$$\text{Hence, Pole strength, } q_m = \frac{m}{2l}$$

$$= \frac{5.0}{0.20} = 25 \text{ Am}$$

96. What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from the midpoint? The magnetic moment of the bar magnet is 0.40 Am^2 .

Ans :

OD 2018

$$\text{Given, } m = 0.40 \text{ Am}^2$$

$$r = 50 \text{ cm} = 0.50 \text{ m}$$

$$2l = 5.0 \text{ cm}$$

Clearly, the magnet is a short magnet ($l \ll r$).

$$1. \quad B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

$$= \frac{10^{-7} \times 0.4}{(0.5)^3} = 3.2 \times 10^{-7} \text{ T}$$

$$2. \quad B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

$$= 6.4 \times 10^{-7} \text{ T}$$

97. A magnetised needle of magnetic moment $4.8 \times 10^{-2} \text{ JT}^{-1}$ is placed at 30° with the direction of uniform magnetic field of magnitude $3 \times 10^{-2} \text{ T}$. What is the torque acting on the needle?

Ans :

SQP 2013, Comp 2001

$$\text{Given, } m = 4.8 \times 10^{-2} \text{ JT}^{-1}$$

$$\theta = 30^\circ, B = 3 \times 10^{-2} \text{ T}$$

$$\text{Hence, Torque, } \tau = mB \sin \theta$$

$$= 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \sin 30^\circ$$

$$= 7.2 \times 10^{-4} \text{ J}$$

98. A short bar magnet placed with its axis at 30° to a uniform magnetic field of 0.2 T experiences a torque of 0.06 Nm.

1. Calculate the magnetic moment of the magnet.
2. Find out what orientation of the magnet corresponds to its stable equilibrium in the magnetic field.

Ans :

Foreign 2010

$$1. \quad \text{Given, } B = 0.2 \text{ T}$$

61. A short bar magnet, placed with its axis making an angle θ with a uniform magnetic field $\vec{B}$, experiences a torque $\vec{\tau}$. What is the magnetic moment of the magnet?

Ans :

Delhi 2017

The torque $\vec{\tau}$ experienced by a bar magnet placed in a uniform magnetic field $\vec{B}$ is given by,

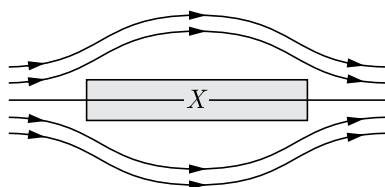
$$\vec{\tau} = \vec{M} \times \vec{B}$$

In magnitude, $\tau = MB \sin \theta$

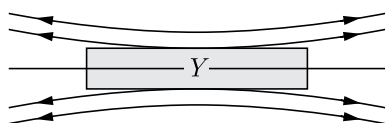
$$M = \frac{\tau}{B \sin \theta}$$

SHORT ANSWER QUESTIONS

62. A uniform magnetic field gets modified as shown in the figure, when two specimens X and Y are placed in it.



(a)



(b)

Ans :

Comp 2019

- X is a diamagnetic substance because in this substance, the magnetic field lines are farther than in air.
 - Y is a ferromagnetic substance because in this substance the field lines are much closer than in air.
63. State two characteristic properties distinguishing behaviour of paramagnetic and diamagnetic materials.

Ans :

Comp 2021

The difference between distinguishing and paramagnetic substance are as follows :

Diamagnetic Substance	Paramagnetic Substance
In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker field, i.e., they move from stronger to weaker magnetic field.	In non-uniform magnetic field, paramagnetic substances move from weaker to stronger part of the magnetic field slowly.
Their permeability is less than one ($\mu < 1$).	Their permeability is slightly greater than one ($\mu > 1$).

64. Write two differences between electromagnet and permanent magnet.

Ans :

OD 2020, SQP 2009

The difference between electromagnet and permanent magnet.

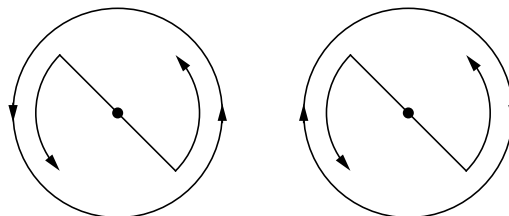
	Electromagnet	Permanent Magnet
1.	It shows temporary magnetism. It produces the magnetic field as long as current flows in its coil.	It shows permanent magnetism i.e., it retains magnetism in it even when the current is switched off.
2.	The polarity of an electromagnet can be changed by reversing the direction of the current.	The polarity of a permanent magnet cannot be changed.

65. How does a circular loop carrying current behave as a magnet?

Ans :

Foreign 2011

The current round in the face of the coil is in anti-clockwise direction, then this behaves like a North pole, whereas when it is viewed from the other side, then current round in it is in clockwise direction necessarily forming South pole of magnet.



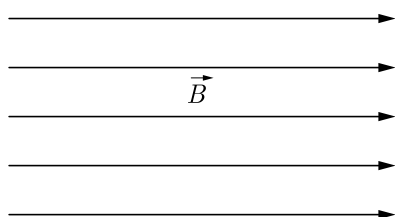
Hence, current loop has both magnetic poles and therefore, behaves like a magnetic dipole.

66. Define uniform magnetic field. How is it represented geometrically?

Ans :

Foreign 2012

A magnetic field is said to be uniform if a unit isolated north pole placed at different points in the field experiences the same force in the same direction. Graphically, a uniform magnetic field is represented by equidistant and mutually parallel lines.



67. 1. How does a diamagnetic material behave when it is cooled to very low temperature?
2. Why does a paramagnetic sample display greater magnetisation when cooled? Explain.

Ans :

OD 2014

1. For diamagnetic substances, the variation of susceptibility is very small, i.e., diamagnetic materials are unaffected by the change in temperature (except bismuth).
2. A paramagnetic sample displays greater tendency when cooled at lower temperatures, the tendency to disrupt the alignment of magnetic dipole decreases due to the reduced random thermal motion of atoms or molecules.
68. A magnetic needle suspended freely in uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet however experiences a force of attraction in addition to torque. Why?

Ans :

SQP 2013

When a magnetic needle is suspended freely in uniform magnetic field, the two poles of the needle will experience equal and opposite forces hence net force is zero. On the other hand, when an iron nail is placed near a bar magnet, the magnetic field of bar magnet is non-uniform and the two poles of the iron nail will experience different force.

69. Why does a paramagnetic sample displays greater magnetism when cooled?

Ans :

Foreign 2014

The paramagnetic material contains atoms possessing permanent magnetic dipole moment. When a paramagnetic material is placed in a magnetic field, these magnetic dipoles get aligned in the direction of magnetic field. The high temperature disrupt this alignment. When a paramagnetic material is cooled,

the thermal agitation trying to disrupt the alignment decreases and thus sample displays greater magnetism.

70. What are artificial magnets? Give some of their advantages over natural magnets.

Ans :

OD 2018

Artificial Magnets

Pieces of iron and other magnetic materials can be made to acquire the properties of natural magnets. Such magnets are called artificial magnets.

Advantages of Artificial Magnets Over Natural Magnets

1. They can be made magnetically much stronger than natural magnets.
 2. They can be made of any convenient size and shape.
 3. Their polarity can be reversed whenever desired.
71. Derive relation between relative magnetic permeability and magnetic susceptibility.

Ans :

Delhi 2012, OD 2017

In SI units, $B = \mu_0(H + I)$

Since, $I = \chi_m H$

Hence, $B = \mu_0(H + \chi_m H)$

$$B = \mu_0 H(1 + \chi_m)$$

Also, $B = \mu H$

Hence, $\mu H = \mu_0 H(1 + \chi_m)$

$$\mu = \mu_0(1 + \chi_m)$$

$$\frac{\mu}{\mu_0} = 1 + \chi_m$$

$$\mu_r = 1 + \chi_m$$

This is the required relation.

72. A magnetic needle is placed on a cork floating on a still lake in the northern hemisphere. Does this needle together with the cork move towards the north of the lake?

Ans :

Delhi 2015

Since magnetic needle does not experience any net force, so the needle and the cork will not move towards the north of the lake. But, if the needle is inclined to the magnetic meridian, a torque will act on the needle and after a few oscillations, the needle will set itself along the magnetic meridian.

73. An iron bar is heated to 1000°C and then cooled in a magnetic field free space. Will it retain magnetism?

Ans :

SQP 2007

The iron bar will lose its magnetism as the temperature to which it is heated (i.e. 1000°C) is much above the curie point for iron (i.e. 770°C). So the iron bar

magnet after heating to 1000°C and then cooled, cannot retain its magnetism.

74. Define magnetic dipole and dipole moment. What are its units?

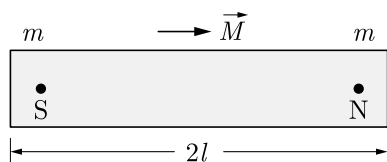
Ans :

SQP 2010

Magnetic Dipole : An arrangement of two magnetic poles of equal and opposite strength separated by a finite distance is said to constitute a magnetic dipole.

Common examples of magnetic dipoles are compass needle, a bar magnet, a current loop, an electron revolving in a circular or elliptical path etc.

Magnetic Dipole Moment : Consider a magnetic consisting of two poles N and S each having pole strength m as shown in the figure. The distance between two poles is called magnetic length.



Magnetic dipole moment is the product of strength of either pole and the magnetic length.

i.e., $\vec{M} = m(2\vec{l})$

The direction of magnetic dipole moment is from south to north pole.

Unit : In SI the unit of dipole moment is ampere metre² (Am^2).

75. State four basic properties of magnets.

Ans :

SQP 2007

Basic properties of magnets are as follows:

1. **Attractive Property :** A magnet attracts small pieces of iron, nickel, cobalt, etc.
 2. **Directive Property :** A freely suspended magnet aligns itself nearly in the geographic north-south direction.
 3. **Like Poles Repel and Unlike Poles Attract :** This is a fundamental law of magnetic poles.
 4. **Magnetic Poles Exist in Pairs :** Isolated magnetic poles do not exist. If we break a magnet into two pieces, we get two smaller dipole magnets.
76. Two identical iron bars A and B are given: one of which definitely known to be magnetised. How would you ascertain, whether or not both are magnetised. If only one is magnetised?

Ans :

SQP 2008

Since repulsion is the sure test of magnetisation, because if bar A attracts the other bar B , then both may be magnets (i.e. their unlike poles are facing each other) or one bar may be a simple piece of iron, but if

there is a force of repulsion between them, then both bars must be magnetised.

To know which one is magnetised, place the bar A on the table and bring one of bar B near the two ends and at the middle of bar A . If there is force of attraction only at the ends of bar A , then bar A is magnetised and B not magnetised and if there is force of attraction both at the ends as well as at the middle, then bar B is magnetised and A is unmagnetised.

77. Define magnetic field and magnetic intensity (or magnetic field at a point). What is the S.I. unit of magnetic intensity?

Ans :

OD 2016

Magnetic Field

The space around a magnet or a current carrying conductor in which the magnetic effect can be felt is called the magnetic field.

Magnetic Intensity

The strength of magnetic field or magnetic intensity at a point is the force experienced by a unit north pole placed at that point. The direction of the field is the direction in which this pole begins to move if free to do so.

Thus magnetic intensity is a vector quantity and has both magnitude and direction.

The S.I. unit of magnetic intensity is tesla or ampere meter.

If a magnetic pole of strength m units placed at a point where the magnetic intensity is $\vec{B}$ then it experiences a force of $m\vec{B}$.

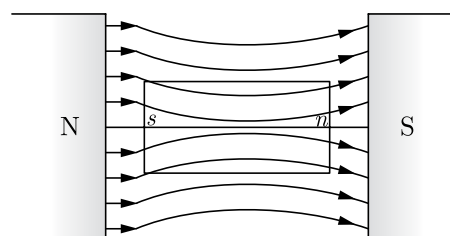
78. Give some important properties of ferromagnetic substances.

Ans :

SQP 2014, Comp 2003

Properties of ferromagnetic substances:

1. They are strongly attracted by magnets.
2. As shown in figure, the lines get highly concentrated in ferromagnetic material so that magnetic induction $B \gg B_0$.



Highly Concentrated Lines of Force in a Ferromagnetic Rod

52. Magnetic field lines can be entirely confined with the core of toroid, but not within a straight solenoid, why?

Ans :

Foreign 2017

The field lines cannot be entirely confined to the two ends of a straight solenoid. If they were so, the magnetic flux through the cross-section at each end would have been non-zero; this is denied by Gauss theorem of magnetism. For a toroid, this difficulty does not arise because it is endless.

53. Magnetic field lines show the direction (at every point) along which a small magnetised needle aligns (at the point). Do the magnetic field lines also represent the 'lines of force'?

Ans :

OD 2012

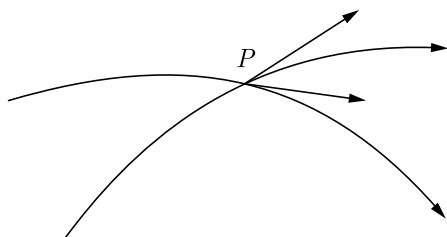
No, the magnetic field lines certainly represent the direction of magnetic field, but not the direction of force; this is because force is always perpendicular to magnetic field $\vec{B}$. Therefore, it is misleading to call magnetic field lines as the lines of force.

54. Why cannot two magnetic lines of forces due to a bar magnet cross each other?

Ans :

Comp 2017, OD 2013

Because if they cross at any point (say P), there would be two tangents at point P and hence two directions of magnetic fields at the same point as shown in the figure, but magnetic field has only one direction.



55. The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What is the cause of this damping?

Ans :

Comp 2018

As the copper plates oscillate in the magnetic field between the two plates of the magnet, there is a continuous change of magnetic flux linked with the pendulum. Due to this, eddy currents are set up in the copper plate which try to oppose the motion of the pendulum according to the Lenz's law and finally bring it to rest.

56. Out of the two magnetic materials, A has relative permeability slightly greater than unity while B has less than unity. Identify the nature of the materials A and B . Will their susceptibilities be positive or negative?

Ans :

Delhi 2017

The nature of the material A is paramagnetic and its susceptibility χ_m is positive.

The nature of the material B is diamagnetic and its susceptibility χ_m is negative.

57. What is the basic difference between magnetic and electric lines of force?

Ans :

Delhi 2011, 06

The basic difference between magnetic and electric lines of force is that whereas magnetic lines of force are closed continuous curves, the electric lines of force are discontinuous. They start from the positive charge and end at the negative charge. On the contrary, magnetic lines of force exist even inside the body of the magnet and are therefore closed continuous curves.

58. The relative magnetic permeability of a magnetic material is 800. Identify the nature of magnetic material and state its two properties.

Ans :

Delhi 2018

Ferromagnetic substances, as these substances have very high magnetic permeability.

Properties :

1. High retentivity.
2. High susceptibility.

59. Why does paramagnetic substance move from weaker to stronger parts of non-uniform magnetic field?

Ans :

OD 2018, Comp 2006

The atoms of paramagnetic substances possess permanent magnetic dipoles and these dipoles are randomly distributed in the absence of external magnetic field. But when the paramagnetic substance is subjected to a non-uniform magnetic field, each atom experiences a torque tendency to align its magnetic dipole moment along the direction of magnetic field, and thus a paramagnetic substance moves from weaker to stronger magnetic field.

60. Why does a magnetic dipole possess potential energy, when placed at some inclination with the direction of the field?

Ans :

Comp 2017

When a magnetic dipole is placed in a uniform magnetic field, it aligns itself along the direction of the magnetic field in equilibrium. If a magnetic dipole is displaced from equilibrium position, a restoring torque acts on the dipole which brings it back. So if a dipole is placed at some inclination with the field, work has to be done against the restoring force. This work done is stored in the form of potential energy in the magnetic dipole.

A paramagnetic material tends to move from weaker field to stronger field regions of the magnetic field.

So, the number of lines of magnetic field increases when passing through it.

Magnetic dipole moments are induced in the direction of magnetic field.

Paramagnetic materials has a small positive susceptibility.

A diamagnetic material tends to move from stronger field to weaker field region of the magnetic field. So, the number of lines of the magnetic field passing through it decreases.

Magnetic dipole moments are induced in the opposite direction of the applied magnetic field.

Diamagnetic materials has a negative susceptibility in the range $(-1 \leq \chi < 0)$.

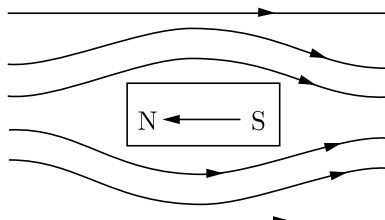
83. Explain the following

- (1) Why do magnetic field lines form continuous closed loops?
- (2) Why are the field lines repelled (expelled) when a diamagnetic material is placed in an external uniform magnetic field?

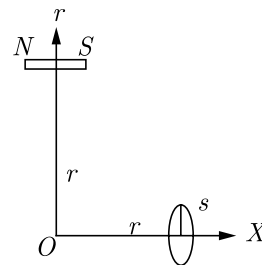
Ans :

OD 2012

- (1) Magnetic lines of force form continuous closed loops because a magnet is always a dipole and as a result, the net magnetic flux of a magnet is always zero.
- (2) When a diamagnetic substance is placed in an external magnetic field, a feeble magnetism is induced on opposite direction. So, magnetic lines of force are repelled.



84. A small magnet of magnetic moment M , is placed at a distance r from the origin O with its axis parallel to X -axis as shown. A small coil, if one turn is placed on the X -axis, at the same distance from the origin, with the axis of the coil coinciding with X -axis. For what value of current in the coil does a small magnetic needle, kept at origin, remains undeflected? What is the direction of current in the coil?



Ans :

Comp 2010

This happens when magnetic field of bar magnet is equal and opposite to the magnetic field of coil

$$|\vec{B}_m| = |\vec{B}_c|$$

$$\frac{\mu_0 M}{4\pi r^3} = \frac{\mu_0 I a^2}{2r^3}$$

$$I = \frac{2M}{4\pi a^2}$$

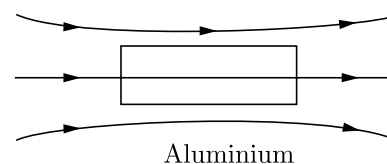
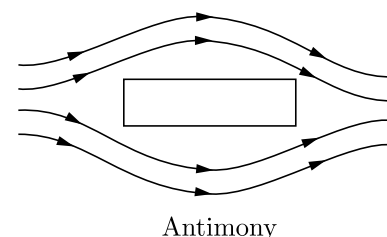
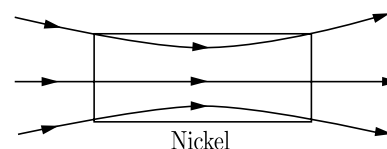
Current is in anti-clockwise sense, as seen from the origin.

85. Three identical specimens of a magnetic materials nickel, antimony and aluminium are kept in a non-uniform magnetic field. Draw the modification in the field lines in each case. Justify your answer.

Ans :

Delhi 2017, SQP 2013

The modifications are shown in the figure.



It happens because

- (i) nickel is a ferromagnetic substance.
- (ii) antimony is a diamagnetic substance.
- (iii) aluminium is a paramagnetic substance.

Right Hand Thumb Rule : If we curl the fingers of the right hand in the direction of current in the loop, then the extended thumb gives the direction of the magnetic moment associated with the loop.

2. **Dimensions of Magnetic Dipole Moment :** For any current loop,

Magnetic dipole moment = Current $\times$ Area of the loop

Hence, $[m] = [A][L^2] = [AL^2]$

SI unit of magnetic dipole moment is Am^2 . It is defined as the magnetic moment associated with one turn loop of area one square metre when a current of one ampere flows through it.

91. Explain how does an atom behave as a magnetic dipole. Derive an expression for the magnetic dipole moment of the atom.

or

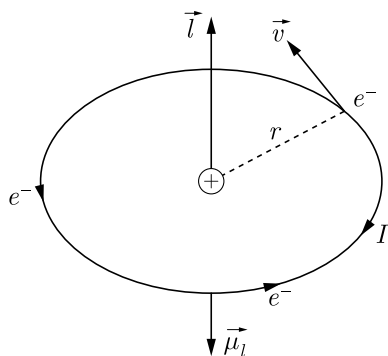
Deduce the expression for the magnetic dipole moment of an electron orbiting around the central nucleus.

Ans :

Comp 2018

Magnetic Dipole Moment of a Revolving Electron

In hydrogen-like atoms, an electron revolves around the nucleus. Its motion is equivalent to a current loop which possesses a magnetic dipole moment $= IA$. As shown in figure, consider an electron revolving anticlockwise around a nucleus in an orbit of radius r with speed v and time period T .



Orbital Magnetic Moment of a Revolving Electron

$$\begin{aligned}\text{Current, } I &= \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} \\ &= \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}\end{aligned}$$

Area of the current loop,

$$A = \pi r^2$$

The orbital magnetic moment of the electron is

$$\mu_l = IA = \frac{ev}{2\pi r} \cdot \pi r^2$$

or

$$\mu_l = \frac{evr}{2} \quad \dots(1)$$

According to right hand thumb rule, the direction of the magnetic dipole moment of the revolving electron will be perpendicular to the plane of its orbit and in the downward direction.

Also, the angular momentum of the electron due to its orbital motion is,

$$l = m_e vr \quad \dots(2)$$

The direction of $\vec{l}$ is normal to the plane of the electron orbit and in the upward direction.

Dividing Eq. (1) by (2), we get

$$\frac{\mu_l}{l} = \frac{\frac{evr}{2}}{m_e vr} = \frac{e}{2m_e}$$

The above ratio is a constant called gyromagnetic ratio. Its value is $8.8 \times 10^{10} \text{ Ckg}^{-1}$. So,

$$\mu_l = \frac{e}{2m_e} l$$

Vectorially, $\vec{\mu}_l = -\frac{e}{2m_e} \vec{l}$

The negative sign shows that the direction of $\vec{l}$ is opposite to that of $\vec{\mu}_l$. According to Bohr's quantisation condition, the angular momentum of an electron in any permissible orbit is,

$$l = \frac{nh}{2\pi}, \quad (\text{where, } n = 1, 2, 3, \dots)$$

Hence, $\mu_l = n \left(\frac{eh}{4\pi m_e} \right)$

This equation gives orbital magnetic moment of an electron revolving in n^{th} orbit.

92. Derive an expression for the magnetic field intensity at a point on the equatorial line of a bar magnet. What is the direction of this field?

or

Derive an expression for the magnetic field due to a magnetic dipole in broad-side on position at a distance r from its centre. The length of the dipole is $2l$ and its magnetic moment is m .

Ans :

Delhi 2020, Comp 2002

Magnetic field of a bar magnet at an equatorial point (broadside-on position). Consider a bar magnet NS of length $2l$ and of pole strength q_m . Suppose the magnetic field is to be determined at a point P lying on the equatorial line of the magnet NS at a distance from its centre, as shown in figure.

and
$$v_d = \frac{1}{2\pi} \sqrt{\frac{(M_1 + M_2) B_H}{(I_1 + I_2)}}$$

From Eq. (1) and (2), we get

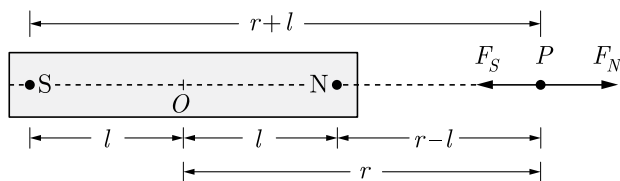
$$\begin{aligned} \frac{T_s}{T_d} &= \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \\ \frac{M_1}{M_2} &= \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} \\ &= \frac{v_s^2 + v_d^2}{v_s^2 - v_d^2} \end{aligned}$$

89. Derive an expression for the magnetic field intensity at a point on the axis of a bar magnet. What is the direction of the field?

Ans :

Comp 2020

Magnetic field of a bar magnet at an axial point (end-on position). Let NS be a bar magnet of length $2l$ and of pole strength q_m . Suppose the magnetic field is to be determined at a point P . Which lies on the axis of the magnet at a distance r from its centre, as shown in figure.



Imagine a unit north pole placed at point P . Then from Coulomb's law of magnetic forces, the force exerted by the N -pole of strength q_m on unit north pole will be

$$F_N = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{(r-l)^2}, \text{ along } \vec{NP}$$

Similarly, the force exerted by S -pole on unit north pole is

$$F_S = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{(r+l)^2}, \text{ along } \vec{PS}$$

Therefore, the strength of the magnetic field $\vec{B}$ at point P is

B_{axial} = Force experienced by a unit north-pole at point P

$$\begin{aligned} &= F_N - F_S \\ &= \frac{\mu_0 q_m}{4\pi} \left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right] \\ &= \frac{\mu_0 q_m}{4\pi} \cdot \frac{4rl}{(r^2 - l^2)^2} \end{aligned}$$

But $q_m \cdot 2l = m$, is the magnetic dipole moment, so

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$$

For a short bar magnet, $l \ll r$, therefore, we have

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}, \text{ along } \vec{NP} \quad \dots(1)$$

Clearly, the magnetic field at any axial point of magnetic dipole is in the same direction as that of its magnetic dipole moment.

90. 1. A current loop behaves as a magnetic dipole. Obtain an expression for the magnetic dipole moment of a circular loop. State the rule used to find the direction of the magnetic dipole moment.
2. Obtain the dimensions and units of magnetic dipole moment.

Ans :

OD 2016, Delhi 2008

1. **Current Loop as a Magnetic Dipole :** The magnetic field produced at a large distance r from the centre of a circular loop (or radius a) along its axis is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2IA}{r^3} \quad \dots(1)$$

On the other hand, the electric field of an electric dipole at an axial point lying far away from it is given by

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} \quad \dots(2)$$

where p is the electric dipole moment.

On comparing equations (1) and (2), we note that both B and E have same distance dependence ($\frac{1}{r^3}$). Moreover, they have same direction at any far away point, not just on the axis.

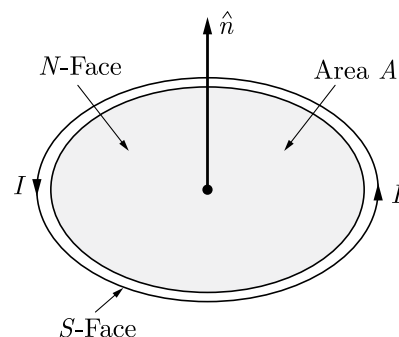
This suggests that a circular current loop behaves as a magnetic dipole of magnetic moment,

$$m = IA$$

In vector notation,

$$\vec{m} = I\vec{A}$$

This result is valid for planer current loop of any shape. Thus the magnetic dipole moment of any current loop is equal to the product of the current and its loop area. Its direction is defined to be normal to the plane of the loop in the sense give by right hand thumb rule, as shown in figure.



Current Loop as a Magnetic Dipole

3. When placed in a non-uniform magnetic field, a ferromagnetic substance tends to move quickly from weaker to the stronger parts of the field.
 4. When freely suspended, a rod of ferromagnetic material quickly aligns itself parallel to the magnetic field.
 5. Their relative permeability is large, of the order of thousands.
 6. Their susceptibility is large and positive.
 7. Ferromagnetism decreases with the increase of temperature.
79. State Gauss's law of magnetism. What are its important consequences?

Ans :

Delhi 2005

Gauss's Law in Magnetism

This law states that the net magnetic flux through any closed surface is zero. Or, the surface integral of a magnetic field over a closed surface is zero. Mathematically,

$$\phi_B = \oint_S \vec{B} \cdot d\vec{S} = 0$$

Consequences of Gauss's Law

1. Gauss's law indicates that there are no sources or sinks of magnetic field inside a closed surface. This implies that isolated magnetic poles (i.e., monopoles) do not exist.
 2. The magnetic poles always exist as unlike pairs of equal strengths.
 3. If a number of magnetic lines of force enter a closed surface, then an equal number of lines of force must leave that surface.
80. Distinguish between diamagnetic and ferromagnetic materials in terms of
- (1) Susceptibility and
 - (2) their behaviour in a non-uniform magnetic field.

Ans :

Foreign 2011

- (1) **Susceptibility for diamagnetic material :** It is independent of magnetic field and temperature (except for bismuth at low temperature).

Susceptibility for ferromagnetic material : The susceptibility of ferromagnetic materials decreases steadily with increase in temperature. At the temperature, the ferromagnetic materials become paramagnetic.

- (2) Behaviour in non-uniform magnetic field
Diamagnetic materials are feebly repelled, whereas ferromagnetic materials are strongly attracted by non-uniform field i.e., diamagnetic move in the direction of decreasing field, whereas ferromagnetic feel force in the direction of increasing field intensity.

81. Distinguish between soft and hard ferromagnetic materials. Give examples of each type.

Ans :

Delhi 2017

Ferromagnetic materials can be divided into two categories:

1. **Soft Ferromagnetic Materials or Soft Ferromagnets :** These are the ferromagnetic materials in which the magnetisation disappears on the removal of the external magnetising field. They have low retentivity, low coercivity, and low hysteresis loss. But they have high relative magnetic permeability. They are used as cores of solenoids and transformers.
Examples: Soft iron, mu metal, stalloy, etc.
2. **Hard Ferromagnetic Materials or Hard Ferromagnets :** These are the ferromagnetic materials which retain magnetisation even after the removal of the external magnetising field. They have high retentivity, high coercivity and large hysteresis loss. They are used for making permanent magnets.
Examples: Steel, alnico, lodestone, ticonal, etc.

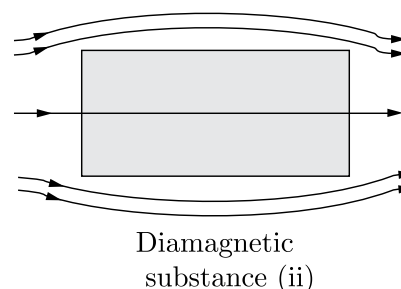
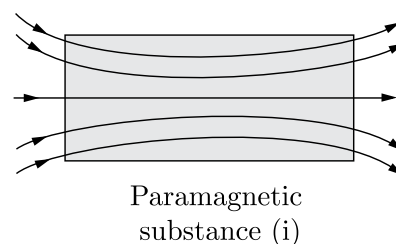
82. Show diagrammatically the behaviour of magnetic field lines in the presence of (i) paramagnetic and (ii) diamagnetic substances. How does now explain this distinguishing feature?

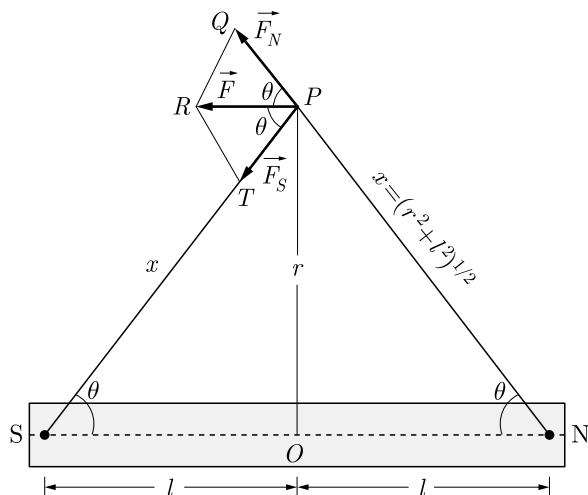
or

Draw the magnetic field lines distinguishing between diamagnetic and paramagnetic materials. Give a simple explanation to account for the difference in the magnetic behaviour of these materials.

Ans :

OD 2014, Comp 2005





Magnetic Field of a Bar Magnet at an Equatorial Point

Imagine a unit north-pole placed at point P . Then from Coulomb's law of magnetic forces, the force exerted by the N -pole of the magnet on unit north-pole is,

$$F_N = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \text{ along } \vec{NP}$$

Similarly, the force exerted by the S -pole of the magnet on unit north-pole is

$$F_S = \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2}, \text{ along } \vec{PS}$$

As the magnitudes of F_N and F_S are equal, so their vertical components get cancelled while the horizontal components add up along PR .

Hence the magnetic field at the equatorial point P is,

$$B_{\text{equa}} = \text{Net force on a unit } N\text{-pole placed at point } P$$

$$= F_N \cos \theta + F_S \cos \theta \quad [\text{Here } F_N = F_S]$$

$$= 2F_N \cos \theta$$

$$= 2 \cdot \frac{\mu_0}{4\pi} \cdot \frac{q_m}{x^2} \cdot \frac{1}{x}$$

$$\text{or } B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} \quad [\text{since } x = (r^2 + l^2)^{1/2}]$$

where, $m = q_m \cdot 2l$ is the magnetic dipole moment.

Again for a short magnet, $l \ll r$, so we have

$$B_{\text{equa}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}, \text{ along } \vec{PR} \quad \dots(2)$$

Clearly, the magnetic field at any equatorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment.

On comparing Eqs. (1) and (2), we note that the magnetic field at a point at a certain distance on the

axial line of a short magnet is twice of that at the same distance on its equatorial line.

93. (i) Derive an expression for the torque on a magnetic dipole placed in a magnetic field and hence define magnetic dipole moment.
 (ii) Write the SI unit of magnetic moment.
 (ii) When is the torque on a magnet (a) minimum and (b) maximum?

Ans :

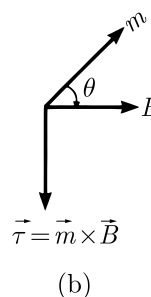
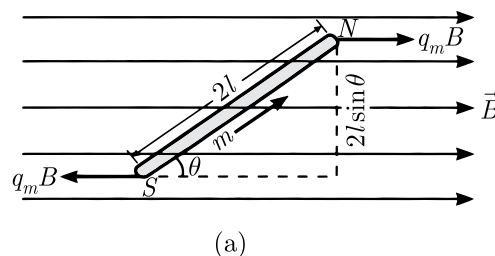
OD 2019

- (i) Derive an expression for the torque on a magnetic dipole

Torque on a magnetic dipole in a uniform magnetic field. Consider a bar magnet NS of length $2l$ placed in a uniform magnetic field $\vec{B}$. Let q_m be the pole strength of its each pole. Let the magnetic axis of the bar magnet make an angle θ with the field $\vec{B}$, as shown in Figure (a).

Force on N -pole = $q_m B$, along $\vec{B}$

Force on S -pole = $q_m B$, opposite to $\vec{B}$



The forces on the two poles are equal and opposite. They form a couple. Moment of couple or torque is given by,

$$\tau = \text{Force} \times \text{perpendicular distance}$$

$$= q_m B \times 2l \sin \theta = (q_m \times 2l) B \sin \theta$$

$$\tau = mB \sin \theta \quad \dots(1)$$

where, $m = q_m \times 2l$, is the magnetic dipole moment of the bar magnet. In vector notation,

$$\vec{\tau} = \vec{m} \times \vec{B} \quad \dots(2)$$

The direction of the torque $\vec{\tau}$ is given by the right hand screw rule as indicated in Figure (b).

86. A coil of N turns and radius R carries a current I . It is unwound and rewound to make a square coil of side a having same number of turns (N). Keeping the current I same, find the ratio of the magnetic moments of the square coil and the circular coil.

Ans :

Delhi 2015

The magnetic moment of a current loop is given by the relation $M = NIA$

For the circular loop $M_c = NI\pi R^2$... (1)

Now, When the coil is unwound and rewound to make a square coil, then

$$\begin{aligned} M_s &= NI a^2 = NI(\pi R/2)^2 \\ &= NI\pi^2 R^2/4 \end{aligned} \quad \dots (2)$$

From eqs. (1) and (2), we have

$$\frac{M_s}{M_c} = \frac{NI\pi^2 R^2/4}{NI\pi R^2} = \frac{\pi}{4}$$

LONG ANSWER QUESTIONS

87. Find time period of a bar magnet oscillating freely in a uniform magnetic field $\vec{B}$

Ans :

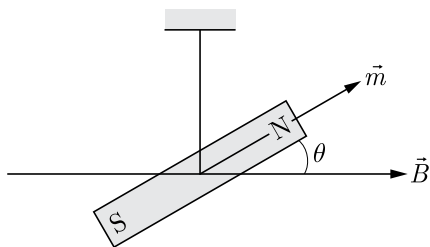
Comp 2021

Oscillations of a Freely Suspended Magnet in a Magnetic Field

In the position of equilibrium, the magnetic dipole lies along $\vec{B}$. When it is slightly rotated from this position and released, it begins to vibrate about the field direction under the restoring torque,

$$\tau = -mB \sin \theta$$

The negative sign indicates that the direction of torque τ is such so as to decrease θ .



For small angular displacement θ , $\sin \theta \approx \theta$.

Hence, $\tau = -mB\theta$

If I is the moment of inertia of the magnet, then the deflecting torque on the magnet is,

$$\tau = I\alpha = I \frac{d^2\theta}{dt^2}$$

In the equilibrium condition,

Deflecting torque = Restoring torque

Hence, $I \frac{d^2\theta}{dt^2} = -mB\theta$

or $\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta = -\omega^2\theta$

i.e. angular acceleration $\frac{d^2\theta}{dt^2} \propto$ angular displacement θ .

Hence, the oscillations of a freely suspended magnetic dipole in a uniform magnetic field are simple harmonic. The time period of oscillation is given by.

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mB}}$$

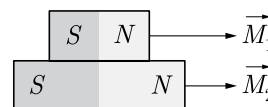
88. How are the magnetic moment of two bar magnet compared without measuring their moment of inertia?

Ans :

Delhi 2021, Foreign 2010

Let us consider two magnets whose moment of inertia are I_1 and I_2 and magnetic moments are M_1 and M_2 respectively.

Sum Position



Net magnetic moment,

$$M_s = M_1 + M_2$$

Net moment of inertia,

$$I_s = I_1 + I_2$$

Time period of oscillation of this pair in earth's magnetic field B_H .

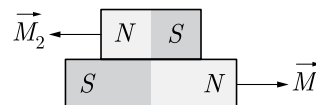
$$\begin{aligned} T_s &= 2\pi \sqrt{\frac{I_s}{M_s B_H}} \\ &= 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 + M_2) B_H}} \end{aligned} \quad \dots (1)$$

Frequency,

$$v_s = \frac{1}{2\pi} \sqrt{\frac{(M_1 + M_2) B_H}{I_s}}$$

Difference Position

Net magnetic moment,



$$M_d = M_1 - M_2$$

Net moment of inertia,

$$I_d = I_1 + I_2$$

and

$$\begin{aligned} T_d &= 2\pi \sqrt{\frac{I_d}{M_d B_H}} \\ &= 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 - M_2) B_H}} \end{aligned} \quad \dots (2)$$

$$\theta = 30^\circ$$

$$\tau = 0.06 \text{ Nm}$$

$$\begin{aligned} \text{Magnetic moment, } m &= \frac{\tau}{B \sin \theta} = \frac{0.06}{0.2 \sin 30^\circ} \\ &= \frac{0.06}{0.2 \times 0.5} = 0.6 \text{ Am}^2 \end{aligned}$$

2. The P.E. of a magnetic dipole in a uniform magnetic field is

$$U = -mB \cos \theta$$

In stable equilibrium, the P.E. is minimum. So

$$\cos \theta = 1$$

$$\theta = 0^\circ$$

Hence the bar magnet will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to the magnetic field $\vec{B}$.

99. A planar loop of irregular shape encloses an area of $7.5 \times 10^{-4} \text{ m}^2$ and carries a current of 12 A. The sense of flow of current appears to be clockwise to an observer. What is the magnitude and direction of the magnetic moment vector associated with the current loop?

Ans :

OD 2015

$$\text{Given, } A = 7.5 \times 10^{-4} \text{ m}^2$$

$$\text{and } I = 12 \text{ A}$$

Magnetic moment associated with the loop is,

$$\begin{aligned} m &= IA \\ &= 12 \times 7.5 \times 10^{-4} \\ &= 9.0 \times 10^{-3} \text{ JT}^{-1} \end{aligned}$$

Applying right hand rule, the direction of magnetic moment is along the normal to the plane of the loop away from the observer.

100. A current of 6 A is flowing through a 10 turn circular coil of radius 7 cm. The coil lies in the x - y plane. What is the magnitude and direction of the magnetic dipole moment associated with it?

If this coil were to be placed in a uniform external magnetic field directed along the x -axis, in which plane would the coil lie, when in equilibrium? (Take $\pi = 22/7$)

Ans :

Delhi 2017

Magnetic dipole moment,

$$\begin{aligned} m &= NIA = NI \times \pi r^2 \\ &= 10 \times 6 \times \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \\ &= 0.77 \text{ Am}^2 \end{aligned}$$

The direction of magnetic dipole moment is perpendicular to the plane of the coil. Hence it is

along z -axis.

Torque on the current loop of magnetic moment m is,

$$\tau = mB \sin \alpha$$

where, α is angle between $\vec{m}$ and $\vec{B}$. For stable equilibrium torque is zero, so $\alpha = 0^\circ$. For this $\vec{B}$ should be perpendicular to the plane of the coil. Hence the coil will lie in y - z plane in the condition of stable equilibrium.

101. An electron in an atom revolves around the nucleus in an orbit of radius $0.5 \text{ \AA}$. Calculate the equivalent magnetic moment if the frequency of revolution of the electron is 10^{10} MHz .

Ans :

Foreign 2011

The electron revolving around the nucleus in a circular orbit is equivalent to a current loop.

Its magnetic moment is,

$$m = IA = e\nu \times \pi r^2$$

$$\text{Given, } e = 1.6 \times 10^{-19} \text{ C}$$

$$\nu = 10^{10} \text{ MHz} = 10^{16} \text{ Hz}$$

$$r = 0.5 \text{ \AA} = 0.5 \times 10^{-10} \text{ m}$$

Therefore,

$$\begin{aligned} m &= 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2 \\ &= 1.257 \times 10^{-23} \text{ Am}^2 \end{aligned}$$

102. The electron in the hydrogen atom is moving with a speed of $2.3 \times 10^6 \text{ ms}^{-1}$ in an orbit of radius $0.53 \text{ \AA}$. Calculate the magnetic moment of the revolving electron.

Ans :

Delhi 2016

$$\text{Given, } v = 2.3 \times 10^6 \text{ ms}^{-1}$$

$$r = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

Orbital magnetic moment of the electron,

$$\begin{aligned} \mu_l &= \frac{evr}{2} \\ &= \frac{1.6 \times 10^{-19} \times 2.3 \times 10^6 \times 0.53 \times 10^{-10}}{2} \\ &= 9.75 \times 10^{-24} \text{ Am}^2 \end{aligned}$$

103. A compass needle whose magnetic moment is 60 Am^2 pointing geographical north at a certain place where the horizontal component of earth's magnetic field is $40 \mu\text{Wb/m}^2$ experiences a torque of $1.2 \times 10^{-3} \text{ Nm}$. What is the declination of the place?

Ans :

Foreign 2009, Comp 2002

In stable equilibrium, a compass needle points along magnetic north and experiences no torque. When

it is turned through declination α , it points along geographic north and experiences torque,

$$\tau = mB \sin \alpha$$

Hence,
$$\sin \alpha = \frac{\tau}{mB}$$

$$= \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2}$$

$$\alpha = 30^\circ$$

- 104.** Obtain the earth's magnetisation. Assume that the earth's field can be approximated by a giant bar magnet of magnetic moment $8.0 \times 10^{22} \text{ Am}^2$. The earth's radius is 6400 km.

Ans :

SQP 2005

Given, magnetic moment,

$$m = 8.0 \times 10^{22} \text{ Am}^2$$

Radius of the earth,

$$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$$

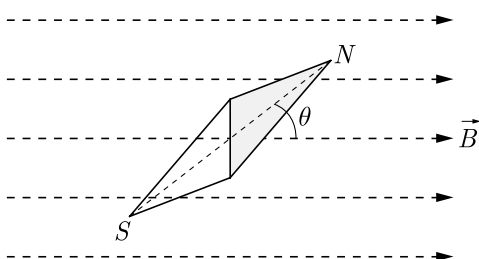
Magnetisation, $M = \frac{m}{V}$

$$= \frac{m}{\frac{4}{3}\pi r^3}$$

$$= \frac{8.0 \times 10^{22} \times 3}{4 \times 3.14 \times (6.4 \times 10^6)^3}$$

$$= 72.9 \text{ Am}^{-1}$$

- 105.** In figure, a magnetic needle is free to oscillate in a uniform magnetic field. The magnetic needle has magnetic moment 6.7 Am^2 and moment of inertia $I = 7.5 \times 10^{-6} \text{ kg m}^2$. It performs 10 complete oscillations in 6.70s. What is the magnitude field?



Ans :

OD 2016

Given, $T = \frac{6.70 \text{ s}}{10} = 0.67 \text{ s}$

$$m = 6.7 \text{ Am}^2$$

$$I = 7.5 \times 10^{-6} \text{ kg m}^2$$

As, $T = 2\pi \sqrt{\frac{I}{mB}}$

$$T^2 = 4\pi^2 \frac{I}{mB}$$

The magnitude of the magnetic field is,

$$B = \frac{4\pi^2 I}{mT^2}$$

$$= \frac{4 \times 9.87 \times 7.5 \times 10^{-6}}{6.7 \times (0.67)^2}$$

$$= 9.8 \times 10^{-5} \text{ T}$$

- 106.** A bar magnet of magnetic moment 6.4 A-m^2 is placed in a uniform magnetic field of 0.4 T . What is the torque acting on the magnet, when its axis makes an angle of 60° with the magnetic field?

Ans :

Comp 2007

Magnetic moment, $M = 6.4 \text{ A-m}^2$

Magnetic field, $B = 0.4 \text{ T}$

and angle between bar magnet and magnetic field,

$$\theta = 60^\circ$$

Torque acting on the bar magnet in uniform field,

$$\tau = MB \sin \theta$$

$$= 6.4 \times 0.4 \times \sin 60^\circ$$

$$= 6.4 \times 0.4 \times 0.866 = 2.2 \text{ N-m}$$

- 107.** A magnet of magnetic moment M is freely suspended in a uniform magnetic field of strength B . Calculate the work done in rotating the magnet through 60° to 90° ?

Ans :

Comp 2018

Magnetic moment = M

Strength of magnetic field = B

Initial angle of rotation = 60°

Final angle of rotation = 90°

Work done in rotating the magnet in uniform magnetic field,

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

$$= MB(\cos 60^\circ - \cos 90^\circ)$$

$$= MB\left(\frac{1}{2} - 0\right) = \frac{MB}{2}$$

- 108.** Work done in turning a bar magnet of magnetic moment M through 90° from the meridian, is n times the corresponding work done to turn it through an angle of 60° . What is the value of n ?

Ans :

Delhi 2016, OD 2011

Magnetic moment = M

Initial angle through which magnet is turned,

$$\theta_1 = 90^\circ$$

and final angle through which magnet is turned,

$$\theta_2 = 60^\circ$$

Work done in turning the bar magnet,

$$W = MB(1 - \cos \theta)$$

Therefore,

$$\frac{W_1}{W_2} = \frac{1 - \cos \theta_1}{1 - \cos \theta_2} = \frac{1 - \cos 90^\circ}{1 - \cos 60^\circ}$$

$$= \frac{1 - 0}{1 - 0.5} = \frac{1}{0.5} = 2$$

$$W_1 = 2W_2$$

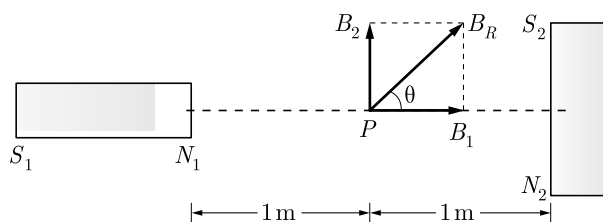
$$\frac{W_1}{W_2} = 2 \quad \left(\text{where, } n = \frac{W_1}{W_2}\right)$$

- 109.** Two identical magnetic dipoles of magnetic moments 1.0 Am^2 each are placed at a separation of 2 m with their axes perpendicular to each other. What is the resultant magnetic field at a point mid-way between the dipoles?

Ans :

Comp 2019

The situation is shown in figure.



The magnetic fields of the two magnets at the midpoint P are,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3} = \frac{10^{-7} \times 2 \times 1}{1^3}$$

$$= 2 \times 10^{-7} \text{ T} \quad (\text{in horizontal direction})$$

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{r^3} = 10^{-7} \text{ T} \quad (\text{in vertical direction})$$

Hence, $B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{5} \times 10^{-7} \text{ T}$

If the resultant field B_R makes angle θ with B_1 , then

$$\tan \theta = \frac{B_2}{B_1} = \frac{10^{-7}}{2 \times 10^{-7}} = 0.5$$

Hence, $\theta = 26.57^\circ$

- 110.** The effective length of a magnet is 31.4 cm , and its pole strength is 0.5 A-m . If it is bent in the form of a semicircle. What is the new magnetic moment?

Ans :

OD 2010

Initial effective length of magnet or initial distance between the poles,

$$d_1 = 31.4 \text{ cm} = 0.314 \text{ m}$$

and pole strength of the magnet,

$$m = 0.5 \text{ A-m}$$

Initial magnetic moment,

$$M_1 = m \times d_1$$

$$= 0.5 \times 0.314 = 0.157 \text{ A-m}^2$$

When a magnet is bent to form a semicircle, then

distance between the poles

= Diameter of semicircle

$$d_2 = \frac{2d_1}{\pi} = \frac{2 \times 0.314}{3.14} = 0.2 \text{ m}$$

Magnetic moment of magnet

M = Pole strength

$\times$ Distance between the poles

$$= m \times d$$

i.e.,

$$= M \propto d$$

Therefore, $\frac{M_1}{M_2} = \frac{d_1}{d_2} = \frac{0.314}{0.2} = 1.57$

where M_2 = New Magnetic moment of magnet

$$M_2 = \frac{M_1}{1.57}$$

$$= \frac{0.157}{1.57} = 0.1 \text{ A-m}^2$$

- 111.** A current of 7.0 A is flowing in a plane circular coil of radius 1.0 cm having 100 turns. The coil is placed in a uniform magnetic field of 0.2 Wbm^{-2} . If the coil is free to rotate, what orientations would correspond to its (1) stable equilibrium and (2) unstable equilibrium? Calculate the potential energy of the coil in these cases.

Ans :

SQP 2006, Comp 2015

Given,

$$N = 100$$

$$A = 7.0 \text{ A}$$

$$r = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$$

$$B = 0.2 \text{ Wbm}^{-2}$$

Magnetic moment associated with the coil is,

$$m = NIA = NI \times \pi r^2$$

$$= 100 \times 7.0 \times \frac{22}{7} \times (1.0 \times 10^{-2})^2$$

$$= 0.22 \text{ Am}^2$$

1. The stable equilibrium corresponds to $\vec{m}$ parallel to $\vec{B}$. The potential energy is then minimum.

$$U_{\min} = -mB \cos 0^\circ$$

$$= -0.22 \times 0.2 \times 1$$

$$= -0.044 \text{ J}$$

2. The unstable equilibrium corresponds to $\vec{m}$ anti-parallel to $\vec{B}$. The potential energy is then maximum.

$$U_{\max} = -mB \cos 180^\circ$$

$$= -0.22 \times 0.2 \times (-1)$$

$$= +0.044 \text{ J}$$

112. A short bar magnet placed with its axis at 30° experiences a torque of 0.016 Nm in an external field of 800 G .

1. What is the magnetic moment of the magnet?
2. What is the work done by an external force in moving it from its most stable to most unstable position?
3. What is the work done by the force due to the external magnetic field in the process mentioned in part (2)?
4. The bar magnet is replaced by a solenoid of cross-sectional area $2 \times 10^{-4} \text{ m}^2$ and 1000 turns, but the same magnetic moment. Determine the current flowing through the solenoid.

Ans :

OD 2014

1. Given,

$$\theta = 30^\circ$$

$$B = 800 \text{ G} = 800 \times 10^{-4} \text{ T}$$

$$\tau = 0.016 \text{ Nm}$$

$$\begin{aligned} \text{Magnetic moment, } m &= \frac{\tau}{B \sin \theta} \\ &= \frac{0.016}{800 \times 10^{-4} \times \sin 30^\circ} \\ &= 0.40 \text{ Am}^2 \end{aligned}$$

2. For most stable position, $\theta = 0^\circ$ and for most unstable position $\theta = 180^\circ$. So the required work done by the external force,

$$\begin{aligned} W &= -mB(\cos 180^\circ - \cos 0^\circ) \\ &= 2mB \\ &= 2 \times 0.40 \times 800 \times 10^{-4} \\ &= 0.064 \text{ J} \end{aligned}$$

3. Here the displacement and the torque due to the magnetic field are in opposition. So the work done by the magnetic field due to the external magnetic field is

$$W_B = 0.064 \text{ J}$$

4. Given,

$$A = 2 \times 10^{-4} \text{ m}^2$$

$$N = 1000$$

Magnetic moment of solenoid,

$$m_s = m = 0.40 \text{ Am}^2$$

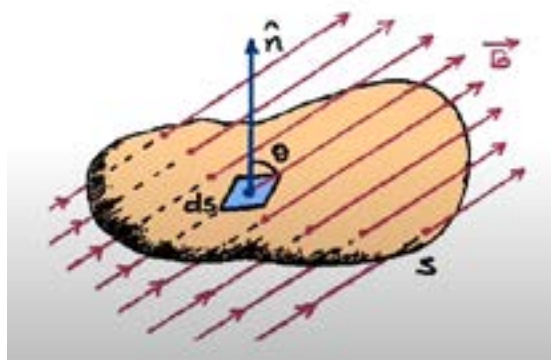
$$\text{But, } m_s = NIA$$

$$\begin{aligned} \text{Hence, Current, } I &= \frac{m_s}{NA} \\ &= \frac{0.40}{1000 \times 2 \times 10^{-4}} \\ &= 2 \text{ A} \end{aligned}$$

CASE BASED QUESTIONS

113. A physics teacher explain Gauss's theorem in electrostatics and Gauss's theorem in magnetism to his students in a class. He tells them that total normal electric flux over a closed surface in vacuum is $\phi_e = \frac{Q}{\epsilon_0}$, where Q is algebraic sum of charges inside

the surface. Further, total normal magnetic flux over a closed surface in vacuum is always zero. The teacher emphasises that this is because in magnetism, poles always exist in unlike pairs of equal strength i.e., isolated magnetic poles called monopoles not exist.



Read the above passage and answer the following questions

- (i) What are the implications of Gauss's theorem in magnetism in day-to-day life?
- (ii) Two magnetic dipoles of moments $5 \text{ A} - \text{m}^2$ and $3 \text{ A} - \text{m}^2$ oriented along opposite directions are enclosed in a surface. What is total normal magnetic flux over the surface?
- (iii) Two point charges $+4q$ and $-q$ are enclosed in a surface in vacuum and third charge $5q$ lies outside the surface. What is total normal electric flux over the surface?

Ans :

- (i) In day-to-day life, we can visualize North pole of a magnetic dipole as source and South pole of the dipole as sink. When source and sink (of magnetic flux) having the same strength are enclosed by a surface, the total normal magnetic flux from the surface (i.e., net out come) will be zero. Hence, to achieve success in any sphere of life, we must identify the sinks and plug them properly. Arrange to have as many sources as possible and success will yours.
- (ii) As stated in theorem, total normal magnetic flux over the surface would always be zero, it being zero for every individual magnetic dipole.

$$(iii) \quad \phi_e = \frac{\sum q_{\text{inside}}}{\epsilon_0} = \frac{4q - q}{\epsilon_0} = \frac{3q}{\epsilon_0}$$

The charge $5q$ outside the surface does not affect the electric flux.

- 114.** Mr. Rajesh the chief development officer, in southern railway went on an official tour to attend a seminar on fast moving trains. He met his friend Ontosaki in Tokyo after he finished his seminar there. His friend explained to Rajesh how Japanese people are concentrating on energy conservation and saving of fossil fuels using Maglev trains. Mr. Rajesh travelled from Tokyo to Osaka in Maglev train and found that sound is less, travelling is smooth and understood in what way we are lagging behind Japanese in mass transporting systems. This works on the principle of Meissner's effect.

- (i) What is Meissner's effect?
- (ii) Write the value of magnetic permeability for perfect diamagnetism.

Ans :

- (i) When a superconductor is cooled in a magnetic field below its critical temperature the magnetic field lines are expelled showing diamagnetic property. This is called Meissner's effect.
- (ii) Diamagnetic materials have magnetic permeability of less than 1.

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CHAPTER 6

Electromagnetic Induction

SUMMARY

1. MAGNETIC FLUX

The total number of magnetic field lines crossing through any surface normally when it placed in a magnetic field is known as the magnetic flux through that surface.

2. FARADAY'S LAW OF EMI

Faraday gave two laws of EMI :

1. **First Law** : An emf is induced in a circuit when the magnetic flux linked with circuit changes.
2. **Second Law** : The magnitude of induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit.

3. INDUCED EMF AND CURRENT

Induced emf, $e = -N \frac{d\phi_B}{dt}$

Induced current, $I = \frac{e}{R} = \frac{-N}{R} \frac{d\phi_B}{dt}$

Here, R = resistance of the circuit

and N = number of turns

4. LENZ'S LAW

According to this law, the polarity of induced emf is such that it tends to produce a current which oppose the change in magnetic flux produced it.

5. FLEMING'S RIGHT HAND RULE

If we stretch the thumb, the forefinger, the central finger of our right hand in such a way that all three are mutually perpendicular to each other, then if the thumb represents the direction of force, forefinger represent the direction of magnetic field, then the central finger will represent the direction of induced current.

6. INDUCED CURRENT IN A CIRCUIT

If induced current is produced in a coil rotated in a uniform magnetic field, then

$$I = I_0 \sin \omega t$$

7. MOTIONAL EMF AND FARADAY'S LAW

If e is the induced emf, then according to Faraday's law,

$$e = (-d\phi/dt) = -Blv$$

8. ENERGY CONSIDERATION

Power required to move conductor in a uniform magnetic field perpendicular is,

$$P = \frac{B^2 l^2 v^2}{R}$$

Here, R = resistance of the circuit through which current is flowing.

B = a uniform magnetic field

l = length of the conductor

v = speed

9. INDUCTANCE

It is the ratio of the flux to the current. It depends on the geometry of the coil and intrinsic material properties.

10. SELF-INDUCTANCE

It is the property of a coil by virtue of which it opposes any changes in the strength of current flowing through it by inducing an emf in itself. When current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $e = -L \frac{dI}{dt}$, where L is the self-inductance of the coil. If coil of N turns and area A is rotated with v revolution per second in a uniform magnetic field B , then the induced emf is,

$$\begin{aligned} e &= e_0 \sin \omega t \\ &= NAB \omega \sin \omega t \end{aligned}$$

$$= NBA(2\pi v) \sin(2\pi v) t.$$

11. SELF-INDUCTANCE OF A LONG SOLENOID

Self-inductance of a long solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

Where N = Number of turns in solenoids

A = Area of cross-section of a coil.

I = current flowing in solenoid.

12. MUTUAL INDUCTANCE

The phenomenon according to which an opposing emf is produced as a result of change in current or magnetic flux linked with a neighbouring coil.

13. MUTUAL INDUCTANCE OF TWO LONG COAXIAL SOLENOIDS

Mutual inductance of two long coaxial solenoids is given by,

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

14. AC GENERATOR

A generator produces electrical energy from mechanical work just opposite of what a motor does.

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OBJECTIVE QUESTIONS

1. Lenz's law is associated with principle of conservation of
 - (a) charge
 - (b) mass
 - (c) energy
 - (d) momentum

Ans :

OD 2016, Comp 2015

In each case whenever there is relative motion between a coil and the magnet a force begins to act which opposes the relative motion therefore to maintain the relative motion, a mechanical work must be done. This work appears in the form of electric energy of coil. Thus Lenz's law is based on principle of conservation of energy.

Thus (c) is correct option.

2. S.I. unit of self-inductance is
 - (a) coulomb
 - (b) volt
 - (c) ohm
 - (d) henry

Ans :

SQP 2010

Self inductance, $L = \frac{e}{dI/dt}$

$$\text{S.I Unit of } L = \frac{1 \text{ V}}{1 \text{ A s}^{-1}}$$

$$= 1 \text{ V s A}^{-1} = 1 \text{ henry}$$

Thus (d) is correct option.

3. To convert mechanical energy into electrical energy one can use
 - (a) D.C. dynamo
 - (b) A.C. dynamo
 - (c) Motor
 - (d) Transformer

Ans :

OD 2009

An AC dynamo or generator produces electrical energy from mechanical work, just the opposite of what a motor does. In it a shaft is rotated by some mechanical means such as an engine or a turbine starts working and an emf is induced in the coil.

Thus (b) is correct option.

4. The Lenz's law gives
 - (a) direction of induced E.M.F.
 - (b) magnitude of induced E.M.F.
 - (c) direction of induced current
 - (d) magnitude of induced current

Ans :

Foreign 2014

Lenz's law gives the direction of the induced current in electromagnetic induction.

According to this law, the direction of induced current is such that it opposes the cause which produces it.

Thus (c) is correct option.

5. The working of dynamo is based on the principle of
- heating effect of current
 - electromagnetic induction
 - magnetic induction
 - electric induction

Ans :

Delhi 2011

The working of dynamo or AC generator is based on the principle of electromagnetic induction. When a closed coil is rotated in uniform magnetic field with its axis perpendicular to the magnetic field, the magnetic flux linked with coil changes and an induced emf is set up in the coil hence a current is set up in it.

Thus (b) is correct option.

6. Faraday's laws are the consequence of the conservation of
- charge
 - energy
 - magnetic field
 - both (b) and (c)

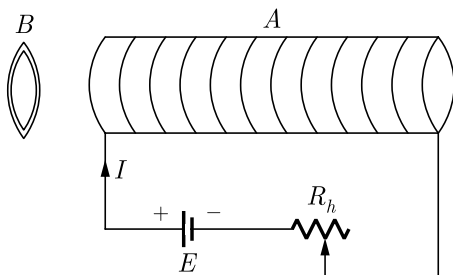
Ans :

Foreign 2013

Faraday's laws involve conversion of mechanical energy into electrical energy. Thus they are in accordance with the law of conservation of energy.

Thus (b) is correct option.

7. An aluminium ring B faces an electromagnet A . Which of the following statement is correct?



- if I increases, A will repel B .
- if I decreases, A will repel B .
- if I increases, A will attract B .
- whether I increases or decreases B will not experience any force.

Ans :

OD 2012

when circuit is closed, increasing current in the coils of electromagnet A produces time varying magnetic flux which magnetised it.

We also know from Lenz's law that when time varying magnetic flux links with a nearby ring, then direction of induced current (or E.M.F.) in the ring will be such that it opposes the cause which produces it.

Thus (a) is correct option.

8. A small piece of metal wire is dragged across the gap between the poles of a magnet in 0.4 s. If change in magnetic flux in the wire is 8×10^{-4} Wb, then E.M.F. induced in the wire is

- 8×10^{-3} V
- 6×10^{-3} V
- 4×10^{-3} V
- 2×10^{-3} V

Ans :

SQP 2009, OD 2014

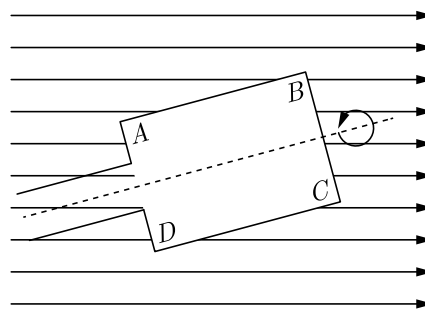
Time-taken $dt = 0.4$ s and change in magnetic flux $d\phi = 8 \times 10^{-4}$ Wb.

E.M.F. induced in the wire,

$$E = \frac{d\phi}{dt} = \frac{8 \times 10^{-4}}{0.4} \\ = 2 \times 10^{-3} \text{ V}$$

Thus (d) is correct option.

9. A rectangular coil $ABCD$ is rotated anticlockwise with a uniform angular velocity about the axis shown in the figure. Initially, the axis of rotation of the coil as well as the magnetic field B were horizontal. The induced E.M.F. in the coil would be maximum when plane of the coil



- is horizontal
- is at right angle to the magnetic field
- makes an angle of 30° with the horizontal
- makes an angle of 45° with the direction of magnetic field

Ans :

Delhi 2008

E.M.F. induced in the coil when rotated in magnetic field,

$$E = E_0 \sin \theta \\ = E_0 \sin \omega t$$

where, θ = angle between plane of coil and vertical

Therefore, for maximum induced E.M.F., the value of $\sin \theta$ should be maximum.

Thus, $\sin \theta = 1$

$$\theta = 90^\circ$$

Therefore induced E.M.F. (E) in coil $ABCD$ would be

- (c) winding the coils on wooden cores
- (d) decreasing the no. of turns in the coils

Ans :

Delhi 2014

Mutual inductance of coils,

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

Therefore, it can be increased by increasing no. of turns in the coils and decreasing the length of coils.

Thus (b) is correct option.

ASSERTION AND REASON

- 20. Assertion :** Lenz's law violates the principle of conservation of energy.

Reason : Induced emf always opposes the change in magnetic flux responsible for its production.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Lenz's law (that the direction of induced emf is always such as to oppose the change that cause it) is direct consequence of the law of conservation of energy.

Thus (a) is correct option.

- 21. Assertion :** Faraday's laws are consequence of conservation of energy.

Reason : In a purely resistive ac circuit, the current lags behind the emf in phase.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

In purely resistive circuit, the current and emf are in the same phase.

Thus (c) is correct option.

- 22. Assertion :** An emf $\vec{E}$ is induced in a closed loop where magnetic flux is varied. The induced $\vec{E}$ is not a conservative field.

Reason : The line integral $\vec{E} \cdot d\vec{l}$ around the closed loop is non-zero.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

According to Faraday's law of electromagnetic induction

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

So, (E) is non-conservative field as in conservative field line integral over a closed loop is zero.

Thus (a) is correct option.

VERY SHORT ANSWER QUESTIONS

- 23.** Derive dimension and SI unit of magnetic flux.

Ans :

OD 2010

We have

$$\phi = B \cdot A$$

Since,

$$B = \frac{F}{qv \sin \theta}$$

$$\phi = \frac{F \cdot A}{qv \sin \theta}$$

Here $\sin \theta$ is the dimensionless quantity.

$$\begin{aligned} \phi &= \frac{[M^1 L^1 T^{-2}] [L^2]}{[AT] [LT^{-1}]} \\ &= [M^1 L^2 T^{-2} A^{-1}] \end{aligned}$$

The S.I. Unit of magnetic flux is weber (wb).

- 24.** On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend?

Ans :

OD 2020

Depends on the time rate of change in magnetic flux (or simply change in Magnetic flux)

$$|\varepsilon| = \frac{\Delta \phi}{\Delta t}$$

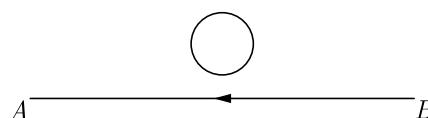
- 25.** Why is the core of a transformer laminated?

Ans :

Foreign 2013, Delhi 2003

The core of a transformer is laminated because of preventing eddy current being produced in the core.

- 26.** The electric current flowing in a wire in the direction from B to A decreasing. Find out the direction of the induced current in the metallic loop kept the wire as shown in the figure.



- (a) currents in the coils
- (b) materials of the wires of the coils
- (c) relative position and orientation of the coils
- (d) rates at which the currents are changing in the coils

Ans :**SQP 2005**

Mutual inductance of the pair of coils depends upon the geometry of the coils, distance between the coils, relative position and orientation of the coils, no. of turns in the coils, permeability of the medium in which the coils wound and degree of coupling i.e., the extent to which the magnetic flux primary current links the secondary.

Thus (c) is correct option.

- 15.** Which of the following does not obey the phenomenon of mutual induction?

- (a) dynamo
- (b) transformer
- (c) induction coil
- (d) electric heater

Ans :**Delhi 2017, Comp 2011**

Phenomenon according to which an opposing E.M.F. is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil, is called mutual induction.

Since electric heater is based on heating effect of current, therefore it does not obey the phenomenon of mutual induction.

And dynamo, transformer and induction coils are based on the phenomenon of the mutual induction.

Thus (d) is correct option.

- 16.** The mutual inductance, when the magnetic flux changes by 5×10^{-2} Wb and current change by 0.01 A, is

- (a) 0.2 H
- (b) 2.5 H
- (c) 5 H
- (d) 10 H

Ans :**OD 2004**

Change in magnetic flux,

$$d\phi = 5 \times 10^{-2} \text{ Wb}$$

Change in current, $dI = 0.01 \text{ A}$

$$\begin{aligned} \text{Mutual inductance, } M &= \frac{d\phi}{dI} \\ &= \frac{5 \times 10^{-2}}{0.01} = 5 \text{ H} \end{aligned}$$

Thus (c) is correct option.

- 17.** A metallic rod of length l is placed normal to the magnetic field B and revolved in a circular path about one of the ends with angular speed ω . The potential difference across the ends will be

- (a) $B\omega^2 l$
- (b) $\frac{1}{2} B\omega l$
- (c) $\frac{1}{2} B\omega l^2$
- (d) $\frac{1}{8} B\omega^3 l$

Ans :**SQP 2002**

Given, Length of rod $= l$

Magnetic field $= B$

and Angular speed $= \omega$

Change in magnetic flux linked with the rod when it rotates in circular path,

$$\begin{aligned} d\phi &= B \times A \\ &= B \times \pi l^2 \end{aligned}$$

and time-taken to complete one-revolution,

$$dt = \frac{2\pi}{\omega}$$

Therefore, induced E.M.F. in the metallic rod,

$$E = \frac{d\phi}{dt} = \frac{B \times \pi l^2}{\frac{2\pi}{\omega}} = \frac{1}{2} B\omega l^2$$

Thus (c) is correct option.

- 18.** A 2 m long solenoid with radius 2 cm and 2000 turns has a another solenoid of 1000 turns wound closely near its mid-point. The mutual inductance of solenoids is

- (a) 0.8 mH
- (b) 1.6 mH
- (c) 3.2 mH
- (d) 6.4 mH

Ans :**Foreign 2003**

Given,

Length of each solenoid, $l = 2 \text{ m}$

Radius of first solenoid, $r = 2 \text{ cm} = 0.02 \text{ m}$

No. of turns in first solenoid, $N_1 = 2000$

No. of turns in second solenoid, $N_2 = 1000$

Area of solenoid,

$$\begin{aligned} A &= \pi r^2 = \pi \times (0.02)^2 \\ &= 4\pi \times 10^{-4} \text{ m}^2 \end{aligned}$$

Therefore, mutual inductance of two solenoids,

$$\begin{aligned} M &= \frac{\mu_0 N_1 N_2 A}{l} \\ &= \frac{(4\pi \times 10^{-7}) \times 2000 \times 1000 \times (4\pi \times 10^{-4})}{2} \\ &= 1.6 \times 10^{-3} \text{ H} = 1.6 \text{ mH} \end{aligned}$$

Thus (b) is correct option.

- 19.** The mutual inductance of two coils can be increased by
- (a) increasing the length of coils
 - (b) increasing the no. of turns in the coils

maximum when plane of coil is horizontal (i.e., makes an right angle to the vertical).

Thus (a) is correct option.

10. An infinitely long conducting cylinder is kept parallel to a uniform magnetic field B directed along positive z -axis. The direction of induced current on the surface of cylinder as seen from the z -axis will be
- zero
 - clockwise of the positive z -axis
 - along the magnetic field
 - anticlockwise of the positive z -axis

Ans :

Foreign 2015

As the magnetic field is uniform along the positive z -axis, therefore no change in the magnetic flux through the conducting cylinder.

Thus no current will be induced on the surface of it.

Therefore, direction of induced current on the surface of cylinder, as seen from the z -axis will be zero.

Thus (a) is correct option.

11. If a wire of length 2 m is moving with a velocity of 1 m-s^{-1} perpendicular to a magnetic field of 0.5 T , then E.M.F. induced in the wire will be
- 0.2 V
 - 0.5 V
 - 1 V
 - 2 V

Ans :

Delhi 2002

Length of wire, $l = 2 \text{ m}$

Velocity of wire, $v = 1 \text{ m-s}^{-1}$

Magnetic field, $B = 0.5 \text{ T}$

E.M.F. induced in the wire,

$$E = Bvl = 0.5 \times 1 \times 2 = 1 \text{ Volt}$$

Thus (c) is correct option.

12. The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in a time Δt . The total electric charge Q that passes any point in the circuit during the time Δt is represented by

- $Q = \frac{\Delta\phi}{\Delta t}$
- $Q = \frac{\Delta\phi}{R}$
- $Q = R \cdot \frac{\Delta\phi}{\Delta t}$
- $Q = \frac{1}{R} \cdot \frac{\Delta\phi}{\Delta t}$

Ans :

SQP 2016

Resistance of circuit = R ;

Change in magnetic flux = $\Delta\phi$;

Time-taken = Δt

and total charge that passes any point in the circuit = Q .

E.M.F. induced in the circuit,

$$E = \frac{\Delta\phi}{\Delta t}$$

Therefore, current induced in the circuit,

$$I = \frac{\text{Induced e.m.f.}}{\text{Resistance of circuit}}$$

$$= \frac{\frac{\Delta\phi}{\Delta t}}{R} = \frac{\Delta\phi}{R \Delta t}$$

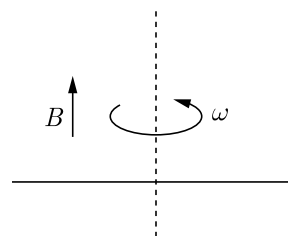
We also know that total electric charge that passes any point in the circuit during the time,

$$\Delta t(Q) = I \cdot \Delta t$$

$$= \frac{\Delta\phi}{R \Delta t} \times \Delta t = \frac{\Delta\phi}{R}$$

Thus (b) is correct option.

13. A conducting rod of length $2l$ is rotating with a constant angular speed ω about its perpendicular bisector as shown in the figure. A uniform magnetic field B exists parallel to the axis of rotation. The E.M.F. induced between two ends of the rod is



- zero
- $B\omega l^2$
- $\frac{1}{2}B\omega l^2$
- $\frac{1}{8}B\omega l^2$

Ans :

Foreign 2007, OD 2009

Given, Length of rod = $2l$

Angular speed = ω

and Uniform magnetic field = B

Induced E.M.F. in a small element dx of the rod from its centre = $Bvdx$.

Therefore, induced E.M.F. between the two ends of the rod,

$$E = \int_{-l}^{+l} Bvdx$$

$$= \int_{-l}^{+l} B\omega x dx$$

$$= B\omega \left[\frac{x^2}{2} \right]_{-l}^{+l}$$

$$= B\omega \left[\frac{l^2}{2} - \frac{(-l)^2}{2} \right] = 0$$

Thus (a) is correct option.

14. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon

Ans :**Delhi 2018**

According to Lenz's law, the direction of induced current will oppose the cause of its production. So, the current in loop will induce in such a way that it will support the current flowing in the wire i.e., in the same direction. So, the direction of current in the loop will be clockwise.

27. How can the self-inductance of a given coil having N number of turns, area of cross-section A and length l be increased?

Ans :**OD 2019**

The self-inductance can be increased by the help of electric fields. It does not depend on the current through circuit but depends upon the permeability of material from which the core is made up of.

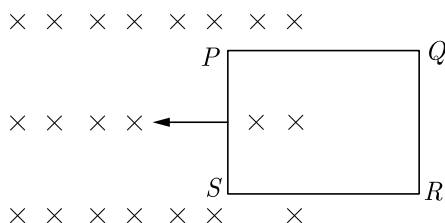
28. Write Lenz's law of electromagnetic induction.

Ans :**Foreign 2016, 13**

Lenz's Law states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it. i.e., it opposes the change in magnetic flux.

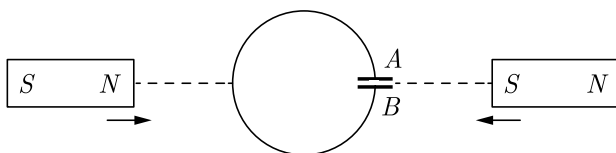
$$e = \frac{-d\phi}{dt}$$

29. The closed loop $PQRS$ of wire is moved into a uniform magnetic field at right angles to the plane of the paper as shown in the figure. Predict the direction of the induced current in the loop.

**Ans :****Comp 2020**

Since, magnetic flux increasing when the loop moves into uniform magnetic field. So, the induced current should oppose this increase. Thus, flow will be from $QPSRQ$. i.e., anti-clockwise.

30. Predict the polarity of the capacitor in the situation described below :

**Ans :****Delhi 2011**

The polarity of plate A will be positive with respect to plate B in the capacitor.

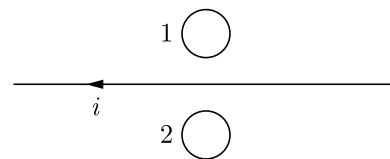
31. A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why ? give reason.

Ans :**SQP 2013**

When the current begins to grow through the electromagnet, the magnetic flux through the disc begins to increase. This sets up eddy current in the disc in the same direction as that of the electromagnetic current.

Thus, if the upper surface of electromagnet acquires N -polarity, the lower surface of the disc also acquires N -polarity. As, same magnetic poles repel each other, the light metallic disc is thrown up.

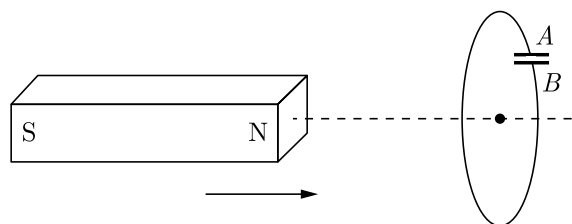
32. Predict the direction of induced current in metal rings 1 and 2 when current i in the wire is steadily decreasing?

**Ans :****OD 2019, SQP 2007**

Current in the wire is steadily decreasing, so the induced current in rings 1 and 2 will flow in such a way that it opposes the decreases of current.

So, it will flow in same direction. Now, from the figure. It is clear that the direction of induced current in ring 1 is clockwise and ring 2 is anti-clockwise.

33. In the given figure, a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.

**Ans :****Delhi 2011**

Here, the North pole is approaching the magnet, so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like North pole, so south pole developed in loop when viewed from right hand side of the loop. The flow of induced current is clockwise, hence A acquires positive polarity and B requires negative.

34. Define mutual inductance between two long coaxial solenoids. Find out the expression for the mutual inductance of inner solenoid of length l having the

or
$$L = \frac{\mu_0 N^2 A}{l}$$

If core of any other magnetic material, μ is placed,

Then,

$$\mu = \mu_r \mu_0$$

$[\mu_r = \text{relative magnetic permeability}]$

Hence,
$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$

51. Two concentric circular coils C_1 and C_2 radius r_1 and r_2 ($r_1 < r_2$) respectively are kept coaxially. If current is passed through C_2 , then find an expression for mutual inductance between the two coils.

Ans :

OD 2017, Comp 2000

Let current I_2 passes through the coil C_2 .

Magnetic field at centre due to current loop C_2

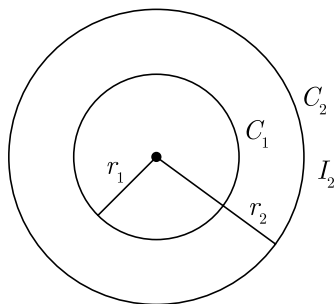
$$B_2 = \frac{\mu_0 N_2 I_2}{2r_2}$$

N_2 = number of turns in coil C_2 .

Magnetic flux linked with coil C_2

$$\phi_1 = N_1 B_2 A_1$$

$$\phi_1 = N_1 \left(\frac{\mu_0 N_2 I_2}{2r_2} \right) (\pi r_1^2)$$



But,

$$\phi = \mu I_2$$

$$M I_2 = \frac{\mu_0 N_1 N_2 I_2}{2r_2} \pi r_1^2$$

$$M = \frac{\mu_0 \pi N_1 N_2 r_1^2}{2r_2}$$

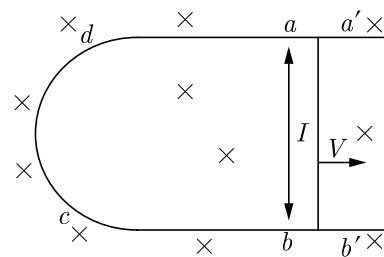
52. (i) A rod of length l is moved horizontally with a uniform velocity v in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.
- (ii) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain.

Ans :

OD 2016

- (i) Consider a straight conductor moving with velocity v and U shaped conductor placed in

perpendicular magnetic field as shown in the figure.



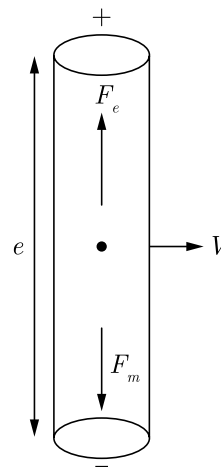
Let conductor shifts from ab to $a' b'$ in time dt , then change in magnetic flux

$$\begin{aligned} d\phi &= B \times \text{change in area} \\ &= B \times (\text{area } a' b' ab) \\ &= B \times (l \times v dt) \end{aligned}$$

$$\frac{d\phi}{dt} = Bvl$$

Hence, induced emf $|\varepsilon| = \frac{d\phi}{dt} = Bvl$

- (ii) During motion, free e^- are shifted at one end due to magnetic force so due to polarisation of rod electric field is produced which applies electric force on free e^- on opposite direction.



At equilibrium of Lorentz force,

$$F_e + F_m = 0$$

$$qE + q(v \times B) = 0$$

$$E = -v \times B$$

$$= B \times v$$

$$|E| = |Bv \sin 90|$$

$$\frac{dv}{dr} = Bv$$

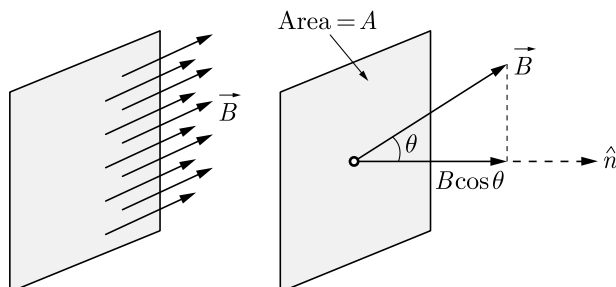
$$P_D = Bvl$$

38. Define the term magnetic flux. Is it a scalar or vector quantity?

Ans :

OD 2021

The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.



Magnetic Flux Through a Given Surface Area

As shown in figure, if the magnetic field $\vec{B}$ makes an angle θ with the normal drawn to a surface area A , then the flux linked with this area is,

$$\begin{aligned}\phi &= \text{Normal component of } B \\ &\quad \times \text{Surface area} \\ &= B \cos \theta \times A \\ \phi &= BA \cos \theta = \vec{B} \cdot \vec{A}\end{aligned}$$

Here the direction of vector $\vec{A}$ is the direction of the outward drawn normal to the surface. Clearly, magnetic flux is a scalar quantity.

39. Name and define the SI and CGS units of magnetic flux. Write the relation between them.

Ans :

Delhi 2020

SI Unit of Magnetic Flux

The SI unit of magnetic flux is weber (Wb). One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m^2 .

$$1 \text{ weber} = 1 \text{ tesla} \times 1 \text{ metre}^2$$

$$\text{or} \quad 1 \text{ Wb} = 1 \text{ Tm}^2$$

CGS Unit of Magnetic Flux

The CGS unit of magnetic flux is Maxwell (Mx). One maxwell is the flux produced when a uniform magnetic field of one gauss acts normally over an area of 1 cm^2 .

$$1 \text{ maxwell} = 1 \text{ gauss} \times 1 \text{ cm}^2$$

$$1 \text{ Mx} = 1 \text{ G cm}^2$$

Relation between Weber and Maxwell

$$1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2$$

$$= 10^4 \text{ G} \times 10^4 \text{ cm}^2$$

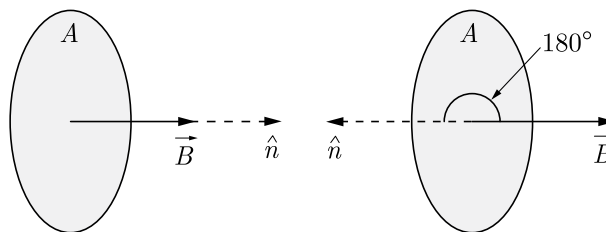
$$1 \text{ Wb} = 10^8 \text{ maxwell}$$

40. When is the magnetic flux taken as positive and negative?

Ans :

OD 2020, Comp 2007

A normal to a plane can be drawn from either side. If the normal drawn to a plane points out in the direction of the field, then $\theta = 0^\circ$ and the flux is taken as positive. If the normal points in the opposite direction of the field, then $\theta = 180^\circ$ and the flux is taken as negative.



$$\theta = 0^\circ, \phi = +BA$$

$$\theta = 180^\circ, \phi = -BA$$

(a) Positive Flux

(b) Negative Flux

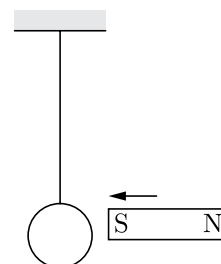
41. Give the three possible ways of producing an induced emf in a coil giving an example in each case.

Ans :

SQP 2010

As $\phi = BA \cos \theta$, so the magnetic flux linked with a loop can be changed and hence induced emf can be produced by three methods :

1. By changing the magnetic field B e.g., by moving a bar magnet towards or away from a closed coil.
 2. By changing the area of a closed loop e.g., by moving one arm of a rectangular conductor in a perpendicular magnetic field.
 3. By changing in relative orientation θ of the loop and the magnetic field e.g., by rotating a closed coil about an axis perpendicular to the magnetic field.
42. When a magnet is moved towards a suspended wire loop as shown in figure then evaluate the direction of induced current in loop. State the law used by you for this evaluation.



Ans :

OD 2019

When south pole of magnet moves towards the coil, the induced current flows clockwise in the coil when seen from its right hand side. Lenz's law evaluate the

radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length.

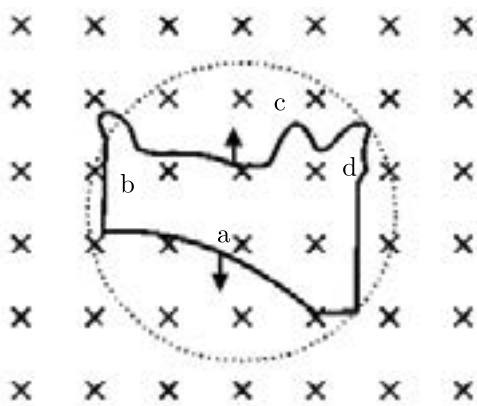
Ans :

Delhi 2010

The phenomenon of generation of induced emf in a long solenoid due to a change of current in another neighbouring coaxial solenoid is known as mutual induction. The mutual induction for the pair of the solenoid is

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

35. A flexible wire of irregular shape, $abcd$ as shown in the figure, turns into a circular shape when placed in a region field which is directed normal to the plane of the loop away from the reader. Predict the direction of the induced current in the wire.



Ans :

Delhi 2021, SQP 2014

The wire is expanding to form a circle, which means that force is acting outwards on each part of the wire because of the magnetic field (acting in the downwards direction). The direction of the induced current should be opposite such that it will produce magnetic field in upward direction (towards the reader). Hence, the force on the wire will be towards inward direction, i.e., induced current is flowing in anti-clockwise direction in the loop from $cbadc$.

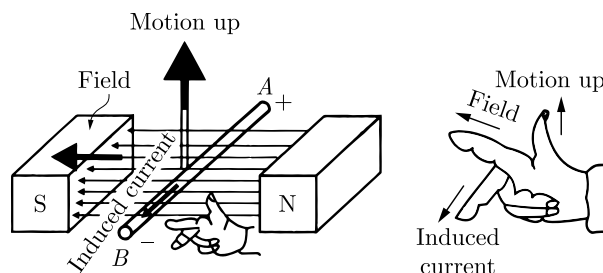
SHORT ANSWER QUESTIONS

36. State a rule to determine the direction of current induced due to the motion of a conductor in a perpendicular magnetic field.

Ans :

Foreign 2015

Fleming's right hand rule gives the direction of induced current set up in a conductor moving perpendicular to a magnetic field and can be stated as follows:

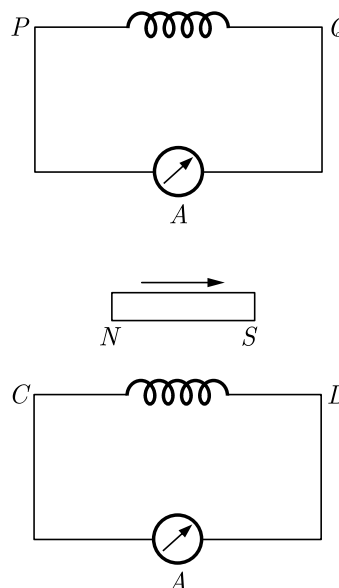


Fleming's Right Hand Rule

If we stretch the thumb and the first two fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor; then the central finger points in the direction of current induced in the conductor.

This rule is also called the dynamo or generator rule and has wide practical applications.

37. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD . Predict the directions of induced current in each coil.



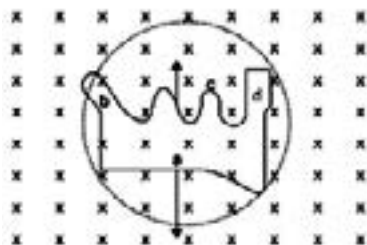
Ans :

Comp 2021

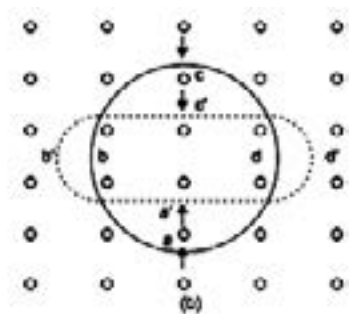
From the figure, it is clear that North pole of the magnet is moving away from coil PQ , so the direction of current at end Q will flow in such a way that it will oppose the away moment of North pole, so it has to act as South pole. Hence, the direction of current will be anti-clockwise. Again, the South pole is approaching towards coil CD , so end C of the coil will act as South pole (to oppose the approaching of South pole). Hence, the direction of current will be clockwise.

The near face of coil C_2 will also become S -pole to oppose the approach of magnet, so the current in coil C_2 will also be clockwise.

48. Use Lenz's law to determine the direction of induced current in the situation described by following figures.



(a)



(b)

- (a) A wire of irregular shape turning into a circular shape.
 (b) A circular loop being deformed into a narrow straight wire.

Ans :

Comp 2014

- (a) For the given periphery the area of a circle is maximum. When a coil takes a circular shape, the magnetic flux linked with coil increases, so current induced in the coil will tend to decrease the flux and so will produce a magnetic field upward. As a result the current induced in the coil will flow anti-clockwise i.e., along $adcb$.
 (b) For given periphery the area of circle is maximum. When circular coil takes the shape of narrow, straight wire, the magnetic flux linked with the coil decreases, so current induced in the coil will tend to oppose the decrease in magnetic flux, hence it will produce upward magnetic field, so current induced in the coil will flow anti-clockwise i.e., along $a'd'c'b'$.

49. In an experimental arrangement of two coils C_1 and C_2 placed coaxially parallel to each other, find out the expression for the emf induced in the coil C_1 (of N_1 turns) corresponding to the change of current I_2 in the coil C_2 (of N_2 turns)

Ans :

Foreign 2015

Let ϕ_1 be the flux through coil C_1 (of N_1 turns) when current in coil C_2 is I_2 . Then, we have

$$N_1\phi_1 = MI_2 \quad \dots(1)$$

For current varying with time.

$$\frac{d(N_1\phi_1)}{dt} = \frac{d(MI_2)}{dt} \quad \dots(2)$$

Since induced emf in coil C_1 is given by

$$\varepsilon_1 = -\frac{d(N_1\phi_1)}{dt}$$

Hence, From (2)

$$-\varepsilon_1 = M\left(\frac{dI_2}{dt}\right)$$

$$\varepsilon_1 = -M\frac{dI_2}{dt} \quad [\text{from (1)}]$$

It shows that varying current in a coil induces emf in the neighbouring coil.

50. Find an expression of self induction of an ideal solenoid.

Ans :

Delhi 2014, Foreign 2003

A long solenoid is one whose length is very large as compared to its area of cross-section. The magnetic field B at any point inside such a solenoid is practically constant and is given by

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \quad \dots(1)$$

$$\left[\text{Since, } n = \frac{N}{l} \right]$$

where, μ_0 = magnetic permeability of free space.

N = total number of turns in the solenoid,

l = length of the solenoid

n = number of turns per unit length.

Hence, Magnetic flux through each turn of the solenoid,

$$\phi = B \times \text{area of the each turn}$$

$$\phi = \left(\mu_0 \frac{N}{l} I \right) A$$

where, A = area of each turn of the solenoid

Total magnetic flux linked with the solenoid

= Flux through each turn

× Total number of turns

$$N\phi = \mu_0 \frac{N}{l} IA \times N \quad \dots(2)$$

If, L is coefficient of self-inductance of the solenoid, then

$$N\phi = LI \quad \dots(3)$$

From equation (2) and (3), we get

$$LI = \mu_0 \frac{n}{l} IA \times N$$

$$= N_2 \left(\frac{\mu_0 N_1 I_1}{2r_1} \right) A_2 = \frac{\mu_0 N_1 N_2 A_2}{2r_1} I_1$$

By definition, mutual inductance,

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A_2}{2r_1}$$

But,

$$A_2 = \pi r_2^2$$

$$M = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$$

Special case : If both coils have one turn each; then $N_1 = N_2 = 1$, so mutual inductance

$$M = \frac{\mu_0 \pi r_2^2}{2r_1}$$

LONG ANSWER QUESTIONS

57. (i) Define coefficient of self-induction. Obtain an expression for self-inductance of a long solenoid of length l area of cross section A having N -turns.
- (ii) Calculate the self-inductance of a coil using the following data obtained when an AC source of frequency $\left(\frac{200}{\pi}\right)$ Hz and a DC source is applied across the coil.

AC Source		
S.No.	V (Volts)	I (A)
1	3.0	0.5
2	6.0	1.0
3	9.0	1.5

DC Source		
S.No.	V (Volts)	I (A)
1	4.0	1.0
2	6.0	1.5
3	8.0	2.0

Ans :

OD 2023

(i) The coefficient of self induction of a coil is numerically equal to the emf induced in the coil, when the rate of change of current in the coil is unity. The SI unit of the coefficient of self induction is Henry (H).

Derivation : Total number of turn in solenoid, $N = nl$
Magnetic field inside, the long solenoid,

$$B = \mu_0 ni$$

Flux through one turn,

$$\phi_1 = BA = \mu_0 niA$$

Thus, total flux through N turns

$$\phi_E = N\phi_1 = nl \times \mu_0 niA$$

$$= \mu_0 n^2 l A i$$

Using $\phi_i = Li$, where L is the self inductance of the coil

$$\mu_0 n^2 l A i = Li$$

$$L = \mu_0 n^2 A l$$

$$= \mu_0 n^2 (nl) A$$

$$= \mu_0 \frac{N^2}{l} A$$

(ii) When an D.C. source is applied,

$$R = \frac{V}{I} = \frac{4}{1} = \frac{6.0}{1.5} = \frac{8.0}{2.0} = 4 \Omega$$

When AC source 3 V is applied,

$$I = \frac{3}{\sqrt{4^2 + \omega^2 L^2}}$$

$$0.5 = \frac{3}{\sqrt{16 + (2\pi f L)^2}}$$

$$\sqrt{16 + \left(\frac{2\pi \times 200 L}{\pi}\right)^2} = 6$$

Squaring both sides, we get

$$16 + 160000 L^2 = 6^2$$

$$160000 L^2 = 36 - 16$$

$$L^2 = \frac{20}{160000} = \frac{1}{8000} = 0.000125$$

$$L^2 = 125 \times 10^{-6}$$

$$L = 11.2 \times 10^{-3} H$$

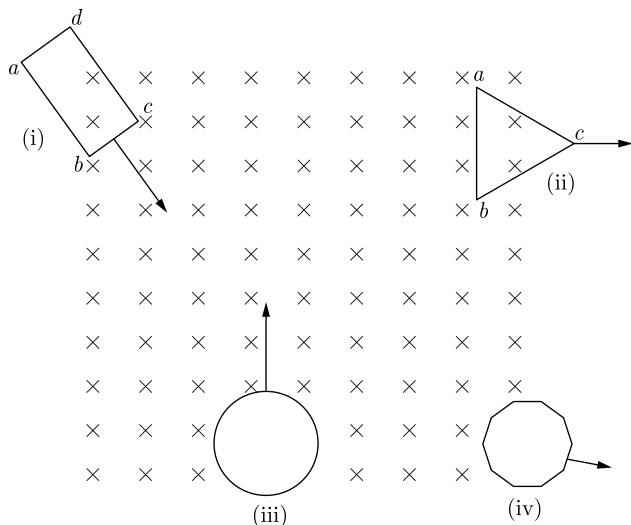
58. (i) With the help of a labelled diagram describe the principle and working of an ac generator. Hence, obtain an expression for the instantaneous value of the emf generated.
- (ii) The coil of an ac generator consists of 100 turns of wire each of area 0.5 m^2 . The resistance of the wire is 100Ω . The coil is rotating in a magnetic field of 0.8 T perpendicular to its axis of rotating in at a constant angular speed of $60 \text{ radian per second}$. Calculate the maximum emf generated and power dissipated the coil.

Ans :

OD 2023, SQP 2017

(i) **Principle** AC generator works on the principle of Faraday's law of electromagnetic induction. Whenever a coil is rotated in a uniform magnetic field about an axis perpendicular to the field, the magnetic flux linked with coil changes and an induced emf is set up across its ends.

53. Figure shows planar loops of different shapes moving out of or into a region of magnetic field which is directed normal to the plane of loops downwards. Determine the direction of induced current in each loop using Lenz's law.



Ans :

Delhi 2013

- (a) In fig. (i) the rectangular loop $abcd$ and in fig. (iii) circular loop are entering the magnetic field, so the flux linked with them increases. The direction of induced currents in these coils, will be such as to oppose the increase of magnetic flux, hence, the magnetic field due to current induced will be upward, i.e., currents induced will flow anti-clockwise.
- (b) In fig. (ii), the triangular loop abc and in fig. (iv) the zig-zag shaped loop are emerging from the magnetic field. Therefore magnetic flux linked with these loops decreases. The currents induced in them will tend to increase the magnetic field in downward direction, so the currents will flow clockwise.
54. (i) Define mutual inductance. Write its SI unit.
(ii) A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the charge of flux linkage with the other coil ?

Ans :

SQP 2007

- (i) Mutual inductance of two coils in the magnetic flux linked with the secondary coil when a unit current flows through the primary coil,

i.e., $\phi_2 = MI_1$ or $M = \frac{\phi_2}{I_1}$

SI unit of mutual inductance is Henry (H).

- (ii) Change of flux for small change in current

$$d\phi = MdI$$

$$= 1.5 (20 - 0)$$

$$= 30 \text{ Weber}$$

55. Starting from the expression for the energy $W = \frac{1}{2} LI^2$, stored in a solenoid of self-inductance L to build up the current I , obtain the expression for the magnetic energy in terms of the magnetic field B , area A and length l of the solenoid having n number of turns per unit length. Hence, show that the energy density is given by $B^2/2\mu_0$.

Ans :

OD 2013, Comp 2006

Energy stored in the magnetic field,

$$W = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{L} \cdot \frac{B^2 L^2}{\mu_0^2 N^2}$$

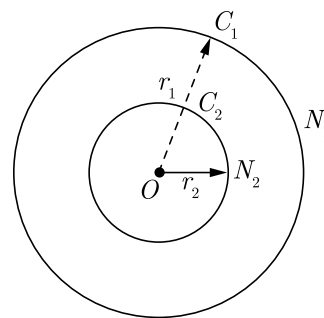
$$= \frac{\mu_0^2}{2\mu_0} (AL)^2 \left[L = \frac{\mu_0 N^2 A}{L}, B = \frac{\mu_0 NL}{L} \right]$$

Energy density,

$$u_B = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{B^2}{2\mu_0} \quad [\text{Hence, } B = AL]$$

56. Two concentric circular coils, one of small radius r_2 and the other of large radius r_1 , such that $r_2 \ll r_1$ are placed co-axially with centers coinciding. Obtain the mutual inductance of the arrangement.



Ans :

Comp 2015

Mutual Inductance of two plane coils : Consider two concentric circular plane coils C_1 and C_2 placed very near to each other. The number of turns in the primary coil is N_1 and radius is r_1 ; while the number of turns in the secondary coil is N_2 and its radius is r_2 . If I_1 is the current in the primary coil, then magnetic field produced at its centre,

$$B_1 = \frac{\mu_0 N_1 I_1}{2r_1}$$

If we suppose the magnetic field to be uniform over the entire plane of secondary coil, then total effective magnetic flux linkage with secondary coil.

$$\phi_2 = N_2 B_1 A_2$$

direction of induced current in loop.

Lenz's law states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.

43. State the Faraday's law's of electromagnetic induction.

Ans :

SQP 2008

On the basis of his experiment, Faraday gave the following two laws :

1. Whenever magnetic flux linked with a circuit changes, an emf is induced in it which lasts, so long as change in flux continuous.
2. The emf induced in loop or closed circuit is directly proportional to the rate of change of magnetic flux linked with the loop

$$\text{i.e., } e \propto \frac{d\phi}{dt}$$

$$\text{or } e = N \frac{d\phi}{dt}$$

where, N = number of turns in the coil

44. State Lenz's law. A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends ? Justify your answer.

Ans :

Comp 2016

Lenz's Law states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it. i.e., it opposes the change in magnetic flux.

$$e = \frac{-d\phi}{dt}$$

Yes, emf will be induced in the rod as there is change in magnetic flux.

When a metallic rod held horizontally along East-west direction, is allowed to fall freely under gravity i.e., fall from North to south, the intensity of magnetic lines of the Earth's magnetic field changes through it, i.e., the magnetic flux changes and hence emf induced in it.

45. A metallic rod of length L is rotated with angular frequency of ω with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius L , about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring.

Ans :

SQP 2006

$$\text{Angular velocity of rod, } \omega = \frac{2\pi}{T}$$

where, T = time period

Hence, charge in flux in one revolution

$$= BA$$

$$= B(\pi L^2)$$

According to Faraday's law of EMI, magnitude of induced emf

$$\begin{aligned} \varepsilon &= \frac{\Delta \phi}{\Delta T} = \frac{B\pi L^2}{T} \\ &= \frac{B\pi L^2}{\left(\frac{2\pi}{\omega}\right)} \quad \left[\text{since } T = \frac{2\pi}{\omega}\right] \\ \varepsilon &= \frac{1}{2} B\omega L^2 \end{aligned}$$

46. How does the mutual inductance of a pair of coils change, when

1. distance between the coils is increased and
2. number of turns in the coils is increased?

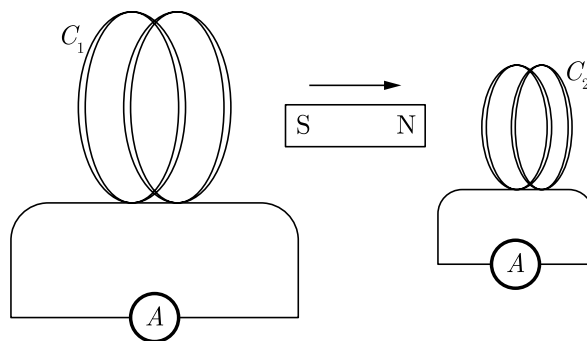
Ans :

Comp 2011, OD 2013

1. $\phi \cdot A = MI$, with the increase in the distance between the coils the magnetic flux linked with the secondary coil decreases and hence, the mutual inductance of the two coils will decrease with the increase of separation between them.
2. Mutual inductance of two coils can be found out by $M = \mu_0 N_1 N_2 A l$, i.e.,

$M \propto N_1 N_2$, so, with the increase in number of turns mutual inductance increases.

47. A magnet is quickly moved in the direction indicated by an arrow between two coils C_1 and C_2 as shown in the figure. What will be the direction of induced current in each coil as seen from the magnet? Justify your answer.

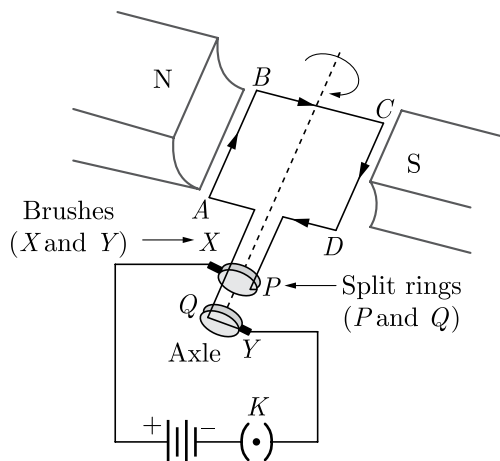


Ans :

Foreign 2009

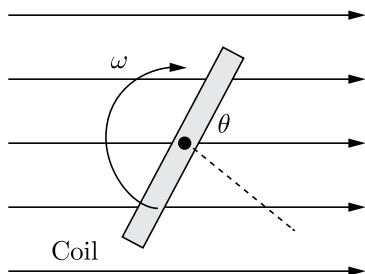
According to Lenz's law, the direction of induced current is such that it opposes the relative motion between coil and magnet.

As N -pole of the magnet is moving away from the coil C the near face of coil C_1 will become S -pole, so the direction of current in coil C_1 , will be clockwise.



Working : When the coil starts rotating with the arm AB moving up and the arms CD moving down, cutting the magnetic lines of force, then according to Fleming's right-hand rule and the induced current is set up in these arms along the direction of AB and CD. So an effective induced current flowing in the direction of ABCD is obtained. If there are large number of turns in the coil, the current generated in each turn adds upto give a large quantity of current through the coil. After half rotation of the coil, its arm CD starts moving up and AB moving down. As a result the direction of the induced currents gets reversed in the coil in the direction of DCBA. Thus, after every half rotation the polarity of the current in the respective arm changes. Such a current, which changes its direction after equal intervals of time, is called alternating current. To get a direct current a split ring commutator must be used in place of slip ring commutator.

Derivation : Consider a coil of N turns and area A being rotated at a constant angular velocity (ω) in a magnetic field of flux density B , its axis being perpendicular to the field. When the normal to the coil is at angle θ to the field, the flux through the coil is



$$\phi = BAN \cos \theta = BAN \cos (wt)$$

Since,

$$\theta = wt$$

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(BAN \cos \theta)$$

$$= BAN \omega \sin \omega t$$

$$(ii) \text{ Here, } N = 100, A = 0.5 \text{ m}^2, R = 100 \Omega$$

$$\omega = 60 \text{ rad/s}, B = 0.8 \text{ T}$$

As induced emf is given by $NBA \cos \omega t$

$$\text{Max. emf } e_0 = NBA \omega$$

$$= 100 \times 0.8 \times 0.5 \times 60$$

$$= 2400 \text{ V}$$

$$\text{Induced current (max.) } I_0 = \frac{e_0}{R}$$

$$= \frac{2400}{100} = 24 \text{ A}$$

Power

$$= e_0 I_0 = 2400 \times 24$$

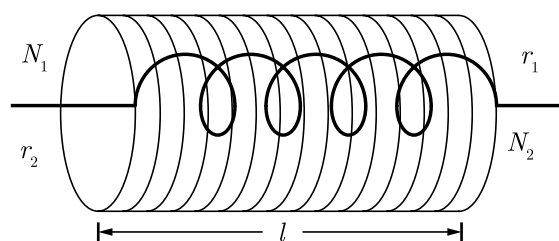
$$= 57600 = 5.76 \times 10^4 \text{ W}$$

59. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length round one over the other.

Ans :

Delhi 2021, OD 2014

Let a current of i_1 ampere flow in the primary coil. Let, due to this current, the magnetic flux linked with each turn of the secondary coil be ϕ_2 . If N_2 be the number of turns in the secondary coil, then the number of flux-linkages in the coil will be $N_2 \phi_2$. This number is proportional to the current i_1 flowing in the primary coil, that is



$$N_2 \phi_2 \propto i_1 \text{ or } N_2 \phi_2 = M i_1$$

Where, M is constant, called the coefficient of mutual induction or mutual inductance of the two coils. From the equation, we have,

$$M = \frac{N_2 \phi_2}{i_1}$$

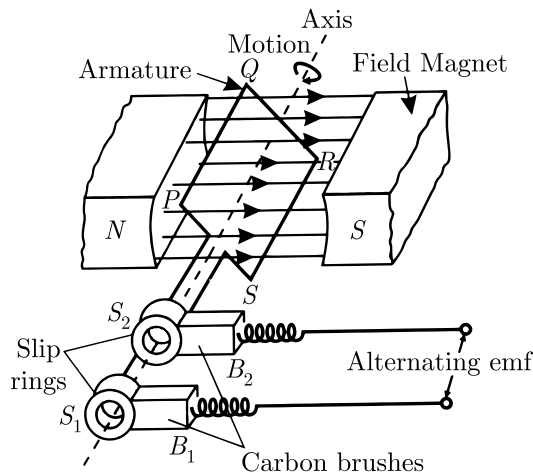
If $i_1 = 1$, then $M = N_2 \phi_2$

Mutual Inductance of Two Long Coaxial Solenoids

Consider two long solenoids S_1 and S_2 of same length (l), such that solenoid (S_2) surrounds solenoid (S_1) incompletely.

Principle

The working of an ac generator is based on the principle of electromagnetic induction. When a closed coil is rotated in a uniform magnetic field with its axis perpendicular to the magnetic field, the magnetic flux linked with the coil changes and an induced emf and hence a current is set up in it.



Construction

It consists of the following main parts:

1. **Field Magnet** : N-S is either a strong permanent magnet or an electromagnet.
2. **Armature** : PQRS is a rectangular armature coil. It consists of a large number of turns of insulated copper wire wound on a soft iron cylindrical core. It can be rotated about an axis perpendicular to the magnetic field of the field magnet.
3. **Slip Rings** : These are two brass rings S_1 and S_2 rigidly connected to the two ends of the armature coil. As the armature coil rotates slip rings also rotate about the same axis of rotation.
4. **Brushes** : These are two graphite rods B_1 and B_2 which remain fixed in their positions but maintain sliding contacts with the slip rings S_1 and S_2 . Through these brushes, the current induced in the armature coil is fed to the external circuit.

Working

As the armature coil rotates, the magnetic flux linked with it changes and so an induced current flows through it. Initially, suppose the coil is in the vertical position and is rotated clockwise. Its side PQ moves downward and SR moves upward. By Fleming's right hand rule, the current flows from Q to P and from S to R. So during the first half rotation of the coil, the current flows in the direction SRQP. During the second half rotation, the direction of current is reversed, i.e., it flows along PQRS. The direction of current in the external circuit is reversed after every

half cycle. Hence alternating current is produced by the generator.

Expression for Induced emf

Let, N = number of turns in the coil
 A = face area of each turn
 B = magnitude of the magnetic field
 θ = angle which normal to the coil makes with field $\vec{B}$ at any instant t
 ω = the angular velocity with which coil rotates

Then magnetic flux linked with the coil at any instant t will be,

$$\begin{aligned}\phi &= NBA \cos \theta \\ &= NBA \cos \omega t\end{aligned}$$

Induced emf will be,

$$\begin{aligned}e &= -\frac{d\phi}{dt} \\ &= -\frac{d}{dt}(NBA \cos \omega t) \\ &= NBA\omega \sin \omega t\end{aligned}$$

or $e = e_0 \sin \omega t$

where, $e_0 = NBA\omega$ = peak value of induced emf.

Clearly induced emf varies sinusoidally with time.

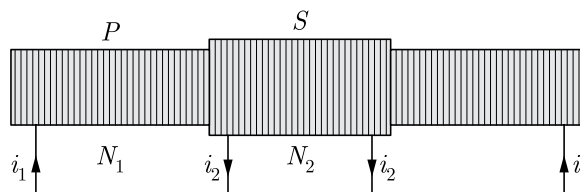
69. Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns.

Ans :

Delhi 2017, OD 2008

Mutual Inductance of Two Coaxial Solenoids

Let us consider a long air-cored solenoid P (primary) of length l and area of cross-section A , having N_1 turns (figure). Let a shorter secondary coil S having N_2 turns be wound closely over the central portion of the primary P . Let, i_1 be the current in the primary. The magnitude of the magnetic field $\vec{B}$ inside the primary is, in SI unit, given by,

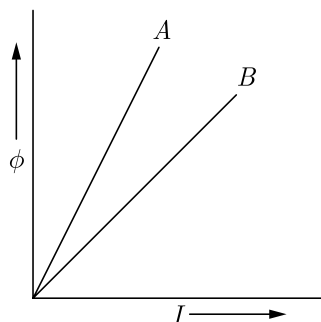


$$B = \mu_0 \frac{N_1}{l} i_1$$

The magnetic flux linked with the primary due to the field $\vec{B}$ is also linked with the secondary because the secondary is wound closely over the central portion of

61. (a) How does the mutual inductance of a pair of coils change when
- distance between the coils is increased and
 - number of turns in the coils is increased?
- (b) A plot of magnetic flux (ϕ) versus current (I), is shown in the figure for two inductors A and B . Which of the two has large value of self-inductance?
- (c) How is the mutual inductance of a pair of coils affected when
- separation between the coils is increased?
 - the number of turns in each coil is increased?
 - a thin iron sheet is placed between the two coils, after factors remaining the same?

Justify your answer in each case.



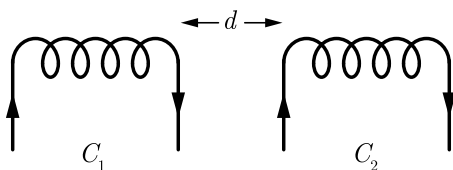
Ans :

Foreign 2005

- (a) (i) Mutual inductance decreases.
(ii) Mutual inductance increases.

Concept

- (i) If distance between two coils is increased as shown in figure.



It causes decrease in magnetic flux linked with the coil C_2 . Hence, induced emf in coil C_2 decreases by relation $\varepsilon_2 = \frac{-d\phi_2}{dt}$. Hence, mutual inductance decreases.

- (ii) From relation $M_{21} = \mu_0 N_1 N_2 AI$, if number of turns in one of the coils or both increases, means mutual inductance will increase.

(b)
$$\phi = LI = \frac{\phi}{I} = L$$

The slope of $\frac{\phi}{I}$ of straight line is equal to self-inductance L . It is larger for inductor A , therefore inductor A has longer value of self inductance L .

- (c) (i) When the relative distance between the coil is increased, the leakage of flux increases which reduces the magnetic coupling of the coils. So magnetic flux linked with all the turns decreases. Therefore, mutual inductance will be decreased.
(ii) Mutual inductance for a pair of coil is given by

$$M = K\sqrt{L_1 L_2}$$

where, $L = \frac{\mu N^2 A}{l}$ and L is called self inductance.

Therefore, when the number of turns in each coil increases, the mutual inductance also increases.

- (iii) When a thin iron sheet is placed between the two coils, the mutual inductance increases because

$M \propto$ permeability. The permeability of the medium between coils increases.

62. Define the term self-inductances of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I through it.

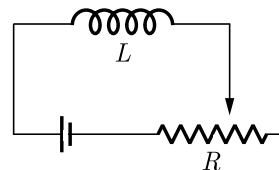
Ans :

OD 2013, Comp 2005

- (i) **Self-inductance** : Self-inductance is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.

The induced emf is also called back emf. When the current in a coil is switched on, the self-induction opposes the growth of the current and when the current is switched off, the self-induction opposes the decay of the current,

So, self-induction is also called the inertia of electricity



- (ii) **Self-inductance of long solenoid** : A long solenoid is one whose length is very large as compared to its area of cross-section.

The magnetic field (B) at any point inside such a solenoid is practically constant and is given by

$$B = \frac{\mu_0 NI}{l} \quad \dots(1)$$

where, μ_0 = absolute magnetic permeability of free space,

N = total number of turns in the solenoid.

and l = length of the solenoid

Hence, Magnetic flux through each turn of the solenoid,

$$\phi = B \times \text{area of the each turn}$$

$$\phi = \left(\mu_0 \frac{N}{l} I \right) A$$

where, A = area of each turn of the solenoid.

Total magnetic flux linked with the solenoid.

= flux through each turn $\times$ total number of turns

$$N\phi = \mu_0 \frac{N}{l} I A \times N \quad \dots(2)$$

If L is coefficient of self-inductance of the solenoid, then

$$N\phi = LI \quad \dots(3)$$

From eqs. (2) and (3), we get

$$LI = \mu_0 \frac{N}{l} I A \times N$$

$$\text{or} \quad L = \frac{\mu_0 N^2 A}{l}$$

If core is of any other magnetic material μ is placed, then

$$\mu = \mu_0 \mu_r$$

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

The magnitude of emf is given by

$$e = L \frac{dI}{dt}$$

Multiplying (I) to both sides, we get

$$eI dt = LI dI \quad \dots(4)$$

$$\text{But,} \quad I = \frac{dq}{dt} \text{ or } Idt = dq$$

Also, work done (dW) = voltage (e) $\times$ charge (dq)

$$\text{or} \quad dW = e \times dq = eIdt \quad \dots(5)$$

Subtracting eq. (5) in eq. (4), we get,

$$dW = LI dI \quad \dots(6)$$

Total work done in increasing the current from zero to I_0 , by integrating both sides of eq. (5) we get,

$$\int_0^W dW = \int_0^{I_0} LI dI$$

$$W = \frac{1}{2} LI_0^2$$

This work done in increasing the current flowing through the inductor is stored in the potential energy (U) in the magnetic field of inductor.

$$U = \frac{1}{2} LI_0^2$$

63. What is self-induction? Define self-inductance. Give its units and dimensions.

Ans :

OD 2018

When a changing current flows through a coil, the magnetic flux linked with the coil changes and an opposing emf is induced in the coil. This emf is called self-induced emf or back emf and the phenomenon is known as self-induction.

Self-inductance or Coefficient of Self-induction

At any instant,

Magnetic flux linked with a coil $\propto$ Current through the coil.

$$\text{or} \quad \phi \propto I$$

$$\phi = LI$$

The proportionality constant L is called self-inductance. The induced emf set up in the coil will be,

$$e = - \frac{d\phi}{dt}$$

$$= - L \frac{dI}{dt}$$

$$\text{If,} \quad \frac{dI}{dt} = 1$$

$$\text{Then,} \quad e = -L$$

Thus, the self-inductance of a coil may be defined as the induced emf set up in the coil due to a unit rate of change of current through it.

Units of Self-inductance

We have,

$$L = \frac{e}{dI/dt}$$

$$\text{Hence, SI unit of } L = \frac{1 \text{ V}}{1 \text{ As}^{-1}}$$

$$= 1 \text{ VsA}^{-1} = 1 \text{ henry (H)}$$

The self-inductance of a coil is said to be one henry if an induced emf of one volt is set up in it when the current in it changes at the rate of one ampere per second.

$$\text{Also,} \quad L = \frac{\phi}{I}$$

So, SI unit of self-inductance is WbA^{-1} .

$$\text{Hence, } 1 \text{ henry, } H = 1 \text{ VsA}^{-1}$$

$$= 1 \text{ WbA}^{-1}$$

Dimensions of Self-inductance

$$\text{Now} \quad L = \frac{\phi}{I} = \frac{BA}{I}$$

$$= \frac{F}{qv \sin \theta} \cdot \frac{A}{I}$$

Hence, Dimensions of

$$L = \frac{MLT^{-2} \cdot L^2}{C \cdot LT^{-1} \cdot A} = \frac{ML^2T^{-2}}{A \cdot A}$$

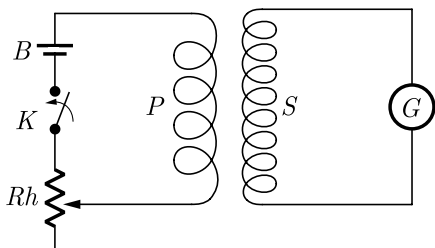
$$= [ML^2T^{-2}A^{-2}]$$

64. What is meant by mutual induction? Define the term mutual inductance. Give its units and dimensions.

Ans :

Foreign 2013

Mutual induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil.



Mutual Induction

As shown in figure, consider two coils P and S placed close to each other. If the current in the coil P is varied, the magnetic flux linked with the coil S changes which induces an emf and hence a current in it, as is seen from the deflection in the galvanometer. This coil P is called the primary coil and coil S , the secondary coil, because it is the former which causes an induced emf in the latter.

Mutual inductance or coefficient of mutual induction. At any instant,

Magnetic flux linked with the secondary coil

$$\propto \text{Current in the primary coil}$$

$$\text{i.e., } \phi \propto I$$

$$\text{or } \phi = MI$$

The proportionality constant M is called the mutual inductance or coefficient of mutual induction of the two coils. Any change in the current I sets up an induced emf in the secondary coil which is given by,

$$e = -\frac{d\phi}{dt} = -M \cdot \frac{dI}{dt}$$

$$\text{If } \frac{dI}{dt} = 1$$

$$\text{Then, } e = -M$$

The mutual inductance of two coils may be defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.

Unit of Mutual Inductance

$$\text{We have, } M = \frac{e}{dI/dt}$$

$$\text{Hence, SI unit of } M = \frac{1 \text{ V}}{1 \text{ As}^{-1}}$$

$$= 1 \text{ VsA}^{-1} = 1 \text{ henry (H)}$$

The mutual inductance of two coils is said to be one henry if an induced emf of one volt is set up in one coil when the current in the neighbouring coil changes at the rate of 1 ampere per second.

65. Define self inductance and mutual inductance. Find an expression for mutual inductance of two long coaxial solenoids.

Ans :

Delhi 2016, Foreign 2002

Self Inductance

When a current flows in a coil, it gives rise to a magnetic flux through the coil itself. As the strength of current changes, the linked magnetic flux changes and an opposing emf is induced in the coil. This emf is called self-induced emf or back emf and the phenomenon is known as self-induction.

$$\text{Self inductance, } L = \frac{e}{dI/dt}$$

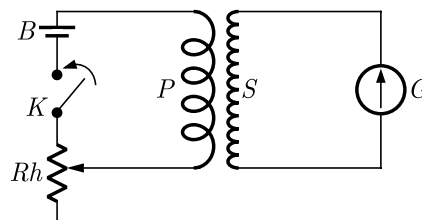
$$\text{S.I. unit of } L = \frac{1 \text{ V}}{1 \text{ As}^{-1}}$$

$$= 1 \text{ VsA}^{-1} = 1 \text{ henry (H)}$$

Mutual induction

Mutual induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil.

As shown in figure consider two coil P and S placed close to each other.



The coil P is called the primary coil and coil S , the secondary coil, because it is the former which causes an induced emf in the latter.

At any instant,

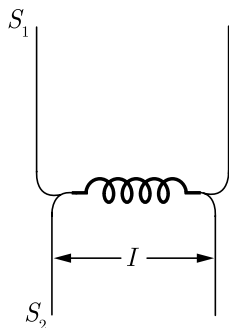
Magnetic flux linked with the secondary coil

$$\propto \text{current in the primary coil}$$

$$\text{i.e., } \phi \propto I$$

$$\phi = MI \quad \dots(1)$$

The proportionality constant M is called the mutual inductance or coefficient of mutual induction of the two coil. Any change in the current I sets up an induced emf in the secondary coil which is given by,



Let n_1 , be the number of turns per unit length of S_2 , n_2 be the number of turns per unit length of S_1 , I_1 be current passed through solenoid S_1 and ϕ_{21} be flux linked with S_2 due to current flowing through S_1 .

$$\phi_{21} \propto I_1 \text{ or } \phi_{21} = M_{21} I_1$$

Where M_{21} is the coefficient of mutual induction of the two solenoids.

When current is passed through solenoid (S_1) an emf is induced in solenoid (S_2).

Magnetic field produced inside solenoid (S_1) on passing current through it.

$$B_1 = \mu_0 n_1 I_1$$

Magnetic flux linked with each turn of solenoid (S_2) will be equal to B_1 times the area of cross-section of solenoid (S_2).

Magnetic flux linked with each turn of the solenoid

$$S_1 = B_1 A.$$

Therefore, total magnetic flux linked with the solenoid (S_2),

$$\begin{aligned} \phi_{21} &= B_1 A \times n_2 l \\ &= \mu_0 n_1 I_1 \times A \times n_2 l \\ \phi_{21} &= \mu_0 n_1 n_2 A l I_1 \\ M_{21} &= \mu_0 n_1 n_2 A l \end{aligned} \quad \dots(1)$$

Similarly, the mutual inductances between the two solenoids, when current is passed through solenoid (S_2) and induced emf is produced in solenoid (S_1) and is given by

$$\begin{aligned} M_{12} &= \mu_0 n_1 n_2 A l \\ M_{12} &= M_{21} = M \quad (\text{say}) \end{aligned}$$

Hence, coefficient of mutual induction between the two long solenoids

$$M = \mu_0 n_1 n_2 A l.$$

We can rewrite eq. (1) as

$$\begin{aligned} M &= \mu_0 \left(\frac{N_1}{l} \right) \left(\frac{N_2}{l} \right) \pi r_1^2 \times l \\ M &= \frac{\mu_1 N_1 N_2 A}{l} \end{aligned}$$

60. Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf.

Ans :

Comp 2020, SQP 2011

Self Inductance : Using formula $\phi = LI$, if $I = 1$ Ampere then $L = \phi$

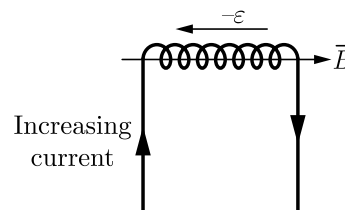
Self inductance of the coil is equal to the magnitude of the magnetic flux linked with the coil, when a unit current flows through it.

OR

Using formula, $|-e| = L \frac{dI}{dt}$

If, $\frac{dI}{dt} = 1 \text{ A/s}$ then $L = |-e|$

Expression for magnetic energy :



When a time varying current flows through the coil, back emf ($-e$) produces, which opposes the growth of the current flow, it means some work needs to be done against induced emf in establishing a current I . This work done will be stored as magnetic potential energy for the current I at any instant, the rate of work done is

$$\frac{dW}{dt} = (-e) I,$$

where e is self induced emf.

If we ignore the resistive losses and consider only the inductive effect, then using

$$\begin{aligned} |e| &= L \frac{dI}{dt} \\ \frac{dW}{dt} &= L \left(\frac{dI}{dt} \right) I \\ dW &= LI dI \end{aligned}$$

Hence, total amount of work done in establishing the current I ,

$$W = \int dW = \int_0^I LI dI = \frac{1}{2} LI^2$$

Thus, the energy required to build up the current I is

$$W = \frac{1}{2} LI^2$$

This energy is stored as magnetic potential energy in the inductor.

Thus, energy, $U = \frac{1}{2} LI^2$

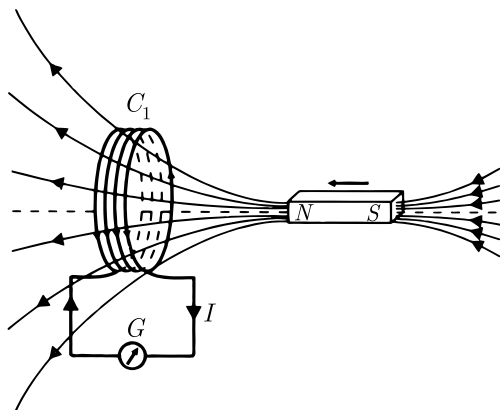
67. Describe the various experiments performed by Faraday and Henry which ultimately led to the discovery of the phenomenon of electromagnetic induction.

Ans :

OD 2019

Faraday's Experiments

Experiment 1 : Induced emf with a stationary coil and moving magnet.

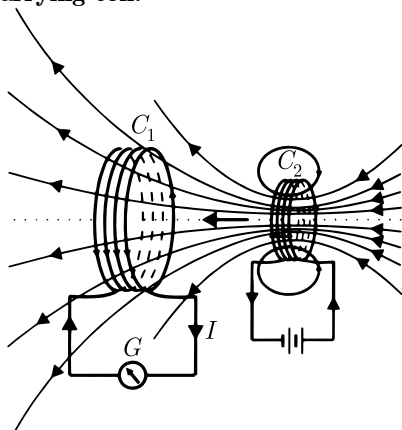


When the Bar Magnet is Pushed Towards the Coil, the Pointer in the Galvanometer G Deflects

As shown in figure, take a coil C_1 connected to a galvanometer G . Whenever the N -pole of a bar magnet is moved towards or away from the coil, a current is induced in the coil as is shown by deflection in the galvanometer. Faster the motion of the magnet, larger is the current induced in the coil. The galvanometer does not show any deflection when the magnet is held stationary.

The motion of the magnet towards or away from coil C_1 changes the magnetic flux linked with coil C_1 . The change in the magnetic flux induces an emf in coil C_1 which causes a current to flow through C_1 .

Experiment 2 : Induced emf due to the motion of current carrying coil.

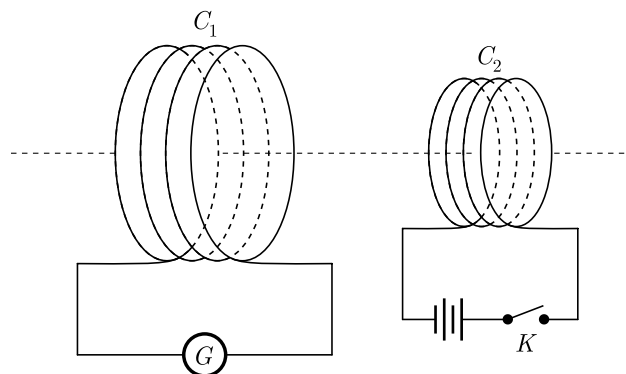


Current is Induced in Coil C_1 due to Motion of Current Carrying Coil C_2

Figure shows a coil C_1 connected to a galvanometer and coil C_2 connected to a battery. The steady current in coil C_2 produces a steady magnetic field. As the coil is moved towards C_1 , the galvanometer shows a deflection. This indicates an induced current flowing through C_1 . When C_2 is moved away from C_1 , the direction of current through C_1 is reversed.

Due to the relative motion between coils C_1 and C_2 , the magnetic flux linked with C_1 changes and an emf is induced in it.

Experiment 3 : Induced emf by changing current in the neighbouring coil.



Induced emf by Change of Current in the Neighbouring Coil

Figure shows a coil C_1 connected to a galvanometer and another coil C_2 connected to a battery through a tapping key K . When the key K is pressed, the galvanometer shows a momentary deflection. As the key is kept pressed continuously, the galvanometer shows no deflection. When the key is released, a momentary deflection is observed again, but in the opposite direction.

When the key K is pressed, the current through C_2 increases from zero to a maximum value in a short time. The flux through C_1 increases. This induces an emf in C_1 . When the key is kept pressed continuously, the current through C_2 becomes constant. There is no change in flux linked with C_1 and hence induced current becomes zero. When the key is released, the current through C_2 decreases, the flux linked with C_1 decreases and hence induced current in C_1 flows in the opposite direction.

68. With the help of a labelled diagram, explain the principle, construction and working of an ac generator. Derive the expression for induced emf.

Ans :

Comp 2018, Delhi 2012

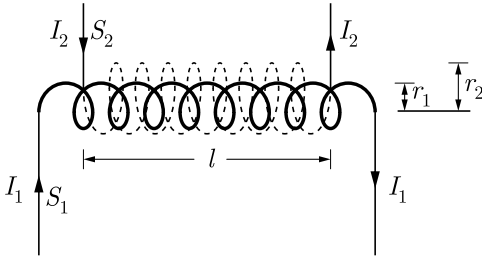
AC Generator

It is a device which converts mechanical energy into alternating form of electrical energy.

$$\varepsilon = -\frac{d\phi}{dt} = -M \cdot \frac{dI}{dt} \quad \dots(2)$$

Mutual Inductance of Two Long Solenoids

As shown in Figure consider two long co-axial solenoids S_1 and S_2 , with S_2 wound over S_1 .



Let, l = length of each solenoid
 r_1, r_2 = radii of the two solenoids
 $A = \pi r_1^2$
 = area of cross-section of inner solenoid S_1
 N_1, N_2 = number of turns in the two solenoids.

First we pass a time varying current I_2 through S_2 .

The magnet field set up inside S_2 due to I_2 is,

$$B_2 = \mu_0 n_2 I_2$$

where, $n_2 = N_2/l$
 = the number of turn per unit length of S_2 .

Total magnetic flux linked with the inner solenoid S_1 is,

$$\phi_1 = B_2 A N_1 = \mu_0 n_2 I_2 \cdot A N_1$$

Hence, Mutual inductance of coil 1 with respect to coil 2 is,

$$\begin{aligned} M_{12} &= \frac{\phi_1}{I_2} = \mu_0 n_2 A N_1 \\ &= \frac{\mu_0 N_1 N_2 A}{l} \end{aligned}$$

We now consider the flux linked with the outer solenoid S_2 due to the current I_1 in the inner solenoid S_1 . The field B_1 due to I_1 is constant inside S_1 but zero in the annular region between the two solenoids.

$$\text{Hence } B_1 = \mu_0 n_1 I_1$$

where, $n_1 = N_1/l$
 = the number of turn per unit length of S_1

Total flux linked with the outer solenoid S_2 is,

$$\begin{aligned} \phi_2 &= B_1 A N_2 \\ &= \mu_0 n_1 I_1 \cdot A N_2 \\ &= \frac{\mu_0 N_1 N_2 A l}{l} \end{aligned}$$

Hence, Mutual inductance of coil 2 with respect to coil 1 is,

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Clearly, $M_{12} = M_{21} = M$ (say)

$$\text{Hence, } M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2$$

$$Al = \mu_0 n_1 n_2 \pi r_1^2 l$$

Thus, the mutual inductance of two coil is the property of their combination. It does not matter which one of them functions as the primary or the secondary coil. This fact is known as reciprocity theorem.

66. A metallic rod of length l and resistance R is rotated with a frequency v , with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l , about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere.

- Derive the expression for the induced emf and the current in the rod.
- Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.
- Hence, obtain the expression for the power required to rotate the rod.

Ans :

Foreign 2014

- In the one revolution change of area, $dA = \pi l^2$
 Hence, change of magnetic flux,

$$d\phi = B \cdot dA = B d,$$

$$A \cos 0^\circ = B\pi l^2$$

Period of revolution is T .

- Induced emf,

$$e = \frac{B\pi l^2}{T} = B\pi l^2 v$$

- Induced current in the rod,

$$I = \frac{e}{R} = \frac{\pi v B l^2}{R}$$

- Force acting on the rod,

$$F = IlB = \frac{\pi v B^2 l^3 v}{R}$$

The external force required to rotate the rod opposes the Lorentz force acting on the rod/ external force acts in the direction opposite to the Lorentz force.

- Power required to rotate the rod.

$$P = Fv = \frac{\pi v B^2 l^3 v}{R}$$

the primary. Thus, the magnetic flux linked with each turn of the secondary is,

$$\phi_2 = BA = \mu_0 \frac{N_1}{l} i_1 A$$

where, A is the area of cross-section of the secondary (although it is almost the same as of the primary). By definition, the mutual inductance of the two solenoids is,

$$M = \frac{N_2 \phi_2}{i_1}$$

Putting the above value of ϕ_2 , we have

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

If n_1 be the number of turns per unit length in the primary, then, $n_1 = \frac{N_1}{l}$, and we have

$$M = \mu_0 n_1 N_2 A$$

If an iron core be placed inside the primary, the magnitude of M would greatly increase.

70. 1. What is induced emf? Write Faraday's law of electromagnetic induction. Express it mathematically.
2. A conducting rod of length l , with one end pivoted, is rotated with a uniform angular speed ' ω ' in a vertical plane, normal to a uniform magnetic field ' B '. Deduce an expression for the emf induced in this rod.
- If resistance of rod is R , what is the current induced in it.

Ans :

Foreign 2009

1. **Induced EMF :** The emf developed in a coil due to change in magnetic flux linked with the coil is called the induced emf.

Faraday's Law of Electromagnetic Induction : On the basis of experiments, Faraday gave two laws of electromagnetic induction.

- (a) When the magnetic flux linked with a coil or circuit changes, an emf is induced in the coil. If coil is closed, the current is also induced. The emf and current last so long as the change in magnetic flux lasts. The magnitude of induced emf is proportional to the rate of change of magnetic flux linked with the circuit. Thus, if $\Delta\phi$ is the change in magnetic flux linked in time Δt , then rate of change of flux is $\frac{\Delta\phi}{\Delta t}$.

So, emf induced,

$$e \propto \frac{\Delta\phi}{\Delta t}$$

- (b) The emf induced in the coil (or circuit) opposes the cause producing it.

$$\text{Hence, } e \propto -\frac{\Delta\phi}{\Delta t}$$

Here the negative sign shows that the induced emf e opposes the change in magnetic flux

$$\text{Hence, } e = -K \frac{\Delta\phi}{\Delta t}$$

where, K is constant of proportionality which depends on units chosen for ϕ , t and e . In SI system the unit of flux ϕ is weber, unit of time t is second and unit of emf e is volt and $K = 1$,

$$e = -\frac{\Delta\phi}{\Delta t} \quad \dots(1)$$

If the coil contains N -turns of insulated wire, then the flux linked with each turn will be same and the emf induced in each turn will be in the same direction. Hence the emfs of all turns will be added. Therefore, the emf induced in the whole coil,

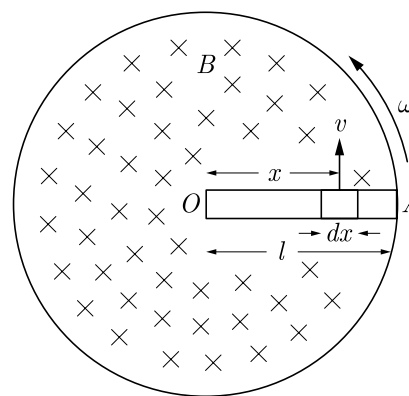
$$e = -N \frac{\Delta\phi}{\Delta t} = -\frac{\Delta(N\phi)}{\Delta t} \quad \dots(2)$$

$N\phi$ is called the effective magnetic flux or the number of flux linkages in the coil and may be denoted by Φ .

2. Expression for Induced emf in a Rotating ROD

: Consider a metallic rod OA of length l , which is rotating with angular velocity ω in a uniform magnetic field B , the plane of rotation being perpendicular to the magnetic field. A rod may be supposed to be formed of a large number of small elements.

Consider a small element of length dx at a distance x from centre. If v is the linear velocity of this element, then area swept by the element per second $= v dx$.



The emf induced across the ends of element,

$$de = B \frac{dA}{dt} = Bv dx$$

But, $v = x\omega$

Hence, $de = Bx\omega dx$

Hence, The emf induced across the rod,

$$\begin{aligned}
 e &= \int_0^l B \omega dx \\
 &= B\omega \int_0^l x dx \\
 &= B\omega \left[\frac{x^2}{2} \right]_0^l \\
 &= B\omega \left[\frac{l^2}{2} - 0 \right] \\
 &= \frac{1}{2} B\omega l^2
 \end{aligned}$$

Current induced in rod,

$$I = \frac{e}{R} = \frac{1}{2} \frac{B\omega l^2}{R}$$

If circuit is closed, power dissipated,

$$= \frac{e^2}{R} = \frac{B^2 \omega^2 l^4}{4R}$$

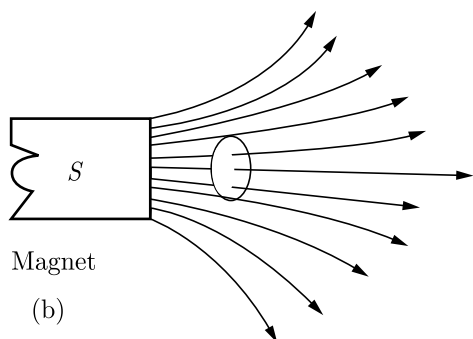
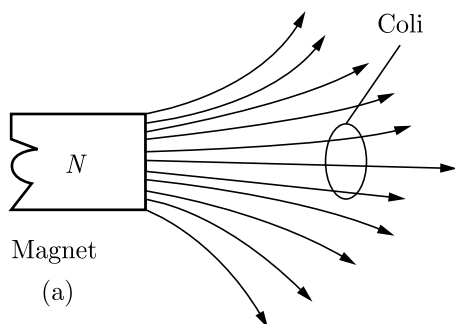
71. (a) State Lenz's law. Give one example to illustrate the law. "The Lenz's law is a consequence of the principle of conservation of energy." Justify the statement.
- (b) Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns.

Ans :

Comp 2012, Delhi 2010

- (a) **Lenz's law** : The direction of the induced emf, or the current, in any circuit is such as to oppose the cause that produces it.

Example & Explanation

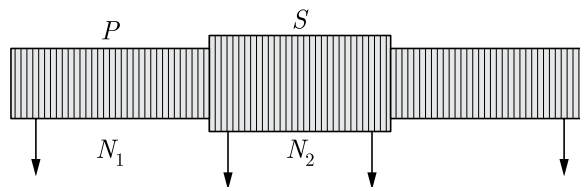


In the magnet and coil experiment, the direction of the induced current is in accordance to Lenz's law, that is, it opposes the motion of the magnet which

produces it when the north pole of the magnet is moved towards the coil, the induced current flows in a direction so that the near (left) face of the coil acts as a magnetic north pole (fig. a). The repulsion between the two poles opposes the motion of the magnet towards the coil. Similarly, when the north pole of the magnet is moved away from the coil, the direction of the induced current is such as to make the near face of the coil a south pole (fig. b). The attraction between the two pole opposes the motion of the magnet away from the coil. In either case, therefore, work has to be done in moving the magnet. It is this mechanical work which appears as electrical energy in the coil.

If the direction of the induced current were such as not to oppose the motion of the magnet, then we would be obtaining electrical energy continuously without doing any work, which is impossible. Thus, Lenz's law is in accordance with the principle of conservation of energy.

- (b) **Mutual Inductance of two Coaxial Solenoids** : Let us consider a long air-cored solenoid P (primary) of length l and area of cross-section A , having N_2 turns he would closely over the central portion of the primary P . Let i_1 be the current in the primary. The magnitude of the magnetic field $\vec{B}$ inside the primary is, in SI unit, given by



The magnetic flux linked with the primary due to the field $\vec{B}$ is also linked with the secondary because the secondary is wound closely over the current portion of the primary. Thus, the magnetic flux linked with each turn of the secondary is

$$\phi_2 = BA = \mu_0 \frac{N_1}{l} i_1 A$$

Where, A is the area of cross-section of the secondary (although it is almost the same as of the primary).

By definition, the mutual inductance of the two solenoids is

$$M = \frac{N_2 \phi_2}{i_1}$$

Putting the above value of ϕ_2 , we have

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

If n_1 be the number of turns per unit length in the

primary, then $n_1 = \frac{N_1}{l}$, and we have

$$M = \mu_0 n_1 N_2 A$$

If an iron core be placed inside the primary, the magnitude of M would greatly increase.

72. 1. A rod of length l is moved horizontally with a uniform velocity ' V ' in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.
2. How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain.

Ans :

OD 2009

1. Suppose a rod of length ' l ' moves with velocity V inward in the region having uniform magnetic field B . Initial magnetic flux enclosed in the rectangular space is $\phi = Blx$.

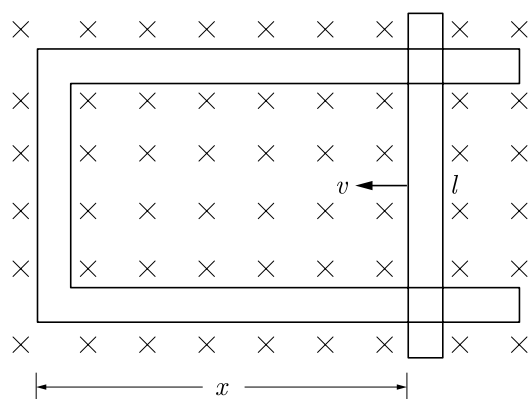
As the rod moves with velocity,

$$v = \frac{dx}{dt}$$

Using Lenz's law,

$$\begin{aligned} e &= -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx) \\ &= Bl\left(-\frac{dx}{dt}\right) \end{aligned}$$

Hence, $e = Blv$



2. Suppose any arbitrary charge ' q ' in the conductor of length l moving inward in the field as shown in figure, the charge q also moves with velocity v in the magnetic field B .

The Lorentz force on the charge ' q ' is F .

$$F = qvB \text{ and its direction is downwards}$$

So, work done in moving the charge ' q ' along the conductor of length l .

$$W = F.l$$

$$W = qvBl$$

Since emf is the work done per unit charge,

$$\text{Hence, } e = \frac{W}{q} = Blv$$

This equation gives emf induced across the rod.

73. (i) How is magnetic flux linked with the armature coil changed in a generation?
- (ii) Derive the expression for maximum value of the induced emf and state the rule that gives the direction of the induced emf.
- (iii) Show the variation of the emf generated versus time as the armature is rotated with respect to the direction of the magnetic fields.

Ans :

OD 2016, SQP 2012

The direction of flow of current in resistance R get changed alternatively after every half cycle.

Thus, AC is produced in coil.

Let at any instant coil magnetic flux linked with the armature coil is G . and $\theta = \omega t$ is the angle made by area vector of coil with magnetic field.

$$\phi = NBA \cos \theta$$

$$= NBA \cos \omega t$$

$$\frac{d\phi}{dt} = -NBA\omega \sin \omega t$$

$$\frac{d\phi}{dt} = NBA\omega \sin \omega t$$

By Faraday's law of EMI, $e = -\frac{d\phi}{dt}$

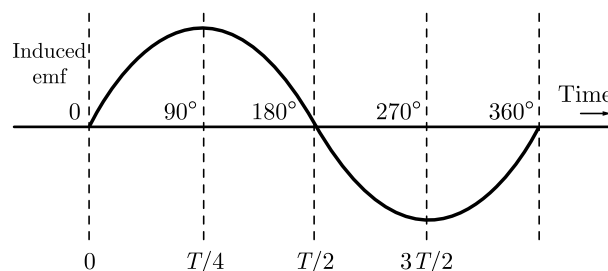
Induced emf in coil is given by,

$$e = NBA\omega \sin \omega t$$

$$e = e_0 \sin \omega t$$

where, $e_0 = NBA\omega = \text{peak value of induced emf.}$

The mechanical energy spend in rotating the coil in magnetic field appears in the form of electrical energy.



74. Derive expression for self inductance of a long air-cored solenoid of length l , cross-sectional area A and having number of turns N .

Ans :

SQP 2007

Self-Inductance of a long air-cored solenoid : Consider a long air solenoid having n number of turns per unit

length. If current in solenoid is I , then magnetic field within the solenoid,

$$B = \mu_0 n I \quad \dots(1)$$

where, $\mu_0 = 4\pi \times 10^{-7}$ henry/metre is the permeability of free space.

If A is cross-sectional area of solenoid, then effective flux linked with solenoid of length ' l '; $\phi = NBA$ where $N = nl$ is the number of turns in length ' l ' of solenoid.

Hence, $\phi = (nl BA)$

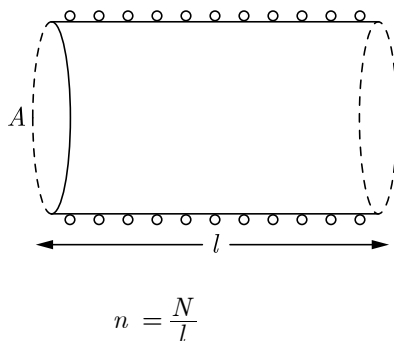
Substituting the value of B from (1)

$$\phi = nl(\mu_0 nl) A = \mu_0 n^2 A l \quad \dots(2)$$

Hence, self-inductance of air solenoid

$$L = \frac{\phi}{I} = \mu_0 n^2 A l \quad \dots(3)$$

If N is total number of turns in length l , then



Hence, self inductance

$$\begin{aligned} L &= \mu_0 \left(\frac{N}{l} \right)^2 A l \\ &= \frac{\mu_0 N^2 A}{l} \quad \dots(4) \end{aligned}$$

Remark- If solenoid contains a core of ferromagnetic substance of relative permeability μ_r , then self-inductance,

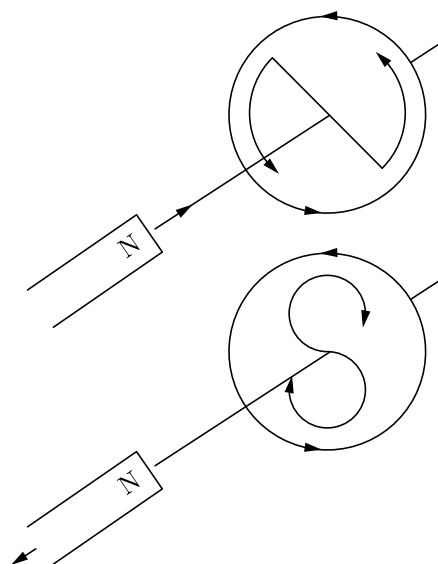
$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$

75. (a) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce an induced current which opposes the change of magnetic flux that produces it.
- (b) The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of
- Magnetic flux versus the current
 - Induced emf versus $\frac{dI}{dt}$
 - Magnetic potential energy stored versus the current.

Ans :

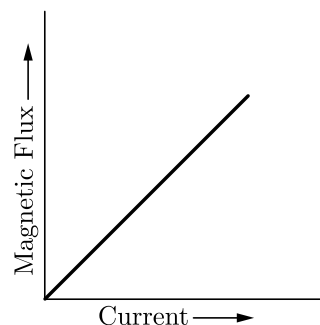
Comp 2013

- (a) When the North pole of a bar magnet moves towards the closed coil, the magnetic flux through the coil increases. This produces an induced emf which produces (or tend to produce if the coil is open) an induced current in the anti-clockwise sense. The anti-clockwise sense corresponds to the generation of North pole which opposes the motion of the approaching N pole of the magnet. The face of the coil, facing the approaching magnet, then has the same polarity as that of the approaching pole of the magnet. The induced current, therefore, is seen to oppose the change of magnetic flux that produces it.

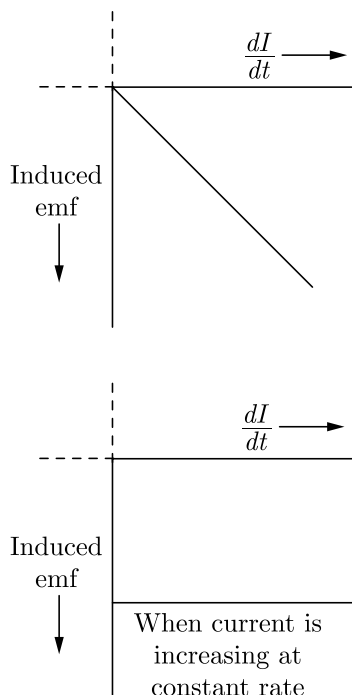


When a North pole of a magnet is moved away from the coil, the current (I) flows in the clockwise sense which corresponds in the generation of South pole. The induced South pole opposes the motion of the receding North pole.

- (b) (i) Magnetic flux versus the current

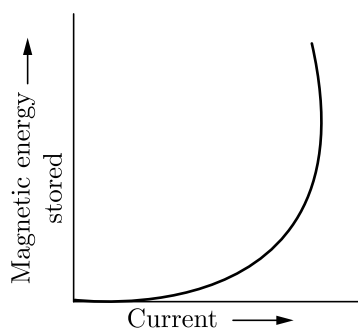


- (ii) Induced emf versus $\frac{dI}{dt}$



$\frac{dI}{dt}$ is positive and e is negative and constant.

(iii) Magnetic energy stored



$$U = \frac{1}{2} LI^2 = U \propto I^2$$

76. A rectangular coil of N turns, area A is held in a uniform magnetic field B . If the coil is rotated at a steady angular speed ω , deduce an expression for the induced emf in the coil at any instant of time.

or

A rectangular coil of area A , having number of turns N is rotated at f revolutions per second in a uniform magnetic field B , the field being perpendicular to the coil. Prove that the maximum emf induced in the coil is $2\pi fNBA$.

Ans :

SQP 2011, OD 2009

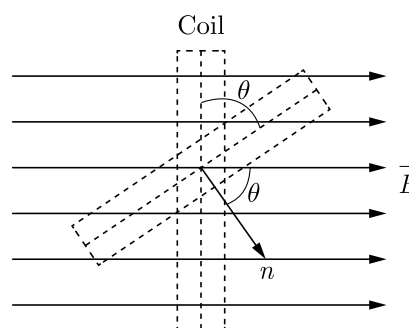
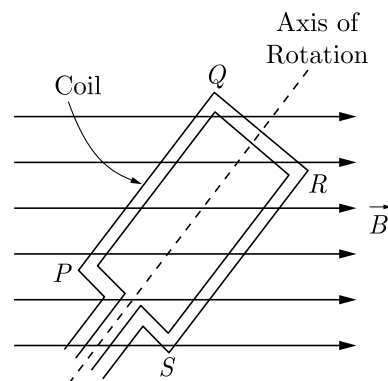
Induced emf by changing relative orientation of the coil and the magnetic field : Theory of AC generator.

Consider a coil $PQRS$ free to rotate in a uniform

magnetic field $\vec{B}$. The axis of rotation of the coil is perpendicular to the field $\vec{B}$. The flux through the coil, when its normal makes an angle θ with the field, is given by,

$$\phi = BA \cos \theta$$

where A is the face area of the coil.



Rotating Coil in a Magnetic Field

If the coil rotates with an angular velocity ω and turns through an angle θ in time t , then

$$\theta = \omega t$$

Hence,

$$\phi = BA \cos \omega t$$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by,

$$\begin{aligned} e &= -\frac{d\phi}{dt} \\ &= -\frac{d}{dt}(BA \cos \omega t) \\ &= BA\omega \sin \omega t \end{aligned}$$

If the coil has N turns, then the total induced emf will be,

$$e = NBA\omega \sin \omega t$$

Thus the induced emf varies sinusoidally with time t . The value of induced emf is maximum when $\sin \omega t = 1$ or $\omega t = 90^\circ$. Denoting this maximum value by e_0 .

$$V = 200 \text{ Volt}$$

$$L = ?$$

Hence, Induced emf,
$$e = -L \frac{\Delta I}{\Delta t}$$

$$200 = -L \left(\frac{-2}{10 \times 10^{-3}} \right)$$

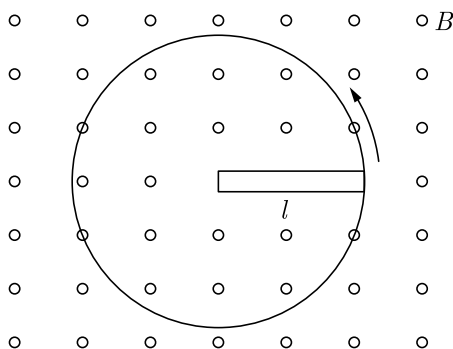
$$200 = L \times 2 \times 10^2$$

$$L = 1 \text{ H}$$

81. A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angle to the horizontal component of the Earth's magnetic field, $0.3 \times 10^{-4} \text{ Wb} \cdot \text{m}^{-2}$. Find the instantaneous value of the emf induced in the rod.

Ans :

Comp 2020



Given,
$$B = 0.3 \times 10^{-4} \text{ Wb} \cdot \text{m}^{-2}$$

$$v = 5 \text{ m/s}$$

$$\theta = 90^\circ$$

$$l = 10 \text{ m}$$

$$e = Bvl \sin \theta$$

$$= 0.3 \times 10^{-4} \times 10 \times 5 \times \sin 90^\circ$$

$$= 15 \times 10^{-4} \text{ V}$$

Hence,
$$e = 1.5 \times 10^{-3} \text{ V} = 1.5 \text{ mV}$$

82. A wheel with 8 metallic spokes each 20 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the plane is 0.4 G and the angle of dip is 60° . Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased?

Ans :

Delhi 2021

If a rod of length l rotates with angular speed ω in uniform magnetic field B

$$e = \frac{1}{2} B l^2 \omega$$

In case of Earth's magnetic field $B_H = |B_e| \cos \theta$

and $B_v = |B_e| \sin \theta$

$$e = \frac{1}{2} |B_e| \cos \theta l^2 \omega$$

$$= \frac{1}{2} \times 0.4 \times 10^{-10} \times \cos 60^\circ \times (0.5)^2 \times 2\pi v$$

$$= \frac{1}{2} \times 0.4 \times 10^{-4} \times \frac{1}{2} \times (0.5)^2 \times 2\pi \times \left(\frac{120}{60} \right)$$

$$= 10^{-5} \times 0.25 \times 2 \times 3.14 \times 2$$

$$= 3.14 \times 10^{-5} \text{ Volt.}$$

Induced emf is independent of the number of spokes i.e., it will remain same.

83. There are two coils A and B separated by some distance. If a current of 2 A flows through A , a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B , What is the flux through A ?

Ans :

Delhi 2020, OD 2012

Applying the mutual inductance of coil A with respect to coil B .

$$M_{21} = \frac{N_2 \phi_2}{I_1}$$

Therefore, we have

$$\text{Mutual inductance} = \frac{10^{-2}}{2} = 5 \text{ mH}$$

Again applying this formula for other case,

$$N_1 \phi_1 = M_{12} I_2$$

$$= 5 \text{ mH} \times 1 \text{ A} = 5 \text{ mWb}$$

84. A toroidal solenoid with air core has an average radius of 15 cm, area of cross-section 12 cm^2 and has 1200 turns. Calculate the self-inductance of the toroid. Assume the field to be uniform across the cross-section of the toroid.

Ans :

Comp 2017

Given,
$$r = 15 \text{ cm} = 0.15 \text{ m}$$

$$A = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

and
$$N = 1200$$

As we know that,

$$\text{Self inductance, } L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 A}{2\pi r}$$

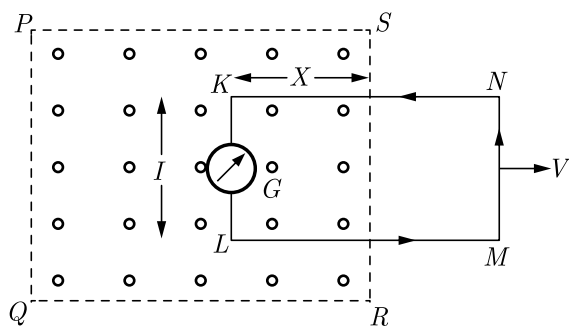
$$= \frac{4\pi \times 10^{-7} \times (1200)^2 \times (12 \times 10^{-4})}{2\pi \times 0.15}$$

$$= 2.3 \times 10^{-3} \text{ H}$$

85. The current flowing in the two coils of self-inductance $L_1 = 16 \text{ mH}$ and $L_2 = 12 \text{ mH}$ are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of

has free charges and Lorentz force acting on them causes a current to be set up in the conductor. In the calculations above, $e = Blv$ is the expression for motional emf.

Induced emf by Changing the Area A : Suppose a uniform magnetic field B perpendicular to the plane of the paper and directed outwards (shown by dots in fig.) is confined in region $PQRS$. Let a rectangular loop of wire $KLMN$ be held partially in the magnetic field in the plane of the paper. Let $KL = l$. Suppose x is the portion of the loop in the field at any instant t . When the loop is moved with a velocity in the plane of the paper, area of the loop associated with the magnetic field changes. An induced emf is set up in the wire. This is detected by deflection in the galvanometer G connected in the loop.



To calculate the emf induced, suppose in a small time Δt , the loop is moved out of magnetic field through a small distance Δx .

Hence, decrease in area of the loop $= -l\Delta x$
decrease in magnetic flux linked with the loop

$$d\phi = Bl \Delta x$$

As induced emf, $e = d\phi/dt$

$$e = + \frac{Bl \Delta x}{\Delta t} Blv$$

i.e., $e = Blv$

- (b) **Energy Consideration :** If R is resistance of the rectangular loop, which does not change with the motion of the loop, then current I in the loop is

$$I = \frac{e}{R} = \frac{Blv}{R}$$

The magnitude of force on arm KL is

$$\begin{aligned} F &= BIl \\ &= B \left(\frac{Blv}{R} \right) l = \frac{B^2 l^2 v}{R} \end{aligned}$$

Power required to push the loop

$$\begin{aligned} P &= Fv \\ &= \frac{B^2 l^2 v \times v}{R} = \frac{B^2 l^2 v^2}{R} \end{aligned}$$

This is the energy spent/sec., mechanically in moving the loop, which is dissipated as Joule's heat loss/sec.

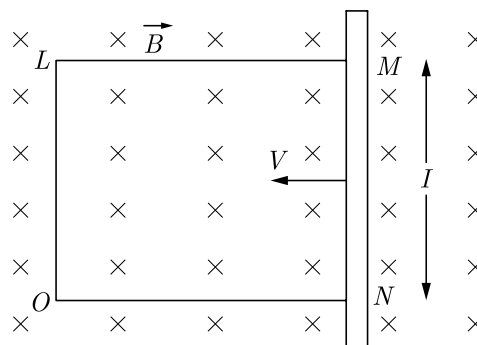
$$\begin{aligned} P &= I^2 R \\ &= \left(\frac{Blv}{R} \right)^2 R = \frac{B^2 l^2 v^2}{R} \end{aligned}$$

Thus, mechanical energy needed to move the arm KL is converted into electrical energy (induced e.m.f.) and then to thermal energy.

- (c) The induced emf is produced because component H will be interchanged and higher potential will be at East side of wire.

NUMERICAL QUESTIONS

79. A rectangular conductor $LMNO$ is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20 cm is moved towards left with a velocity of 10 ms^{-1} , calculate the emf induced in the arm. Given the resistance of the arm to be 5Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm.



Ans :

OD 2021, SQP 2004

Induced arm in a moving rod in a magnetic field is given by

$$\varepsilon = -Blv$$

Since the rod is moving to the left so

$$\varepsilon = +Blv = 0.5 \times 0.2 \times 10 = 1 \text{ V}$$

Current in the rod $I = \frac{\varepsilon}{R} = \frac{1}{5} = 0.2 \text{ A}$

80. Current in a circuit falls steadily from 2.0 A to 0.0 A in 10 ms. If an average emf of 200 V is induced, then calculate the self-inductance of the circuit.

Ans :

OD 2015

Given,

$$\Delta I = -2 \text{ A}$$

$$\Delta t = 10 \times 10^{-3} \text{ s}$$

We have, $e_0 = NBA\omega = 2\pi fNBA$

Hence, $e = e_0 \sin \omega t = e_0 \sin 2\pi ft$

where, f = frequency of rotation of the coil

7. Explain how does the emf vary when the coil turns through an angle of 2π ? What is the instantaneous value of induced emf when the plane of the coil makes an angle of 60° with the magnetic lines.

Ans :

Variation of induced emf in a rotating coil.

Figure shows how the induced emf e between the two terminals of the coil varies with time.

We consider the following special cases :

1. When $\omega t = 0^\circ$, the plane of the coil is perpendicular to $\vec{B}$,

$$\sin \omega t = \sin 0^\circ = 0, \text{ so that } e = 0$$

2. When $\omega t = \frac{\pi}{2}$, the plane of the coil is parallel to field $\vec{B}$,

$$\sin \omega t = \sin \frac{\pi}{2} = 1, \text{ so that } e = e_0$$

3. When $\omega t = \pi$, the plane of the coil is again perpendicular to B ,

$$\sin \omega t = \sin \pi = 0, \text{ so that } e = 0$$

4. When $\omega t = \frac{3\pi}{2}$, the plane of the coil is again parallel to $\vec{B}$,

$$\sin \omega t = \sin \frac{3\pi}{2} = -1, \text{ so that } e = -e_0$$

5. When $\omega t = 2\pi$, the plane of the coil again becomes perpendicular to $\vec{B}$ after completing one rotation,

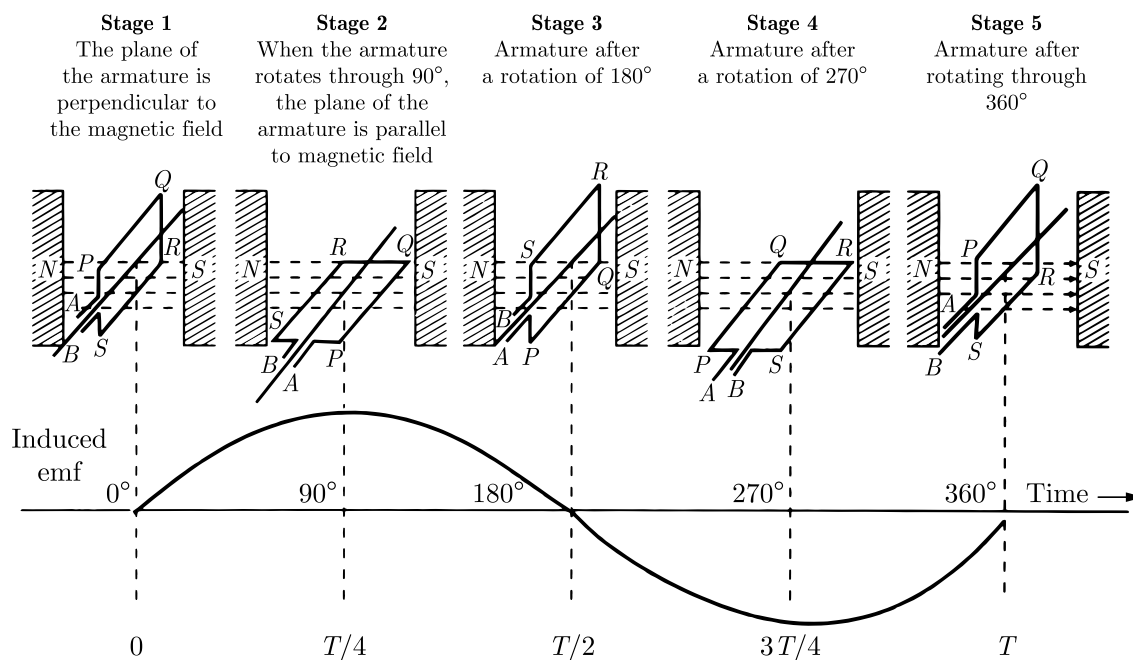
$$\sin \omega t = \sin 2\pi = 0, \text{ so that } e = 0$$

As the coil continues to rotate in the same sense, the same cycle of changes repeats again and again. As shown in Figure (b), the graph between emf e and time t is a sine curve. Such an emf is called sinusoidal or alternating emf. Both the magnitude and direction of this emf change regularly with time.

When the plane of the coil makes an angle of 60° with the field $\vec{B}$, the normal to the plane of the coil makes an angle of 30° with the field B .

Then instantaneous value of induced emf is,

$$e = e_0 \sin 30^\circ = \frac{e_0}{2}$$



78. (a) Starting from the expression for the Lorentz magnetic force acting on the free charge carriers of a conductor moving in a perpendicular magnetic field, obtain the expression for the motional emf induced.
- (b) deduce the expressions for the power delivered by the source and the power dissipated as Joule heat.
- (c) A straight wire extending from East to West falls with a speed v at right angles to the horizontal

component of the Earth's magnetic field. Which end of the wire would be at the higher electrical potential and why?

Ans :

Foreign 2012, Comp 2003

- (a) When the magnetic field is uniform and time independent, and the conductor is moving in such a field, then the emf, induced in the conductor is also called motional emf. In fact, the conductor

- Induced voltages
- The current and
- The energies stored in the coil at a given instant.

Ans :

Delhi 2018

Given, $L_1 = 16 \text{ mH}$

$$L_2 = 12 \text{ mH}$$

- Induced voltage is given by,

$$v = L \frac{dI}{dt}$$

$$v \propto L$$

$$\frac{v_1}{v_2} = \frac{L_1}{L_2} \quad \left(\text{as } \frac{dI}{dt} \text{ is same} \right)$$

$$\frac{v_1}{v_2} = \frac{16}{12} = \frac{4}{3} \quad \dots(1)$$

- The current is given by,

$$\text{Power, } P = Iv$$

$$\text{Now, } \frac{I_1}{I_2} = \frac{v_2}{v_1} \quad (\text{as } P \text{ is the same})$$

$$\frac{I_1}{I_2} = \frac{3}{4} \quad \dots(2)$$

- Energy stored is given by, coil 1 and coil 2.

$$E_1 = \frac{1}{2} L_1 I_1^2 \quad \dots(3)$$

$$E_2 = \frac{1}{2} L_2 I_2^2 \quad \dots(4)$$

On dividing Eqs, (3) and (4), we get

$$\begin{aligned} \frac{E_1}{E_2} &= \frac{L_1 I_1^2}{L_2 I_2^2} \\ &= \frac{16}{12} \times \frac{9}{16} \quad [\text{From Eq. (2)}] \end{aligned}$$

$$\frac{E_1}{E_2} = \frac{3}{4}$$

- 86.** In a closed circuit of resistance 10Ω , the linked flux varies with time according to relation $\phi = 6t^2 - 5t + 1$. At $t = 0.25$ second, What is the current (in Ampere) flowing through the circuit?

Ans :

OD 2016

$$\text{Given, } \phi = 6t^2 - 5t + 1$$

$$\begin{aligned} \text{Now } e &= \frac{-d\phi}{dt} \\ &= \frac{-d(6t^2 - 5t + 1)}{dt} \\ &= -12t + 5 \end{aligned}$$

$$\begin{aligned} \text{Here, } t &= 0.25 \\ &= -12 \times 0.25 + 5 \\ &= -3 + 5 = 2 \text{ Volt} \end{aligned}$$

$$\text{Now, Current, } I = \frac{e}{R}$$

$$\begin{aligned} \text{Here, } R &= 10 \Omega \\ &= \frac{2}{10} = 0.2 \text{ A} \end{aligned}$$

- 87.** A circular ring of area of 0.5 m^2 is situated in a non-uniform magnetic field of 4 T changing in 2 s and making an angle of 60° with the magnetic field. What is the magnitude of the induced E.M.F.?

Ans :

Comp 2019, OD 2015

$$\text{Area of ring, } A = 0.5 \text{ m}^2$$

$$\text{Magnetic field, } B = 4 \text{ T}$$

and angle which the ring makes with the magnetic field,

$$\theta = 60^\circ$$

Change in magnetic flux,

$$\begin{aligned} d\phi &= BA \cos \theta \\ &= 4 \times 0.5 \times \cos 60^\circ \\ &= 4 \times 0.5 \times 0.5 = 1 \text{ Wb} \end{aligned}$$

Therefore induced E.M.F. in the ring,

$$E = \frac{d\phi}{dt} = \frac{1}{2} = 0.5 \text{ Volt}$$

- 88.** A rectangular coil of 300 turns has an area of 25 cm^2 . The coil rotates with a speed of 50 c.p.s. in a uniform magnetic field of $4 \times 10^{-2} \text{ T}$, about an axis perpendicular to the field. What is the value of peak induced E.M.F. in it?

Ans :

Delhi 2019

Given,

$$\text{Number. of turns in coil, } N = 300$$

$$\begin{aligned} \text{Area of coil, } A &= 25 \times 10 = 250 \text{ cm}^2 \\ &= 250 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\text{Angular frequency of coil, } f = 50 \text{ c.p.s.}$$

and angle between axis of coil and magnetic field,

$$\theta = 90^\circ$$

Angular speed of coil,

$$\begin{aligned} \omega &= 2\pi f = 2\pi \times 50 \\ &= 100\pi \text{ rad-s}^{-1} \end{aligned}$$

Therefore value of peak induced E.M.F. in the coil,

$$\begin{aligned} E &= NAB\omega \sin \theta \\ &= 300 \times (250 \times 10^{-4}) \times (4 \times 10^{-2}) \\ &\quad \times 100\pi \times \sin 90^\circ \\ &= 30\pi \times 1 = 30\pi \text{ Volt} \end{aligned}$$

89. The magnetic flux linked with a coil is given by the equation: $\phi = 3t^2 + 4t + 9$ Wb. What is the magnitude of E.M.F. induced at $t = 2$ s?

Ans :

SQP 2010

Equation of magnetic flux,

$$\phi = 3t^2 + 4t + 9 \text{ Wb}$$

Time, $t = 2$ s

E.M.F. induced in the coil,

$$\begin{aligned} E &= \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 4t + 9) \\ &= 6t + 4 \end{aligned}$$

Therefore, E.M.F. induced in the coil at $t = 2$ s.

$$E_2 = (6 \times 2) + 4 = 16 \text{ Volt}$$

90. A train is moving in N-S direction with a velocity 108 km-h^{-1} . If vertical component of earth's magnetic field is $8 \times 10^{-5} \text{ T}$, What is the amount of E.M.F. generated between two wheels ? (Given : Length of axle = 2 m)

Ans :

Foreign 2011

Velocity of train, $v = 108 \text{ km-h}^{-1}$

$$= 30 \text{ m-s}^{-1}$$

Vertical component of earth's magnetic field,

$$B_V = 8 \times 10^{-5} \text{ T and}$$

Length of axle, $l = 2 \text{ m}$

Amount of E.M.F. generated between two wheels of train.

$$\begin{aligned} E &= B_V vl \\ &= (8 \times 10^{-5}) \times (30) \times 2 \\ &= 4.8 \times 10^{-3} \text{ V} = 4.8 \text{ mV} \end{aligned}$$

91. A helicopter rises vertically with a velocity of 10 m-s^{-1} . If helicopter has length of 10 m and horizontal component of earth's magnetic field is $2 \times 10^{-3} \text{ T}$. What is the E.M.F. induced between tip of nose and tail of the helicopter?

Ans :

OD 2014

Velocity of helicopter, $v = 10 \text{ m-s}^{-1}$

Length of helicopter, $l = 10 \text{ m}$

and horizontal component of earth's magnetic field,

$$B_H = 2 \times 10^{-3} \text{ T}$$

E.M.F. induced between tip of nose and tail of helicopter.

$$\begin{aligned} E &= B_H vl \\ &= (2 \times 10^{-3}) \times 10 \times 10 \\ &= 0.2 \text{ V} \end{aligned}$$

92. If 10 A current changes in one second through a coil and the E.M.F. induced is 10 V. What is self-inductance of the coil?

Ans :

Delhi 2017

Change in current, $dI = 10 \text{ A}$

Time-taken, $dt = 1 \text{ s}$

E.M.F. induced in coil, $E = 10 \text{ Volt}$

Rate of change of current,

$$\frac{dI}{dt} = \frac{10}{1} = 10 \text{ A-s}^{-1}$$

Therefore, self-inductance of the coil,

$$L = \frac{E}{\left(\frac{dI}{dt}\right)} = \frac{10}{10} = 1 \text{ H}$$

93. The average E.M.F. induced in a coil when current changes from 0 to 3A in 0.5 s, is 15 V. What is the self-inductance of the coil?

Ans :

Foreign 2008, Delhi 2005

Initial current, $I_1 = 0 \text{ A}$

Final current, $I_2 = 3 \text{ A}$

Time-taken, $dt = 0.5 \text{ s}$

Induced E.M.F., $E = 15 \text{ Volt}$

Rate of change of current,

$$\begin{aligned} \frac{dI}{dt} &= \frac{I_2 - I_1}{dt} \\ &= \frac{3 - 0}{0.5} = 6 \text{ A-s}^{-1} \end{aligned}$$

Therefore, self-inductance of the coil,

$$L = \frac{E}{\left(\frac{dI}{dt}\right)} = \frac{15}{6} = 2.5 \text{ H}$$

94. A horizontal electric pole wire 200 m long is lying along the earth's magnetic field at a height of 10 m. If horizontal component of the earth's magnetic field is $0.32 \times 10^{-4} \text{ T}$, what is E.M.F. induced in the wire, on striking the ground?

Ans :

OD 2018

Length of wire, $l = 200 \text{ m}$

Height of wire from the ground,

$$h = 10 \text{ m}$$

Initial speed of wire,

$$u = 0 \quad (\text{because is at rest})$$

and horizontal component of earth's magnetic field,

$$B_H = 0.32 \times 10^{-4} \text{ T}$$

Relation for the final speed of wire on striking the ground v is,

$$v^2 = u^2 + 2gh$$

$$= (0)^2 + 2 \times 9.8 \times 10 = 196$$

$$v = \sqrt{196} = 14 \text{ m-s}^{-1}$$

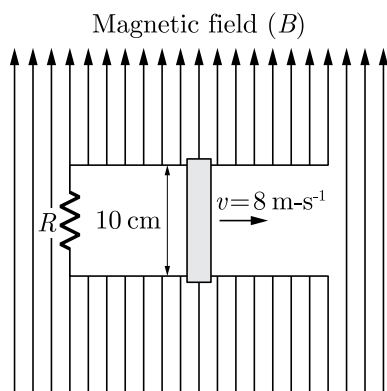
Since, horizontal component of the earth's magnetic field is normal to wire and direction of motion of wire, therefore E.M.F. induced in the wire on striking the ground.

$$E = B_H vl$$

$$= (0.32 \times 10^{-4}) \times 14 \times 200$$

$$= 0.09 \text{ Volt}$$

95. A metal rod makes contact and completes the circuit as shown in the figure. The direction of motion of rod is perpendicular to the magnetic field of 0.5 T. If the resistance R is 2Ω , what is the force needed to move the rod as indicated with a constant velocity of 8 m-s^{-1} ?



Ans :

SQP 2013

Magnetic field, $B = 0.5 \text{ T}$

Resistance, $R = 2\Omega$

Velocity of rod, $v = 8 \text{ m-s}^{-1}$

Length of rod, $l = 10 = 0.1 \text{ m}$

E.M.F. induced in the rod,

$$E = Bvl$$

$$= 0.5 \times 8 \times 0.1$$

$$= 0.4 \text{ Volt}$$

Therefore, induced current in the rod,

$$I = \frac{E}{R} = \frac{0.4}{2} = 0.2 \text{ A}$$

Force acting on the rod in magnetic field,

$$F = BIl$$

$$= 0.5 \times 0.2 \times 0.1$$

$$= 0.01 \text{ N}$$

96. An E.M.F. of 20 mV is induced in a coil, when the

rate of change of current 4 A-s^{-1} . What is the self-inductance of the solenoid?

Ans :

Comp 2010

Given,

Induced E.M.F., $E = 20 \text{ mV}$

$$= 20 \times 10^{-3} \text{ Volt}$$

and rate of change of current,

$$\frac{dI}{dt} = 4 \text{ A-s}^{-1}$$

Self-inductance of the coil,

$$L = \frac{E}{\left(\frac{dI}{dt}\right)}$$

$$= \frac{20 \times 10^{-3}}{4}$$

$$= 5 \times 10^{-3} \text{ H} = 5 \text{ mH}$$

97. When the current in a coil is changed from 10 A in one direction to 10 A in opposite direction in 0.5 s, an E.M.F. of 1 V is induced in it. What is the self-inductance of the coil?

Ans :

OD 2018, Delhi 2009

Initial current, $I_1 = 10 \text{ A}$

Final current, $I_2 = -10 \text{ A}$

(Negative sign due to opposite direction);

Time-taken, $dt = 0.5 \text{ s}$

Induced E.M.F. in coil, $E = 1 \text{ Volt}$

Rate of change of current,

$$\frac{dI}{dt} = \frac{I_1 - I_2}{dt}$$

$$= \frac{10 - (-10)}{0.5} = 40 \text{ A-s}^{-1}$$

Therefore, self-inductance of the coil,

$$L = \frac{E}{\left(\frac{dI}{dt}\right)}$$

$$= \frac{1}{40}$$

$$= 0.025 \text{ H} = 25 \text{ mH}$$

98. A 100 mH coil carries a current of 10 A. What is the magnetic energy stored in the coil?

Ans :

SQP 2006

Inductance of coil, $L = 100 \text{ mH} = 0.1 \text{ H}$

Current, $I = 10 \text{ A}$

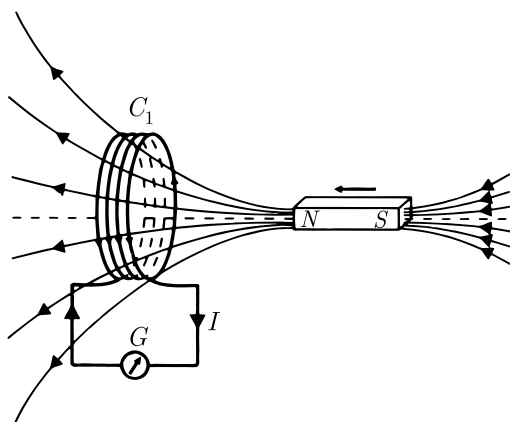
Magnetic energy stored in the coil,

$$u = \frac{1}{2} LI^2$$

$$= \frac{1}{2} \times 0.1 \times (10)^2 = 5 \text{ J}$$

CASE BASED QUESTIONS

99. Faraday's second law of EMI says that magnitude of emf induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit, i.e., $\varepsilon \propto \frac{d\phi}{dt}$. It means if the magnetic flux linked with a circuit changes at a faster rate, the induced emf is larger and vice-versa.

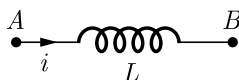


Read the above passage and answer the following question.

- (i) Coil A is held in a very strong magnetic field and coil B is held in a weak magnetic field. In which cell do you expect larger induced emf?
- (ii) A coil is moved in a uniform magnetic field. When do you expect induced emf in the coil?

Ans :

- (i) Induced emf in both the coils is zero. This is because magnetic flux linked with A or B larger and magnetic flux linked with B is smaller, but amount of magnetic flux linked with the coil does not induce emf in the coil. The magnetic flux must change to induce emf.
 - (ii) Induced emf would develop in the coil, when it is entering the magnetic field or leaving the magnetic field, i.e., when the coil is partially in magnetic field and partially out of magnetic field. This is because magnetic flux linked with the coil would change only in these situations. When the entire coil is moving in the magnetic field, magnetic flux linked with the coil is consistent, but it is not changing. Therefore, there is no induced emf in the coil.
100. An inductor is simply a coil or a solenoid that has a fixed inductance. It is referred to as a choke. The usual circuit rotation for an inductor is as shown.



Let a current i flows through the inductor from A to B. Whenever, electric current changes through it, a back emf is generated. If the resistance of inductor is assumed to be zero (ideal inductor) then induced emf in it is given by

$$e = V_B - V_A = -L \frac{di}{dt}$$

Thus, potential drops across an inductor as we moves the direction of current. But potential also drops across a pure resistor when we move in the direction of the current. The main difference between a resistor and an inductor is that while a resistor opposes the current through it, an inductor opposes the change in current through it.

Now answer the following questions.

- (i) How does inductor behave when a steady current flow through it?
- (ii) How does inductor behave when a steadily increasing current flows through it?
- (iii) How does inductor behave when a steadily decreasing current flows through it?
- (iv) Name the phenomenon in which change in current in a coil induces EMF in a coil itself?

Ans :

- (i) As electric current is steady therefore

$$\frac{di}{dt} = 0;$$

induced emf $= e = 0$ and the inductor behaves as short circuit.

- (ii) In the expression

$$e = -L \frac{di}{dt}$$

as $\frac{di}{dt}$ is positive EMF is negative, that is $V_B < V_A$ That is back EMF is generated that opposes the increase in current.

- (iii) $\frac{di}{dt}$ is negative, therefore EMF is positive, that is $V_B > V_A$ Forward EMF is generated that opposes fall in current.
- (iv) Self induction.

101. A small town with a demand of 800 KW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.



- (i) Estimate the line power loss in the form of heat.
- (ii) How much power must the plant supply, assuming there is negligible power loss due to leakage?
- (iii) Characterise the step up transformer at the plant.

Ans :

$$\text{Line resistance} = 30 \times 0.5 = 15 \Omega$$

rms current in the line

$$800 \times 1000 \text{ W} / 4000 \text{ V} = 200 \text{ A}$$

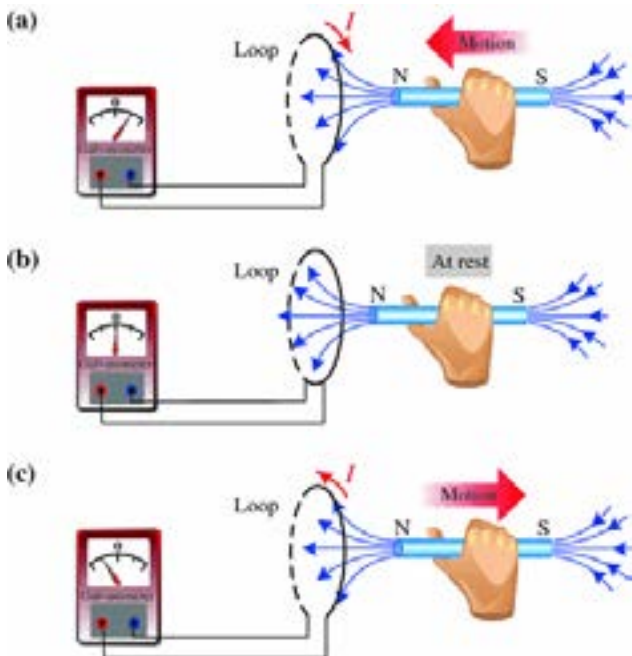
- (i) Line power loss $= (200 \text{ A})^2 \times 15 \Omega = 600 \text{ KW}$.
- (ii) Power supply by the plant

$$800 \text{ KW} + 600 \text{ KW} = 1400 \text{ KW}$$

- (iii) Voltage drop on the line $= 200 \text{ A } 15 \Omega$
 $= 3000 \text{ Volt}$

The step-up transformer at the plant is 440 Volt-7000 Volt.

- 102.** Faraday established that an emf can be induced in a coil by changing the amount of magnetic flux (ϕ) linked with the coil. As $\phi = BA \cos \theta$, therefore three methods of inducing emf are by changing magnetic field E or by changing area A or by changing relative orientation θ of the coil w.r.t. the magnetic field.



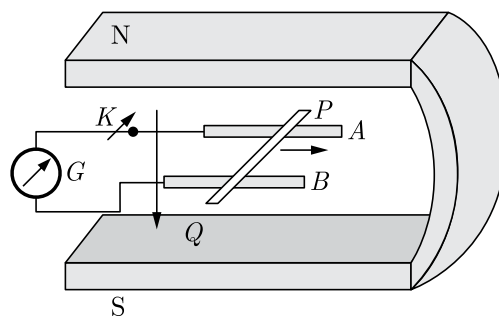
- (i) Have you heard of a battery-less flashlight? How does it work?
- (ii) Can you operate a typical cell phone from the hip movements of the person?
- (iii) What do you know about gas-electric hybrid autos?

Ans :

- (i) Yes, a flashlight without a battery can give us light for a few minutes. It is based on EMI. On shaking the flashlight, its permanent internal magnet passes through a coil, inducing some current. Thus, current is converted into AC and is used for charging a capacitor. The charged capacitor then send current through Light Emitting Diode (LED), which gives us light.
- (ii) Yes, a backpack hybrids is used to convert the hip movements of the person into electricity and produce 7 W of electric power to operate a typical cell phone.
- (iii) Gas electric hybrid autos are the latest innovations in the auto industry. The petrol engine is made smaller than the conventional one through the use of EMI some energy is supplied to batteries and electric motor. The purpose behind is to get more mileage for every litre of petrol consumed. This would make the auto cost effective.

- 103.** Given figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K . Length of the rod $= 15 \text{ cm}$, $B = 0.50 \text{ T}$, resistance of the closed loop containing the rod $= 9.0 \text{ m}\Omega$. Assume the field to be uniform.

- (i) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.



- (ii) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?
- (iii) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.
- (iv) What is the retarding force on the rod when K is closed?
- (v) How much power is required (by an external agent) to keep the rod moving at the same speed ($= 12 \text{ cm/sec}$) when K is closed? How much power is required when K is open?

- (vi) How much power is dissipated as heat in the closed circuit? What is the source of this power?
- (vii) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Ans :

- (i) $EMF = vBL = 0.12 = 0.50 \times 0.15 = 9.0 \text{ mV}$,
 P positive end and Q negative end.
- (ii) Yes. When K is closed, the excess charge is maintained by the continuous flow of current.
- (iii) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite signs at the ends of the rod.
- (iv) Retarding force $= IBL$
 $9 \text{ mV} / 9 \text{ m}\Omega \times 0.5 \text{ T} \times 0.15 \text{ m} = 75 \times 10^{-3} \text{ N}$
- (v) Power expended by an external agent against the above retarding force to keep the rod moving uniformly at 12 cm s^{-1}

$$= 75 \times 10^{-3} \times 12 \times 10^{-2}$$

$$= 9.0 \times 10^{-3} \text{ W}$$

When K is open, no power is expended.

- (vi) $I^2 R = 1 \times 1 \times 9 \times 10^{-3}$
 $= 9.0 \times 10^{-3} \text{ W}$

The source of this power is the power provided by the external agent as calculated above.

- (vii) Zero motion of the rod does not cut across the field lines

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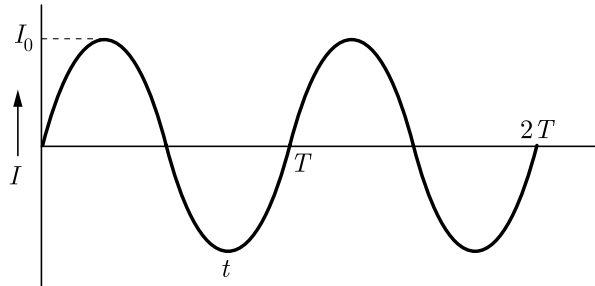
CHAPTER 7

Alternating Current

SUMMARY

1. ALTERNATING CURRENT (AC)

It is the current which varies in both magnitude as well as direction alternatively and periodically as shown in the figure.



$$I = I_0 \sin \omega t \text{ or } I = I_0 \cos \omega t$$

where, I_0 = peak value of AC

2. MEAN AND RMS VALUE OF ALTERNATING CURRENT

An electrical device reads root mean square value as,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

Then mean value of alternating current over complete cycle is zero.

$$(I_{\text{mean}})_{\text{full cycle}} = 0$$

While for cycle it is,

$$(I_{\text{mean}})_{\text{half cycle}} = \frac{2I_0}{\pi} = 0.636 I_0$$

3. ALTERNATING EMF OR VOLTAGE

It is the emf which varies in both magnitude as well as direction alternatively and periodically. The instantaneous alternating emf is given by

$$V = V_0 \sin \omega t$$

$$V = V_0 \cos \omega t$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707$$

$$V_{rms} = 7.07\% \text{ of } V_0$$

$$V_{av} = \frac{2V_0}{\pi} = 0.637$$

$$V_{av} = 6.37\% \text{ of } V_0$$

4. REACTANCE

The opposition offered by inductance and capacitance or both in AC circuit is called reactance. It is denoted by X_C or X_L . The opposition due to inductor alone is called the inductive reactance while that due to capacitance alone is called the capacitive reactance.

Inductive reactance,

$$X_L = \omega L = 2\pi fL \quad (\because \omega = 2\pi f)$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

5. POWER

In an AC circuit, both emf and current change continuously w.r.t. time, so in circuit we have to calculate average power in complete cycle ($O_{(\text{zero})} \rightarrow T$)

$$P = V_{rms} I_{rms} \cos \phi$$

where, $\cos \phi$ = power factor

5.1 Purely Resistive Circuit

If a circuit contains pure resistance, then difference $\phi = 0$ i.e., current and voltage are in the same phase impedance,

5.2 Purely Inductive Circuit

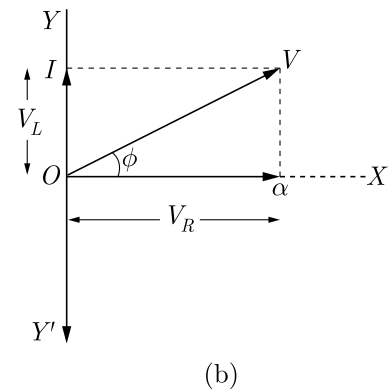
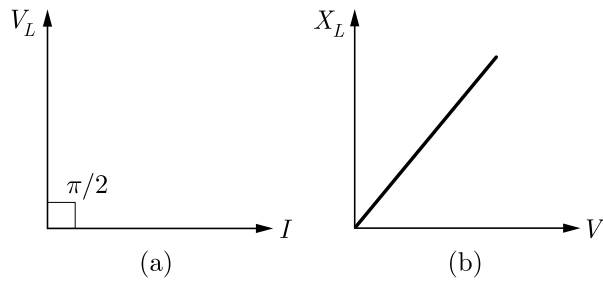
If a circuit contains pure inductance, then $\phi = \frac{\pi}{2}$, i.e., current lags behind the applied voltage by an angle $\frac{\pi}{2}$.

If,
$$V = V_0 \sin \omega t$$

Then,
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

In this case inductive reactance, $X_L = \omega L$.

The inductive reactance increases with the increase of frequency of AC linearly [Figure (b)].



5.3 Purely Capacitive Circuit

If circuit contains pure capacitance, then $\phi = \frac{\pi}{2}$, i.e., current leads the applied voltage by angle $\frac{\pi}{2}$.

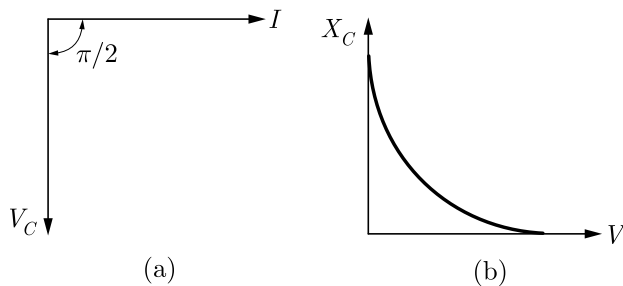
i.e., $V = V_0 \sin \omega t$

and $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

Capacitive reactance,

$$X_C = \frac{1}{\omega C}$$

Clearly, capacitive reactance (X_C) is inversely proportional to the frequency v [Figure (b)].



1. Impedance,

$$Z = \sqrt{R^2 + X_L^2} = \frac{V_{rms}}{I_{rms}}$$

2. For the phase angle,

$$\tan \phi = \frac{X_L}{R} = \frac{\omega L}{R}$$

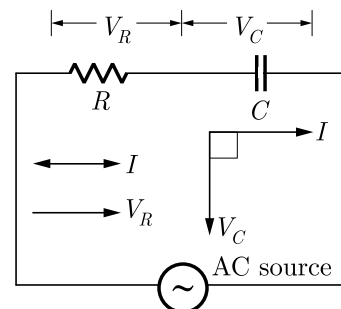
Voltage leads current by phase ϕ .

3. If $V = V_0 \sin \omega t$, then

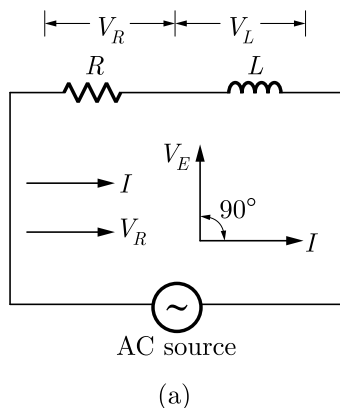
$$I = I_0 \sin(\omega t - \phi)$$

5.5 R-C Series AC Circuit

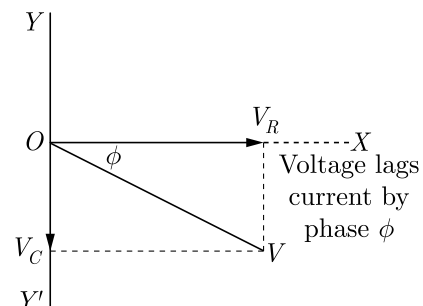
When the inductance C and resistance R are connected in series to an AC source of voltage, then the circuit is called an RC circuit. As they are connected in series, they both will have the same amount of current flowing through them.



(a) Circuit diagram



(a)



(b) Phasor diagram

Then,

1. Impedance,

$$Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2}$$

2. For the phase angle,

$$\tan \phi = \frac{X_C}{R} = \frac{1}{\omega CR}$$

3. If $V = V_0 \sin \omega t$, then

$$I = I_0 \sin(\omega t + \phi)$$

4. Power factor,

$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

5.6 L-C Series AC Circuit

In an LC circuit, a pure inductor L is connected in series to a pure capacitor C connected to an AC source.

1. Impedance,

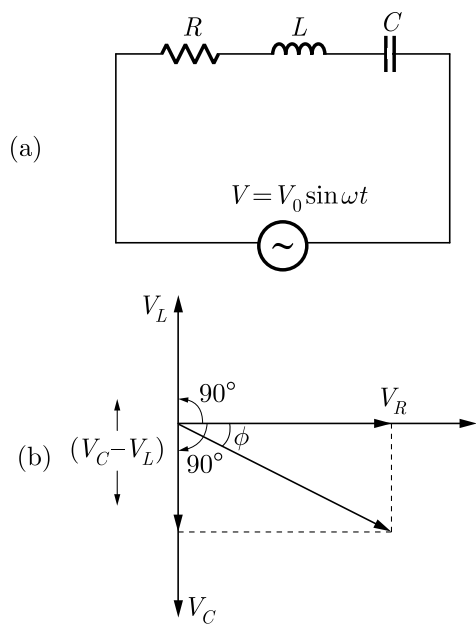
$$Z = \frac{V_{rms}}{I_{rms}} = X_L - X_C$$

2. Phase difference between voltage and current is $\frac{\pi}{2}$.

3. Power factor, $\cos \phi = 0$.

5.7 L-C-R Series Circuit

An LCR series circuit, also known as an RLC circuit, is an electrical circuit that consists of a resistor R , capacitor C , and inductor L connected in series.



If a circuit contains inductance L , capacitance C and resistance R , connected in series to an alternating voltage,

$$V = V_0 \sin \omega t$$

Then impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$

and phase, $\phi = \tan^{-1} \frac{X_C - X_L}{R}$

Net Voltage, $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

5.8 Resonant Circuits

In series LCR circuit, when phase ϕ between current and voltage is zero, the circuit is said to be resonant circuit.

In resonant circuit,

$$X_C = X_L$$

or $\frac{1}{\omega C} = \omega L$

$$\omega = \frac{1}{\sqrt{LC}}$$

Resonant angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Linear frequency, $f = \frac{\omega_r}{2\pi}$

$$= \frac{1}{2\pi\sqrt{LC}}$$

At resonant frequency, $\phi = 0, V = V_R$.

6. AC GENERATOR

It is a device used to convert mechanical energy into electrical energy and is based on the phenomenon of electromagnetic induction. If a coil of N turns, area A is rotated at frequency f in uniform magnetic field of induction B , then motional emf in coil (if initially it is perpendicular to field) is,

$$e = NBA \omega \sin \omega t$$

with $\omega = 2\pi f$

Peak emf, $e_0 = NBA\omega$

7. TRANSFORMER

A transformer is a device which converts low AC voltage into high AC voltage and vice-versa. It works on the principle of mutual induction. If N_p and N_s are the number of turns in primary and secondary coil, V_p and I_p are voltage and current in primary coil, then voltage V_s and current I_s in secondary coil will be,

$$V_s = \left(\frac{N_s}{N_p}\right) V_p$$

and

$$I_s = \left(\frac{N_p}{N_s} \right) I_p$$

Step up transformer increase the voltage while step down transfer decrease the voltage.

1. In step up transformer $V_s > V_p$, so $N_s > N_p$
2. In step down transformer $V_s < V_p$, so $N_s < N_p$
3. Efficiency of a transformer is given by,

$$\eta = \frac{V_s I_s}{V_p I_p}$$

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OBJECTIVE QUESTIONS

1. In AC circuit, the current and voltage are given by $i = 5 \cos \omega t$ and $V = 200 \sin \omega t$ respectively. Power loss in the circuit is

- (a) 20 W (b) 40 W
(c) 1000 W (d) Zero

Ans :

OD 2018

We have $i = 5 \cos \omega t = 5 \sin(\omega t + \frac{\pi}{2})$

and $V = 200 \sin \omega t$

Power loss in circuit,

$$P = (V_{\text{rms}} \cdot i_{\text{rms}})$$

where, θ is the phase difference between current and voltage.

$$\theta = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Hence,

$$P = (V_{\text{rms}} \cdot i_{\text{rms}})$$

$$P = 0 \quad (\text{Since, } \cos \frac{\pi}{2} = 0)$$

Thus (d) is correct option.

2. If the equation of an electric current is $I = 0.6 \sin 100\pi t$, the frequency of electric current is

- (a) 50π (b) 50
(c) 100π (d) 100

Ans :

SQP 2017

Electric current, $I = 0.6 \sin 100\pi t$... (1)

The standard equation of electric current is given by,

$$I = I_0 \sin \omega t \quad \dots (2)$$

After comparing equation (1) and (2), we get

$$\omega = 100\pi$$

Since, $\omega = 2\pi f$

$$\text{Now, } 2\pi f = 100\pi = \frac{100}{2} = 50 \text{ Hz}$$

Thus (b) is correct option.

3. The unit of reactance is
(a) Ohm (b) Farad
(c) Ampere (d) Mho

Ans :

OD 2017, 2016, 2013

The property due to which a capacitor or inductor opposes the flow of electric current is known as reactance. The unit of reactance is Ohm.

Thus (a) is correct option.

4. The relation between peak current, I_0 and root mean square current, I_{rms} is

- (a) $I_0 = \sqrt{2} I_{\text{rms}}$ (b) $I_0 = I_{\text{rms}}$

$$(c) I_0 = 2I_{\text{rms}} \quad (d) I_0 = \frac{I_{\text{rms}}}{\sqrt{2}}$$

Ans :

Delhi 2017

We have $I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I^2 dt} \quad \dots(1)$

Here, $I = I_0 \sin \omega t$

Now,
$$\begin{aligned} \int_0^T I^2 dt &= \int_0^T I_0^2 \sin^2 \omega t dt \\ &= I_0^2 \int_0^T \frac{1 - \cos 2\omega t}{2} dt \\ &= \frac{I_0^2}{2} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \\ &= \frac{I_0^2}{2} \left[(T - 0) - \frac{1}{2\omega} \left[\sin \frac{4\pi}{T} t \right]_0^T \right] \\ &= \frac{I_0^2}{2} \left[T - \frac{1}{2\pi} (\sin 4\pi - \sin 0) \right] \\ &= \frac{I_0^2 T}{2} \end{aligned}$$

From equation (1), we get

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{T} \times \frac{I_0^2 T}{2}} \\ I_{\text{rms}} &= \frac{I_0}{\sqrt{2}} \\ I_0 &= \sqrt{2} I_{\text{rms}} \end{aligned}$$

Thus (a) is correct option.

5. The power factor of L - R circuit is

$$\begin{aligned} (a) R + \omega L & \quad (b) \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \\ (c) R\sqrt{R^2 + \omega^2 L^2} & \quad (d) \omega L/R \end{aligned}$$

Ans :

OD 2016

Power factor of L - R circuit is given by,

$$\begin{aligned} \cos \phi &= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} \\ &= \frac{R}{\sqrt{R^2 + (\omega L)^2}} \quad (\text{Since, } X_L = \omega L) \\ &= \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \end{aligned}$$

Thus (b) is correct option.

6. If N_1 and N_2 are numbers of turns in primary and secondary coils of a step-up transformer. Then

$$\begin{aligned} (a) N_1 > N_2 & \quad (b) N_2 > N_1 \\ (c) N_1 = N_2 & \quad (d) N_1 = 0 \end{aligned}$$

Ans :

Foreign 2016

In Step up transformer, $N_2 > N_1$ i.e., the turns ratio is greater than 1 and therefore $e_2 > e_1$. The output voltage is greater than input voltage.

Here, N_1 = number of turns in primary coil

N_2 = number of turns in secondary coil

Thus (b) is correct option.

7. Impedance of L - R circuit is

$$\begin{aligned} (a) R^2 + \omega^2 L^2 & \quad (b) \sqrt{R^2 + \omega L} \\ (c) R + \omega L & \quad (d) \sqrt{R^2 + \omega^2 L^2} \end{aligned}$$

Ans :

SQP 2015

Impedance of L - R circuit is given by,

$$Z = \sqrt{R^2 + X_L^2}$$

Here, $X_L = \omega L$

Hence,
$$\begin{aligned} Z &= \sqrt{R^2 + (\omega L)^2} \\ &= \sqrt{R^2 + \omega^2 L^2} \end{aligned}$$

Thus (d) is correct option.

8. The ratio of peak value and *r.m.s* value of AC is

$$\begin{aligned} (a) 2 & \quad (b) \sqrt{2} \\ (c) \frac{1}{\sqrt{2}} & \quad (d) \frac{1}{2} \end{aligned}$$

Ans :

Delhi 2015, OD 2013

The rms value of current is $\frac{1}{\sqrt{2}}$ times of the peak value of current i.e.,

$$\begin{aligned} I_{\text{rms}} &= \frac{I_0}{\sqrt{2}} \\ \frac{I_0}{I_{\text{rms}}} &= \sqrt{2} \end{aligned}$$

where, I_0 = peak value of current

I_{rms} = rms value of current

Thus (b) is correct option.

9. The peak voltage of an AC is 440 volt. Its virtual voltage is

$$\begin{aligned} (a) 220 \text{ V} & \quad (b) 440 \text{ V} \\ (c) 220\sqrt{2} \text{ V} & \quad (d) 440\sqrt{2} \text{ V} \end{aligned}$$

Ans :

OD 2012

We have $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

Here, $V_0 = 440 \text{ Volt}$

Hence,
$$\begin{aligned} V_{\text{rms}} &= \frac{440}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= 220\sqrt{2} \text{ Volt} \end{aligned}$$

Thus (c) is correct option.

10. An AC source of angular frequency ω is connected in a series LCR circuit. The peak value of current will be maximum when

- (a) 15 V (b) 20 V
(c) 25 V (d) 50 V

Ans :

Foreign 2005

Potential difference across the inductance,

$$V_L = 15 \text{ Volt}$$

and potential difference across the resistance,

$$V_R = 20 \text{ Volt}$$

Total potential difference across LR-circuit,

$$\begin{aligned} V &= \sqrt{(V_L)^2 + (V_R)^2} \\ &= \sqrt{(15)^2 + (20)^2} \\ &= \sqrt{625} = 25 \text{ Volt} \end{aligned}$$

Thus (c) is correct option.

23. The impedance of a circuit, when a resistance R and an inductor of inductance L are connected in series in an AC circuit of frequency f is

- (a) $\sqrt{R + 2\pi^2 f^2 L^2}$ (b) $\sqrt{R + 4\pi^2 f^2 L^2}$
(c) $\sqrt{R^2 + 4\pi^2 f^2 L^2}$ (d) $\sqrt{R^2 + 2\pi^2 f^2 L^2}$

Ans :

SQP 2016

$$\text{Resistance} = R$$

$$\text{Inductance} = L$$

and Frequency of current $= f$

Angular frequency of AC source,

$$\omega = 2\pi f$$

Impedance of the circuit

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L)^2} \\ &= \sqrt{R^2 + (\omega L)^2} \\ &= \sqrt{R^2 + (2\pi f L)^2} \\ &= \sqrt{R^2 + 4\pi^2 f^2 L^2} \end{aligned}$$

Thus (c) is correct option.

24. A coil of resistance 300Ω and inductance 3 H is connected across an AC source of frequency $\frac{100}{2\pi} \text{ Hz}$. The phase difference between the voltage and current in the circuit is

- (a) 30° (b) 45°
(c) 60° (d) 90°

Ans :

SQP 2002, Delhi 2012

Resistance of coil, $R = 300 \Omega$ Inductance of coil, $L = 3 \text{ H}$ Frequency, $f = \frac{100}{2\pi} \text{ Hz}$

Angular frequency of AC source,

$$\begin{aligned} \omega &= 2\pi f = 2\pi \times \frac{100}{2\pi} \\ &= 100 \text{ rad-s}^{-1} \end{aligned}$$

Phase difference between the voltage and current in the LR-circuit,

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{\omega L}{R}\right) \\ &= \tan^{-1}\left(\frac{100 \times 3}{300}\right) \\ &= \tan^{-1}(1) = 45^\circ \end{aligned}$$

Thus (b) is correct option.

25. An alternating current of 1.5 mA and angular frequency 300 rad-s^{-1} flows through $10 \text{ K}\Omega$ resistor and $0.5 \mu\text{F}$ capacitor in series. The r.m.s voltage across the capacitor is

- (a) 5 V (b) 10 V
(c) 20 V (d) 40 V

Ans :

Foreign 2007

R.M.S. value of alternating current,

$$I_{\text{r.m.s.}} = 1.5 \text{ mA}$$

Angular frequency, $\omega = 300 \text{ rad-s}^{-1}$ Resistance of resistor, $R = 10 \text{ K}\Omega = 10 \times 10^3 \Omega$

and capacitance of capacitor,

Reactance of the capacitor,

$$\begin{aligned} X_C &= \frac{1}{\omega C} \\ &= \frac{1}{300 \times (0.5 \times 10^{-6})} \\ &= 6.67 \times 10^3 \Omega \end{aligned}$$

Therefore, r.m.s. voltage across capacitor,

$$\begin{aligned} V_C &= I_{\text{rms}} \times X_C \\ &= (1.5 \times 10^{-3})(6.67 \times 10^3) \\ &= 10 \text{ Volt} \end{aligned}$$

Thus (b) is correct option.

26. A circuit consists of a resistor, an inductor and a capacitor connected in series. If resistance of the resistor is 25Ω and the value of inductive reactance is equal to capacitive reactance, then net impedance of the circuit will be

- (a) 5Ω (b) 25Ω
(c) 125Ω (d) 625Ω

Ans :

Delhi 2005

Resistance, $R = 25 \Omega$

According to the question,

Inductive reactance = capacitive reactance

i.e., $X_L = X_C$

- (c) AC can not pass through DC ammeter
 (d) average value of current for a complete cycle is zero

Ans :

Foreign 2001

Alternating current can not be measured by DC ammeter, because average value of the current for a complete cycle is zero. That is why, DC ammeter records zero reading.

Thus (d) is correct option.

- 16.** In a circuit, the current lags behind the voltage by a phase difference of $\frac{\pi}{2}$ radians. The circuit contains which of the following?
 (a) only R (b) only L
 (c) only C (d) R and C

Ans :

SQP 2015

In an AC circuit containing only inductance L , the current lags behind the voltage by a phase difference of $\frac{\pi}{2}$ radians.

Thus (b) is correct option.

- 17.** In an AC circuit containing only capacitor, the current
 (a) leads the voltage by 90°
 (b) leads the voltage by 180°
 (c) lags behind the voltage by 90°
 (d) remains in phase with the voltage

Ans :

OD 2005, Foreign 2003

In an AC circuit containing only capacitor, the current leads the voltage by $\frac{\pi}{2}$ radians (*i.e.*, 90°).

Thus (a) is correct option.

- 18.** The value of peak AC in a 220 V mains is
 (a) 220 V (b) $110\sqrt{2}$ V
 (c) $220\sqrt{2}$ V (d) $440\sqrt{2}$ V

Ans :

Delhi 2014

r.m.s. voltage, $V_{\text{r.m.s.}} = 220$ Volt

Value of peak voltage of AC,

$$\begin{aligned} V_0 &= \sqrt{2} \times V_{\text{r.m.s.}} \\ &= \sqrt{2} \times 220 \\ &= 220\sqrt{2} \text{ Volt} \end{aligned}$$

Thus (c) is correct option.

- 19.** In a purely resistive AC circuit, the current
 (a) is in phase with the e.m.f.
 (b) leads the e.m.f. by a difference of π radians phase
 (c) leads the e.m.f. by a phase difference of $\frac{\pi}{2}$ radians
 (d) lags behind the e.m.f. by phase difference of $\frac{\pi}{4}$ radians

Ans :

OD 2003

In a purely resistive AC circuit, the current is in phase with the e.m.f. (*i.e.*, angle between current and e.m.f. is 0°).

Thus (a) is correct option.

- 20.** The value of current flowing through an inductor of inductance 1 H and having negligible resistance when connected to an AC source of 200 V and 50 Hz, is
 (a) 0.64 A (b) 1.64 A
 (c) 2.64 A (d) 3.64 A

Ans :

SQP 2004

Inductance, $L = 1$ H

Voltage, $V = 200$ Volt

Frequency, $f = 50$ Hz

Angular frequency,

$$\begin{aligned} \omega &= 2\pi f = 2 \times 3.14 \times 50 \\ &= 314 \text{ rad-s}^{-1} \end{aligned}$$

Therefore, reactance of the inductor,

$$X_L = \omega L = 314 \times 1 = 314 \Omega$$

Current flowing through the inductor,

$$I = \frac{V}{X_L} = \frac{200}{314} = 0.64 \text{ A}$$

Thus (a) is correct option.

- 21.** Reactance of a capacitor of capacitance C for an alternating current of frequency $\frac{400}{\pi}$ Hz is 25Ω . The value of C is
 (a) $25 \mu\text{F}$ (b) $50 \mu\text{F}$
 (c) $75 \mu\text{F}$ (d) $100 \mu\text{F}$

Ans :

Foreign 2006, SQP 2010

Capacitance of capacitor = C

Frequency of current, $f = \frac{400}{\pi}$ Hz

Reactance of capacitor, $X_C = 25 \Omega$

Reactance of the capacitor,

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{2\pi f C} \\ C &= \frac{1}{2\pi f X_C} \\ &= \frac{1}{2\pi \times \frac{400}{\pi} \times 25} \\ &= 50 \times 10^{-6} \\ F &= 50 \mu\text{F} \end{aligned}$$

Thus (b) is correct option.

- 22.** In an AC circuit, the potential difference across an inductance and resistance joined in series are 15 V and 25 V respectively.

The standard equation of e.m.f. in an AC circuit,

$$E = E_0 \cos(\omega t)$$

Comparing the given equation with the standard equation, we get:

$$E_0 = 4$$

and

$$\omega = 100$$

Impedance of LR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L)^2} \\ &= \sqrt{R^2 + (\omega L)^2} \\ &= \sqrt{R^2 + \omega^2 L^2} \\ &= \sqrt{(4)^2 + [(100)^2 \times (0.03)^2]} \\ &= \sqrt{16 + 9} = \sqrt{25} = 5 \Omega \end{aligned}$$

Therefore, amplitude of current in the circuit,

$$I_0 = \frac{E_0}{Z} = \frac{4}{5} = 0.8 \text{ A}$$

Thus (b) is correct option.

- 31.** In a series combination, $R = 300 \Omega$, $L = 0.9 \text{ H}$, $C = 2 \mu\text{F}$, $\omega = 1000 \text{ rad} \cdot \text{s}^{-1}$. The impedance of the LCR-circuit is

- (a) 400Ω (b) 500Ω
(c) 900Ω (d) 1300Ω

Ans :

Foreign 2011

Resistance, $R = 300 \Omega$

Inductance, $L = 0.9 \text{ H}$

Capacitance, $C = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$

Angular velocity, $\omega = 1000 \text{ rad} \cdot \text{s}^{-1}$

Reactance of inductor,

$$\begin{aligned} X_L &= \omega L \\ &= 1000 \times 0.9 = 900 \Omega \end{aligned}$$

and reactance of capacitor,

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{1000 \times (2 \times 10^{-6})} \\ &= 500 \Omega \end{aligned}$$

Therefore, impedance of LCR-circuit

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(300)^2 + (900 - 500)^2} \\ &= \sqrt{90000 + 160000} \\ &= \sqrt{250000} = 500 \Omega \end{aligned}$$

Thus (b) is correct option.

- 32.** In a series LCR-circuit, resistance $R = 10 \Omega$ and the impedance $Z = 20 \Omega$. The phase difference between the current and the voltage is

- (a) 30° (b) 45°
(c) 60° (d) 90°

Ans :

OD 2007, SQP 2002

Resistance, $R = 10 \Omega$

Impedance, $Z = 20 \Omega$

Impedance in the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ 20 &= \sqrt{(10)^2 + (X_L - X_C)^2} \\ 400 &= 100 + (X_L - X_C)^2 \\ (X_L - X_C)^2 &= 400 - 100 = 300 \\ X_L - X_C &= \sqrt{300} \\ &= 10\sqrt{3} \end{aligned}$$

Phase difference between current and voltage,

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{X_L - X_C}{R} \right) \\ &= \tan^{-1} \left(\frac{10\sqrt{3}}{10} \right) \\ &= \tan^{-1}(\sqrt{3}) = 60^\circ \end{aligned}$$

Thus (c) is correct option.

- 33.** If a circuit consists of inductance L , capacitance C and resistance R , then the resonant frequency of an LCR-circuit is

- (a) $\pi\sqrt{LC}$ (b) $2\pi\sqrt{LC}$
(c) $2\pi\sqrt{\frac{1}{LC}}$ (d) $\frac{1}{2\pi\sqrt{LC}}$

Ans :

Delhi 2014

Inductance = L

Capacitance = C

and Resistance = R

Resonant frequency of an LCR circuit,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Thus (d) is correct option.

- 34.** In a LCR-circuit, the potential difference between the terminals of the inductance is 60 V , between terminals of the capacitor is 30 V and that between the terminals of the resistance is 40 V . The supply voltage will be
- (a) 25 V (b) 50 V
(c) 100 V (d) 200 V

Ans :

SQP 2002

Potential difference across inductance,

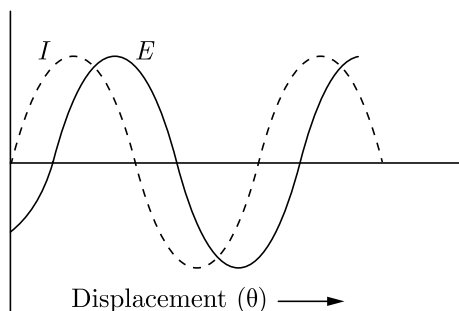
$$V_L = 60 \text{ Volt}$$

Net impedance of the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(25)^2 + (X_L - X_C)^2} \\ &= 25 \Omega \end{aligned}$$

Thus (b) is correct option.

27. When an AC source of e.m.f. $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. E and the current I in the circuit is observed to be $\frac{\pi}{4}$ rad, as shown in the diagram. If the circuit consists possibly only of RC or RL or LC in series, then relationship between the two elements is



- (a) $R = 1 \text{ k}\Omega$, $C = 1 \mu\text{F}$ (b) $R = 1 \text{ K}\Omega$, $L = 1 \text{ H}$
 (c) $R = 1 \text{ K}\Omega$, $C = 10 \mu\text{F}$ (d) $R = 1 \text{ K}\Omega$, $L = 10 \text{ H}$

Ans :

Foreign 2011

Equation for e.m.f., $E = E_0 \sin(100t)$

and phase difference between e.m.f. and current,

$$\phi = \frac{\pi}{4} \text{ rad} = 45^\circ$$

Standard equation of e.m.f. of AC source,

$$E = E_0 \sin(\omega t)$$

Comparing the given equation with the standard equation, we get

$$\omega = 100 \text{ rad-s}^{-1}$$

We also know from the given figure that the current leads e.m.f. by,

$$\frac{\pi}{4} \text{ rad} = 45^\circ$$

Therefore circuit consists of only RC-circuit, in which the phase difference,

$$\theta = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$$45^\circ = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$$\left(\frac{1}{\omega CR}\right) = \tan 45^\circ = 1$$

$$\frac{1}{CR} = \omega = 100$$

$$CR = 10^{-2}$$

This condition is satisfied when,

$$R = 1 \text{ k}\Omega$$

and

$$C = 10 \mu\text{F}$$

Thus (c) is correct option.

28. In an LCR series AC circuit, the voltage across each of the components, L , C and R is 50 V. The voltage across the LC combination will be
 (a) zero (b) 50 V
 (c) $50\sqrt{2}$ V (d) 100 V

Ans :

OD 2007, Foreign 2009

Voltage across inductor, $V_L = 50$ Volt

Voltage across capacitor, $V_C = 50$ Volt

Voltage across resistance, $V_R = 50$ Volt

Voltage the LC combination

$$V_L - V_C = 50 - 50 = 0$$

Thus (a) is correct option.

29. In an experiment, 200 V AC is applied at the ends of an LCR-circuit. The circuit consists of an inductive reactance of 50Ω , capacitive reactance of 35Ω and ohmic resistance of 20Ω . The impedance of the circuit is
 (a) 15Ω (b) 25Ω
 (c) 35Ω (d) 45Ω

Ans :

Delhi 2016

Supply voltage, $V = 200$ Volt

Inductive reactance, $X_L = 50 \Omega$

Capacitive reactance, $X_C = 35 \Omega$

Ohmic resistance, $R = 20 \Omega$

Impedance of the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(20)^2 + (50 - 35)^2} \\ &= \sqrt{400 + 225} = \sqrt{625} = 25 \Omega \end{aligned}$$

Thus (b) is correct option.

30. In an AC circuit, the equation of e.m.f is: $E = 4 \cos(100t)$. The amplitude of current in an LR-circuit of inductance 0.03 H and resistance 4Ω is
 (a) $\frac{4}{\sqrt{7}} \text{ A}$ (b) $\frac{4}{7} \text{ A}$
 (c) 0.4 A (d) 0.8 A

Ans :

Foreign 2008

Equation of e.m.f., $E = 4 \cos(100t)$

Inductance, $L = 0.03 \text{ H}$

Resistance, $R = 4 \Omega$

- (a) $\omega < \frac{1}{\sqrt{LC}}$ (b) $\omega < \sqrt{LC}$
 (c) $\omega = \frac{1}{\sqrt{LC}}$ (d) $\omega > \frac{1}{\sqrt{LC}}$

Ans :

OD 2010

For maximum value of current,

$$\begin{aligned}X_L &= X_C \\ \omega L &= \frac{1}{\omega C} \\ \omega^2 &= \frac{1}{LC} \\ \omega &= \frac{1}{\sqrt{LC}}\end{aligned}$$

Thus (c) is correct option.

11. If L be the inductance, R resistance and C be the capacitance of a capacitor, then dimensional formulae of $\frac{L}{R}$ and RC are

- (a) $M^0 L T^{-1} M L^0 T^{-1}$ (b) $M^0 L^0 T, M L T^0$
 (c) $M^0 L^0 T, 1$ (d) $M^0 L^0 T, M^0 L^0 T$

Ans :

Delhi 2009

Dimension of inductance,

$$\begin{aligned}L &= \frac{\text{EMF}}{\text{current/time}} \\ &= \frac{[M L^2 T^{-3} A^{-1}]}{(A T^{-1})} \\ &= [M L^2 T^{-2} A^{-2}] \quad \dots(1)\end{aligned}$$

Dimension of resistance,

$$\begin{aligned}R &= \frac{\text{Potential difference}}{\text{current}} \\ &= \frac{M L^2 T^{-3} A^{-1}}{A} \\ &= [M L^2 T^{-3} A^{-2}] \quad \dots(2)\end{aligned}$$

Dimension of capacitance,

$$\begin{aligned}C &= \frac{\text{Charge}}{\text{Potential difference}} \\ &= \frac{A T}{M L^2 T^{-3} A^{-1}} \\ &= [M^{-1} L^{-2} T^4 A^2] \quad \dots(3)\end{aligned}$$

After dividing equation (1) by equation (2), we get

$$\begin{aligned}\frac{L}{R} &= \frac{[M L^2 T^{-2} A^{-2}]}{[M L^2 T^{-3} A^{-2}]} \\ &= [T] = [M^0 L^0 T]\end{aligned}$$

After multiplying equation (2) with (3), we get

$$\begin{aligned}RC &= [M L^2 T^{-3} A^{-2}] [M^{-1} L^{-2} T^4 A^2] \\ &= [T] = [M^0 L^0 T]\end{aligned}$$

Thus (d) is correct option.

12. If ϕ is the phase difference between AC current and $e.m.f.$, then the value of power factor is

- (a) $\tan \phi$ (b) $\cos^2 \phi$
 (c) $\sin \phi$ (d) $\cos \phi$

Ans :

Foreign 2015

Power factor is the magnitude of difference in the phase of voltage to current wave form. It is denoted by $\cos \phi$.

Thus (d) is correct option.

13. A hotwire ammeter reads 10 A in an AC circuit. The peak value of the current is

- (a) $5\pi A$ (b) $10\sqrt{2} A$
 (c) $\frac{10}{\sqrt{2}} A$ (d) $\frac{5}{\sqrt{2}} A$

Ans :

SQP 2005, Delhi 2007

Reading of hotwire ammeter or r.m.s. value of current,

$$I_{r.m.s.} = 10 A$$

Peak value of current,

$$\begin{aligned}I_0 &= \sqrt{2} \times I_{r.m.s.} \\ &= \sqrt{2} \times 10 = 10\sqrt{2} A\end{aligned}$$

Thus (b) is correct option.

14. An alternating current is given by: $I = I_1 \cos \omega t + I_2 \sin \omega t$. The root mean square current is given by

- (a) $\frac{(I_1 + I_2)}{\sqrt{2}}$ (b) $\frac{(I_1 + I_2)^2}{2}$
 (c) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (d) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$

Ans :

OD 2003

Equation of alternating current,

$$I = I_1 \cos \omega t + I_2 \sin \omega t$$

Resultant or maximum current,

$$I_0 = \sqrt{I_1^2 + I_2^2}$$

Therefore, r.m.s. current,

$$\begin{aligned}I_{r.m.s.} &= \frac{I_0}{\sqrt{2}} \\ &= \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}} \\ &= \sqrt{\frac{I_1^2 + I_2^2}{2}}\end{aligned}$$

Thus (c) is correct option.

15. Alternating current can not be measured by DC ammeter, because

- (a) AC changes direction
 (b) DC ammeter will get damaged

Potential difference across capacitor,

$$V_C = 30 \text{ Volt}$$

and potential difference across resistance,

$$V_R = 40 \text{ Volt}$$

Supply voltage in LCR-circuit,

$$\begin{aligned} V &= \sqrt{V_R^2 + (V_L - V_C)^2} \\ &= \sqrt{(40)^2 + (60 - 30)^2} \\ &= \sqrt{1600 + 900} \\ &= \sqrt{2500} = 50 \text{ Volt} \end{aligned}$$

Thus (b) is correct option.

- 35.** Quality factor of LCR-circuit having resistance R and inductance L at resonance angular frequency ω is given by

(a) $\frac{\omega L}{R}$ (b) $\frac{R}{\omega L}$
 (c) $\left(\frac{\omega L}{R}\right)^{1/2}$ (d) $\left(\frac{\omega L}{R}\right)^2$

Ans :

Foreign 2008

Resistance = R and Inductance = L .

and resonance angular frequency = ω .

Applied e.m.f. = IR .

and potential difference developed across the inductance

$$= I\omega L$$

Therefore, quality factor,

$$Q = \frac{I\omega L}{IR} = \frac{\omega L}{R}$$

Thus (a) is correct option.

- 36.** For an AC circuit, average power is 60 W. What will be the power factor for the same circuit, if its apparent power is 100 W ?

(a) 0.6 (b) 1.6
(c) 2.4 (d) 3.2

Ans :

OD 2007

$$\text{Average power} = 60 \text{ W}$$

$$\text{and Apparent power} = 100 \text{ W}$$

$$\begin{aligned} \text{Power factor, } \cos \phi &= \frac{\text{Average power}}{\text{Apparent power}} \\ &= \frac{60}{100} = 0.6 \end{aligned}$$

Thus (a) is correct option.

- 37.** In an AC circuit, voltage applied is $V = 100 \sin(100t)$. If impedance of the circuit is 110Ω and phase difference between current and voltage is 60° , then power in the circuit is
- (a) 55 W (b) 110 W

(c) 220 W

(d) 330 W

Ans :

SQP 2006

$$\text{Equation of voltage, } V = 220 \sin(100t)$$

$$\text{Impedance of circuit, } Z = 110 \Omega$$

and Phase difference between current and voltage,

$$\phi = 60^\circ$$

Standard equation for the voltage is,

$$V = V_0 \sin(\omega t)$$

Comparing the given equation with the standard equation, we get

$$V_0 = 220 \text{ Volt}$$

Therefore peak value of current,

$$I_0 = \frac{V_0}{Z} = \frac{220}{110} = 2 \text{ A}$$

Power in the circuit,

$$\begin{aligned} P &= V_{\text{r.m.s.}} I_{\text{r.m.s.}} \cos \phi \\ &= \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \phi \\ &= \frac{220}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos 60^\circ \\ &= 220 \times 0.5 = 110 \text{ W} \end{aligned}$$

Thus (b) is correct option.

- 38.** In an AC circuit, equation for the voltage is $V = 100 \sin(100t)$ volt and that for current is $I = 100 \sin(100t + \pi/3)$ mA. The power dissipated in the circuit is

(a) 2.5 W (b) 5 W
(c) 10 W (d) 20 W

Ans :

Delhi 2016, OD 2005

Equation of voltage,

$$V = 100 \sin(100t)$$

and equation of current,

$$I = 100 \sin(100t + \pi/3) \text{ mA}$$

Standard equations of voltage and current are,

$$V = V_0 \sin(\omega t)$$

and

$$I = I_0 \sin(\omega t + \phi)$$

Comparing the given equation with the standards equations, we get,

$$V_0 = 100 \text{ Volt}$$

$$I_0 = 100 \text{ mA} = 0.1 \text{ A}$$

and

$$\phi = \frac{\pi}{3} = 60^\circ$$

We also know that power dissipated in the circuit,

$$P = V_{\text{r.m.s.}} I_{\text{r.m.s.}} \cos \phi$$

$$\begin{aligned}
 &= \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \phi \\
 &= \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \cos 60^\circ \\
 &= 5 \times \frac{1}{2} = 2.5 \text{ W}
 \end{aligned}$$

Thus (a) is correct option.

39. Which of the following quantity is increased in a step-down transformer?

(a) current (b) voltage
(c) power (d) frequency

Ans :

Foreign 2009

In a step-down transformer, the output voltage is decreased. As a result of this, the output current is increased.

Thus (a) is correct option.

40. The core of a transformer is laminated, so as to

(a) make it light weight
(b) make it robust and strong
(c) increase the secondary voltage
(d) reduce energy loss due to eddy current

Ans :

Delhi 2014, OD 2011

The varying magnetic flux link through a metallic conductor, sets up eddy current in the conductor and this eddy current is converted into heat energy which leads to the wastage of energy.

That is why, the core of a transformer is laminated to reduce the eddy current so as to reduce the energy loss.

Thus (d) is correct option.

41. In an ideal transformer, voltage and current in primary are 200 V and 2 A respectively. If voltage in the secondary is 2000 V, then current in the secondary is

(a) 20 A (b) 10 A
(c) 2 A (d) 0.2 A

Ans :

SQP 2002

Voltage in primary, $V_P = 200 \text{ Volt}$

Current in primary, $I_P = 2 \text{ A}$

Voltage in secondary, $V_S = 2000 \text{ Volt}$

Relation between currents and voltages in primary and secondary is,

$$\begin{aligned}
 \frac{I_P}{I_S} &= \frac{V_S}{V_P} \\
 I_S &= I_P \times \frac{V_P}{V_S} \\
 &= 2 \times \frac{200}{2000} = 0.2 \text{ A}
 \end{aligned}$$

where, I_S = current in secondary

Thus (d) is correct option.

42. The ratio of number of turns of primary coil to secondary coil in a transformer is 2:3. If a cell of 6 V is connected across the primary coil, then voltage across the secondary coil will be

(a) 3 V (b) 6 V
(c) 9 V (d) 12 V

Ans :

OD 2008

Number of turns in primary,

$$N_P = 2 \text{ N}$$

Number of turns in secondary coil,

$$N_S = 3 \text{ N}$$

Supply voltage on primary coil,

$$V_P = 6 \text{ Volt}$$

Relation between voltages and number of turns in primary and secondary is,

$$\begin{aligned}
 \frac{V_P}{V_S} &= \frac{N_P}{N_S} \\
 V_S &= V_P \times \frac{N_S}{N_P} \\
 &= 6 \times \frac{3 \text{ N}}{2 \text{ N}} = 9 \text{ Volt}
 \end{aligned}$$

where, V_S = voltage across secondary coil

Thus (c) is correct option.

43. A step-up transformer has transformation ratio of 5:3. What is the voltage in the secondary, if voltage in the primary is 60 V?

(a) 180 V (b) 100 V
(c) 60 V (d) 20 V

Ans :

SQP 2016

Number of turns in secondary,

$$N_S = 5 \text{ N}$$

Number of turns in primary,

$$N_P = 3 \text{ N}$$

Voltage in primary,

$$V_P = 60 \text{ Volt}$$

Relation between voltages and number of turns in primary and secondary is,

$$\begin{aligned}
 \frac{V_S}{V_P} &= \frac{N_S}{N_P} \\
 V_S &= V_P \times \frac{N_S}{N_P} \\
 &= 60 \times \frac{5 \text{ N}}{3 \text{ N}} = 100 \text{ Volt}
 \end{aligned}$$

where, V_S = voltage in secondary

Thus (b) is correct option.

44. A step-up transformer operates on a 230 V line supplies a current of 2 A. The ratio of primary and secondary winding is 1:25. The current in the primary is

- (a) 15 A (b) 50 A
(c) 75 A (d) 125 A

Ans :

Foreign 2004, Delhi 2015

Voltage in primary, $V_P = 230$ Volt

Current in secondary, $I_S = 2$ A

number of windings in primary,

$$N_P = N$$

number of windings in secondary,

$$N_S = 25 N$$

Relation between currents and number of turns in primary and secondary coil is,

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$I_P = I_S \times \frac{N_S}{N_P}$$

$$= 2 \times \frac{25 N}{N} = 50 \text{ A}$$

where, I_P = current in primary

Thus (b) is correct option.

45. In a transformer, the output current and voltage are respectively 4 A and 20 V. If ratio of number of turns in primary to secondary is 2 : 1, what is the input current and voltage?

- (a) 1 A and 20 V (b) 2 A and 40 V
(c) 4 A and 80 V (d) 8 A and 160 V

Ans :

OD 2009

Output current in secondary,

$$I_S = 4 \text{ A}$$

Output Voltage in secondary,

$$V_S = 20 \text{ Volt}$$

number of turns in primary, coil

$$N_P = 2 N$$

and number of turns in secondary, coil

$$N_S = N$$

Relation between currents and number of turns in primary and secondary coil is,

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$I_P = I_S \times \frac{N_S}{N_P}$$

$$= 4 \times \frac{N}{2N} = 2 \text{ A}$$

where,

I_P = current in primary

Relation between voltage and number of turns in primary and secondary coil is:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

or

$$V_P = V_S \times \frac{N_P}{N_S} = 20 \times \frac{2 N}{N}$$

$$= 40 \text{ Volt}$$

where,

V_P = input voltage in primary

Thus (b) is correct option.

46. The best material for the core of a transformer is

- (a) soft iron (b) mild steel
(c) hard steel (d) stainless steel

Ans :

Delhi 2016

Transformer core is made by placing soft iron strips one above the other. These strips are insulated from each other to reduce eddy current.

Thus (a) is correct option.

ASSERTION AND REASON

47. **Assertion :** In series LCR circuit resonance can take place.

Reason : Resonance takes place if inductance and capacitive resistances are equal and opposite.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

Ans :

In series resonance circuit, current becomes maximum because total impedance becomes zero. In case of LC circuit,

$$\text{Total impedance} = \omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

Thus (a) is correct option.

The transformation ratio is defined as the ratio of the secondary voltage (E_2) to primary voltage. (E_1) It is denoted by K .

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

where, N_1 and N_2 are the number of turns in primary and secondary coil.

1. If $N_2 > N_1$ i.e., $K > 1$, then transformer is called step-up transformer.
2. If $N_2 < N_1$ i.e., $K < 1$, then transformer is called step down transformer.

65. Write down the equation of induced current at any instant in L-C-R circuit when $X_L > X_C$ or circuit is inductive.

Ans :

Delhi 2021

Equation of induced current at any instant in LCR circuit is given by,

$$I = \frac{E}{Z}$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ [$X_L > X_C$]

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Since, $X_L = \omega L$

and $X_C = \frac{1}{\omega C}$

$$I = \frac{E}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

where, E = net emf of the circuit

R = Resistance in the circuit

X_L = Inductive reactance

ω = Angular frequency

66. Define mean value and root mean square value of alternating current.

Ans :

Delhi 2020 OD, 2007

Mean Value of Alternating Current, AC

It is defined as the value of AC which would send same amount of charge through a circuit in half cycle that is sent by steady current in the same time. It is denoted by I_m or I_{av} .

Root mean Square Value of AC

It is defined as that average value of alternating current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the resistor and in the same time during a complete cycle.

67. What is transformer? What do you mean by its efficiency?

Ans :

Comp 2019

Transformer is a device, which is used to increase or decrease the alternating voltage.

Efficiency of Transformer

The ratio of output power to input power is known as efficiency of transformer

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

The efficiency, $\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\%$

The efficiency of real transformers is fairly high (90-99%) through not 100%.

68. Explain why current flows through an ideal capacitor when it is connected to an AC source, but not when it connected to a DC source in a steady state.

Ans :

Delhi 2016

For AC source, circuit is complete due to the presence of displacement current in the capacitor. For steady DC, there is no displacement current, therefore, circuit is not complete.

Mathematically, Capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$

So, capacitor allows easy path for AC source.

For DC, $f = 0$

$X_C = \text{infinity}$

So, Capacitor block DC.

69. An applied voltage signal consists of a superposition of a DC voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show the DC signal will appear across C and the AC voltage across L .

Ans :

Foreign 2011

Inductive reactance, $X_L = \omega L$

Capacitive reactance, $X_C = \frac{1}{\omega C}$

For DC in steady state $\omega = 0$; $X_L = 0$, $X_C = \infty$ (for DC).

Thus, the inductive reactance has zero value for DC and very high value for high frequency AC. On the other hand capacitance offers infinite reactance to DC and very low reactance to high frequency AC.

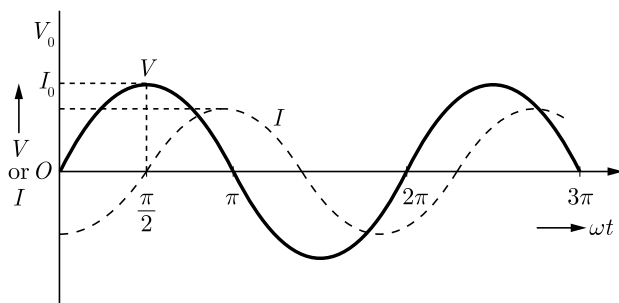
So, when inductor and capacitor are connected in series, the capacitor gives easy path to AC and does not allow DC while inductor gives easy path to DC and offers very high opposition to AC. Hence, DC signal will appear across C and AC voltage across L .

70. An inductor L or reactance X_L is connected in series with a bulb B to an AC source as shown in figure.

Ans :

OD 2021

Graphs of V and i versus ωt for this circuit is shown below :



55. What is the function of a step-up transformer?

Ans :

Delhi 2019

Step-up transformer converts low alternating voltage into high alternating voltage and high alternating current into low alternating current. The secondary coil of step-up transformer has greater number of turns than the primary ($N_s > N_p$).

56. What is inductive reactance?

Ans :

Comp 2015, Foreign 2009

The opposing nature of inductor to flow of alternating current is called inductive reactance.

$$X_L = \omega L$$

ω = average frequency

L = self Inductance

57. Why is core of transformer laminated?

Ans :

Delhi 2018

The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core, because of its insulation properly.

58. Name the energy losses in a transformer.

Ans :

OD 2013

Energy loss in transformers are :

1. Flux leakage
2. Resistance to the windings
3. Eddy currents
4. Hysteresis.

59. What is wattless current?

Ans :

Foreign 2011

When pure inductor and/or pure capacitor is connected to AC source, the current flows in the circuit, but with no power loss; the phase difference between voltage and current is $\frac{\pi}{2}$. Such a current is called the wattless current.

60. What is the function of a step-up transformer?

Ans :

SQP 2014

Step-up transformer converts low alternating voltage into high alternating voltage and high alternating current into low alternating current. The secondary coil of step-up transformer has greater number of turns than the primary ($N_s > N_p$).

61. Mention the two important properties of the material suitable for making core of a transformer.

Ans :

Delhi 2017

Two characteristic properties:

1. Low hysteresis loss
2. Low coercivity.

62. Define Capacitor reactance. Write its SI units.

Ans :

OD 2010

Capacitor reactance is the resistance offered by a capacitor, when it is connected to an electric circuit. It is given by,

$$X_C = \frac{1}{\omega C}$$

where, ω = angular frequency of the source

C = Capacitance of the capacitor

The SI unit of capacitor reactance is ohm (Ω).

63. In any AC circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

Ans :

Comp 2013

Yes, in any AC circuit, the applied instantaneous voltage is equal to the algebraic sum of instantaneous voltage across the series elements, because instantaneous voltages are in the same phase.

This is not true in the case of rms voltages, because they are usually not in the same phase.

SHORT ANSWER QUESTIONS

64. What is transformer? What is transformation ratio?

Ans :

Comp 2021

Transformer

It is a device, which is used to increase or decrease the alternating voltage.

The transformers are of the following types

1. Step-up transformer
2. Step-down transformer

Transformation Ratio

- 48. Assertion :** Long distance power transmission is done at high voltage.

Reason : At high voltage supply power losses are less.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

$$\text{Power loss} = I^2 R = \left(\frac{P}{V}\right)^2 R$$

where P = transmitted power

Thus (d) is correct option.

- 49. Assertion :** Ohm's law cannot be applied to a.c. circuit.

Reason : Resistance offered by capacitor for a.c. source depends upon the frequency of the source.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Thus (d) is correct option.

- 50. Assertion :** The resistance offered by an inductor in a d.c. circuit is always constant.

Reason : The resistance of inductor in steady state is non-zero.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

Ans :

Resistance offered by an inductor in a d.c. circuit at $t = 0$ is infinity, which decreases to zero at steady state.

Thus (d) is correct option.

- 51. Assertion :** A laminated core is used in transformers to increase eddy currents.

Reason : The efficiency of a transformer increases with increase in eddy currents.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

- (c) Assertion is correct but Reason is incorrect.

- (d) Assertion is incorrect but Reason is correct.

Ans :

Large eddy currents are produced in non-laminated iron core of the transformer by the induced emf, as the resistance of bulk iron core is very small. By using thin iron sheets as core the resistance is increased. Laminating the core substantially reduces the eddy currents. Eddy current heats up the core of the transformer. More the eddy currents greater is the loss of energy and the efficiency goes down.

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

- 52.** The current flowing through a pure inductance 2 mH is $I = (15 \cos 300t) A$. What is the (i) r.m.s. and (ii) average value of current for a complete cycle?

Ans :

Delhi 2021

Current flowing through the inductor,

$$I = 15 \cos 300t$$

Comparing with

$$I = I_0 \sin \omega t$$

Here, peak value of current,

$$I_0 = 15 A$$

- (i) For complete cycle, rms value of current

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} A$$

- (ii) For complete cycle average value of current is zero

$$\text{i.e., } I_{av} = 0$$

- 53.** Why is the use of AC voltage preferred over DC voltage? Give two reasons.

Ans :

Comp 2021

The use of AC voltage is preferred over DC voltage because of

- (i) the loss of energy in transmitting the AC voltage over long distance with the help of step-up transformers is negligible as compared to DC voltage.
 (ii) AC voltage can be stepped up and stepped down as per the requirement by using a transformer.

- 54.** An AC source of voltage $V = V_0 \sin \omega t$ is connected to an ideal inductor. Draw graphs of voltage V and current I versus ωt .

Explain briefly how does the brightness of the bulb change when (1) number of turns of the inductor is reduced (2) a capacitor of reactance $X_C = X_L$ is included in the circuit.

Ans :

Comp 2017

1. When the number of turns in the inductor is reduced, the self-inductance of the coil decrease; So, impedance of circuit reduces and current in the circuit ($I = \frac{E}{Z}$) increases. Thus, the brightness of the bulb increases.
2. When capacitor of reactance $X_C = X_L$ is introduced, the net reactance of circuit becomes zero, so impedance of circuit decrease; it becomes $Z = R$; So current in circuit increases; hence brightness of bulb increases. Thus, brightness of bulb in both cases increases.

- 71.** State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distance done with the use of transformers?

Ans :

Delhi 2014, SQP 2004

Principle of Transformer

A transformer is based on the principle of mutual induction, i.e., whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

Power Transmission

Electric power is transmitted over long distances at high voltage. So step-up transformers are used at power stations to increase the voltage of power whereas a series of step-down transformers are used to decrease the voltage upto 220 V.

- 72.** State the principle of working of a transformer. Can a transformer be used to step-up or step-down a DC voltage? Justify your answer.

Ans :

OD 2019

Working of a transformer is based on the principle of mutual induction. Transformer cannot step-up or step-down a DC voltage.

Reason : No change in magnetic flux.

Explanation : When DC voltage source is applied across a primary coil of a transformer, the current in primary coil remains same, so there is no change in magnetic flux associated with it and hence no voltage is induced across the secondary coil.

- 73.** Why an inductor is an easy path for DC and resistive path for AC?

Ans :

Delhi 2009

The inductor reactance (resistance offered by inductor) of an inductor is given by,

$$X_L = \omega L = 2\pi\nu L$$

where, ν is the frequency of AC source.

For DC

$$\nu = 0$$

Hence,

$$X_L = 2\pi \cdot 0 \cdot L = 0$$

i.e. the inductor offers no resistance to DC.

For AC

ν is finite

Hence,

$$X_L \propto \nu$$

So larger the value of ν , more will be the effective resistance offered by the inductor.

Hence an inductor is an easy path for DC but a resistive path for AC.

- 74.** Prove that an ideal capacitor in an AC circuit does not dissipate power.

Ans :

OD 2018

Since, average power consumption in an AC circuit is given by,

$$P_{av} = V_{rms} \times I_{rms} \times \cos \phi$$

But in pure capacitive circuit, phase difference between voltage and current is,

$$\phi = \frac{\pi}{2}$$

Hence,

$$P_{av} = V_{rms} \times I_{rms} \times \cos \frac{\pi}{2}$$

$$P_{av} = 0 \quad \left(\text{since } \cos \frac{\pi}{2} = 0 \right)$$

Thus, no power is consumed in pure capacitive AC circuit.

- 75.** What do you mean by lag and lead in alternating current?

Ans :

SQP 2012

In a DC supply the e.m.f. and current are always in the same phase and there is nothing like 'lag' or 'lead' but in an AC supply, although the e.m.f. and current have the same frequency, yet they may be out of phase with each other, i.e. the peak value of e.m.f. and peak value of current may not occur at the same time. This brings in the idea of 'lag' and 'lead' in an AC circuit. If the peak value of e.m.f. occurs before the peak value of current, the e.m.f. is said to 'lead' the current is said to 'lag' behind the e.m.f. by a certain angle of fraction of the time period. The exact value of 'lag' or 'lead' depends on the nature of circuit.

- 76.** Show that a capacitor is an easy path for AC but a block for DC (i.e. offers infinite resistance for DC).

Ans :

Comp 2007

The capacitive reactance X_C (resistance offered by a capacitor) is given by,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

where C is the capacitance of the capacitor and ν is the frequency of AC source.

For DC

$$\nu = 0$$

Hence,
$$X_C = \frac{1}{0} = \infty$$

i.e. capacitor offers infinite resistance to DC in other words it blocks DC.

For AC, ν is finite

So,
$$X_C \propto \frac{1}{\nu}$$

So larger the value of ν , smaller will be the value of X_C . Hence capacitor is an easy path for AC.

So a capacitor is an easy path for AC but a block for DC.

- 77.** An alternating voltage of frequency f is applied across LCR circuit. Let f_r be the resonance frequency for the circuit. Will the current in the circuit lag, lead or remain in phase with the applied voltage when

1. $f > f_r$
2. $f < f_r$?

Explain your answer in each case.

Ans :

Delhi 2017, OD 2012

In an LCR circuit, the phase angle ϕ is given by,

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{2\pi fL - \frac{1}{2\pi fC}}{R}$$

1. When $f > f_r$, then $X_L > X_C$ and the circuit is inductive dominant circuit and $\tan \phi$ is positive. So the current will lag behind the applied voltage by a phase ϕ .
2. When $f < f_r$, then $X_L < X_C$ and the circuit is capacitive dominant circuit, and $\tan \phi$ is negative, so the current will lead the applied voltage by a phase ϕ .

The current will remain in phase if $f = f_r$, then $X_L = X_C$ and the circuit is purely resistive and hence current and voltage are in phase.

- 78.** State the factors on which induced e.m.f. in a coil rotating in a uniform magnetic field depends.

Ans :

Comp 2016

Since magnitude of induced e.m.f. is given by,

$$e = nBA\omega$$

So induced e.m.f. depends upon

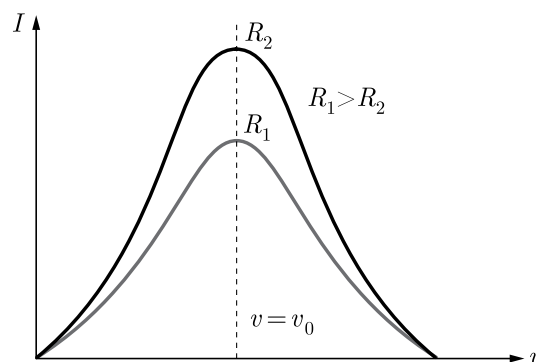
1. Number of turns of the coil (n).
2. Face area of the coil (A).
3. Magnetic field strength (B).
4. The angular velocity of the coil (ω).

- 79.** State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing the variation of current with frequency of an AC source in series LCR circuit.

Ans :

OD 2019

The condition for series resonance is,



$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

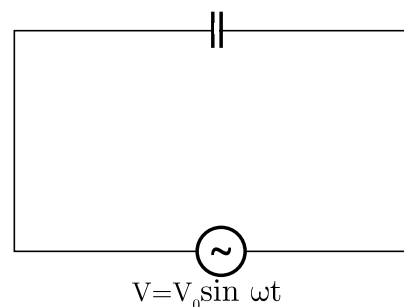
$$2\pi\nu_0 = \frac{1}{\sqrt{LC}}$$

(where ν_0 is resonance frequency)

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

The graph between I and ν is as shown in figure.

- 80.** Show that the current leads the voltage in phase by $\frac{\pi}{2}$ in an AC circuit containing an ideal capacitor.



Ans :

SQP 2012

The instantaneous voltage,

$$V = V_0 \sin \omega t \quad \dots(1)$$

Let q be the charge on capacitor and I , the current in the circuit at any instant, then instantaneous potential difference,

$$V = \frac{q}{C} \quad \dots(2)$$

From eqs. (1) and (2), we get

$$\frac{q}{C} = V_0 \sin \omega t$$

$$q = CV_0 \sin \omega t$$

The instantaneous current,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$= CV_0 \frac{d}{dt}(\sin \omega t)$$

$$= CV_0 \omega \cos \omega t$$

$$I = \frac{V_0}{\frac{1}{\omega C}} \cos \omega t$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Hence, the current leads the applied voltage in phase by $\frac{\pi}{2}$.

81. Sketch a graph to show how the reactance of (1) a capacitor (2) an inductor varies as a function of frequency.

or

Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied AC source.

Ans :

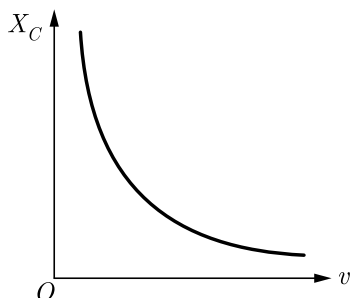
Foreign 2013, Delhi 2005

- (1) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

i.e., $X_C \propto \frac{1}{\nu}$

More the frequency, lesser the X_C and vice-versa. The graph between X_C and ν is as shown in Figure (a).



- (2) Inductive reactance,

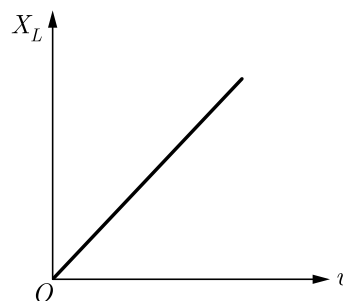
$$X_L = \omega L$$

$$X_L = 2\pi\nu L$$

$$X_L \propto \nu$$

i.e. inductive reactance is directly proportional to frequency.

The graph between X_L and ν is as shown in Figure (b).



82. (i) An alternating voltage $V = V_m \sin \omega t$ applied to a series LCR circuit derives a current given by $I = I_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.
- (ii) For circuit used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

Ans :

SQP 2011

- (i) Let at any instant, the current and voltage in an LCR series AC circuit is given by

$$I = I_0 \sin \omega t$$

$$V = V_0 \sin(\omega t + \phi)$$

The instantaneous power is given by

$$P = VI = V_0 \sin(\omega t + \phi) I_0 \sin \omega t$$

$$P = \frac{V_0 I_0}{2} [2 \sin \omega t \sin(\omega t + \phi)]$$

$$P = VI$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)] \dots(1)$$

$$[\text{Since, } 2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$$

Work done for a very small time interval dt is given by

$$dW = P dt$$

$$dW = VI dt$$

Hence, Total work done over T , a complete cycle is given by

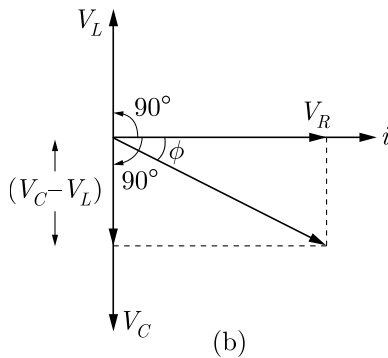
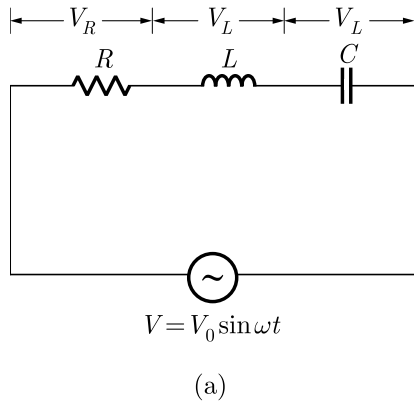
$$W = \int_0^T VI dt$$

But, $P_{av} = \frac{W}{T} = \frac{\int_0^T VI dt}{T}$

$$P_{av} = \frac{1}{T} \int_0^T VI dt$$

$$= \frac{1}{T} \int_0^T \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)] dt$$

applied across it. [Figure (a)] On account of being in series, the current (i) flowing through all of them is the same.



Suppose, the voltage across resistance R is V_R , voltage across inductance L is V_L and voltage across capacitance C is V_C . The voltage V_R and current i are in the same phase, the voltage V_L will lead the current by angle 90° while the voltage V_C will lag behind the current by angle 90° [Figure (b)] clearly, V_C and V_L are in opposite directions, therefore their resultant potential difference = $V_C - V_L$ (if $V_C > V_L$).

Thus, V_R and $(V_C - V_L)$ are mutually perpendicular and the phase difference between them is 90° . As applied voltage across the circuit is V , the resultant of V_R and $(V_C - V_L)$ will also be V . From figure.

$$V^2 = V_R^2 + (V_C - V_L)^2$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2} \quad \dots(1)$$

But, $V_R = iR$, $V_C = X_C i$
and $V_L = X_L i$

Where, $X_C = \frac{1}{\omega C}$ = capacitance reactance and $X_L = \omega L$ = inductive reactance.

Hence, $V = \sqrt{(iR)^2 + (iX_C - iX_L)^2}$

Hence, impedances of circuit,

$$Z = \frac{V}{i} = \sqrt{R^2 + (X_C - X_L)^2}$$

i.e., $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$= \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

Instantaneous current,

$$I = \frac{V_0 \sin(\omega t + \phi)}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

Phase difference (ϕ) in series LCR circuit is given by-

$$\begin{aligned} \tan \phi &= \frac{V_C - V_L}{V_R} \\ &= \frac{i_m(X_C - X_L)}{I_m R} \\ &= \frac{(X_C - X_L)}{R} \end{aligned}$$

When current and voltage are in phase

$$\begin{aligned} \phi &= 0 \Rightarrow X_C = X_L \\ &= 0 \Rightarrow X_C = X_L \end{aligned}$$

This condition is called resonance and the circuit is called resonant circuit.

2. Case I

$$X_L = R$$

Hence, $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{R^2 + R^2} = \sqrt{2} R$$

Power factor, $P_1 = \cos \phi$

$$= \frac{R}{Z} = \frac{R}{\sqrt{2} R} = \frac{1}{\sqrt{2}}$$

Case II

$$X_L = X_C$$

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{R^2} = R \end{aligned}$$

Power factor,

$$P_2 = \frac{R}{Z} = \frac{R}{R} = 1$$

Hence, $\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

100. (a) Describe briefly, with the help of a labelled diagram, the working of a step up transformer.
- (b) Write any two sources of energy loss in a transformer.
- (c) A step up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy?

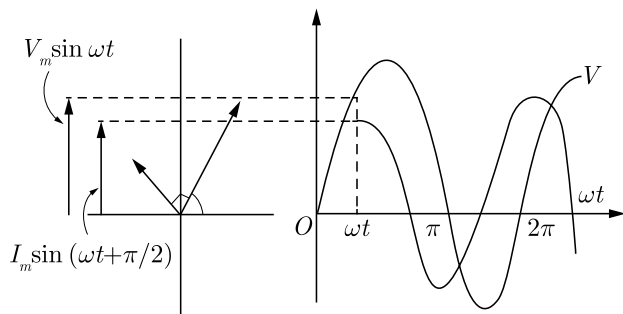
or

Draw a labelled diagram of a step-down transformer. State the principle of its working. Express the turn

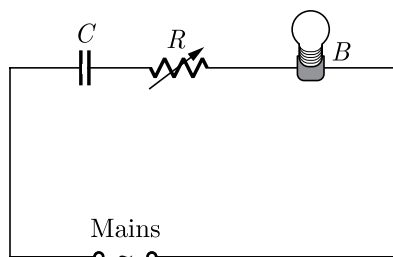
Substituting the value of eqs. (4) and (5) in eq. (3), we get,

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase diagram which shows the current lead the voltage in phase by 90° is given below :



85. A capacitor C , a variable resistor R and a bulb B are connected in series to the AC mains in the circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor keeping resistance R to be the same (ii) the resistance R is increased keeping the same capacitance?



Ans :

Delhi 2016

- (i) As, the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e., $V = \frac{Q}{C}$. As a result, the potential drop across the bulb will increase as they are connected in series. Thus its brightness will increase.
- (ii) As the resistance R is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease.
86. In a series $L - C - R$ circuit, obtain the conditions under which (i) the impedance of circuit is minimum and (ii) wattless current flows in the circuit.

Ans :

OD 2013

- (i) The impedance of a series $L - C - R$ circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Z will be minimum when $\omega L = \frac{1}{\omega C}$, i.e., When the circuit is under resonance.

Hence, for this condition Z will be minimum and equal to R .

- (ii) Average power dissipated through a series $L - C - R$ circuit is given by

$$P_{av} = E_v I_v \cos \phi$$

Where, E_v = rms value of alternating voltage

I_v = value of alternating current

ϕ = phase difference between current and voltage

For wattless current, the power dissipated through the circuit should be zero.

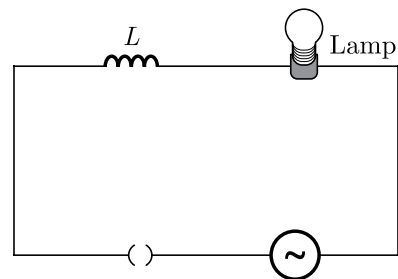
$$\text{i.e.,} \quad \cos \phi = 0$$

$$\cos \phi = \cos \frac{\pi}{2}$$

$$\phi = \frac{\pi}{2}$$

Hence, the condition for wattless current is that the phase difference between the current and the circuit is purely inductive or purely capacitive.

87. (i) When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.
- (ii) A lamp is connected in series with an inductor and an AC source. What happens to the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor? Explain



Ans :

Delhi 2017

$$(i) \quad P_{av} = I_{av} \times e_{av} \cos \phi$$

For an ideal inductor, $\phi = \frac{\pi}{2}$ (The phase difference between voltage and current is $\frac{\pi}{2}$)

$$\text{Hence,} \quad P_{av} = I_{av} \times e_{av} \cos \frac{\pi}{2}$$

$$P_{av} = 0$$

- (ii) When an iron rod is inserted, the inductance increases due to which current decreases and hence the brightness decreases.

[Since, $H = I^2 RT$]

In complete cycle ($0 \rightarrow T$), the total heat produced is H .

After integrating Eq. (1), we get

$$\int dH = \int_0^T I^2 R dt$$

$$H = \int_0^T I^2 R dt$$

Put the value of I in the above equation, we get

$$\begin{aligned} H &= \int_0^T (I_0 \sin \omega t)^2 R dt \\ &= I_0^2 R \int_0^T \sin^2 \omega t dt \\ &= I_0^2 R \int_0^T \left[\frac{1 - \cos 2\omega t}{2} \right] dt \\ &\quad \left[\text{Since, } \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \right] \\ &= \frac{I_0^2 R}{2} \int_0^T (1 - \cos 2\omega t) dt \\ &= \frac{I_0^2 R}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t dt \right] \\ &= \frac{I_0^2 R}{2} \left[t \Big|_0^T - \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T \right] \\ &= \frac{I_0^2 R}{2} \left[(T - 0) - \frac{1}{2\omega} |\sin 2\omega T - \sin 0^\circ| \right] \\ &= \frac{I_0^2 R}{2} \left[T - \frac{1}{2\omega} |\sin 2 \times 2\pi - 0| \right] \\ &\quad \left[\text{Since, } \omega T = 2\pi \right] \\ &= \frac{I_0^2 R}{2} \left[T - \frac{1}{2\omega} |0 - 0| \right] \left[\text{Since, } \sin 4\pi = 0 \right] \\ &= \frac{I_0^2 RT}{2} \quad \dots(2) \end{aligned}$$

If I_{rms} is rms value of alternating current and H is the heat produced by rms current (I_{rms}), then

$$H = I_{\text{rms}}^2 RT \quad \dots(3)$$

On comparing Equation (2) and (3), we get

$$\begin{aligned} I_{\text{rms}}^2 RT &= \frac{I_0^2 RT}{2} \\ I_{\text{rms}}^2 &= \frac{I_0^2}{2} \\ I_{\text{rms}} &= \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \\ I_{\text{rms}} &= 70.7\% \text{ of } I_0 \end{aligned}$$

97. Show diagrammatically two different arrangements used for winding the primary and secondary coils in a transformer. Assuming the transformer to be an ideal one, write the expression for the ratio of its:

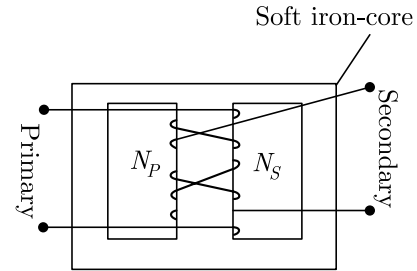
1. Output voltage to input voltage.
2. Output current to input current.

Mention two reasons for energy losses in an actual transformer.

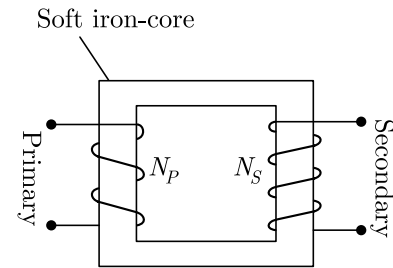
Ans :

OD 2019, Comp 2015

Arrangement of winding of primary and secondary coil in a transformer are shown in fig (a) and (b).



(a) Two coils on to of each other



(b) Two coils on separate limbs of the core

1. Ratio of output voltage to input voltage,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

2. Ratio of output current to input current,

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Reasons for energy losses in a transformer

1. **Joule Heating :** Energy is lost due to heating of primary and secondary windings as heat ($I^2 Rt$).
 2. **Flux Leakage :** Energy is lost due to coupling of primary and secondary coils not being perfect, i.e., whole of magnetic flux generated in primary coil is not linked with the secondary coil.
98. (a) An alternating voltage $V = V_m \sin \omega t$ applied to a series $L - C - R$ circuit drives a current given by $I = I_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.
- (b) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

or

A voltage $V = V_0 \sin \omega t$ is applied to a series $L - C - R$ circuit. Derive the expression for the average power dissipated over a cycle.

Power, $P = VI$

Substituting the value of V and I from eq. (1) and eq. (2), we get

$$P = V_0 \sin \omega t \cdot I_0 \sin(\omega t - \phi)$$

So, the instantaneous power is given by

$$\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$$

- (i) If $\phi = 90^\circ$, then no power is dissipated even though the current flows through the circuit.

$$P_{av} = 0$$

$$\tan \phi = \left(\frac{X_L - X_C}{R} \right)$$

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = \infty$$

(Since, $\tan 90^\circ = \infty$)

- (ii) If $\phi = 0^\circ$, then maximum power is dissipated in the circuit.

$$P_{av} = \text{maximum}$$

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$$

(Since, $\tan 0^\circ = 0$)

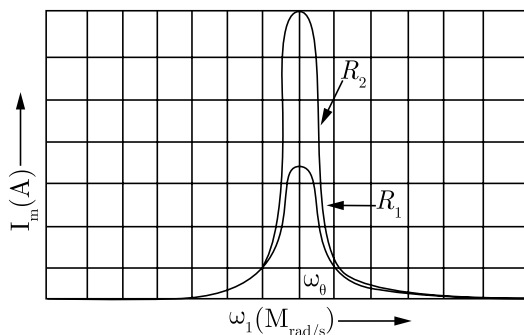
$$X_L = X_C$$

91. In a series LCR circuit connected to an AC source of variable frequency and voltage $V = V_m \sin \omega t$, draw a graph showing the variation of current (i) with angular frequency (ω) for two different values of resistance R_1 and R_2 ($R_1 > R_2$). Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q -factor of the circuit and give its significance.

Ans :

Delhi 2018

Figure shows the variation of i_m with ω in a LCR series circuit for two values of resistance R_1 and R_2 ($R_1 > R_2$).



The condition for resonance in the LCR circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We see that the current amplitude is maximum at the resonant frequency ω . Since $i_m = \frac{V_m}{R}$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1 .

Quality factor or simply the Q -factor of a resonant LCR circuit is defined as the ratio of voltage drop across the capacitor (or inductor) to that of applied voltage.

It is given by $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$

The Q -factor determines the sharpness of the resonance curves. Less sharp the resonance, less is the selectivity of the circuit while higher is the Q , sharper is the resonance curve and lesser will be the loss in energy of the circuit.

92. A series $L-C-R$ circuit is connected to an AC source. Using the phaser diagram. Derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation.

Ans :

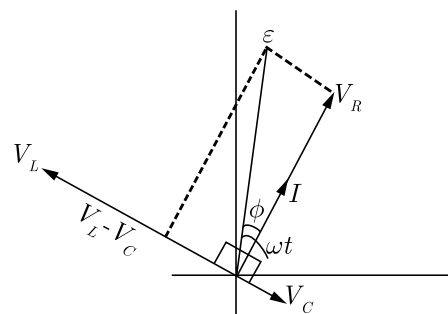
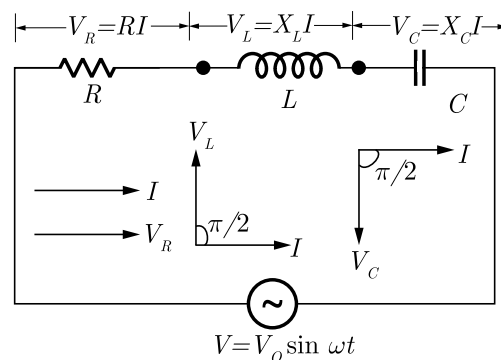
OD 2017, Foreign 2008

Assuming $X_L > X_C$,

$$V_L > V_C$$

Since, Net voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Where, V_L , V_C and V_R are alternating voltages across L , C and R respectively.



$$V_R = IR, V_L = IX_L,$$

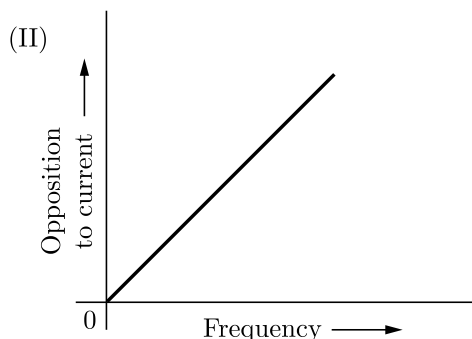
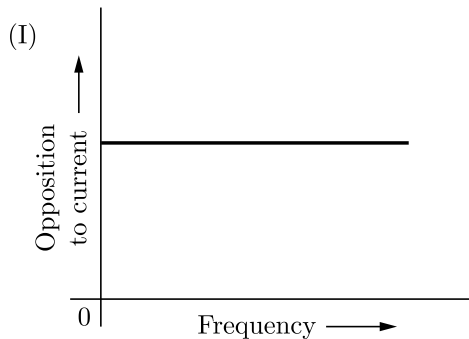
$$\begin{aligned}
 \text{or } P_{av} &= \frac{V_0 I_0}{2T} \left[\int_0^T \cos \phi \, dt - \int_0^T \cos(2\omega t + \phi) \, dt \right] \\
 &= \frac{V_0 I_0}{2T} [\cos \phi (t)_0^T - 0] \quad (\text{By trigonometry}) \\
 P_{av} &= \frac{V_0 I_0}{2T} \times \cos \phi \times T \\
 &= \frac{V_0 I_0}{2} \times \cos \phi \\
 P_{av} &= \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos \phi \\
 P_{av} &= V_{rms} \times I_{rms} \times \cos \phi
 \end{aligned}$$

This is the required expression.

(ii) Since, Power factor, $\cos \phi = \frac{R}{Z}$

Where, R = resistance and Z = impedance. Low power factor ($\cos \phi$) implies lower ohmic resistance and higher power loss as $P_{av} \propto \frac{1}{R}$ in power system (transmission line.)

83. (i) The graphs (I) and (II) represents the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit element corresponding to each graph.
- (ii) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC sources. Which will be ahead in phase in this circuit, voltage or current?



Ans :

SQP 2008

- (i) From graph (I), it is clear that resistance (opposition to current) is not changing with frequency, i.e., resistance does not depend on frequency of applied source, so the circuit element here is pure resistance (R).
- (ii) From graph (II), it is clear that resistance increase linearly with frequency, so the circuit element here is an inductor.

$$\begin{aligned}
 \text{Inductive resistance, } X_L &= 2\pi fL \\
 X_L &\propto f
 \end{aligned}$$

- (i) Impedance offered by the series combination of resistance R and inductor L .

$$\begin{aligned}
 Z &= \sqrt{R^2 + X_L^2} \\
 &= \sqrt{R^2 + (2\pi fL)^2}
 \end{aligned}$$

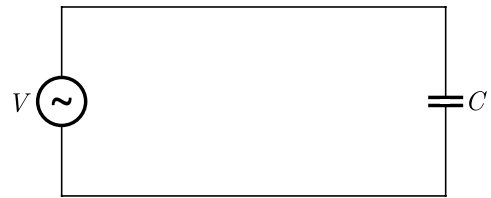
In LR circuit, the applied voltage leads the current in phase by $\frac{\pi}{2}$.

84. Show that the current leads the voltage in phase by $\frac{\pi}{2}$ in an AC circuit containing an ideal capacitor.

Ans :

Foreign 2014, Delhi 2001

Let us consider a capacitor C connected to an AC source as shown below



Let the AC voltage applied be,

$$V = V_m \sin \omega t \quad \dots(1)$$

$$\text{Hence, } V = \frac{q}{C} \quad \dots(2)$$

From eqs. (1) and (2), we have

$$V_m \sin \omega t = \frac{q}{C}$$

$$q = CV_m \sin \omega t$$

$$\text{Also, } i = \frac{dq}{dt}$$

$$i = \frac{d}{dt} (CV_m \sin \omega t)$$

$$i = \omega CV_m \cos \omega t \quad \dots(3)$$

$$\text{Now } \cos \omega t = \sin \omega t + \frac{\pi}{2} \quad \dots(4)$$

$$\text{in the circuit, } V_m = i_m X_c = i_m \frac{1}{\omega C}$$

$$i_m = V_m \omega C \quad \dots(5)$$

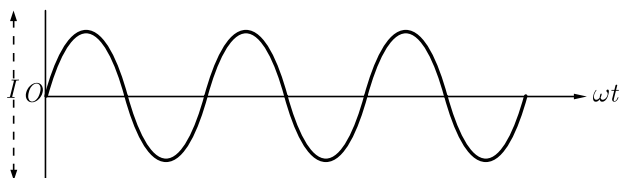
magnitude in the first half cycle and in the opposite direction during the next half cycle.

Thus current whose magnitude changes with time and direction reverses periodically is called alternating current.

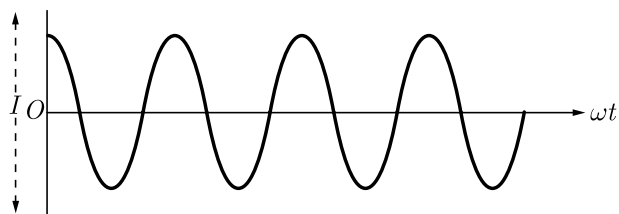
The instantaneous value of AC is given by,

$$I = I_0 \sin \omega t \text{ [Figure (a)]} \quad \dots(1)$$

or $I = I_0 \cos \omega t \text{ [Figure (b)]} \quad \dots(2)$



(a)



(b)

where, I_0 is the peak value of AC.

Similarly, the e.m.f. whose magnitude changes with time and direction reverses periodically is called alternating e.m.f.

The instantaneous alternating e.m.f. is given by,

$$E = E_0 \sin \omega t \quad \dots(3)$$

or $E = E_0 \cos \omega t \quad \dots(4)$

where, $\omega = \frac{2\pi}{T}$ is angular frequency.

Time Period (T)

Time period is the time taken to complete one cycle.

Frequency (ν)

Frequency is the number of cycles per second. So frequency,

$$\nu = \frac{1}{T}$$

Since, $T = \frac{2\pi}{\omega}$

Hence, $\nu = \frac{\omega}{2\pi}$

$$\omega = 2\pi\nu$$

Amplitude (I_0)

Amplitude of alternating current is the peak value of alternating current and is denoted by I_0 .

96. Define alternating current (AC), its peak value and its R.M.S. value. Derive relation between them after evaluating the expression for R.M.S. value.

Ans :

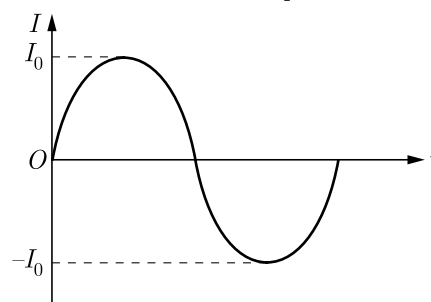
OD 2020

Alternating Current

If the direction of current changes alternatively (periodically) and its magnitude changes continuously with respect to time, then the current is called alternating current. It is sinusoidal (i.e. represented by sine or cosine angles) in nature. Alternating current can be defined as the current whose magnitude and direction changes with time and attains the same magnitude and direction after a definite time interval. It changes continuously between zero and a maximum value and flows in one direction in the first half cycle and in the opposite direction in the next half cycle.

The instantaneous value of AC is given by,

$$I = I_0 \sin \omega t \left[\text{Since, } \omega = \frac{2\pi}{T} = 2\pi\nu \right]$$



Current vs time graph of an AC

where, I = current at any instant t

I_0 = maximum/peak value of AC

ν = frequency

and ω = angular frequency

Root Mean Square (RMS) Value of AC

It is defined as that value of alternating current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle.

It is also called virtual value or effective value of AC.

It is represented by I_{rms} or I_{eff} or I_V . Suppose I is the current which flows in the resistor having resistance R in time T produces heat H .

Instantaneous value of AC,

$$I = I_0 \sin \omega t$$

If dH is small amount of heat produced in time dt in resistor R , then

$$dH = I^2 R dt \quad \dots(1)$$

88. (i) When an AC source is connected to an ideal capacitor, then show that the average power supplied by the source over a complete cycle is zero.
- (ii) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a DC and then an AC source. What happens in each case if the capacitance of the capacitor is reduced?

Ans :

SQP 2006

- (i) When a source of AC is connected to a capacitor of capacitance C_1 the charge on it glows from zero to maximum steady value Q_0 .

The energy stored in a capacitor is

$$E = \frac{1}{2} CV_0^2$$

Where, V_0 is maximum potential difference across the plates of the capacitor.

The alternating voltage applied is

$$E = E_0 \sin \omega t$$

and the current leads the emf by a phase angle of $\frac{\pi}{2}$.

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \\ = I_0 \cos \omega t$$

Hence, work done over a complete cycle is

$$W = \int_0^T E I dt \\ = \int_0^T (E_0 \sin \omega t) (I_0 \cos \omega t) dt \\ = \frac{E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t \\ W = \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t dt \\ = \frac{E_0 I_0}{2} \left[1 - \frac{\cos 2\omega t}{2\omega}\right]_0^T = 0$$

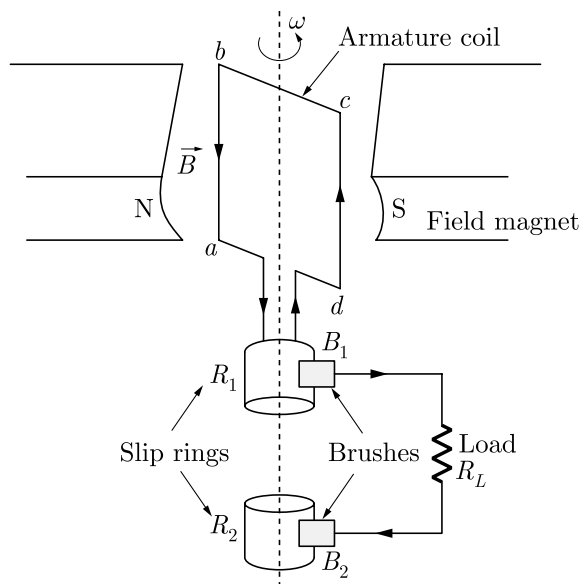
- (ii) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitive reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C_1 , X_C increases. Therefore, the glowing of the bulb reduces.

89. Draw a labelled diagram of an AC generator. Obtain the expression for the emf induced in the rotation coil of N turns each of cross sectional area A , in the presence of a magnetic field B .

Ans :

Delhi 2016, 2012



An AC generator converts mechanical energy into electric energy. It consists of a coil mounted on a rotor shaft. The axis of rotation of coil is perpendicular to magnetic field and the coil (armature) is mechanically rotated in magnetic field. Rotation causes magnetic flux. Hence an emf is induced in coil.

Let ω be angular velocity and θ be the angle between magnetic field and area vector, then $\theta = \omega t$, hence flux is

$$\phi_B = BA \cos \theta \\ = -NBA \frac{d}{dt}(\cos \omega t)$$

Thus, instantaneous emf is,

$$\varepsilon = NBA\omega \sin \omega t$$

If $NAB\omega = \varepsilon_0$

then, $\varepsilon = \varepsilon_0 \sin \omega t$

Since, function varies between ± 1 and emf for $\theta = 0^\circ$ to $\theta = 270^\circ$.

The direction of current change periodically, hence called alternating current.

90. A voltage $V = V_0 \sin \omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle.

Under what condition is (i) no power dissipated even though the current flows through the circuit (ii) maximum power dissipated in the circuit?

Ans :

Comp 2015

Let applied alternating voltage,

$$V = V_0 \sin \omega t \quad \dots(1)$$

Now

$$I = I_0 \sin(\omega t - \phi) \quad (2)$$

Under what condition is (i) no power dissipated even though the current flows through the circuit. (ii) maximum power dissipated in the circuit? Show that power dissipated at resonance is maximum.

Ans :

Delhi 2017

(a) Given, $V = V_m \sin \omega t$ and $i = i_m (\omega t + \phi)$

and instantaneous power, $P = Vi$

$$\begin{aligned} &= V_m \sin \omega t \cdot i_m \sin (\omega t + \phi) \\ &= V_m i_m \sin \omega t \sin (\omega t + \phi) \\ &= \frac{1}{2} V_m i_m [2 \sin \omega t \cdot \sin (\omega t + \phi)] \end{aligned}$$

From trigonometric formula,

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

Hence, instantaneous power,

$$\begin{aligned} P &= \frac{1}{2} V_m i_m [\cos (\omega t + \phi - \omega t) - \cos (\omega t + \phi + \omega t)] \\ &= \frac{1}{2} V_m i_m [\cos \phi - \cos (2\omega t + \phi)] \end{aligned}$$

Average power for complete cycle

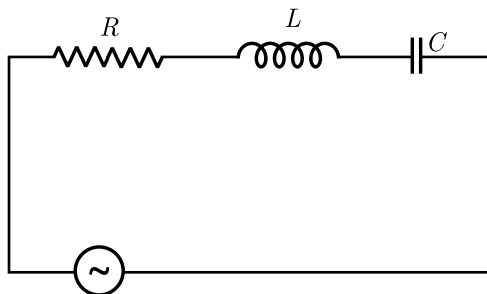
$$P_{av} = \frac{1}{2} V_m i_m [\cos \phi - \cos (2\omega t + \phi)] \quad \dots(1)$$

Where $\cos (2\omega t + \phi)$ is the mean value of $\cos (2\omega t + \phi)$ over complete cycle. But for a complete cycle, $\cos (2\omega t + \phi) = 0$

Since, Average power,

$$\begin{aligned} P_{av} &= \frac{1}{2} V_m i_m \cos \phi \\ &= \frac{V_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \phi \\ P_{av} &= V_{rms} i_{rms} \cos \phi \end{aligned}$$

(b) The power is $P = V_{rms} I_{rms} \cos \phi$. If $\cos \phi$ is small, then current considerably increases when voltage is constant. As we know, power loss is $I^2 R$. Hence, power loss increases.



The voltage $V = V_0 \sin \omega t$ is applied across the series LCR circuit. However due to impedance of the circuit either current lags or leads the voltage by phase opposite so the current in the circuit is given by $I = I_0 \sin (\omega t - \phi)$

Instantaneous power dissipation in the circuit.

$$\begin{aligned} P &= VI = V_0 \sin \omega t \times I_0 \sin (\omega t - \phi) \\ &= \frac{V_0 I_0}{2} \times 2 \sin \omega t \cdot \sin (\omega t - \phi) \\ &= \frac{V_0 I_0}{2} [\cos \phi - \cos (2\omega t - \phi)] \\ &[\cos (A - B) - \cos (A + B) = 2 \sin A \sin B] \end{aligned}$$

Average power loss over one complete cycle.

$$\begin{aligned} P &= \frac{1}{T} \int_0^T P dt \\ &= \frac{V_0 I_0}{2T} \left[\int_0^T \cos \phi dt - \int_0^T \cos (2\omega t - \phi) dt \right] \end{aligned}$$

However, $\int_0^T \cos (2\omega t - \phi) dt = 0$

$$\begin{aligned} &= \frac{V_0 I_0}{2T} \cdot \cos \phi \int_0^T dt \\ &= \frac{V_0 I_0}{2} \cos \phi \\ P_{av} &= \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos \phi \\ P_{av} &= V_{eff} \cdot I_{eff} \cos \phi \end{aligned}$$

- If phase angle $\phi = 90^\circ$ (resistance R is used in the circuit) then no power dissipated.
- If phase angle $\phi = 0^\circ$ or circuit is pure resistive (or $X_L = X_C$) at resonance then,

$$\text{Max power, } P = V_{eff} \times I_{eff} = \frac{V_0 I_0}{2}$$

At resonance, we get

$$\begin{aligned} X_C &= X_L \\ \tan \phi &= 0 \\ \phi &= 0^\circ \end{aligned}$$

$\cos \phi = 1$, its maximum value.

Hence, P has its maximum value at resonance

99. 1. An AC source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L , C and R . Use the phasor diagram to obtain expression for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called?
2. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1 . When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2 . Calculate $\frac{P_1}{P_2}$.

Ans :

OD 2003, Delhi 2020

- Expression for impedance in LCR series circuit. Suppose resistance R , inductance L and capacitance C are connected in series and an alternating source of voltage $V = V_0 \sin \omega t$ is

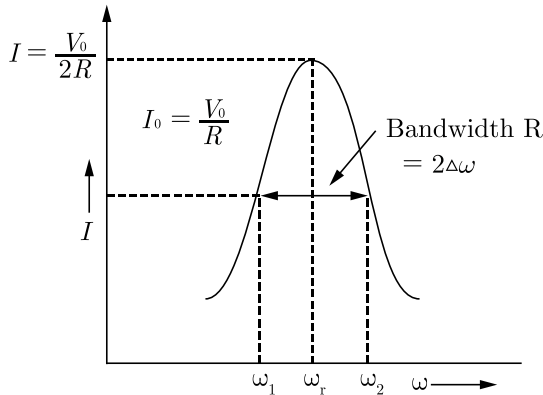
$$V_C = IX_C$$

$$\text{Net voltage, } V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance of LCR circuit,

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$



93. Radio-frequency choke is air cored, whereas audio frequency choke is iron cored. Explain.

Ans :

Delhi 2018

Radio Frequency Choke

The inductive reactance of choke is given by,

$$X_L = 2\pi\nu L$$

Radio frequency is very high frequency i.e. ν is large. So in order to keep X_L low, L should be low. This is possible only by using an air cored ($\mu = 1$) inductance.

Audio Frequency Choke

In audio frequency choke, the frequency is low i.e. ν is small. So to have reasonable value of X_L , L should be high hence iron should be used for which the value of μ is high (because the value of L increases with μ).

LONG ANSWER QUESTIONS

94. What is a choke coil? Discuss its use in AC circuits?

Ans :

Delhi 2021

A choke coil is simply an inductor with large value of self-inductance.

The choke coil consists of a large number of turns of thick copper wire bound closely over a laminated soft iron core. The inductive reactance offered by the coil is $X_L = \omega L$ becomes large. Therefore, magnitude of AC is reduced.

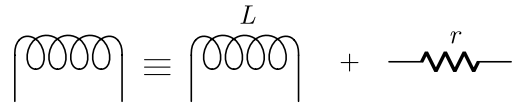
In an ideal choke coil, ohmic resistance = 0. Therefore, no electrical energy/power is wasted. A phase angle of

$\frac{\pi}{2}$ is introduced between current and voltage through an ideal inductor.

Hence, average power over a complete cycle is,

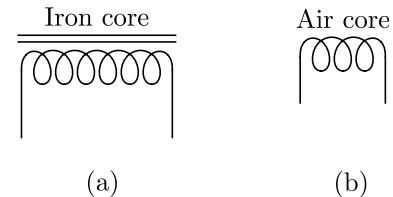
$$P = E_v I_v \cos \phi$$

$$= E_v I_v \cos 90^\circ = 0$$



However, in actual practice, a choke coil of inductance L is linked with a small resistance r . It may be treated as a series combination of L and r . As shown in the figure. Therefore, average power consumed over a complete cycle in the practical choke coil is

$$P = E_v I_v \cos \phi = E_v I_v \frac{r}{\sqrt{r^2 + \omega^2 L^2}}$$



In practice, a low frequency choke is made of insulated copper wire wound over a soft iron core [laminated] called low frequency choke or audio frequency choke figure (a) and a high frequency choke has air as core material figure (b) to eliminate eddy current losses. High frequency chokes are also called radio-frequency chokes.

Air is used as core material because, the virtual value of AC is given by,

$$I_v = \frac{E_v}{\omega L} = \frac{E_v}{2\pi\nu L}$$

So as ν is very high, so L should not be large, hence air cored choke is used.

95. What is alternating current? How alternating e.m.f. and current is represented mathematically. What do you mean by time period, frequency and amplitude of alternating current?

Ans :

Comp 2021, Delhi 2013

The current in which both the magnitude and direction undergo a complete cycle of changes periodically is called alternating current. It increases from zero to a maximum value, then decreases to zero and reverses in direction. It once again becomes maximum in this direction and then reduced to zero. This complete set of variations in magnitude and direction of current or e.m.f. is called a cycle. During one cycle, the current flows in one direction, of course, changing in

ratio in terms of voltages.

Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.

How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 100 W – 500 W refrigerator?

Ans :

Delhi 2016, SQP 2011

- (a) **Transformer :** Transformer is a device by which an alternating voltage may be decreased or increased. It is based on the principle of mutual-induction.

Construction : It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called primary coil and other is called the secondary coil. The terminals of primary coils are connected to AC mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types :

- (i) **Step-up Transformer :** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil (i.e., $N_s > N_p$).
- (ii) **Step-down Transformer :** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (i.e., $N_s < N_p$).

Working : When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_p be the number of turns in primary coil, N_s the number of turns in secondary coil and ϕ the magnetic flux linked with each turn. We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same. According to Faraday's laws the emf induced in the primary coil

$$\varepsilon_p = -N_p \frac{\Delta \phi}{\Delta t} \quad \dots(1)$$

and emf induced in the secondary coil

$$\varepsilon_s = -N_s \frac{\Delta \phi}{\Delta t} \quad \dots(2)$$

From eq. (1) and eq. (2)

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \quad \dots(3)$$

If the resistance of primary coil is negligible, the emf (ε_p) induced in the primary coil, will be equal to the applied potential difference (V_p) across its ends. Similarly if the secondary circuit is open, then the potential difference V_s across its ends will be equal to the emf (ε_s) induced in it; therefore,

$$\frac{V_s}{V_p} = \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} = r(\text{say}) \quad \dots(4)$$

Where $r = \frac{N_s}{N_p}$ is called the transformation ratio. If i_p and i_s are the instantaneous currents in primary and secondary coils and there is no loss of energy.

For about 100% efficiency,

Power in primary = power in secondary

$$V_p i_p = V_s i_s$$

$$\frac{i_s}{i_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{1}{r} \quad \dots(v)$$

In step-up transformer, $N_s > N_p \rightarrow r > 1$; So $V_s > V_p$ and $i_s < i_p$

i.e., Step up transformer increases the voltage.

In step down transformer, $N_s < N_p \rightarrow r < 1$; So $V_s < V_p$ and $i_s > i_p$

i.e., Step-up down transformer decreases the voltage, but increase the current.

The core of a transformer is laminated to reduce the energy losses due to eddy currents, so, that its efficiency may remain nearly 100%.

In a transformer with 100% efficiency (say), input power = output power $dV_p I_p = V_s I_s$.

- (b) The sources of energy loss in a transformer are
 - (i) eddy current losses due to iron core, (ii) flux leakage losses, (iii) copper losses due to heating up of copper wires, (iv) Hysteresis losses due to magnetisation and demagnetisation of core.
- (c) When output voltage increases, the output current automatically decreases to keep the power same. Thus, there is no violation of conservation of energy in a step-up transformer.

We have, $i_p V_p = i_s V_s = 550 \text{ W}$

$$V_p = 220 \text{ Volt}$$

$$i_p = \frac{550}{220} = \frac{5}{2} = 2.5 \text{ A}$$

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Ans :

Delhi 2012

Equation of e.m.f., $E = 4 \cos(100t)$ Inductance, $L = 0.03 \text{ H}$ Resistance, $R = 4 \Omega$

Standard equation of e.m.f. in an AC circuit,

$$E = E_0 \cos(\omega t)$$

Comparing the given equation with the standard equation, we get:

$$E_0 = 4$$

and $\omega = 100$

We also know that impedance of LR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L)^2} \\ &= \sqrt{R^2 + (\omega L)^2} \\ &= \sqrt{R^2 + \omega^2 L^2} \\ &= \sqrt{(4)^2 + [(100)^2 \times (0.03)^2]} \\ &= \sqrt{16 + 9} = \sqrt{25} = 5 \Omega \end{aligned}$$

Therefore, amplitude of current in the circuit,

$$I_0 = \frac{E_0}{Z} = \frac{4}{5} = 0.8 \text{ A}$$

- 125.** A circuit consists of a resistor, an inductor and a capacitor connected in series. If resistance of the resistor is 25Ω and the value of inductive reactance is equal to capacitive reactance, What is the net impedance of the circuit?

Ans :

OD 2017, Comp 2013

Resistance, $R = 25 \Omega$

According to the question,

Inductive reactance = capacitive reactance

i.e., $X_L = X_C$

Net impedance of the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(25)^2 + (X_L - X_L)^2} \\ &= 25 \Omega \end{aligned}$$

- 126.** In an experiment, 200 V AC is applied at the ends of an LCR-circuit. The circuit consists of an inductive reactance of 50Ω , capacitive reactance of 35Ω and ohmic resistance of 20Ω . What is the impedance of the circuit?

Ans :

Comp 2014

Supply voltage, $V = 200 \text{ Volt}$ Inductive reactance, $X_L = 50 \Omega$ Capacitive reactance, $X_C = 35 \Omega$ Ohmic resistance, $R = 20 \Omega$

Impedance of the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(20)^2 + (50 - 35)^2} \\ &= \sqrt{400 + 225} = \sqrt{625} = 25 \Omega \end{aligned}$$

- 127.** In a series combination, $R = 300 \Omega$, $L = 0.9 \text{ H}$, $C = 2 \mu\text{F}$, $\omega = 1000 \text{ rad} - \text{s}^{-1}$. What is the impedance of the LCR-circuit?

Ans :

OD 2016

Resistance, $R = 300 \Omega$ Inductance, $L = 0.9 \text{ H}$ Capacitance, $C = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$ Angular velocity, $\omega = 1000 \text{ rad} - \text{s}^{-1}$

Reactance of inductor,

$$X_L = \omega L = 1000 \times 0.9 = 900 \Omega$$

and reactance of capacitor,

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{1000 \times (2 \times 10^{-6})} \\ &= 500 \Omega \end{aligned}$$

Therefore, impedance of LCR-circuit

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(300)^2 + (900 - 500)^2} \\ &= \sqrt{90000 + 160000} \\ &= \sqrt{250000} = 500 \Omega \end{aligned}$$

- 128.** In a series LCR-circuit, resistance $R = 10 \Omega$ and the impedance $Z = 20 \Omega$. What is the phase difference between the current and the voltage?

Ans :

SQP 2015, OD 2010

Resistance, $R = 10 \Omega$ Impedance, $Z = 20 \Omega$

Impedance in the LCR-circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ 20 &= \sqrt{(10)^2 + (X_L - X_C)^2} \\ 400 &= 100 + (X_L - X_C)^2 \\ (X_L - X_C)^2 &= 400 - 100 = 300 \\ X_L - X_C &= \sqrt{300} = 10\sqrt{3} \end{aligned}$$

Phase difference between current and voltage,

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{X_L - X_C}{R} \right) \\ &= \tan^{-1} \left(\frac{10\sqrt{3}}{10} \right) \\ &= \tan^{-1}(\sqrt{3}) = 60^\circ \end{aligned}$$

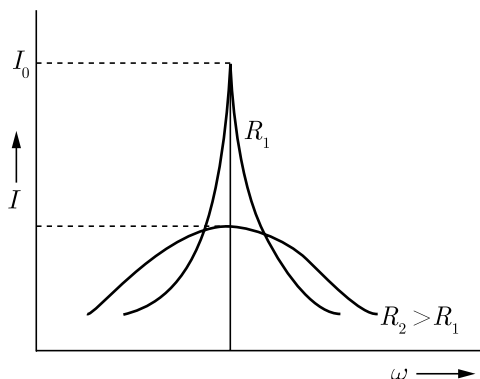
Phase angle between V_R and V_{eff} is

$$\begin{aligned}\cos \phi &= \frac{V_R}{V_{eff}} \\ &= \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}\end{aligned}$$

(i) $I = I_0 \sin(\omega t - \phi)$ For $V_L > V_C$ or $X_L > X_C$

(ii) $I = I_0 \sin(\omega t + \phi)$ For $V_L < V_C$ or $X_L < X_C$

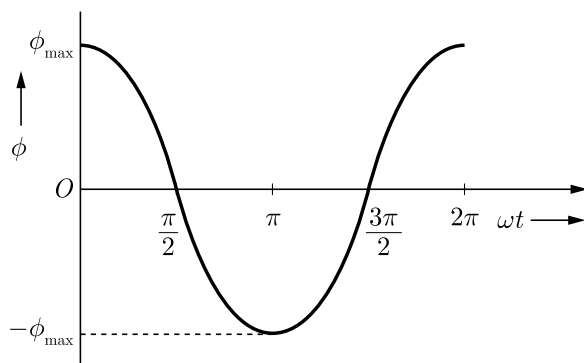
Variation of the current I as a function of angular frequency ω



At resonance, when maximum current flows through the circuit.

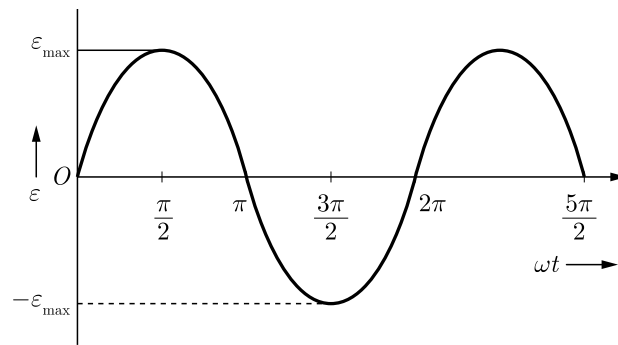
$$\begin{aligned}\omega_r &= \frac{1}{\sqrt{L_1 C_1}} = \sqrt{\frac{1}{L_2 C_2}} \\ L_1 C_1 &= L_2 C_2 \\ \frac{L_1}{L_2} &= \frac{C_2}{C_1}\end{aligned}$$

For fine tuning in the receiver set, combination $L_1 C_1$ and R_1 is better because maximum current flows through the circuit.



From lenz' law, induced emf.

$$\begin{aligned}\varepsilon &= -\frac{d\phi}{dt} = -|B||A|\frac{d}{dt}\cos\omega t \\ &= |B||A|\omega\sin\omega t\end{aligned}$$



103. State the working of AC generator with the help of a labelled diagram.

The coil of an AC generator having N turns, each of area A , is rotated with a constant angular velocity ω . Deduce the expression for the alternating emf generated in the coil.

What is the source of energy generation in this device?

Ans :

Comp 2019

AC Generator

A dynamo or generator is a device which converts mechanical energy into electrical energy.

Principle

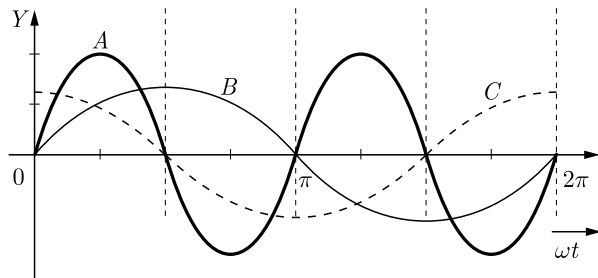
It works on the principle of electromagnetic induction. When a coil rotates continuously in a magnetic field, the effective area of the coil linked normally with the magnetic field lines, changes continuously with time. This variation of magnetic flux with time results in the production of an alternating emf in the coil.

Construction

It consists of the four main parts.

1. **Field Magnet :** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.
2. **Armature :** It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve on an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (i) It serves as a support to coils and (ii) It increases the magnetic field due to air core being replaced by an iron core.
3. **Slip Rings :** The slip rings R_1 and R_2 are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.
4. **Brushes :** There are two flexible metal plates or carbon rods (B_1 and B_2) which are fixed and constantly touch the revolving rings. The output current in external load R_L is taken through these brushes.

101. A device X is connected to an AC source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph :



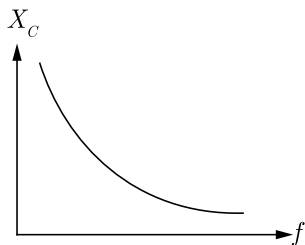
- Identify the device X .
- Which of the curves A , B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.
- How does its impedance vary with frequency of the AC source? Show graphically.
- obtain an expression for the current in the circuit and its phase relation with AC voltage.

Ans :

Comp 2017

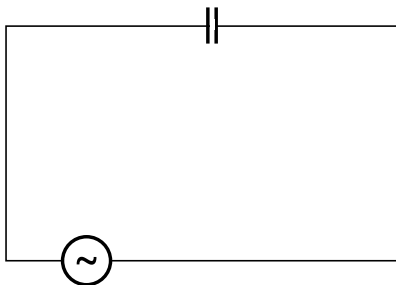
- Device is capacitor.
- $B \rightarrow$ Voltage (Because it is sine wave)
 $C \rightarrow$ Current (Because current leads voltage by $\frac{\pi}{2}$)
 $A \rightarrow$ Power (Average power over one cycle is zero)

(c) $X_C = \frac{1}{\omega C}$



$$X_C = \frac{1}{2\pi f C}$$

(d) $V = V_0 \sin \omega t$



$$C = \frac{q}{v}$$

$$q = cv_0 \sin \omega t$$

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d}{dt}(cv_0 \sin \omega t) \\ &= \omega cv_0 \sin \omega t \\ &= \frac{v_0}{\frac{1}{\omega C}} \cos \omega t \\ i &= \frac{v_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) \\ i &= i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}$$

In pure capacitive circuit leads voltage by $\frac{\pi}{2}$.

102. Using phaser diagram for a series LCR circuit connected to an AC source of voltage $V = V_0 \sin \omega t$, derive the relation for the current flowing in the circuit and the phase angle between the voltage across the resistor and the net voltage in the circuit.

Draw a plot showing the variation of the current I as a function of angular frequency ' ω ' of the applied AC source for the two cases of a series combination of

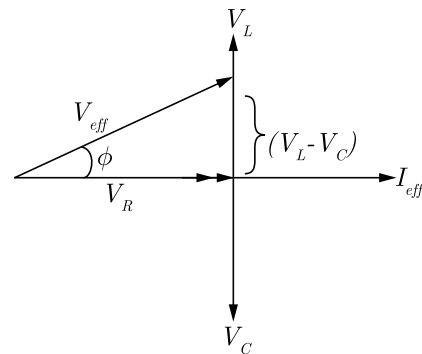
- inductance L_1 , Capacitance C_1 and resistance R_1 and
- inductance L_2 , Capacitance C_2 and resistance R_2 where $R_2 > R_1$

Write the relation between L_1 , C_1 and L_2 , C_2 at resonance. Which one of the two would be better suited for fine tuning in a receiver set? Give reason.

Ans :

Comp 2013, OD 2009

For I_{eff} flow of current through each element R , L and C , effective voltage across the combination can be given as.



$$\begin{aligned} \vec{V}_{eff} &= \hat{i} V_R + \hat{j} (V_L - V_C) \\ |V_{eff}| &= \sqrt{V_R^2 + (V_L - V_C)^2} \\ I_{eff} Z &= \sqrt{(I_{eff} R)^2 + (I_{eff} X_L - I_{eff} X_C)^2} \\ |Z| &= \sqrt{R^2 + (X_L - X_C)^2} \end{aligned}$$

Effective current flow,

$$I_{eff} = \frac{E_{eff}}{Z} = \frac{E_{eff}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{200}{200\sqrt{2}} = \frac{1}{\sqrt{2}}$$

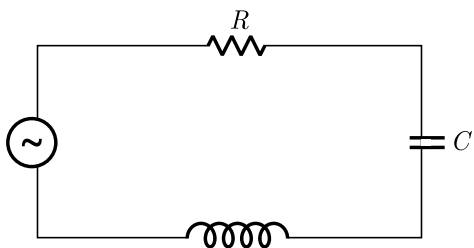
$$\phi = 45^\circ$$

(c) Voltage across resistors,

$$V_R = I_{\text{rms}} R$$

$$\begin{aligned} E_{\text{rms}} &= \frac{E_0}{\sqrt{2}} \Rightarrow \frac{100}{\sqrt{2}} \Rightarrow 50\sqrt{2} \\ &= \frac{50\sqrt{2} \times 200}{200\sqrt{2}} \quad \left[I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} \right] \\ &= 50 \text{ V} \end{aligned}$$

- 106.** The figure shows a series LCR circuit with $L = 100 \text{ H}$, $C = 40 \mu\text{F}$, $R = 60 \Omega$ connected to a variable frequency 240 V source. Calculate



- the angular frequency of the source which drives the circuit at resonance.
- the current at the resonating frequency.
- the rms potential drop across the inductor at resonance.

Ans :

OD 2021

Given,

$$L = 10 \text{ H}, C = 40 \mu\text{F},$$

$$R = 60 \Omega, V_{\text{rms}} = 240 \text{ Volt}$$

(i) resonating angular frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{10 \times 40 \times 10^{-6}}}$$

$$\omega_0 = \frac{1}{20 \times 10^{-3}} = 50 \text{ rad/s}$$

(ii) Current at resonating frequency,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R}$$

$$(\text{Since, at resonance, } Z = R)$$

$$= \frac{240}{60} = 4 \text{ A}$$

(iii) Since inductive reactance, $X_L = \omega L$

At resonance,

$$X_L = \omega_0 L = 50 \times 10 = 500 \Omega$$

Potential drop across to inductor,

$$V_{\text{rms}} = I_{\text{rms}} \times X_L$$

$$= 4 \times 500 = 2000 \text{ Volt}$$

- 107.** (a) For a given AC, $I = I_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} I_m^2 R$.

(b) A light bulb is rated at 100 W for a 220 V AC supply. Calculate the resistance of bulb.

Ans :

Delhi 2021

(a) Average power consumed in resistor R over a complete cycle.

$$\begin{aligned} P_{\text{av}} &= \frac{1}{T} \int_0^T i^2 R dt \\ &= \frac{i_m^2 R}{T} \int_0^T \sin^2 \omega t dt \quad \dots(1) \\ &= \frac{i_m^2 R}{2T} \int_0^T (1 - \cos 2\omega t) dt \\ &= \frac{i_m^2 R}{2T} \left[\int_0^T dt - \int_0^T \cos 2\omega t dt \right] \quad \dots(2) \\ &= \frac{i_m^2 R}{2T} [T - 0] = \frac{i_m^2 R}{2} \end{aligned}$$

(b) In case of AC,

$$\begin{aligned} P_{\text{av}} &= \frac{V_{\text{rms}}^2}{R} = \frac{V_{\text{eff}}^2}{R} \\ R &= \frac{V_{\text{rms}}^2}{P_{\text{av}}} = \frac{220 \times 220}{100} = 484 \Omega \end{aligned}$$

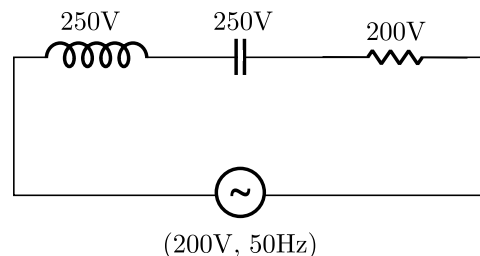
- 108.** A series LCR circuit is connected to an AC source (200 V , 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200 V , 250 V and 250 V .

- The algebraic sum of the voltage across the three elements is greater than the voltage of the source. How is this paradox resolved?
- Given the value of the resistance of R is 40Ω , calculate the current in the circuit.

Ans :

Comp 2020, Delhi 2007

(i) From given parameters $V_R = 200 \text{ V}$, $V_L = 250 \text{ V}$ and $V_C = 250 \text{ V}$



V_{eff} should be given as

$$\begin{aligned} V_{\text{eff}} &= V_R + V_L + V_C \\ &= 200 \text{ V} + 250 \text{ V} + 250 \text{ V} \\ &= 700 \text{ Volt} \end{aligned}$$

However, $V_{\text{eff}} > 200 \text{ V}$ of the AC source.

Working : When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. Considering the armature to be in vertical position and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in fig. In the external circuit, the current is shown in fig. In the external circuit, the current flows along $B_1 R_L B_2$. The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit it flows along $B_2 R_L B_1$. Thus the direction of induced emf and current changes in the external circuit after each half revolution.

Expression for Induced emf

If N is number of turns in coil, f the frequency of rotation, A area of coil and B the magnetic induction, then induced emf.

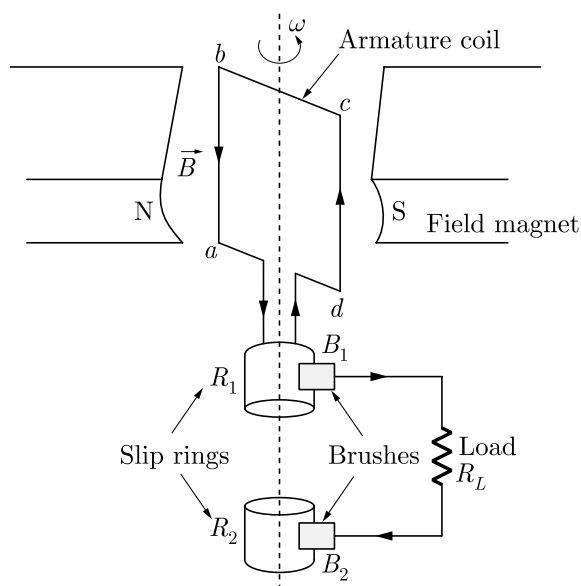
$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\{NBA(\cos 2\pi ft)\}$$

$$= 2\pi NBA f \sin 2\pi ft$$

Obviously, the emf produced is alternating and hence the current is also alternating.

Current produced by an AC generator cannot be measured by moving coil ammeter; because the average value of AC over full cycle is zero.

The source of energy generation is the mechanical energy or rotation of armature coil.



NUMERICAL QUESTIONS

104. An alternating voltage given by $V = 70 \sin 100\pi t$ is connected across a pure resistor of 25Ω . Find
- the frequency of the source.
 - the R.M.S. current through the resistor.

Ans :

Comp 2016

Given, $V = 70 \sin 100\pi t$

On comparing with standard equation,

$$V = V_0 \sin \omega t$$

$$V_0 = 70 \text{ Volt}$$

$$\omega = 100\pi \text{ rad/s}$$

(i) As,

$$\omega = 2\pi v = 100\pi$$

Frequency of the source,

$$U = 50 \text{ Hz}$$

(ii)

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{70}{\sqrt{2}}$$

Hence,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{\left(\frac{70}{\sqrt{2}}\right)}{25}$$

$$= \frac{70}{1.414 \times 25} = 1.98 \text{ A}$$

105. A series CR circuit with $R = 200 \Omega$ and $C = (50/\pi) \mu\text{F}$ is connected across an ac source of peak voltage $\varepsilon_n = 100 \text{ V}$ and frequency $v = 50 \text{ Hz}$. Calculate (a) impedance of the circuit (Z). (b) phase angle (ϕ) and (c) voltage across the resistor.

Ans :

SQP 2023, OD 2012

In the given circuit,

$$R = 200 \Omega$$

$$C = \left(\frac{50}{\pi}\right) \mu\text{F}$$

Peak voltage, $E_0 = 100 \text{ V}$

Frequency, $v = 50 \text{ Hz}$

(a) Impedance, $Z = \sqrt{R^2 + X_C^2}$

$$Z = \sqrt{(200)^2 + \left(\frac{1}{C\omega}\right)^2}$$

$$= \sqrt{(200)^2 + \frac{1}{\left(\frac{50}{\pi} \times 10^{-6} \times 100\pi\right)}}$$

$$[\omega = 2\pi n = 2\pi \times 50 = 100\pi]$$

$$= \sqrt{(200)^2 + (200)^2} = 200\sqrt{2} \Omega$$

(b) Phase angle (ϕ)

$$\cos \phi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{R}{Z}$$

$$\begin{aligned}
 C' &= \frac{1}{L \times 10^6} = \frac{1}{100 \times 10^{-3} \times 10^6} \\
 &= \frac{10}{10^6} = \frac{1}{10^5} = 10^{-5} \\
 C' &= 10^{-5} \text{ F} \\
 &= 10 \times 10^{-6} \text{ F} = 10 \mu\text{F}
 \end{aligned}$$

As, $C' > C$. Hence we have to add an additional capacitor of capacitance $8 \mu\text{F}$ ($10 \mu\text{F} = 2 \mu\text{F}$) in parallel with previous capacitor.

- 110.** A power transmission line needs input power at 2300 V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary winding in order to get output power at 230 V?

Ans :

Delhi 2019

Given $V_P = 2300 \text{ V}$, $N_P = 4000$ turns, $V_S = 230 \text{ V}$, $N_S = ?$

We have,

$$\begin{aligned}
 \frac{V_S}{V_P} &= \frac{N_S}{N_P} \\
 N_S &= \frac{V_S}{V_P} \times N_P \\
 &= \frac{230}{2300} \times 4000 \\
 &= 400 \text{ turns}
 \end{aligned}$$

- 111.** A circuit is set up by connecting inductance $L = 100 \text{ mH}$, resistor $R = 100 \Omega$ and a capacitor of reactance 200Ω in series. An alternating emf of $150\sqrt{2} \text{ V}$, $\frac{500}{\pi} \text{ Hz}$ is applied across this series combination. Calculate the power dissipated in the resistor.

Ans :

OD 2019

Here, $L = 100 \times 10^{-3} \text{ H}$, $R = 100 \Omega$

$X_C = 200 \Omega$, $V_{\text{rms}} = 150\sqrt{2} \text{ Volt}$

$$V = \frac{500}{\pi} \text{ Hz}$$

Inductive reactance,

$$\begin{aligned}
 X_L &= \omega L \\
 &= 2\pi v L \\
 &= 2 \times \pi \times \frac{500}{\pi} \times 100 \times 10^{-3} \\
 &= 100
 \end{aligned}$$

Impedance of circuit,

$$\begin{aligned}
 Z &= \sqrt{R^2 + (X_C - X_L)^2} \\
 &= \sqrt{(100)^2 + (200 - 100)^2} \\
 &= \sqrt{20000} = 100\sqrt{2}
 \end{aligned}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{150\sqrt{2}}{100\sqrt{2}} = \frac{3}{2}$$

$$\text{Power dissipated} = (I_{\text{rms}})^2 R = \frac{9}{4} \times 100 = 225 \text{ W}$$

- 112.** An alternating voltage given by $V = 280 \sin 50\pi t$ is connected across a pure resistor of 40Ω . Find

- (i) the frequency of the source.
(ii) the r.m.s. current through the resistor.

Ans :

OD 2012, Foreign 2003

Given, $V = 280 \sin 50\pi t$, $R = 40 \Omega$

Comparing it with standard equation,

$$V = V_0 \sin \omega t$$

$$V_0 = 280 \text{ Volt}$$

$$\omega = 50\pi \text{ rad/s}$$

(i) As, $\omega = 2\pi v$

$$2\pi v = 50\pi$$

Frequency of AC, $v = \frac{50}{2} = 25 \text{ Hz}$

(ii)

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{280}{\sqrt{2}} \text{ V}$$

Hence, $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{280}{\sqrt{2} \times 40}$

$$I_{\text{rms}} = \frac{280}{1.414 \times 40} = 4.95 \text{ A}$$

- 113.** The power factor of an AC circuit is 0.5. What is the phase difference between voltage and current in the circuit?

Ans :

Delhi 2017

$$\cos \theta = 0.5$$

$$\cos \theta = \cos 60^\circ$$

$$\theta = 60^\circ$$

$$\text{Phase angle} = 60^\circ$$

- 114.** The equation of alternating current is $I = 20 \sin 200 \pi t$. Find out the frequency, peak value and r.m.s. value of the current.

Ans :

SQP 2015

Given, $I = 20 \sin 200 \pi t$... (1)

The standard equation of alternating current is given by,

$$I = I_0 \sin \omega t \quad \dots (2)$$

where, I_0 = Peak value of current

After comparing Eq. (1) with (2), we get

Peak value of current, $I_0 = 20 \text{ A}$

$$\omega = 200 \pi$$

$$2\pi f = 200\pi \quad (\text{Since, } \omega = 2\pi f)$$

This paradox can be solved only by using phaser diagram, as given below.

$$V_{eff} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Since, $V_L = V_C$

so, $V_R = 200 \text{ Volt}$

(ii) Given, $R = 40 \Omega$, so current in the LCR circuit.

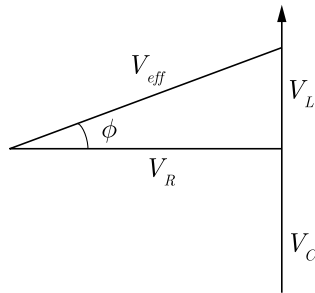
$$I_{eff} = \frac{V_{eff}}{R}$$

$$X_L = X_C$$

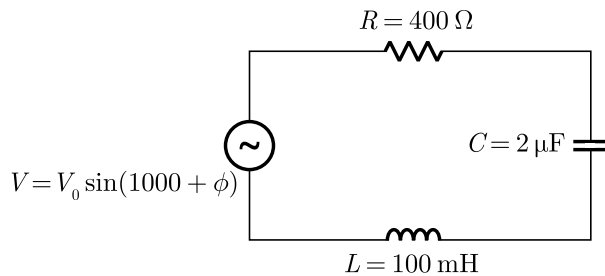
or

$$Z = R$$

$$= \frac{200}{40} = 5 \text{ A}$$



109. (i) Determine the value of phase difference between the current and the voltage in the given series LCR circuit.

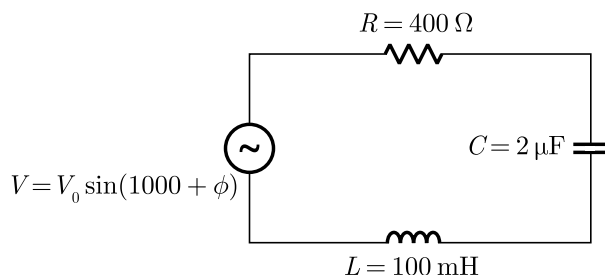


- (ii) Calculate the value of additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity.

Ans :

Delhi 2020

- (i) Consider the given figure



Since, the alternating emf in the above LCR series circuit would be represented by

$$V = V_0 \sin(1000t + \phi)$$

$$\omega = 1000 \text{ Hz}$$

Given, $R = 400 \Omega$, $C = 2 \mu\text{F}$, $L = 100 \text{ mH}$

Hence, capacitive reactance,

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{1000 \times 2 \times 10^{-6}}$$

$$X_C = \frac{10^3}{2}$$

$$X_C = 500 \Omega$$

Since, inductive reactance,

$$X_L = \omega L$$

$$X_L = 1000 \times 100 \times 10^{-3}$$

$$X_L = 100 \Omega$$

So, we can see that $X_C > X_L$, $\tan \phi$ is negative.

Hence, the voltage lags behind the current by a phase angle ϕ . The AC circuit is capacitance dominated circuit.

Since, phase difference,

$$\tan \phi = \frac{X_L - X_C}{R}$$

$$\tan \phi = \frac{100 - 500}{400}$$

$$\tan \phi = \frac{-400}{400}$$

$$\tan \phi = -1$$

$$\tan \phi = -\tan\left(\frac{\pi}{4}\right)$$

$$\phi = -\frac{\pi}{4}$$

This is the required value of the phase difference between the current and the voltage in the given series LCR circuit.

(ii) Suppose, new capacitance of the circuit is C' .

Thus, to have power factor unity

$$\cos \phi' = 1 = \frac{R}{\sqrt{R^2 + (X_L - X'_C)^2}}$$

$$R^2 = R^2 + (X_L - X'_C)^2$$

$$X_L = X'_C \text{ or } \omega L = \frac{1}{\omega C'}$$

$$\omega^2 = \frac{1}{LC'}$$

$$(1000)^2 = \frac{1}{LC'} \quad (\text{Since, } \omega = 1000)$$

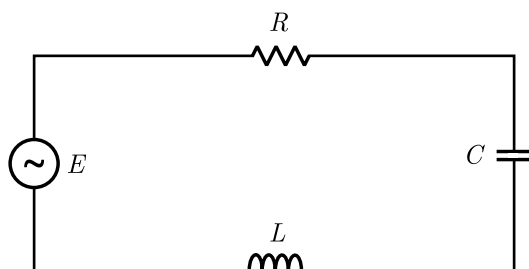
$$= \frac{150\sqrt{2}}{100\sqrt{2}} = \frac{3}{2}$$

$$\text{Power dissipated} = (I_{rms})^2 R$$

$$= \frac{9}{4} \times 100 = 225 \text{ W}$$

- 120.** The figure shows a series LCR circuit connected to a variable frequency 200 V source with $L = 50 \text{ mH}$, $C = 80 \mu\text{F}$ and $R = 40 \Omega$.

1. The source frequency which derives the circuit in resonance.
2. The quality factor Q of the circuit.



Ans :

Comp 2015

Given, $L = 50 \text{ mH} = 50 \times 10^{-3} \text{ H}$

$$C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$$

$$R = 40 \Omega$$

$$V = 200 \text{ Volt}$$

1. In the LCR circuit the resonant angular frequency is given by,

$$\omega_0 = 2\pi v$$

$$v = \frac{\omega}{2\pi}$$

$$v = \frac{500}{2\pi}$$

$$= \frac{250}{\pi} = 79.61 \approx 80 \text{ Hz}$$

2. Quality factor, $Q = \frac{\omega_0 L}{R}$

$$= \frac{500 \times 50 \times 10^{-3}}{40} = 0.625$$

- 121.** Calculate the quality factor of a series LCR circuit with $L = 2.0 \text{ H}$, $C = 2 \mu\text{F}$ and $R = 10 \Omega$. Mention the significance of quality factor in LCR circuit.

Ans :

Delhi 2013

Given, $L = 2.0 \text{ H}$

$$C = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$$

$$R = 10 \Omega$$

Now, $Q\text{-factor} = \frac{1}{R} \sqrt{\frac{L}{C}}$

$$= \frac{1}{10} \sqrt{\frac{2}{2 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 10^{-3}}$$

$$= \frac{1}{10^{-2}} = 100$$

Quality factor is also defined as,

$$Q = 2\pi f \times \frac{\text{Energy stored}}{\text{Power loss}}$$

So, higher the value of Q means the energy loss is at lower rate relative to the energy stored, i.e. the oscillations will die slowly and damping would be less.

- 122.** What is the value of current, flowing through an inductor of inductance 1 H and having negligible resistance when connected to an AC source of 200 V and 50 Hz ?

Ans :

SQP 2007

Inductance, $L = 1 \text{ H}$

Voltage, $V = 200 \text{ Volt}$

Frequency, $f = 50 \text{ Hz}$

Angular frequency,

$$\omega = 2\pi f = 2 \times 3.14 \times 50$$

$$= 314 \text{ rad-s}^{-1}$$

Therefore, reactance of the inductor,

$$X_L = \omega L = 314 \times 1 = 314 \Omega$$

We also know that current flowing through the inductor,

$$I = \frac{V}{X_L} = \frac{200}{314} = 0.64 \text{ A}$$

- 123.** Reactance of a capacitor of capacitance C for an alternating current of frequency $\frac{400}{\pi} \text{ Hz}$ is 25Ω . What is the value of C ?

Ans :

Foreign 2006, OD 2009

Frequency of current, $f = \frac{400}{\pi} \text{ Hz}$

Reactance of capacitor, $X_C = 25 \Omega$

Reactance of the capacitor,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Capacitance of capacitor,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times \frac{400}{\pi} \times 25}$$

$$= 50 \times 10^{-6}$$

$$F = 50 \mu\text{F}$$

- 124.** In an AC circuit, the equation of e.m.f is: $E = 4 \cos(100t)$. What is the amplitude of current in an LR-circuit of inductance 0.03 H and resistance ?

Frequency, $f = 100 \text{ Hz}$

Now $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

Hence, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14 \text{ A}$

- 115.** A light bulb is rated 100 W for 220 V AC supply of 50 Hz. Calculate:

1. The resistance of the bulb.
2. The r.m.s. current through the bulb.

Ans :

Foreign 2010

$$1. \quad R = \frac{V_{\text{rms}}^2}{P} = \frac{220 \times 220}{100} = 484 \Omega$$

$$2. \quad I_{\text{rms}} = \frac{P}{V_{\text{rms}}} = \frac{100}{220} = 0.45 \text{ A}$$

- 116.** A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used.

Ans :

Delhi 2013

Given:

$$P_L = 60 \text{ W}$$

$$I_L = 0.54 \text{ A}$$

$$V_L = \frac{P_L}{I_L} = \frac{60}{0.54} = 111.11 \approx V$$

The transformer is step and have $\frac{1}{2}$ input voltage.

$$\text{Hence, } i_p = \frac{1}{2} \times I_L = \frac{1}{2} \times 0.54 = 0.27 \text{ A}$$

- 117.** An alternating voltage given by $V = 140 \sin 314 t$ is connected across a pure resistor of 50Ω . Find:

1. The frequency of the source.
2. The rms current through the resistor.

Ans :

OD 2017

1. Given:

$$V = 140 \sin 314 t$$

$$\text{and } R = 50 \Omega$$

Comparing it with,

$$V = V_0 \sin \omega t$$

$$\text{Here, } \omega = 314 \text{ rad/s}$$

$$\text{i.e., } 2\pi v = 314$$

$$v = \frac{314}{2\pi} \approx 50 \text{ Hz}$$

Frequency of AC,

$$v = 50 \text{ Hz}$$

2. As,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

$$\text{and } V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

$$\text{Here, } V_0 = 140 \text{ V}$$

$$V_{\text{rms}} = \frac{140}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 70\sqrt{2} \text{ V}$$

$$\text{Hence, } I_{\text{rms}} = \frac{70\sqrt{2}}{R} = \frac{70\sqrt{2}}{50} = 1.9 \text{ A} \approx 2 \text{ A}$$

- 118.** A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate:

1. The resistance of the bulb.
2. The rms current through the bulb.

Ans :

Foreign 2008, SQP 2014

1. $P = 150 \text{ W}$ and $V = 220 \text{ Volt}$

Resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{220 \times 220}{150} = 322.7 \Omega$$

$$\begin{aligned} 2. \text{ As, } I_{\text{rms}} &= \frac{V_{\text{rms}}}{R} \\ &= \frac{220}{322.7} \quad (V_{\text{rms}} = V = 220 \text{ Volt}) \\ &= 0.68 \text{ A} \end{aligned}$$

- 119.** A circuit is set up by connecting inductance $L = 100 \text{ mH}$, resistor $R = 100 \Omega$ and a capacitor of reactance 200Ω in series. An alternating emf of $150 \sqrt{2} \text{ V}$, $\frac{500}{\pi} \text{ Hz}$ is applied across this series combination. Calculate the power dissipated in the resistor.

Ans :

SQP 2011

$$\text{Given, } L = 100 \times 10^{-3} \text{ H, } R = 100 \Omega$$

$$X_C = 200 \Omega$$

$$V_{\text{rms}} = 150\sqrt{2} \text{ Volt}$$

$$v = \frac{500}{\pi} \text{ Hz}$$

$$\text{Inductive reactance, } X_L = \omega L = 2\pi v L$$

$$\begin{aligned} &= 2 \times \pi \times \frac{500}{\pi} \times 100 \times 10^{-3} \\ &= 100 \Omega \end{aligned}$$

$$\begin{aligned} \text{Impedance of circuit, } Z &= \sqrt{R^2 + (X_C - X_L)^2} \\ &= \sqrt{(100)^2 + (200 - 100)^2} \\ &= \sqrt{20000} \\ &= 100\sqrt{2} \end{aligned}$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

- 129.** In a LCR-circuit, the potential difference between the terminals of the inductance is 60 V, between terminals of the capacitor is 30 V and that between the terminals of the resistance is 40 V. What is the supply voltage?

Ans :

Comp 2011

Potential difference across inductance,

$$V_L = 60 \text{ Volt}$$

Potential difference across capacitor,

$$V_C = 30 \text{ Volt}$$

and potential difference across resistance,

$$V_R = 40 \text{ Volt}$$

Supply voltage in LCR-circuit,

$$\begin{aligned} V &= \sqrt{V_R^2 + (V_L - V_C)^2} \\ &= \sqrt{(40)^2 + (60 - 30)^2} \\ &= \sqrt{1600 + 900} \\ &= \sqrt{2500} = 50 \text{ Volt} \end{aligned}$$

- 130.** In an ideal transformer, voltage and current in primary are 200 V and 2 A respectively. If voltage in the secondary is 2000 V, what is the current in the secondary coil of transformer?

Ans :

SQP 2008

Voltage in primary, $V_P = 200 \text{ Volt}$

Current in primary, $I_P = 2 \text{ A}$

Voltage in secondary, $V_S = 2000 \text{ Volt}$

Relation between currents and voltages in primary and secondary is,

$$\begin{aligned} \frac{I_P}{I_S} &= \frac{V_S}{V_P} \\ I_S &= I_P \times \frac{V_P}{V_S} \\ &= 2 \times \frac{200}{2000} = 0.2 \text{ A} \end{aligned}$$

where, I_S = current in secondary

- 131.** The ratio of numbers of turns of primary coil to secondary coil in a transformer is 2 : 3. If a cell of 6 V is connected across the primary coil, What is the voltage across the secondary coil?

Ans :

Comp 2018

No. of turns in primary,

$$N_P = 2 \text{ N}$$

No. of turns in secondary coil,

$$N_S = 3 \text{ N}$$

Supply voltage on primary,

$$V_P = 6 \text{ Volt}$$

Relation between voltages and no. of turns in primary

and secondary is,

$$\begin{aligned} \frac{V_P}{V_S} &= \frac{N_P}{N_S} \\ V_S &= V_P \times \frac{N_S}{N_P} \\ &= 6 \times \frac{3 \text{ N}}{2 \text{ N}} = 9 \text{ Volt} \end{aligned}$$

where, V_S = voltage across secondary coil

- 132.** A step-up transformer has transformation ratio of 5:3. What is the voltage in the secondary, if voltage in the primary is 60 V?

Ans :

OD 2013

No. of turns in secondary, $N_S = 5 \text{ N}$

No. of turns in primary, $N_P = 3 \text{ N}$

Voltage in primary, $V_P = 60 \text{ Volt}$

Relation between voltages and number turns in primary and secondary is,

$$\begin{aligned} \frac{V_S}{V_P} &= \frac{N_S}{N_P} \\ V_S &= V_P \times \frac{N_S}{N_P} \\ &= 60 \times \frac{5 \text{ N}}{3 \text{ N}} = 100 \text{ Volt} \end{aligned}$$

where, V_S = voltage in secondary

- 133.** A step-up transformer operates on a 230 V line supplies a current of 2 A. The ratio of primary and secondary winding is 1:25. What is the current in the primary coil?

Ans :

Foreign 2010, Comp 2005

Voltage in primary, $V_P = 230 \text{ Volt}$

Current in secondary, $I_S = 2 \text{ A}$

No. of windings in primary,

$$N_P = N$$

No. of windings in secondary,

$$N_S = 25 \text{ N}$$

Relation between currents and no. of turns in primary and secondary is,

$$\begin{aligned} \frac{I_P}{I_S} &= \frac{N_S}{N_P} \\ I_P &= I_S \times \frac{N_S}{N_P} \\ &= 2 \times \frac{25 \text{ N}}{N} = 50 \text{ A} \end{aligned}$$

where, I_P = current in primary

- 134.** In a transformer, the output current and voltage are respectively 4 A and 20 V. If ratio of no. of turns in primary to secondary is 2 : 1, what is the input current and voltage?

Ans :

Output current in secondary,

$$I_s = 4 \text{ A}$$

Output Voltage in secondary,

$$V_s = 20 \text{ Volt}$$

No. of turns in primary,

$$N_p = 2 \text{ N}$$

and no. of turns in secondary,

$$N_s = N$$

Relation between currents and number of turns in primary and secondary is,

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$I_p = I_s \times \frac{N_s}{N_p} \\ = 4 \times \frac{N}{2 \text{ N}} = 2 \text{ A}$$

where, I_p = current in primary

We also know that relation between voltage and no. of turns in primary and secondary is:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\text{or} \quad V_p = V_s \times \frac{N_p}{N_s} \\ = 20 \times \frac{2 \text{ N}}{N} = 40 \text{ Volt}$$

where, V_p = input voltage in primary

- 135.** In a series LCR circuit with an AC source of effective voltage 50V, frequency $v = \frac{50}{\pi}$ Hz, $R = 300 \Omega$, $C = 20 \mu\text{F}$ and $L = 1.0 \text{ H}$. Find the r.m.s. current in the circuit.

Ans :

Comp 2017

Given,

$$L = 1.0 \text{ H},$$

$$C = 20 \mu\text{F} = 20 \times 10^{-6} \text{ F}$$

$$R = 300 \Omega$$

$$V_{\text{rms}} = 50 \text{ Volt}$$

and

$$v = \frac{50}{\pi} \text{ Hz}$$

Inductive reactance,

$$X_L = \omega L = 2\pi v L \\ = 2 \times \pi \times \frac{50}{\pi} \times 1 = 100 \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C} \\ = \frac{1}{2 \times \pi \times \frac{50}{\pi} \times 20 \times 10^{-6}}$$

$$= 500 \Omega$$

Impedance of circuit,

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \\ = \sqrt{(300)^2 + (500 - 100)^2} \\ = \sqrt{90000 + 160000} \\ = \sqrt{250000} = 500 \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50}{500} = 0.1 \text{ A}$$

- 136.** Determine the current quality factor at resonance for a series LCR circuit with $L = 1.00 \text{ mH}$ and $C = 1.00 \text{ nF}$ and $R = 100 \Omega$ connected to an AC source having peak voltage of 100 V.

Ans :

OD 2014, Delhi 2009

Given,

$$L = 1.00 \text{ mH} = 1 \times 10^{-3} \text{ H}$$

$$C = 1.00 \text{ nF} = 1 \times 10^{-9} \text{ F}$$

$$R = 100 \Omega$$

$$E_0 = 100 \text{ Volt}$$

Now

$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \\ = \frac{E_0}{Z} \left\{ \begin{array}{l} \text{at resonance } \omega L = \frac{1}{\omega C} \\ \text{Hence, } Z = R \end{array} \right\}$$

Hence,

$$I = \frac{V}{R} = \frac{100}{100}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ = \frac{1.44}{2}$$

$$= 0.707 \text{ A} \quad [\text{Hence } I_0 = IA]$$

$$Q = \frac{1}{2} \sqrt{\frac{L}{C}} \\ = \frac{1}{100} \sqrt{\frac{10 \times 10^{-3}}{1.0 \times 10^{-9}}} \\ = \frac{1}{100} \times 10^3 = 10$$

CASE BASED QUESTIONS

- 137.** A metal detector is an electronic instrument which detects the presence of metal nearby. Metal detectors are useful for finding metal inclusions hidden within objects, or metal objects buried underground. They often consist of a handheld unit with a sensor probe which can be swept over the ground or other objects.



- (i) On what principle does a metal detector work?
- (ii) Why does the detector emit sound when a person carrying any metallic object walks through it?

Ans :

OD 2015

- (i) Metal detector works on the principle of resonance in AC circuits.
- (ii) When a person carrying any metallic object walks through the gate of a metal detector, the impedance of the circuit changes, resulting in significant change in current in the circuit that causes a sound to be emitted as an alarm.

- 138.** Deepak had a high tension tower erected on his farm land. He kept complaining to the authorities to remove it as it was occupying a large portion of his land. His uncle, who was a teacher, explained to him the need for erecting these towers for efficient transmission of power. As Deepak realised its significance, he stopped complaining. Answer the following questions.

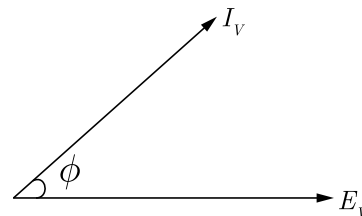


- (i) Why is it necessary to transport power at high voltage?
- (ii) A low power factor implies large power loss. Explain.

Ans :

SQP 2008, OD 2015

- (i) Power loss in the form of heating through wires is given by $I^2 R$.



If the power is transmitted at high voltage then low current flows through wires and hence there is less power loss $P = VI$.

If V increases, I decreases.

- (ii) In AC $P_{av} = E_v I_v \cos \phi$

Where $\cos \phi$ is power factor and ϕ is the phase between E_v and I_v .

Smaller value of $\cos \phi$ will result in higher value of current which will result in more power loss. Hence, low power factor implies large power loss.

- 139.** Rakesh is a student of class x in a school near his village. His uncle gifted him a bicycle with a dynamo fitted in it. He was thrilled to find that while cycling during night, he could light the bulb and see the objects on the road clearly. He, however, did not know how this device works. He asked this question to his teacher. The teacher considered it an opportunity and explained the working of dynamo to the whole class.



- (i) What is the principle of a dynamo?
- (ii) Explain the working of dynamo

Ans :

OD 2015

- (i) It is based on Faraday law of electromagnetic induction.
- (ii) The changing magnetic flux through the coil provides the necessarily induced emf across the ends of the coil.

Working The coil (called armature) is mechanically rotated in the uniform magnetic field by some

external means the rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.

When a coil of N turns, each of area A rotates in uniform magnetic field $\vec{B}$, the magnitude of induced emf can be given as

$$E = NBA\omega \sin \omega t$$

140. A group of students while coming from the school noticed a box marked “Danger H.T. 2200 V” at a substation in the main street. They did not understand the utility of a such a high voltage, while they argued, the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore explained to the whole class.



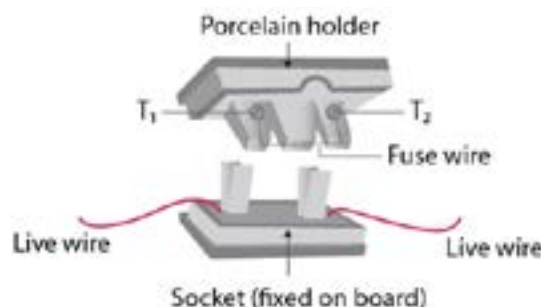
- What device is used to bring the high voltage down to low voltage of AC current and what is the principle of its working?
- Is it possible to use this device for bringing down the high DC voltage to the low voltage? Explain.

Ans :

OD 2015

- Step-down transformer
Mutual induction Whenever an alternative voltage is applied in the primary windings, an emf is induced in the secondary coil.
- No, transformer cannot bring down DC voltage since no mutual induction will take place because flux will not change in DC current.

141. One morning an old man walked bare-foot to replace the fuse wire in kit kat fitted with the power supply mains for his house. Suddenly he screamed and collapsed on the floor. His wife cried loudly for help. His neighbour's son Vishal heard the cries and rushed to the place with shoes on. He took a wooden baton and used it to switch off the main supply.



- What is the voltage and frequency of mains supply in India?
- These days most of the electrical devices we use require AC voltage, Why?
- Can a transformer be used to step up DC voltage?

Ans :

OD 2015

- In most generators, the coil is held stationary and electromagnets are rotated at 50 rev/sec, so the frequency of the current is 50 Hz and voltage supply to the consumer is at 220 V.
- AC supply can be stopped up/stopped down.
 - It can be converted into DC by using full wave rectifiers.
 - Power loss in transmission line is minimum.
- No, because induced emf in the secondary coil of the transformed is only due to time rate of change of magnetic flux.

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CHAPTER 8

Electromagnetic Wave

SUMMARY

1. DISPLACEMENT CURRENT

Ampere's circuital law for conduction current during charging of a capacitor was found inconsistent therefore Maxwell modified Ampere's circuital law by introducing displacement current. It is given by

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

Modified Ampere's circuital law is,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

where, ϕ_E = electric flux

2. CHARACTERISTICS OF ELECTROMAGNETIC WAVES

1. Speed of electromagnetic wave,

$$c = \frac{E_0}{B_0} = \frac{c}{\sqrt{\mu_0 \epsilon_0}}$$

2. The speed of electromagnetic waves in a material medium is given by,

$$v = \frac{1}{\sqrt{\mu \epsilon}} \\ = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{n}$$

where, n is the refractive index.

3. Energy associated with an electromagnetic wave is,

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

4. Linear momentum delivered to the surface, $p = \frac{U}{c}$.
where, U = total energy transmitted by electromagnetic waves and c = speed of electromagnetic wave.

3. ELECTROMAGNETIC SPECTRUM

The electromagnetic waves have a continuous wavelength starting from short gamma rays to long radio-waves. The orderly distribution of wavelength of EM waves is called the electromagnetic spectrum.

The complete spectrum is given in the following table.

	Name	Wavelength Range (m)	Frequency Range (Hz)
1.	Gamma rays	$10^{-13} - 10^{-10}$	$3 \times 10^{21} - 3 \times 10^{18}$
2.	X-rays	$10^{-10} - 10^{-8}$	$3 \times 10^{18} - 3 \times 10^{16}$
3.	Ultraviolet rays	$10^{-8} - 4 \times 10^{-7}$	$3 \times 10^{16} - 7.5 \times 10^{14}$
4.	Visible light	$4 \times 10^{-7} - 7.5 \times 10^{-7}$	$7.5 \times 10^{14} - 4 \times 10^{14}$
5.	Infrared light	$7.5 \times 10^{-7} - 10^{-3}$	$4 \times 10^{14} - 8 \times 10^{14}$
6.	Microwaves	$10^{-3} - 10^{-1}$	$3 \times 10^{11} - 10^{10}$
7.	Radio waves	$10^{-1} - 10^4$	$10^{10} - 3 \times 10^4$

4. ELEMENTARY FACTS ABOUT THE USES OF ELECTROMAGNETIC WAVES

Radio Waves

1. In radio and TV communication.
2. In astronomical field.

Microwaves

1. In RADAR communication.
2. For cooking purpose.
3. In analysis of molecular and atomic structure.

Infrared Waves

1. In knowing molecular structure.
2. In remote control of TV, VCR, etc.

Ultraviolet Rays

1. Used in burglar alarm.
2. To kill germs in water-purifiers.

X-rays

1. In medical diagnosis as they pass through the muscles not through the bones.
2. In detecting faults, cracks, etc., in metal products.

Y-rays

1. As food preservation.
2. In radiotherapy.

16. Which of the following electromagnetic waves have minimum frequency ?

(a) radio-waves (b) ultrasonic-waves
(c) audible-waves (d) micro-waves

Ans :

OD 2004

Wavelength of a wave is inversely proportional to its frequency. Therefore larger the wavelength, smaller is the frequency. Since the audible waves have maximum wavelength, therefore they have minimum frequency.

Thus (c) is correct option.

17. The frequency of X-rays is

(a) 10^{12} Hz (b) 10^{14} Hz
(c) 10^{16} Hz (d) 10^{18} Hz

Ans :

SQP 2011

The range of frequency of X-rays is from 10^{17} to 10^{19} Hz. Therefore frequency of X-rays is 10^{18} Hz.

Thus (d) is correct option.

18. Which of the following have minimum wavelength?

(a) γ -rays (b) X-rays
(c) cosmic rays (d) ultra-violet rays

Ans :

Foreign 2001

The range of wavelengths of γ -rays is 10^{13} to 10^{-10} m; X-rays 10^{-10} to 10^{-8} m; cosmic rays less than 10^{-12} m and ultra-violet rays 10^{-8} to 4×10^{-7} m. Therefore cosmic rays have minimum wavelength.

Thus (c) is correct option.

19. An electromagnetic wave passing through vacuum is described by the equations: $E = E_0 \sin(kx - \omega t)$ and $B = B_0 \sin(kx - \omega t)$. Then

(a) $E_0 k = B_0 \omega$ (b) $E_0 B_0 = \omega k$
(c) $E_0 \omega = B_0 k$ (d) $E_0 = B_0$

Ans :

OD 2007, Comp 2003

Given,

Equations of an electromagnetic wave,

$$E = E_0 \sin(kx - \omega t)$$

and $B = B_0 \sin(kx - \omega t)$

Velocity of light,

$$c = \nu \lambda = \frac{\omega}{2\pi} \lambda = \frac{\omega}{k} \quad (\text{where, } k = \frac{2\pi}{\lambda})$$

We also know that electric field,

$$E_0 = c \times B_0 = \frac{\omega}{k} \times B_0$$

$$E_0 k = B_0 \omega$$

Thus (a) is correct option.

20. Which of the following electromagnetic waves have the smallest wavelength?

(a) γ -rays (b) X-rays
(c) UV waves (d) infra-red rays

Ans :

Foreign 2009

The range of wavelengths of γ -rays is 10^{-13} to 10^{-10} m; X-rays 10^{-10} to 10^{-8} m; UV waves 10^{-8} to 4×10^{-7} m and infra-red rays 8×10^{-7} to 4×10^{-3} m. Therefore, γ -rays have the smallest wavelength.

Thus (a) is correct option.

21. Which of the following radiations have the least wavelength?

(a) X-rays (b) γ -rays
(c) β -rays (d) α -rays

Ans :

SQP 2014

X-rays and γ -rays are electromagnetic waves having range of wavelength 10^{-10} to 10^{-8} m and 10^{-13} to 10^{-10} m respectively. And α -rays and β -rays are of material nature having large wavelength as compared to X-ray and γ -rays which are electromagnetic in nature. Therefore γ -rays have the least wavelength.

Thus (b) is correct option.

22. Which of the following waves have maximum wavelength?

(a) X-rays (b) IR-rays
(c) UV-rays (d) radio-waves

Ans :

OD 2011

The range of wavelengths of X-rays is 10^{-10} to 10^{-8} m; IR-rays 8×10^{-7} to 4×10^{-3} m; UV-rays 10^{-8} to 4×10^{-7} m and radio-waves greater than 10^{-1} m. Therefore radio-waves have maximum wavelength.

Thus (d) is correct option.

ASSERTION AND REASON

23. **Assertion :** X-ray travel with the speed of light.

Reason : X-rays are electromagnetic rays.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

Ans :

All electromagnetic waves have same speed in vacuum. X-ray is a high energy electromagnetic wave.

Thus (a) is correct option.

Ans :**OD 2016**

Displacement current arises when electric field in a region changes with time i.e., it increases or decreases. Thus (d) is correct option.

9. According to the Maxwell's displacement current law, a changing electric field is source of

- (a) an e.m.f. (b) magnetic field
(c) pressure gradient (d) all of these

Ans :**Delhi 2006**

According to the Maxwell's displacement current law that a changing electric field is a source of magnetic field.

Thus (b) is correct option.

10. Cosmic rays are

- (a) high energy radiations
(b) low energy radiations
(c) ultra high energy radiations
(d) very low energy radiations

Ans :**SQP 2017, Foreign 2008**

Cosmic rays are high energy radiations consisting of mainly protons, electrons and some other atomic nuclei.

Thus (a) is correct option.

11. The waves which can not travel in vacuum are

- (a) X-rays (b) radio-waves
(c) infrasonic waves (d) ultra-violet waves

Ans :**OD 2000**

Electromagnetic waves travel in vacuum as well as in a medium, while longitudinal waves need a medium to travel. Since X-rays, radio-waves and ultra-violet waves are electromagnetic waves while infrasonic waves are longitudinal waves, therefore infrasonic waves can not travel in vacuum.

Thus (c) is correct option.

12. Which of the following is/are not electromagnetic waves?

- (a) β -rays (b) X-rays
(c) cosmic rays (d) both a and b

Ans :**Foreign 2015**

β -rays are stream of high energy electrons produced from the nucleus of a radioactive element after decay.

And cosmic rays come from the other space having high energy charged particles like α -particles, protons etc.

Therefore, β -rays and cosmic rays are not electromagnetic waves.

Thus (d) is correct option.

13. Which of the following rays are not electromagnetic waves ?

- (a) γ -rays (b) β -rays
(c) X-rays (d) heat rays

Ans :**Delhi 2009**

β -rays are stream of high energy electrons produced from the nucleus of a radioactive element after decay. Therefore β -rays are not electromagnetic waves.

Thus (b) is correct option.

14. Which radiation in sunlight causes heating effect?

- (a) ultra violet (b) infra-red
(c) visible light (d) all of these

Ans :**OD 2014**

Wavelength of infra-red radiation extends from $8 \times 10^{-7} \text{m}$ to $4 \times 10^{-3} \text{m}$. These radiations have longer wavelength and can penetrate deeply into the materials, causing heating effect.

Thus (b) is correct option.

15. An electromagnetic wave of frequency 3 MHz passes from vacuum into a medium with dielectric constant $k = 4$. Then

- (a) both wavelength and frequency remain unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength is doubled and the frequency remains unchanged

Ans :**Foreign 2010**

Given,

Frequency, $\nu = 3 \text{ MHz}$

and dielectric constant of medium,

$$k = 4$$

Refractive index of medium,

$$\begin{aligned} \mu &= \sqrt{\frac{\epsilon}{\epsilon_0}} \\ &= \sqrt{\frac{k \epsilon_0}{\epsilon_0}} = \sqrt{k} = \sqrt{4} = 2 \end{aligned}$$

and wavelength of electromagnetic wave in medium,

$$\lambda = \frac{\lambda}{\mu} = \frac{\lambda}{2}$$

(where λ = Wavelength of wave in vacuum)

Since, frequency of wave is independent of the medium, therefore it remains unchanged.

Thus (c) is correct option.

OBJECTIVE QUESTIONS

1. The direction of transmission of electromagnetic wave is

- (a) Parallel to $\vec{E}$ (b) Parallel to $\vec{B}$
 (c) Parallel to $\vec{B} \times \vec{E}$ (d) Parallel to $\vec{E} \times \vec{B}$

Ans :

OD 2017

An electromagnetic wave consists of electric and magnetic field oscillating sinusoidally both in time and space. The two fields are perpendicular to each other as well as to the direction of propagation of the wave. Hence the direction of propagation of electromagnetic wave is parallel to $\vec{E} \times \vec{B}$.

Thus (d) is correct option.

2. $\vec{E}$ and $\vec{B}$ represent the electric and the magnetic field of an electromagnetic wave respectively. The direction of propagation of the wave is along

- (a) $\vec{B}$ (b) $\vec{E}$
 (c) $\vec{E} \times \vec{B}$ (d) $\vec{B} \times \vec{E}$

Ans :

OD 2023, Comp 2011

The electric and magnetic field of an electromagnetic wave are perpendicular to each other and their direction of propagation is perpendicular to both the electric and magnetic fields.

If $\vec{E}$ and $\vec{B}$ represent electric and magnetic field vectors of electromagnetic wave, then the direction of propagation of electromagnetic wave is along the direction of vector $\vec{E} \times \vec{B}$.

Thus (c) is correct option.

3. The phase difference between electric and magnetic fields in an electromagnetic wave is

- (a) 0° (b) $\frac{\pi}{2}$
 (c) π (d) anything

Ans :

SQP 2009

In the electromagnetic waves the variations in electric and magnetic fields are in same phase i.e. The phase difference between electric and magnetic field in an electromagnetic wave is 0° . Because both $\vec{E}$ and $\vec{B}$ attain their maximum and minimum value simultaneously.

Thus (a) is correct option.

4. The Maxwell's equations describe the fundamental laws of

- (a) electricity (b) magnetism
 (c) mechanics (d) both (a) and (b)

Ans :

Delhi 2015

Maxwell's equations are as fundamental equations to electromagnetism as Newton's laws are for the study of the mechanics. These equations describe the fundamental laws of electricity as well as magnetism and also predict the existence of electromagnetic waves.

Thus (d) is correct option.

5. The unit of ratio of magnetic field, B and electrical field, $E(\frac{B}{E})$ is

- (a) ms^{-1} (b) sm^{-1}
 (c) ms (d) m^{-2}

Ans :

OD 2017

The speed of electromagnetic wave is given by,

$$c = \frac{E}{B}$$

where, E = Electric field

B = Magnetic field

c = velocity of light

Hence, $\frac{B}{E} = \frac{1}{c} = \frac{1}{\frac{m}{sec}} = sm^{-1}$

Thus (b) is correct option.

6. The idea of displacement current was introduced by

- (a) Hertz (b) Ampere
 (c) Maxwell (d) Farad

Ans :

SQP 2012

Maxwell in 1865 introduced the idea of displacement current.

Thus (c) is correct option.

7. Which scientist experimentally proved the existence of electromagnetic waves?

- (a) Marconi
 (b) Heinrich Rudolf Hertz
 (c) James Clerk Maxwell
 (d) Jagdish Chander Bose

Ans :

Foreign 2002

Heinrich Rudolf Hertz was the first scientist to produce and detect the existence of electromagnetic waves experimentally in 1887.

Thus (b) is correct option.

8. The displacement current arises when electric field in a region

- (a) increases with time
 (b) decreases with time
 (c) attains a constant value
 (d) both (a) and (b)

24. Assertion : Radio waves can be polarised.

Reason : Sound waves in air are longitudinal in nature.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Radio waves can be polarised because they are transverse in nature. Sound waves in air are longitudinal in nature.

Thus (c) is correct option.

25. Assertion : Dipole oscillations produce electromagnetic waves.

Reason : Accelerated charge produces electromagnetic waves.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Hertz produced em waves by oscillating charge between dipole electric field.

A charge moving with non-zero acceleration where both magnetic and electric field are varying emits em waves but this does not explain assertion.

Thus (b) is correct option.

26. Assertion : Environmental damage has increased the amount of ozone in the atmosphere.

Reason : Increase of ozone increases the amount of ultraviolet radiation of earth.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

Ozone layer in the stratosphere helps in protecting life of organism from ultraviolet radiation on earth. Ozone layer is depleted due to of several factors like use of chlorofluoro carbon (CFC) which is the cause of environmental damages.

Thus (a) is correct option.

27. Assertion : Microwaves are better carrier of signals than optical waves.

Reason : Microwaves move faster than optical waves.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

Ans :

The optical waves used in optical fibre communication are better carrier of signals than microwaves. The speed of microwave and optical wave is the same in vacuum.

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

28. What is the frequency of electromagnetic waves produced by oscillating charge of frequency $\nu = 10^5$ Hz?

Ans :

Delhi 2021

The frequency of electromagnetic waves produced by oscillating charge of frequency is radio wave (10^4 Hz to 10^8 Hz)

29. The speed of an electromagnetic wave in a material medium is given by $V = \frac{1}{\sqrt{\mu\epsilon}}$, μ being the permeability of the medium and ϵ its permittivity. How does its frequency change ?

Ans :

OD 2021

The frequency of electromagnetic waves does not change while travelling through a medium.

30. A plane electromagnetic wave travels in vacuum along z -direction. What can you say about the direction of electric and magnetic field vectors ?

Ans :

Comp 2020, OD 2015

Electric field vector along X -axis and magnetic field vector along Y -axis.

31. How are radio waves produced ?

Ans :

Delhi 2017

They are produced by rapid acceleration and decelerations of electrons in aerials.

32. Name the physical quantity which remains same for microwaves of wavelength 1 mm and UV radiations of $1600 \text{ \AA}$ in vacuum.

Ans :

Delhi 2020

Velocity ($c = 3 \times 10^8$ m/s)

This is because both are electromagnetic waves.

$$p = \frac{E}{c}$$

where, p = momentum

E = energy

and c = speed of light

Thus, we are convinced that electromagnetic wave carry energy and momentum.

SHORT ANSWER QUESTIONS

57. How are X-rays produced ? Give any two uses of these.

Ans :

OD 2023

X-rays are produced when high energetic electron beam is made incident on a metallic target of high melting point and high atomic weight.

Uses :

1. **In Engineering :** Using X-rays, cracks produced in buildings or bridges can be detected and the accidents can be avoided.
2. **In radiotherapy :** X-Rays can pass easily through the material of low density like skin, flesh, blood etc. On its basis, Many diseases can be detected cancer is also treated with X-rays.

58. What is displacement current? Why was this concept introduced?

Ans :

Comp 2021

It is that current which comes into existence (in addition to conduction current) whenever the electric field and hence the electric flux changes with time. It is equal to ϵ_0 times the rate of change of electric flux through a given surface.

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 A \frac{dE}{dt}$$

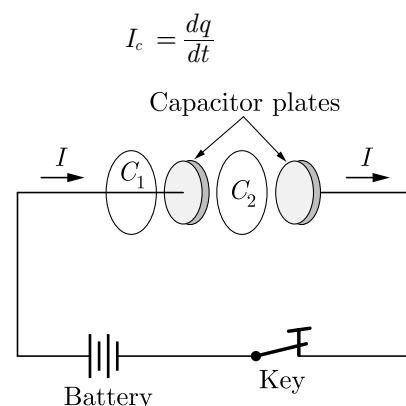
Maxwell introduced the concept of displacement current for the following reasons:

1. To make the Ampere's circuital law logically consistent.
 2. To satisfy the property of continuity of electric current along any closed path.
59. Conduction and displacement currents are individually discontinuous, but their sum is continuous.

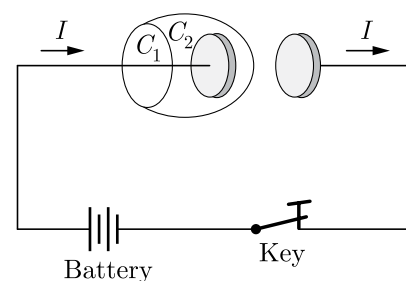
Ans :

OD 2021

In figure (as given below), a current I_c enters one plate and leaves the other plate of the capacitor. The conduction current is not continuous across the capacitor gap as no charge is transported across this gap.



(a)



(b)

A parallel plate capacitor being charged by a battery

The displacement current I_d is zero outside the capacitor plates and in the gap, it has the value,

$$\begin{aligned} \epsilon_0 \frac{d\phi_E}{dt} &= \epsilon_0 \frac{d}{dt}(EA) \\ &= \epsilon_0 \frac{d}{dt}\left(\frac{q}{\epsilon_0}\right) = \frac{dq}{dt} \end{aligned}$$

which is exactly the value of the conduction current in the lead wires. Thus the displacement current satisfies the basic condition that the current is continuous.

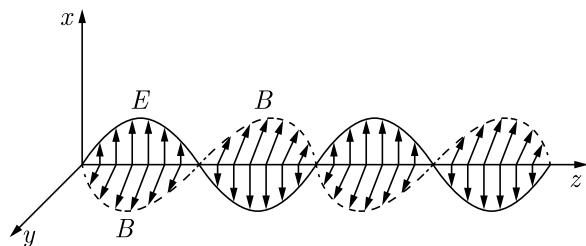
The sum $I_c + \epsilon_0 \frac{d\phi_E}{dt}$ has the same value along the entire path (both inside and outside the capacitor plates), although individually the two currents are discontinuous.

60. State the important consequences of displacement current.

Ans :

Delhi 2020, SQP 2008

The concept of displacement current has made the laws of electricity and magnetism symmetrical. According to Faraday's law of electromagnetic induction, the magnitude of induced emf is equal to the rate of change magnetic flux. But the emf between two points is the work done in taking a unit charge from one point to another against the electrostatic forces. This implies the existence of an electric field in the region. So Faraday's law simply states that a time



48. Write one use each of them:

1. Microwaves
2. γ -rays

Ans :

Foreign 2010

1. **Microwaves** : They are used in RADAR devices.
2. **γ -rays** : It is Used in radio therapy.

49. What is the importance of ozone layer in the atmosphere?

Ans :

Comp 2017

The solar radiation consists of ultraviolet and some other lower wavelength radiations which cause genetic damages to living cells. The ozone layer absorbs these radiations from the sun and prevents them from reaching the earth's surface and causing damage to life. Moreover, it also keeps the earth's surface warm by trapping infrared radiation.

50. Write two main features of LASER rays.

Ans :

Delhi 2015, Foreign 2011

Two main features of LASER rays are following

1. A laser beam is collimated, meaning it consists of waves travelling parallel to each other in single direction.
2. Laser light is coherent, which means all the light wave move in phase together in both time and space.

51. Write any two properties of X-rays.

Ans :

OD 2013

1. The range of wavelength of X-ray is 1 mm to 10^{-3} nm.
2. They give the luminous effect on photographic plates.

52. Write an expression for the momentum carried by an E.M. wave.

Ans :

SQP 2012

An electromagnetic wave transports linear momentum as it travels through space. If an electromagnetic wave transfers a total energy U to a surface in time t , then total linear momentum delivered to the surface is,

$$p = \frac{U}{c} \quad [\text{For complete absorption of energy } U]$$

If the wave is totally reflected, the momentum

delivered will be $\frac{2U}{c}$, because the momentum of the wave will change from p to $-p$.

53. Mention some sources of electromagnetic waves.

Ans :

Delhi 2015

An accelerating charge produces electromagnetic waves. An electric charge oscillating harmonically with frequency ν , generates electromagnetic waves of the same frequency ν .

An electric dipole is a basic source of electromagnetic waves.

An LC -circuit containing inductance L and capacitance C produces electromagnetic waves of frequency,

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

54. Out of electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ in an electromagnetic wave, which is more effective and why?

Ans :

Delhi 2019

Both the electric $\vec{E}$ and magnetic $\vec{B}$ vector are essential in the transmission of EM wave but in the interaction with matter the electric vector $\vec{E}$ is more important.

When a photon impacts a rod or cone on your retina it is the E -vector that donates its energy to the light sensitive molecule, resulting in excitation and hence a signal to your brain. Also in polarization, $\vec{E}$ is important.

55. Explain briefly how electromagnetic waves are produced by an oscillating charge. How is the frequency of electromagnetic waves produced related to that of the oscillating charge?

Ans :

Foreign 2008

An oscillating or accelerated charge is supposed to be sources of an electromagnetic wave. An oscillating charge produces an oscillating electric field in space which further produces an oscillating magnetic field which in turn is a source of electric field. These oscillating electric and magnetic field, hence, keep on regenerating each other and an electromagnetic wave is produced. The frequency of electromagnetic wave = Frequency of oscillating charge.

56. How do you convince yourself that electromagnetic waves carry energy and momentum?

Ans :

Delhi 2015

According to the quantum theory, electromagnetic radiation is made up of massless particles called photons. Momentum of the photon is expressed as

- 33.** To which part of the electromagnetic spectrum does a wave of frequency 5×10^{19} Hz belong ?

Ans :

Comp 2019

A wave of frequency 5×10^{19} Hz belong to γ -rays of electromagnetic spectrum.

- 34.** How are microwaves produced ?

Ans :

SQP 2011

By using a magnetron.

- 35.** To which part of the electromagnetic spectrum does a wave of frequency 3×10^{13} Hz belong ?

Ans :

Comp 2016

The frequency of 3×10^{13} Hz belong to the infrared waves.

- 36.** Name the electromagnetic waves, which (i) maintain the Earth's warmth and (ii) are used in aircraft navigation.

Ans :

Delhi 2012

- (i) Infrared rays
(ii) Microwaves

- 37.** What physical quantity is the same for X -rays of wavelength 10^{-10} m, red light of wavelength $6800 \text{ \AA}$ and radio-waves of wavelength 500 m ?

Ans :

Foreign 2013

X -rays, red light and radio waves are all electromagnetic waves. The speed of propagation in vacuum is the same for all these waves. This speeds is equal to $c = 3 \times 10^8 \text{ m/s}$.

- 38.** Welders wear special goggles or face masks with glass windows to protect their eyes from electromagnetic radiation. Name the radiations and write the range of their frequency.

Ans :

OD 2015

Welders wear special goggles or face mask with glass windows to protect their eyes from ultraviolet rays. The range of UV rays is $4 \times 10^{-7} \text{ m}$ (400 nm) to $6 \times 10^{-10} \text{ m}$ (0.6 nm).

- 39.** Name the physical quantity which remains same for microwaves of wavelength 1 mm and UV radiations of $1600 \text{ \AA}$ in vacuum.

Ans :

Delhi 2010

Velocity.

- 40.** To which part of the electromagnetic spectrum does a wave of frequency 5×10^{19} Hz belong ?

Ans :

OD 2014

X -rays or γ -rays.

- 41.** Write two uses of X -rays.

Ans :

OD 2018

Uses of X -rays :

- (i) In medical diagnosis as they pass through the muscles not through the bones.
(ii) In detecting faults, cracks, etc. in metal products.

- 42.** Do electromagnetic waves carry energy and momentum ?

Ans :

Delhi 2015

Yes. Electromagnetic waves carry energy and momentum.

- 43.** Write an expression for the pressure exerted by an electromagnetic wave.

Ans :

Comp 2017, Delhi 2012

When an electromagnetic wave falls on a surface, it exerts pressure on the surface. This pressure is called radiation pressure. The radiation pressure for an electromagnetic wave of intensity I is given by,

$$P_r = \frac{1}{c}$$

For visible light, $P_r = 7 \times 10^{-6} \text{ Nm}^{-2}$

- 44.** How does oscillating charge produce electromagnetic waves?

Ans :

OD 2013

An oscillating charge produces an oscillating electric field in space, which produces an oscillating magnetic field. The oscillating electric and magnetic fields regenerate each other, and this results in the production of electromagnetic waves in space.

- 45.** Write two uses of infrared rays.

Ans :

Comp 2018

Uses of Infrared rays

- (i) In knowing the molecular structure.
(ii) In remote control of TV, VCR, etc.

- 46.** Arrange the following electromagnetic waves in the descending order of their wavelength.

1. Microwaves
2. Infrared rays
3. Ultraviolet radiation
4. γ -rays

Ans :

Comp 2014

The decreasing order of wavelength of electromagnetic waves are Microwaves $>$ Infrared $>$ Ultraviolet $>$ γ -rays.

- 47.** Sketch a schematic diagram depicting oscillating electric and magnetic fields of an electromagnetic wave propagating along + z-direction.

Ans :

SQP 2009

Electric field is along x-axis and magnetic field is along y-axis.

varying magnetic field gives rise to an electric field.

By symmetry, a time varying electric field should give rise to a magnetic field. This is an important consequence of displacement current which is a source of magnetic field.

Another very important consequence of the symmetry of electricity and magnetism is the existence of electromagnetic waves.

61. State the important properties of displacement current.

Ans :

OD 2020

These are as follows :

1. Displacement current exists whenever there is a change of electric flux. Unlike conduction current, it does not exist under steady conditions.
2. It is not a current. It only adds to current density in Ampere's circuital law. As it produces magnetic field, so it is called a current.
3. The magnitude of displacement current is equal to the rate of displacement of charge from one capacitor plate to the other.
4. Together with the conduction current, displacement current satisfies the property of continuity.

62. State Maxwell's equations.

Ans :

Comp 2019

The whole study of electricity and magnetism can be described mathematically with the help of four fundamental equations, called Maxwell's equations. These are stated as follows:

1. **Gauss Law of Electrostatics :**

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

2. **Gauss Law of Magnetism :**

$$\oint \vec{B} \cdot d\vec{S} = 0$$

3. **Faraday's Law of Electromagnetic Induction :**

$$\begin{aligned} \oint \vec{E} \cdot d\vec{l} &= -\frac{d\phi_B}{dt} \\ &= -\frac{d}{dt} \left[\oint \vec{B} \cdot d\vec{S} \right] \end{aligned}$$

4. **Modified Ampere's Circuital Law :**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right]$$

63. Briefly explain how Maxwell was led to predict the existence of electromagnetic waves.

Ans :

Delhi 2017

According to Faraday's law of electromagnetic induction, **A time-varying magnetic field is a source of changing electric field.**

On the basis of his theoretical studies, Maxwell in 1865, argued that, **A time-varying electric field is a source of changing magnetic field.** This means that the change in either field (electric/magnetic) produces the other field. Maxwell further showed that these variations in electric and magnetic fields occur in mutually perpendicular directions and have wave like properties. He was thus led to the idea that a wave of electric and magnetic fields both varying with space and time should exist, one providing the source of the other. Such a wave is called an electromagnetic wave and it indeed exists.

64. Define intensity of an electromagnetic wave. Obtain an expression for it.

Ans :

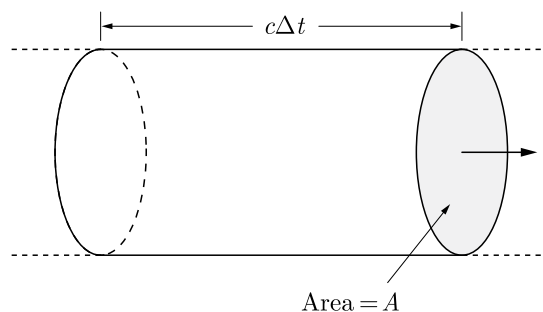
Comp 2018, Foreign 2003

The energy crossing per unit area per unit time in a direction perpendicular to the direction of propagation is called intensity of the wave.

Suppose a plane electromagnetic wave propagates along X-axis with speed c . As shown in figure, consider a cylindrical volume with area of cross-section A and length $c\Delta t$ along the X-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates with speed c . The energy contained is,

$$\begin{aligned} U &= \text{Average energy density} \times \text{volume} \\ &= u \times c\Delta t \times A \end{aligned}$$

Intensity of the wave,



$$\begin{aligned} I &= \frac{\text{Energy}}{\text{Area} \times \text{Time}} \\ &= \frac{U}{A \Delta t} = uc \\ &= \frac{1}{2} \epsilon_0 E_0^2 c = \epsilon_0 E_{\text{rms}}^2 c \end{aligned}$$

Also,

$$\begin{aligned} I &= \frac{1}{2\mu_0} B_0^2 c \\ &= \frac{1}{\mu_0} B_{\text{rms}}^2 c \end{aligned}$$

Thus, the intensity of an electromagnetic wave is proportional to the square of the electric/magnetic field.

$\vec{E}$ is passing through the surface (B); constitute an electric flux

$$\begin{aligned}\phi &= E |A| = \frac{\sigma}{\epsilon_0} |A| \\ &= \frac{Q}{A\epsilon_0} A = \frac{Q}{\epsilon_0} \quad \dots(3)\end{aligned}$$

If the charge on the plate in the tiffin box is changing with time, there must be a current between the plates.

From equation (3), we get

$$I = \frac{dQ}{dt} = \frac{d}{dt}(\epsilon_0 \phi) = \epsilon_0 \frac{d\phi}{dt}$$

This is the missing term in Ampere's circuital law.

The inconsistency may disappear if displacement current is included between the plates.

So, generalised Ampere's circuital law can be given as

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 I(t) + \mu_0 \epsilon_0 \frac{d\phi}{dt} \\ &= \mu_0 I(t) + \mu_0 \epsilon_0 \frac{d\phi}{dt}\end{aligned}$$

81. Answer the following questions

- Name the electromagnetic waves which are used for the treatment of certain forms of cancer. Write their frequency range.
- Thin ozone layer on top of stratosphere is crucial for human survival. Why ?
- Why is the amount of the momentum transferred by the electromagnetic waves incident on the surface so small ?

Ans :

Delhi 2014, OD 2007

- γ -rays are used for the treatment of certain forms of cancer. Its frequency range is 3×10^{19} Hz to 5×10^{22} Hz.
- The thin ozone layer on top of stratosphere absorbs most of the harmful ultraviolet rays coming from the sun towards the earth. They include UVA, UVB and UVC radiations, which can destroy the life system on the earth.
Hence, this layer is crucial for human survival.
- Momentum transferred

$$\begin{aligned}&= \text{Energy} \times \text{Speed of light} \\ &= h\nu c = 10^{-22}\end{aligned}$$

Thus, the amount of the momentum transferred by the electromagnetic waves incident on the surface is very small.

82. When an AC source is connected across a capacitor, current starts flowing through the circuit. Show how Ampere's circuital law is generalized to explain the flow of current through the capacitor. Hence obtain the expression for the displacement current inside the capacitor.

Ans :

Foreign 2007

Maxwell restated Ampere's circuital law as follows

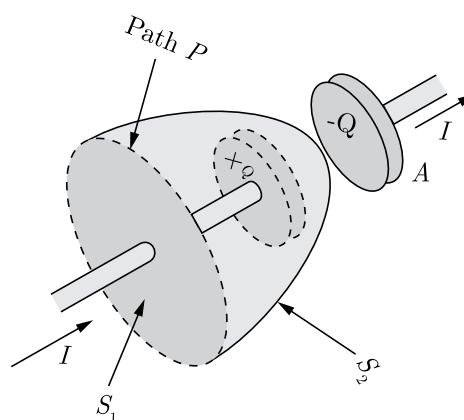
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d) \quad \dots(1)$$

where I_d is called the displacement current. This current arises due to the rate of change of electric flux (ϕ_E), between the plates of the capacitor and is given by

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} \quad \dots(2)$$

Thus, the modified form of Ampere's circuital law, also called Ampere-Maxwell's circuital law, becomes

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$



The term I equals zero if the surface S_2 is chosen and the term I_d equals zero, if S_1 is chosen. The term $\epsilon_0 \frac{d\phi_E}{dt}$ has the dimension of current. The meaning of this expression can be understood by referring to figure.

The electric flux through S_2 is $\phi_E = \int \vec{E} \cdot d\vec{A} = EA$,

where A is the area of the plates and E is the uniform electric field strength between the plates. If Q is the charge on the plates at any instant, then one finds that $E = \frac{Q}{\epsilon_0 A}$.

Therefore, the electric flux through S_2 is simple

$$\phi_E = EA = \frac{Q}{\epsilon_0}$$

Hence, the displacement current I_d through S_2 is

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \frac{dQ}{dt} \quad \dots(3)$$

That is, displacement current is precisely equal to the conduction current I passing through S_1 . The central point of this formalism is the fact that magnetic fields are produced both by conduction currents and by changing electric fields.

$$I_c = I_d$$

The current inside the capacitor,

$$\text{Displacement current } I_c \text{ and } I_d = \varepsilon_0 \frac{d\phi_E}{dt}.$$

70. State two properties of electromagnetic waves. How can we show that electromagnetic waves carry momentum?

Ans :

OD 2017

Properties of electromagnetic waves

1. Transverse nature.
2. Does not get deflected by electric fields or magnetic fields.
3. Same speed in vacuum for all waves.
4. No material medium required for propagation.
5. They get refracted, diffracted and polarised.

Electric charges present on a plane, kept normal to the direction of propagation of an electromagnetic wave can be set and sustained in motion by the electric and magnetic field of the electromagnetic wave. The charges thus acquire energy and momentum from the waves.

71. Arrange the following electromagnetic waves in the order of their increasing wavelength
- (a) Gamma (γ) rays
 - (b) Microwaves
 - (c) X-rays
 - (d) Radio waves

or

How are infrared waves produced ? What role does infrared radiation play in (i) maintaining the Earth's warmth and (ii) physical therapy ?

Ans :

OD 2016, Comp 2009

Gamma (γ) rays, X-rays, Microwaves, Radio-waves.

Infrared rays are produced by hot bodies/vibration of atoms and molecules.

Infrared rays (i) Maintain Earth's warmth through greenhouse effect, (ii) Produce heat.

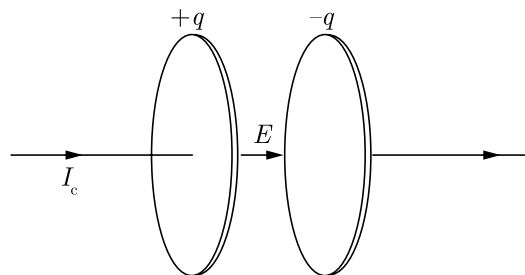
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72. A capacitor made of two parallel plates each of plate area A and separation d , is being charged by an external AC source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor.

Ans :

Delhi 2018

In figure conduction current is flowing in the wires, causes charge on the plates



$$\text{So, } I_c = \frac{dq}{dt} \quad \dots(1)$$

According to Maxwell, displacement current between plates,

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\text{where, } \phi_E = \text{Electric flux} \quad \dots(2)$$

Using Gauss's theorem, if one of the plate is inside the tiffin type Gaussian surface, then

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\text{So, } I_d = \varepsilon_0 \frac{d}{dt} \left(\frac{q}{\varepsilon_0} \right)$$

$$I_d = \frac{dq}{dt} \quad \dots(3)$$

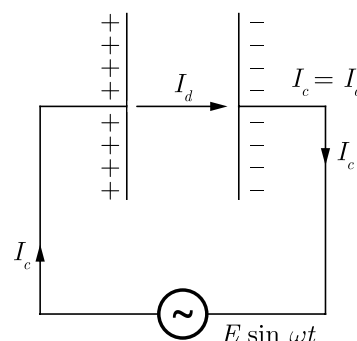
From equation (1) and (3),

Both conduction current and displacement current are equal.

73. When an ideal capacitor is charged AC by a DC battery, no current flows. However, when an AC source is used, the current flows continuously. How does one explain this, based on the concept of displacement current ?

Ans :

OD 2010



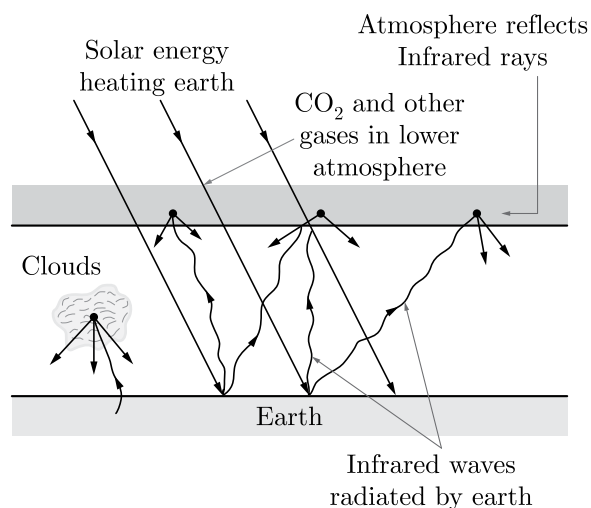
Ideal capacitor means infinite resistance for DC when an AC source is used, the current flow continuously, but we know that the capacitor has dielectric (air) between its plates. So, there is no current and circuit would be incomplete. In real capacitor is charged due

65. What is Greenhouse effect for the atmosphere of the earth and what is its importance?

Ans :

Foreign 2016

This is the phenomenon which keeps the earth's surface warm at night.



The radiation from the sun heats up the earth. Due to its lower temperature, the earth re-radiates it mostly in the infrared region. These infrared radiations cannot pass through the lower atmosphere, they get reflected back by gas molecules. Low lying clouds also reflect them back to the earth. These radiations heat up the objects on the earth's surface and so keep the earth's surface and so keep the earth's surface warm at night.

66. How are infrared waves produced? Why are these referred to as heat waves? Write their one important use.

or

How are infrared waves produced ? Why are these waves referred to as heat waves ? Give any two uses of infrared waves.

Ans :

2023, OD 2019

Infrared waves are produced by hot bodies and molecules. Infrared waves are sometimes referred to as heat waves. This is because water molecules present in most materials readily absorb infrared waves. After absorption, their thermal motion increases, that is they heat up and heat their surroundings.

Infrared lamps are used in physical therapy and in remote control of devices.

67. How are electromagnetic waves produced? What is the source of energy of these waves? Write mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis. Write any two important properties of electromagnetic waves.

Ans :

SQP 2012

Electromagnetic wave produced by oscillating charged particle.

Mathematical expression for electromagnetic wave travel along z-axis.

$$E_x = E_0 \sin(kz - \omega t) \quad [\text{For electric field}]$$

$$B_y = B_0 \sin(kz - \omega t) \quad [\text{For magnetic field}]$$

Properties :

1. Have oscillating electric and magnetic fields along mutually perpendicular direction.
2. Transverse nature.

68. Write the expression for electrical energy density and magnetic energy density associated with electromagnetic wave and show that their ratio is 1.

Ans :

SQP 2011, Comp 2004

Electrical energy density,

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad \dots(1)$$

Magnetic energy density,

$$u_B = \frac{B^2}{2\mu_0} \quad \dots(2)$$

Dividing Eq. (1) by (2),

$$\frac{u_E}{u_B} = (\epsilon_0 \mu_0) \left(\frac{E}{B} \right)^2 \quad \dots(3)$$

Since,

$$E = cB$$

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

Hence,

$$\frac{u_E}{u_B} = \frac{1}{c^2} \left(\frac{E}{B} \right)^2$$

$$\frac{u_E}{u_B} = \frac{1}{c^2} \times c^2 \quad \left[\text{Since, } c = \frac{E}{B} \right]$$

$$= 1$$

69. A capacitor of capacitance C is being charged by connecting it across a DC source along with an ammeter. Will the ammeter show a momentary deflection during the process of charging? If so, how would you explain this momentary deflection and the resulting continuity of current in the circuit? Write the expression for the current inside the capacitor.

Ans :

Delhi 2010

The ammeter will show the momentary deflection.

This momentary deflection occurs due to the fact that the conduction current flows through connection wires during the charging of capacitor. This lead to gathering of charge at two plated and hence varying electric field of increasing nature is produced between the plated which in turn produce displacement current in space between two plates. This maintains the continuity with the conduction current.

displacement current may be zero.

$$\text{So, } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

- (ii) In large region of space, where there is no conduction current, but there is only a displacement current due to time varying electric field (or flux).

$$\text{So, } \oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

- 78.** Write Maxwell's generalization of Ampere's Circuital Law. Show that in the process of charging a capacitor, the current produced within the plates of the capacitor is

$$i = \epsilon_0 \frac{d\phi_E}{dt}$$

Where ϕ_E is the electric flux produced during charging of the capacitor plates.

Ans :

Foreign 2006, Comp 2003

Ampere's circuital law is given by

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

For a circuit containing capacitor, during its charging or discharging the current within the plates of the capacitor varies producing displacement current I_d . Hence, Ampere's circuital law is generalised by Maxwell, given as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C + \mu_0 I_d$$

The electric flux (ϕ_E) between the plates of capacitor changes with time, producing current within the plates which is proportional to $\left(\frac{d\phi_E}{dt}\right)$. Thus, we get,

$$I_C = \epsilon_0 \frac{d\phi_E}{dt} \epsilon$$

- 79.** (i) Identify the part of the electromagnetic spectrum which is
- suitable for radar system used in aircraft navigation.
 - produced by bombarding a metal target by high speed electrons.
- (ii) Why does a galvanometer show a momentary deflection at the time of charging or discharging a capacitor? Write the necessary expression to explain this observation.

Ans :

OD 2018

- (a) Microwaves
 - (b) X-rays
- (ii) Due to conduction current in the connecting wires and the production of displacement current between the plates of capacitor on account of changing electric field.

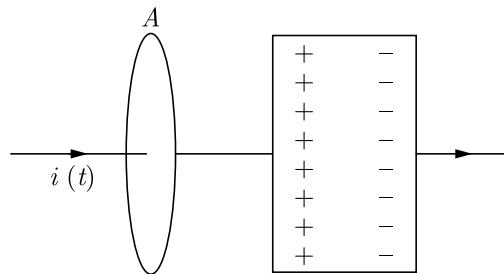
Current inside the capacitor is given by

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

- 80.** Considering the case of a parallel plate capacitor being charged, show how one is required to generalise Ampere's circuital law to include the term due to displacement current.

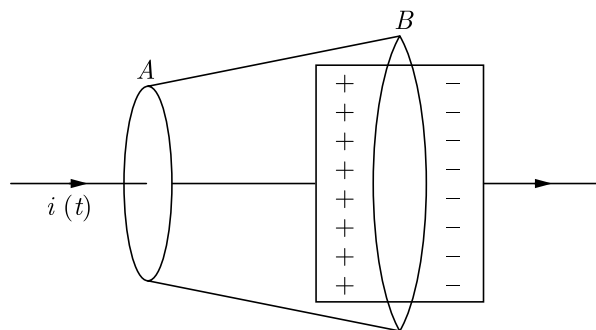
Ans :

Delhi 2012



During charging capacitor C , a time varying current $I(t)$ flows through the conducting wire, so on applying Ampere's circuital law (for loop A)

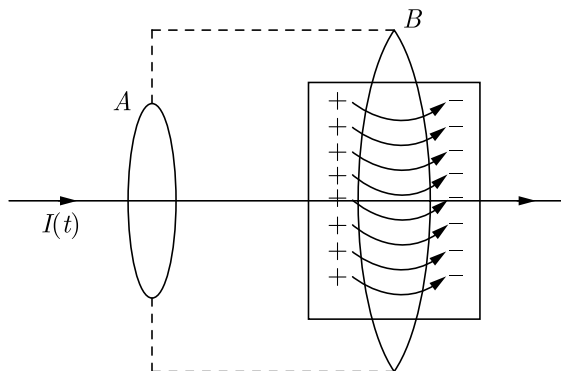
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I(t) \quad \dots(1)$$



Now, we consider a pill box surface enclosing the positively charged plate and nowhere touches the conducting wire,

$$\oint \vec{B} \cdot d\vec{l} = 0 \quad \dots(2)$$

From equation (1) and (2), we have a contradiction



If surface A and B forms a pill box and electric field

thus regenerate each other i.e., they produce electromagnetic waves through space.

- (b) (i) **Microwaves** : In cooking and communication.
 (ii) **Ultraviolet rays** : Photocells and in sterilizing the surgical equipment.
 (iii) **Infrared rays** : Photography and physical therapy.
 (iv) **Gamma rays** : In cancer treatment and study of structure of atomic nucleus.

88. Answer the following questions :

- (i) Name the waves which are produced during radioactive decay of a nucleus. Write their frequency range.
 (ii) Welders wear special glass goggles while working. Why ? Explain.
 (iii) Why are infrared waves often called as heat waves ? Give their one application.

Ans :

OD 2010

- (i) γ -rays are produced during radioactive decay of a nucleus. Its frequency range is from 3×10^{18} to 5×10^{22} Hz.
 (ii) Welders wear special goggles of face mask with glass windows to protect their eyes from ultraviolet rays. The range of UV rays is 4×10^{-7} m to 6×10^{-10} m.
 (iii) Infrared rays are called heat waves because they vary with temperature.
 They are used in physical therapy, weather forecasting.

89. Answer the following questions :

- (i) Show, by giving a simple example, how electromagnetic waves carry energy and momentum.
 (ii) How are microwaves produced ? Why is it necessary in microwaves ovens to select the frequency of microwaves to match the resonant frequency of water molecules ?
 (iii) Write two important uses of infrared waves.

Ans :

Delhi 2014

- (i) Consider a plane perpendicular to the direction of propagation of the wave. An electric charge, on the plane will be set in motion by the electric and magnetic fields of electromagnetic wave, incident on this plane. This illustrates that electromagnetic waves carry energy and momentum.
 (ii) Microwaves are produced by special vacuum tube like the klystron, magnetron and Gunn diode.
 The frequency of microwaves is selected to match the resonant frequency of water molecules, so that energy is transformed efficiently to the kinetic energy of the molecules.

- (iii) (a) Associated with the green house effect.
 (b) In remote switches off household electrical appliances.

90. Identify the electromagnetic waves whose wave lengths vary as

- (a) $10^{-12} \text{ m} < \lambda < 10^{-8} \text{ m}$
 (b) $10^{-13} \text{ m} < \lambda < 10^{-1} \text{ m}$

Ans :

Delhi 2017

- (a) **X-rays** : Use of study atomic structure.
 (b) **Microwaves** : For cooking in microwaves ovens.

91. (i) Which segment of electromagnetic waves has highest frequency ? How are these waves produced ? Give one use of these waves.
 (ii) Which electromagnetic waves lie near the high frequency one of visible part of electromagnetic spectrum ? Give its one use. In what way this component of light has harmful effects on human ?

Ans :

Comp 2016, OD 2001

- (i) Gamma rays have the highest frequency. These are produced during nuclear reactions and also emitted by radioactive nuclei. They are used in medicine to destroy cancer cells.
 (ii) Ultraviolet rays lie near the high frequency end of visible part of electromagnetic spectrum. They are used to sterilise drinking water and surgical instruments. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin.

LONG ANSWER QUESTIONS

92. Is a displacement current associated with a magnetic field? Or, can a changing electric flux induce a magnetic field? Explain it with the help of an example.

Ans :

Delhi 2021

A displacement current produces the same physical effects as the conduction current. Like a conduction current, a displacement current is also associated with a magnetic field. Consider, for example, the charging of a parallel plate capacitor by a constant current I in the connecting wires [Figure (a)]. This increases the charges on the capacitor plates at a steady rate. Consequently, the electric field between the plates also increases at a steady rate. Between the capacitor plates, there exists a displacement current due to time varying electric field.

83. (a) Which one of the following electromagnetic radiations has least frequency
UV radiations, X-rays, Microwaves ?
(b) How do you show that electromagnetic waves carry energy and momentum ?
(c) Write the expression for the energy density of an electromagnetic wave propagating in free space.

Ans :

Delhi 2015

- (a) Microwave.
(b) When a charge oscillates with some frequency, it produces an oscillating electric field and magnetic field in space. So, an electromagnetic wave is produced.

The frequency of the electromagnetic wave is equal to the frequency of oscillation of the charge. Hence, energy associated with the electromagnetic wave comes at the expense of the energy of the source.

If the electromagnetic wave of energy U strikes on a surface and gets completely absorbed, total momentum delivery to the surface is $p = \frac{U}{c}$.

Hence electromagnetic wave also carry momentum.

- (c) The electromagnetic wave consists of oscillating electric and magnetic fields, So net energy density of electromagnetic wave is

$$U = U_E + U_B$$

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 B^2$$

84. Answer the following questions :

- (i) Name the electromagnetic waves which are suitable for RADAR systems used in aircraft navigation. Write the range of frequency of these waves.
(ii) If the earth not have atmosphere, would its average surface temperature be higher or lower than what it is now? Explain.
(iii) An electromagnetic wave exerts pressure on the surface on which it is incident. Justify.

Ans :

OD 2017

- (i) The electromagnetic waves suitable for RADAR system is microwaves. The range of frequency is 3×10^{11} to 1×10^8 Hz.
(ii) The temperature of the earth would be lower because the green house effect of the atmosphere would be absent.
(iii) An electromagnetic waves has momentum,

i.e.,
$$p = \frac{\text{Energy } (E)}{\text{Velocity of light } (c)}$$

When it is incident upon a surface it exerts pressure on it.

85. (a) State clearly how a microwave oven works to heat up a food item containing water molecules.
(b) Why are microwaves found useful for the RADAR systems in aircraft navigation ?

Ans :

OD 2011

- (a) In microwave oven, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves get transferred efficiently to the kinetic energy of the molecules. This kinetic energy raises the temperature of any food containing water.
(b) Microwaves are short wavelength radio waves, with frequency of order of few GHz. Due to short wavelength, they have high penetrating power with respect to atmosphere and less diffraction in the atmospheric layers. So, these waves are suitable for the radar systems used in aircraft navigation.

86. Name the parts of the electromagnetic spectrum which is

- (i) suitable for RADAR systems in aircraft navigations.
(ii) used to treat muscular strain.
(iii) used as a diagnostic tool in medicine.

Write in brief, how these waves can be produced.

Ans :

SQP 2006, OD 2002

The electromagnetic waves suitable for RADAR system is microwaves. The range of frequency is 3×10^{11} to 1×10^8 Hz.

- (i) These rays are produced by special vacuum tubes, namely klystrons, magnetron's and Gunn diodes.
(ii) Infrared waves are used to treat muscular strain. These rays are produced by hot bodies and molecules.
(iii) X-rays are used a diagnostic tool in medicine. These rays are produced when high energy electrons are stopped suddenly on a metal of high atomic number.

87. (a) Describe briefly how electromagnetic waves are produced by oscillating charges.

- (b) Give one use of each of the following :

- (i) Microwaves
(ii) Ultraviolet rays
(iii) Infrared rays
(iv) Gamma rays

Ans :

Comp 2011

- (a) Electromagnetic waves are produced by accelerated charges. An oscillating charge produces an oscillating magnetic field which in its own turn is a source of oscillating electric field. The oscillating electric and magnetic fields

to contribution of varying electric field. The current between the capacitor plates is given by displacement current.

$$D = \epsilon E$$

$$I_d = \epsilon \frac{d\phi}{dt}$$

74. Write the expression for the generalised form of Ampere's circuital law. Discuss its significance and describe briefly how the concept of displacement current is explained through charging/discharging of a capacitor in an electric circuit.

Ans :

Comp 2017, OD 2010

The generalisation in Ampere's circuital law was modified by Maxwell, as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_C + I_d) = \mu_0 I_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

where, $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ is displacement current.

Significance : This expression signifies that the source of magnetic field is not just due to the conduction current in the metallic conductors, but also due to the time rate of change of electric flux called displacement current.

During charging and discharging of a capacitor, electric field between the plates will change. Hence, there will be a change in electric flux, called displacement current, between the plates.

75. Why does a galvanometer when connected in series with a capacitor show a momentary deflection, when it is being charged or discharged ?

How does this observation lead to modifying the Ampere's circuital law ? Hence write the generalised expression of Ampere's law.

Ans :

Foreign 2015

During charging or discharging of the capacitor, displacement current between the plates is produced. Hence, circuit becomes complete and galvanometer shows momentary deflection.

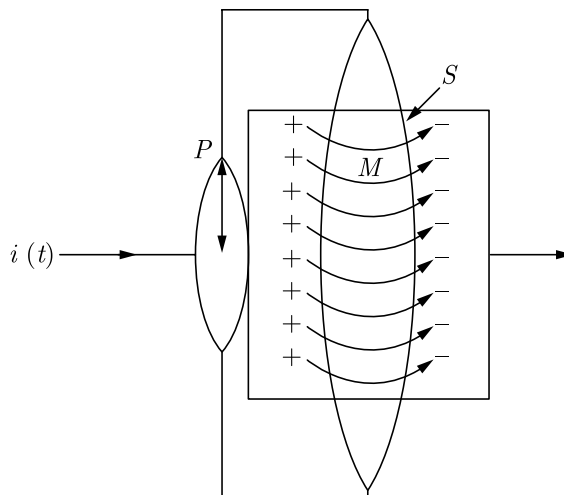
According to Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

At surface P , $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$

At surface S , $\oint \vec{B} \cdot d\vec{l} = 0$

Hence, $\oint_P \vec{B} \cdot d\vec{l} \neq \oint_S \vec{B} \cdot d\vec{l}$



This contradicts Ampere's circuital law. This law must be missing something. Hence the law needs modification. Modified form of Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left[i_C + \epsilon_0 \frac{d\phi_E}{dt} \right]$$

76. Find the wavelength of electromagnetic waves of frequency 6×10^{12} Hz in free space. Give its two applications.

Ans :

SQP 2011

Wavelength, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{6 \times 10^{12}} = 5 \times 10^{-5} \text{ m}$

This wavelength corresponds to infrared waves.

Applications of Infrared Rays

- (i) They are used in green houses to warm the plants.
 - (ii) They are used in taking photographs during fogs.
77. Write the generalised expression for the Ampere's circuital law in terms of the conduction current and the displacement current. Mention the situation when there is

- (i) only conduction current and no displacement current.
- (ii) only displacement current and no conduction current.

Ans :

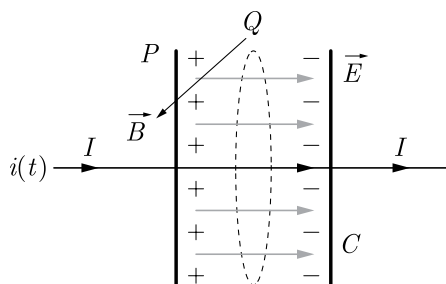
Delhi 2013

Generalised Ampere's circuital Law :

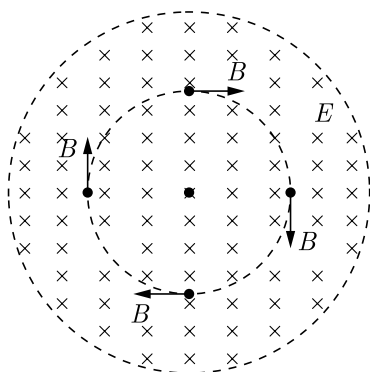
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

Line integral of magnetic field over closed loop is equal to μ_0 times sum of conduction current and displacement current.

- (i) In case of steady electric field in a conducting wire, electric field does not change with time, conduction current exists in the wire but



(a)



(b)

(a) Electric and magnetic fields $\vec{E}$ and $\vec{B}$ at any point Q between the capacitor plates. (b) A cross-sectional view of figure (a)

In such a region, we expect a magnetic field though there is no source of conduction current nearby. Experiments have shown that a magnetic field $\vec{B}$ is indeed induced (say at a point Q) between the capacitor plates and has same magnitude as that just outside (say at point P).

In Figure (a), the direction of $\vec{E}$ is from the positive plate to the negative, whereas the direction of $\vec{B}$ at Q is perpendicular to the plane of paper. Figure (b) shows a cross-sectional loop parallel to the plane of the plates. The field $\vec{E}$ is directed normally into the plane paper, as shown by crosses. The induced field $\vec{B}$ is clockwise along the tangents on a circle in this cross-sectional plane.

93. State and explain Maxwell's modification of Ampere's circuital law.

or

Discuss the inconsistency in Ampere's circuital law. What modification was made by Maxwell in this law?

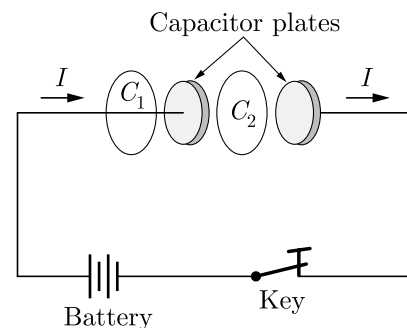
Ans :

OD 2021, Comp 2012

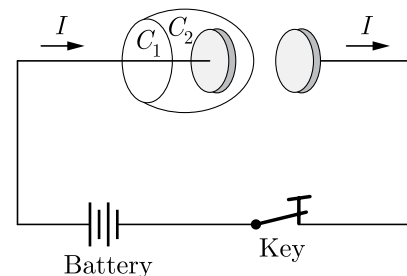
According to Ampere's circuital law, the line integral

of the magnetic field $\vec{B}$ along any closed loop C is proportional to the current I passing through the closed loop, i.e.,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I \quad \dots(1)$$



(a)



(b)

A parallel plate capacitor being charged by a battery

In 1864, Maxwell showed that the Eq. (1) is logically inconsistent. Consider a parallel plate capacitor being charged by a battery, as shown in figure (a). As the charging current I flows, a magnetic field is set up around the capacitor.

Now, the current I flows across the area bounded by loop C_1 . Therefore,

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \mu_0 I \quad \dots(2)$$

But the area bounded by C_2 lies in the region between the capacitor plates, so no current flows across it.

Hence,

$$\oint_{C_2} \vec{B} \cdot d\vec{l} = 0 \quad \dots(3)$$

Imagine the loops C_1 and C_2 to be infinitesimally close to each other, as shown in figure (b). Then we must have,

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \oint_{C_2} \vec{B} \cdot d\vec{l}$$

This result is inconsistent with the Eq. (2) and (3). There must be some missing term in this law.

Maxwell's Modification of Ampere's Law

Maxwell argued that a changing electric field between the capacitor plates must induce a magnetic field. As currents are the usual sources of magnetic fields, a changing electric field must be associated with a current. Maxwell called this current as the displacement current.

If, A be the area of the capacitor plates and q be the charge on the plates at any instant t during the charging process, then the electric field in the gap will be,

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

$$EA = \frac{q}{\epsilon_0}$$

Flux,
$$\phi_E = \frac{q}{\epsilon_0}$$

Hence,
$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{\epsilon_0}\right) = \frac{1}{\epsilon_0} \frac{dq}{dt}$$

But $\frac{dq}{dt}$ is the rate of change of charge on the capacitor plates. It is called displacement current and is given by,

$$I_d = \frac{dq}{dt} = \epsilon_0 \frac{d\phi_E}{dt}$$

This is the missing term in Ampere's circuit law. The total current must be the sum of the conduction current I_c and the displacement current I_d . Thus,

$$I = I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}$$

Hence, the modified form of the Ampere's law is,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right]$$

94. Draw a labelled diagram of Hertz's experiment. Explain how electromagnetic radiations are produced using this set-up.

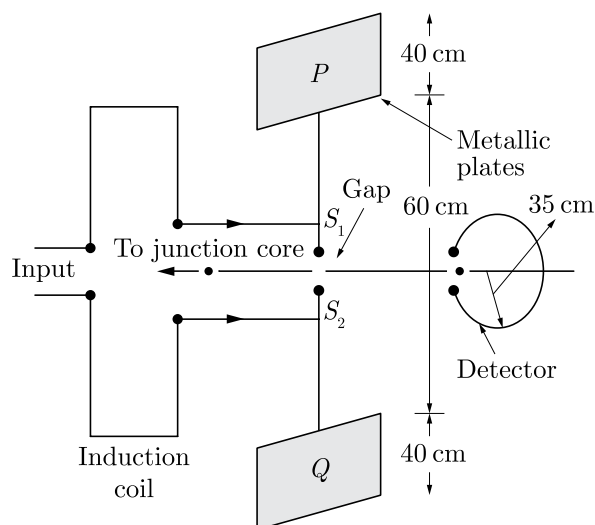
Ans :

OD 2017, Comp 2006

Hertz's experiment was based on the fact that an oscillating electric charge radiates electromagnetic waves and these waves carry energy which is being supplied at the cost of K.E. of the oscillating charge.

Hertz Apparatus : The experimental arrangement used by Hertz for the production and detection of electromagnetic waves in the laboratory, is shown in fig. His experimental arrangement consists of two metal sheets P_1 and P_2 . These sheets are connected to a source of very high voltage (i.e. an induction coil, which can supply a potential difference of several thousand volts). S_1 and S_2 are two metal spheres connected to the metal sheets is kept nearly 60 cm and that between the sphere is normally from 2 cm to 2.5 cm.

The two plates P_1 and P_2 form a capacitor of very low capacitance (c). The circuit containing P_1 and P_2 (being completed by conducting wire), has also some low value of inductance L . It has forms an LC circuit. Detector (D) consisting of a coil to the ends of which two other small metal spheres S_1 and S_2 are connected.



Working of Hertz : apparatus Due to existence of very high voltage, air present in the gap across the plates of spheres S_1 and S_2 gets ionised. Due to presence of the ions or charged particles, the path between the spheres S_1 and S_2 become conducting. As a result of this, very high time-varying current flows across the gap between S_1 and S_2 (as plates P_1 and P_2 form an LC Circuit). Due to this a spark is produced. Since, sheets P_1 , P_2 form an LC-circuit, hence, electromagnetic waves of frequency $f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$ are radiated.

Function of the detector : D Hertz detected the electromagnetic waves by means of a detector D kept at suitable distance from the conducting spheres S_1, S_2 . Detector D is made of two similar conducting spheres S_1 and S_2 . Joined to the ends of a coil to form another LC circuit. The frequency of this LC circuit is made equal to the frequency of electromagnetic waves reaching it. The frequency can be adjusted by changing the distance between S_1 and S_2 . Hertz placed the detector in such a way that the magnetic lines of force produced by the oscillating electric field across the gap between S_1 and S_2 are normal to the plane of coil (C). When magnetic lines of force cut the detector coil an emf is induced in it. Hence, air in the gap between S_1 and S_2 gets ionised. A conducting path becomes available for the induced current it flow across the gap. Thus, the spark is produced between

S_1 and S_2 Hertz also observed that the spark across S_1 and S_2 was greatest when the $S_1'S'_2$ and S_1S_2 were parallel to each other. This clearly established that electromagnetic waves produced were polarise i.e. $\vec{E}$ and $\vec{B}$ always lie in one plane.

95. What is an electromagnetic wave? How can we express mathematically a plane electromagnetic wave propagating along X-axis? Also represent it graphically.

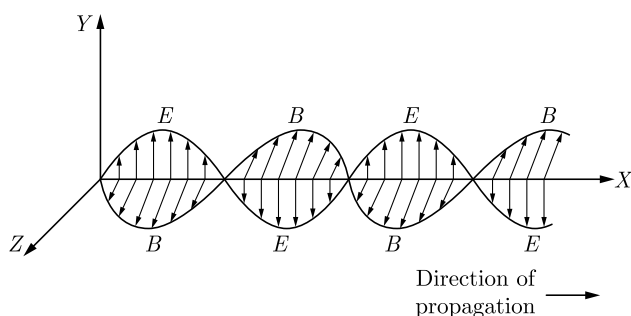
Ans :

Comp 2020

Electromagnetic Wave

An electromagnetic wave is a wave radiated by an accelerated charge and which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave.

Mathematical Representation of Electromagnetic Waves



A plane electromagnetic wave travelling along X-axis

Figure shows a plane electromagnetic wave of frequency ν and wavelength λ travelling along X-axis with speed c . The electric field $\vec{E}$ oscillates along Y-axis while the magnetic field $\vec{B}$ oscillates along Z-axis. The electric field vector can be represented mathematically as follows:

$$\begin{aligned}\vec{E} &= E_y \hat{j} = E_0 \sin(kx - \omega t) \hat{j} \\ &= E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right] \hat{j} \\ &= E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \hat{j} \quad \dots(1)\end{aligned}$$

where, $k = \frac{2\pi}{\lambda}$ is the propagation constant of the wave

$$\text{and} \quad \omega = 2\pi\nu$$

$$\text{Hence,} \quad E_x = E_z = 0$$

The magnetic field vector may be represented as,

$$\vec{B} = B_z \hat{k} = B_0 \sin(kx - \omega t) \hat{k}$$

$$\begin{aligned}&= B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \nu t\right)\right] \hat{k} \\ &= B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \hat{k} \quad \dots(2)\end{aligned}$$

$$\text{Hence,} \quad B_x = B_y = 0$$

Here, E_0 and B_0 are the amplitudes of the electric field $\vec{E}$ and magnetic field $\vec{B}$, respectively. They are related as,

$$\frac{E_0}{B_0} = c$$

The Eq. (1) and (2) show that the variations in electric and magnetic fields are in same phase.

As the electric and magnetic fields in an e.m. wave are always perpendicular to each other and also perpendicular to the direction of wave propagation, so e.m. waves are transverse in nature.

Maxwell showed that the speed of an e.m. wave in free space is given by,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.0 \times 10^8 \text{ ms}^{-1}$$

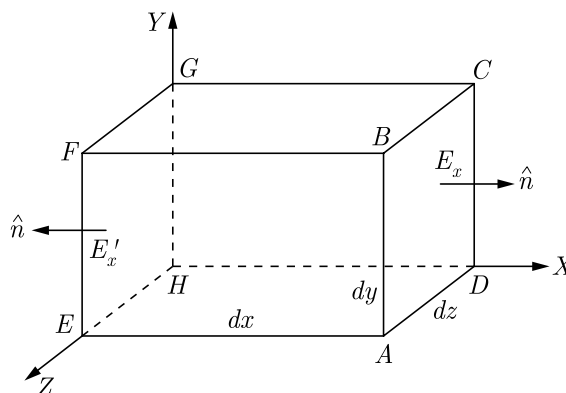
which is the speed of light in vacuum. This fact led Maxwell to predict that light is an electromagnetic wave.

96. Prove mathematically that electromagnetic waves are transverse in nature.

Ans :

Delhi 2016, SQP 2009

Consider a plane electromagnetic wave travelling in the X-direction. The associated wave-front lies in the YZ-plane and ABCD is a portion of it at any time t . The electric and magnetic fields at time t will be zero to the right of ABCD. To the left of ABCD, they depend on x and t .



Consider the closed surface enclosed by the parallelepiped ABCDEFGH of sides dx , dy and dz . The total electric flux through this closed surface must be zero as it does not enclose any charge.

By Gauss's theorem,

CASE BASED QUESTIONS

- 102.** Seema noticed that some scientists have predicted that a global nuclear war on the Earth would be followed by a server 'nuclear winter' (Nuclear winter is the term for a theory describing the climatic effects of nuclear war.) with devastating effect on life on the Earth. She was interested to know the facts and so she went to her friend Reema. Reema explained him all the concerning facts properly.

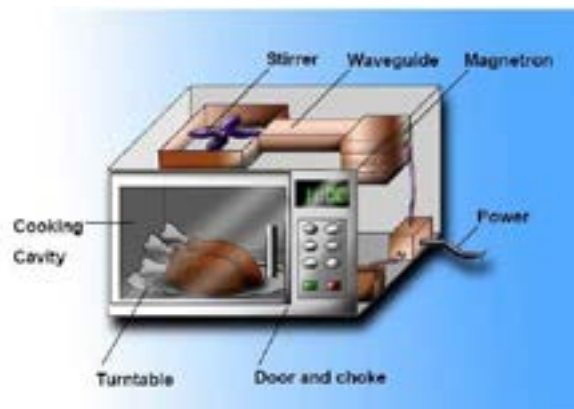


- What is the effect of a nuclear winter?
- What might be the basis of this prediction?

Ans :

- The conditions of semi-darkness, killing frosts and sub-freezing temperatures, combined with high doses of radiation from nuclear fallout, would interrupt plant photosynthesis and could thus destroy much of the Earth's vegetation and animal life.
 - A global nuclear war would make the substantial parts of the sky getting covered by very minute particles. They will be responsible for preventing Sun rays from reaching the Earth for a very long time resulting in 'nuclear winter'.
- 103.** Microwave oven, also called electronic oven, appliance that cooks food by means of high-frequency electromagnetic waves called microwaves. A microwave oven is a relatively small, boxlike oven that raises the temperature of food by subjecting it to a high-frequency electromagnetic field. The microwaves are absorbed by water, fats, sugars, and certain other molecules, whose consequent vibrations produce heat. The heating thus occurs inside the food, without warming the surrounding air; this greatly reduces cooking time, and baking and other cooking tasks that require hours in a conventional oven can be completed in minutes in a microwave oven. Microwave ovens generate radiation at a frequency of about 2,450 megahertz by means of a magnetron, which is a kind of electron tube.

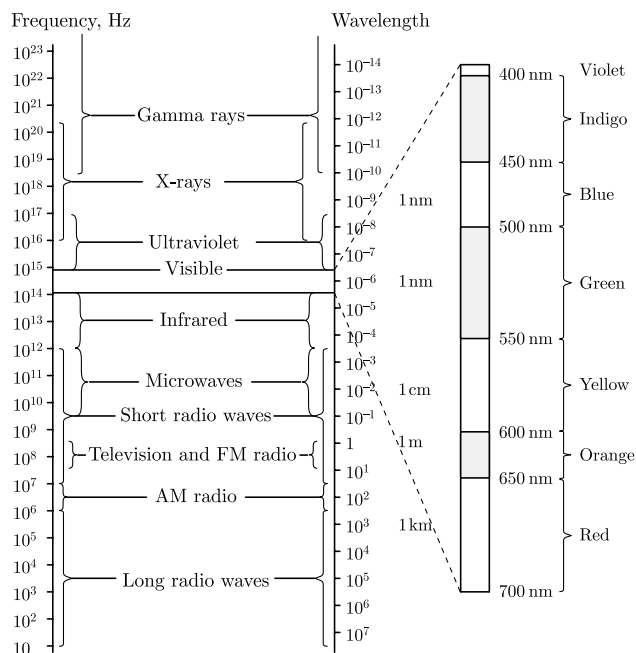
Monika wanted to gift her aunt a microwave oven so that her aunt enjoys some rest from tough household activities. She requested her aunt and convinced her about details and significance of microwave oven .



- Why wave-guide is used in microwave frequency?
- How does microwave oven work? Explain briefly.

Ans :

- Wave-guides are used at frequencies in the microwave range. At microwave frequencies (above 1GHz to 100 GHz) the losses in the two line transmission system will be very high and hence it cannot be used at those frequencies. Hence microwave signals are propagated through the wave-guides in order to minimize the losses.
 - Microwave oven works because microwaves of a particular wavelength are very strongly absorbed by water. The energy carried by the microwaves heats the water up. Generally all food items contain a lot of water, they can be heated and cooked very quickly in microwave oven. In this way the microwaves heat the food item throughout its volume, not just from the outside as in a normal oven.
- 104.** Kavita's aunt was advised by her doctor to have an ECG (Electrocardiograph) for heart. Her aunt did not know much about the details and significance of this test. She also felt that it was too expensive and thought of postponing it. When Kavita learnt about her aunt's problem, she took the help of her parents, friends and responsible neighbours and arranged for the cost of test. Then she told her aunt that an ECG of heart would enable the doctor to know the condition of her heart without causing any harm to her. Her aunt was convinced and got the required ECG of her heart done. The resulting information greatly helped her doctors to treat her well.



Electromagnetic spectrum

NUMERICAL QUESTIONS

99. A parallel plate capacitor is charged to $60 \mu\text{C}$. Due to a radioactive source, the plate loses charge at the rate of $1.8 \times 10^{-8} \text{ C-s}^{-1}$. What is the magnitude of displacement current?

Ans :

Delhi 2020

Given,

Charge on capacitor, $q = 60 \mu\text{C} = 60 \times 10^{-6} \text{ C}$
and rate of losses of charge,

$$\frac{dq}{dt} = 1.8 \times 10^{-8} \text{ C-s}^{-1}$$

Electric field between the parallel plate capacitor,

$$E = \frac{q}{\epsilon_0 A}$$

Therefore electric flux through the plate,

$$\begin{aligned} \phi_E &= EA \\ &= \frac{q}{\epsilon_0 A} \times A = \frac{q}{\epsilon_0} \end{aligned}$$

We also know that magnitude of displacement current,

$$\begin{aligned} I_d &= \epsilon_0 \frac{d\phi_E}{dt} \\ &= \epsilon_0 \frac{d}{dt} \left(\frac{q}{\epsilon_0} \right) \\ &= \epsilon_0 \times \frac{1}{\epsilon_0} \frac{dq}{dt} = \frac{dq}{dt} \end{aligned}$$

$$= 1.8 \times 10^{-8} \text{ C-s}^{-1}$$

100. A parallel plate capacitor consists of two circular plates each of radius 2 cm separated by distance of 0.1 mm. If rate of change of potential difference is $5 \times 10^{13} \text{ V-s}^{-1}$, Find out the value of displacement current ?

Ans :

Delhi 2019

Given,

Radius of each plates, $r = 2 \text{ cm} = 0.02 \text{ m}$

Distance between the plates,

$$d = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$$

and rate of change of potential difference,

$$\frac{dV}{dt} = 5 \times 10^{13} \text{ V-s}^{-1}$$

Area of circular plate,

$$\begin{aligned} A &= \pi r^2 = \pi (0.02)^2 \\ &= 1.26 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Therefore displacement current,

$$\begin{aligned} I_d &= \epsilon_0 \frac{d\phi_E}{dt} \\ &= \epsilon_0 \frac{d}{dt} (EA) \\ &= \epsilon_0 \frac{d}{dt} \left(\frac{V}{d} \times A \right) \\ &= \frac{\epsilon_0 A}{d} \frac{dV}{dt} \\ &= \frac{(8.85 \times 10^{-12})(1.26 \times 10^{-3})}{0.1 \times 10^{-3}} \times (5 \times 10^{13}) \\ &= 5.6 \times 10^3 \text{ A} \end{aligned}$$

(where, ϵ_0 = Absolute electric permittivity of free space equal to $8.85 \times 10^{-12} \text{ C}^2\text{-N}^{-1}\text{-m}^{-2}$)

101. In an electromagnetic wave, the electric and magnetic fields are 100 V-m^{-1} and 0.265 A-m^{-1} . What is the maximum energy flows ?

Ans :

Comp 2018, OD 2013

Given,

Intensity of electric field,

$$E_0 = 100 \text{ V-m}^{-1}$$

and Intensity of magnetic field,

$$H_0 = 0.625 \text{ A-m}^{-1}$$

The maximum energy flow,

$$\begin{aligned} S &= E_0 \times H_0 \\ &= 100 \times 0.265 \\ &= 26.5 \text{ W-m}^2 \end{aligned}$$

$$\oint_{ABCDEFGH} \vec{E} \cdot \vec{dS} = 0$$

$$\left[\oint_{ABCD} \vec{E} \cdot \vec{dS} + \oint_{EFGH} \vec{E} \cdot \vec{dS} \right] + \left[\oint_{ABFE} \vec{E} \cdot \vec{dS} + \oint_{DCGH} \vec{E} \cdot \vec{dS} \right] + \left[\oint_{AEHD} \vec{E} \cdot \vec{dS} + \oint_{BFGC} \vec{E} \cdot \vec{dS} \right] = 0$$

The paired integrals are the contributions from the faces normal to X , Y and Z -axis, respectively. Since E does not depend on y and z , the contributions from the faces normal to Y and Z -axis cancel out in pairs, so the above equation becomes,

$$\oint_{ABCD} \vec{E} \cdot \vec{dS} + \oint_{EFGH} \vec{E} \cdot \vec{dS} = 0 \quad \dots(1)$$

Let, E_x and E'_x be the x components of the electric field at the faces $ABCD$ and $EFGH$ respectively. The outward normals on these faces are oppositely directed (along X -axis), therefore,

$$\oint_{ABCD} \vec{E} \cdot \vec{dS} = E_x \cdot dy dz$$

$$\oint_{EFGH} \vec{E} \cdot \vec{dS} = -E'_x dy dz$$

Hence, Eq. (1) becomes

$$E_x \cdot dy dz - E'_x \cdot dy dz = 0$$

or

$$E_x = E'_x$$

i.e., the component of the electric field along the direction of propagation is constant. But a constant or static field cannot produce a wave, so this constant must be equal to zero, i.e., $E_x = 0$.

Similarly, we can prove that $B_x = 0$. Thus the electric or magnetic fields have no component along the direction of propagation. Or, in an electromagnetic wave both the electric and magnetic fields are perpendicular to the direction of propagation, i.e., the electromagnetic waves are transverse in nature.

97. Obtain expression for the energy density of an electromagnetic wave. In an electromagnetic wave, show that the average energy density of the E field equals the average energy density of the B field.

Ans :

Comp 2019

Electromagnetic waves carry energy as they travel through space. Energy density of an E.M. wave is the energy in unit volume of the space through which the wave travels.

In free space, the energy density of a static field E is,

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

Again in free space, the energy density of a static magnetic field is,

$$\mu_B = \frac{1}{2\mu_0} B^2$$

The total energy density of the static electric and magnetic fields will be,

$$u = u_E + u_B$$

$$= \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

But in an electromagnetic wave, both E and B fields vary sinusoidally in space and time. The average energy density u of E.M. wave can be obtained by replacing E and B by their R.M.S. values in the above equation. Thus,

$$u = \frac{1}{2} \epsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} B_{\text{rms}}^2$$

$$= \frac{1}{4} \epsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

$$\left[\text{since, } E_{\text{rms}} = \frac{E_0}{\sqrt{2}}, B_{\text{rms}} = \frac{B_0}{\sqrt{2}} \right]$$

Moreover,

$$E_0 = cB_0$$

and

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

Therefore,

$$u_E = \frac{1}{4} \epsilon_0 E_0^2 = \frac{1}{4} \epsilon_0 (cB_0)^2$$

$$= \frac{1}{4} \epsilon_0 \cdot \frac{B_0^2}{\mu_0 \epsilon_0}$$

$$= \frac{1}{4\mu_0} B_0^2 = u_B$$

Hence, in an electromagnetic wave, the average energy of the E field equals the average energy density of the B field. It may be noted that,

$$u = \frac{1}{4} \epsilon_0 E_0^2 + \frac{1}{4} \epsilon_0 E_0^2$$

$$= \frac{1}{2} \epsilon_0 E_0^2 = \epsilon_0 E_{\text{rms}}^2$$

Also,

$$u = \frac{1}{4\mu_0} B_0^2 + \frac{1}{4\mu_0} B_0^2$$

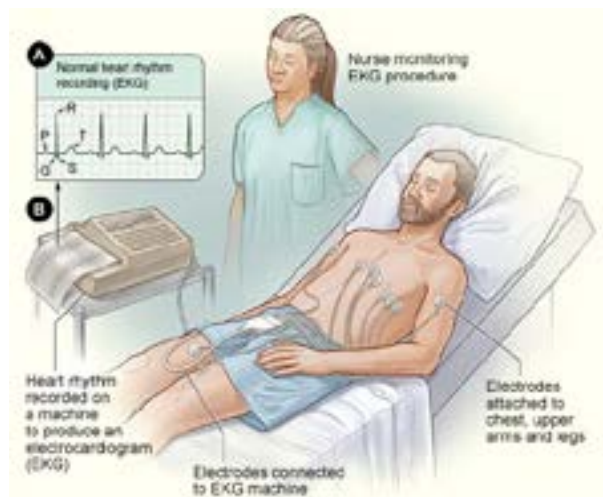
$$= \frac{1}{2\mu_0} B_0^2 = \frac{1}{\mu_0} B_{\text{rms}}^2$$

98. What is electromagnetic spectrum? Draw a sketch of electromagnetic spectrum indicating the frequency and wavelength ranges of its different parts.

Ans :

Delhi 2015, Comp 2011

The orderly distribution of the electromagnetic waves in accordance with their wavelength or frequency into distinct groups having widely differing properties is called electromagnetic spectrum. As shown in figure, the main parts of the e.m. spectrum are γ -rays, X-rays, ultraviolet rays, visible light, infrared rays, microwaves and radio waves in the order of increasing wavelength from $10^{-2} \text{ \AA}$ or 10^{-12} m to 10^6 m .



- (i) What do you mean by ECG?
- (ii) A plane electromagnetic wave of frequency 25 MHz travels in free space along the x -direction. At a particular point space and time, the electric vector $\vec{E} = 6.3 \text{ V/m} \hat{j}$. Calculate B at this point.

Ans :

- (i) An electrocardiogram (ECG) is a simple test that can be used to check your heart's rhythm and electrical activity. Sensors attached to the skin are used to detect the electrical signals produced by your heart each time it beats.
- (ii) In given condition, magnetic vector $\vec{B}$ must be along z -axis with,

$$B_0 = \frac{E_0}{c} = \frac{6.3}{3 \times 10^8}$$

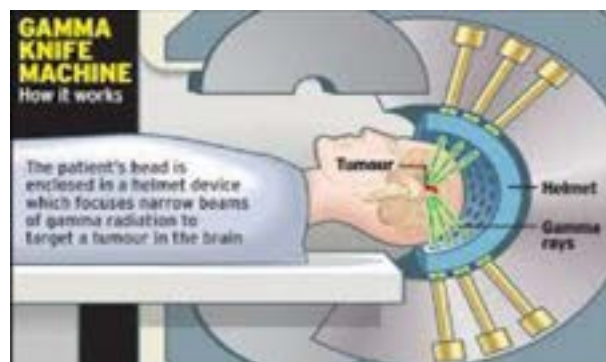
$$= 2.1 \times 10^{-8} \text{ T}$$

Thus, $B = 2.1 \times 10^{-8} \text{ T} \hat{k}$

- 105.** A gamma ray (γ) is a packet of electromagnetic energy (photon) emitted by the nucleus of some radionuclide following radioactive decay. Gamma photons are the most energetic photons in the electromagnetic spectrum. Gamma rays are a form of electromagnetic radiation (EMR). They are similar to X-rays, distinguished only by the fact that they are emitted from an excited nucleus. Electromagnetic radiation can be described in terms of a stream of photons, which are massless particles each travelling in a wave-like pattern and moving at the speed of light. Each photon contains a certain amount (or bundle) of energy, and all electromagnetic radiation consists of these photons. Gamma-ray photons have the highest energy in the EMR spectrum and their waves have the shortest wavelength. Gamma-ray photons generally have energies greater than 100 keV. For comparison, ultraviolet radiation has energy that falls in the range from a few electron volts to about 100 eV and does

not have enough energy to be classified as ionising radiation. The high energy of gamma rays enables them to pass through many kinds of materials, including human tissue. Very dense materials, such as lead, are commonly used as shielding to slow or stop gamma rays.

Gamma rays are used in radiotherapy to treat cancer. They are used to spot tumors, they kill the living cells and damage malignant tumor.



- (i) What is the source of gamma rays ?
- (ii) How is wavelength of gamma rays ?
- (iii) What is the correct order of alpha, beta and gamma rays according to the wave length ?

Ans :

- (i) Radioactive decay of nucleus.
- (ii) High.
- (iii) Gamma > beta > alpha.

- 106.** Mohit fractured his leg by chance while playing. His parents took him to a doctor for treatment. The doctor advised that an X-rays of the leg needed to be done for diagnosing the problem. On seeing X-rays photograph. Mohit was amazed and excited to know that the photograph was so clear. He requested the doctor to explain how it happened. The doctor gave a brief explanation of his question happily and appreciated Mohit's nature.

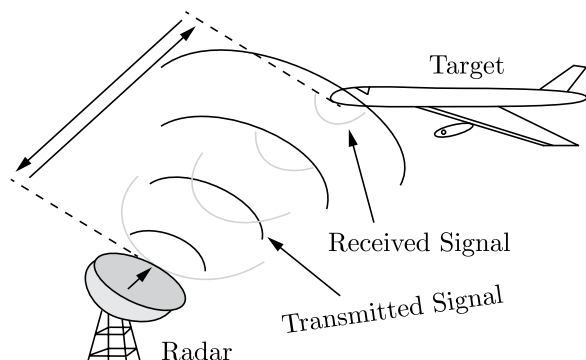


- (i) Which property of X -rays make it suitable for use in diagnosing and identifying the fracture in bones ?
- (ii) Write two other uses of X -rays ?

Ans :

- (i) The property of X -rays to pass through flesh but unable to pass through bones is used to take X -rays photograph.
- (ii) Two other uses of X -ray are :
- Study of crystal structures.
 - In engineering-for detecting cracks, faults etc.

107. Microwave is a form of electromagnetic radiation with wavelengths ranging from about one meter to one millimetre corresponding to frequencies between 300 MHz and 300 GHz respectively. Microwave are used in aircraft navigation. A radar guns out short bursts of microwave and it reflect back from oncoming aircraft and are detected by receiver in gun. The frequency of reflected wave used to compute speed of aircraft.



- How are microwave produced?
- Why microwave use for aircraft navigation?
- Which is use of microwave?
- Where do microwave fall in electromagnetic spectrum?

Ans :

- Klystron and magnetron valve.
- Due to low wavelength
- Studying details of atoms and molecule.
- Between infrared and radio wave.

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CHAPTER 9

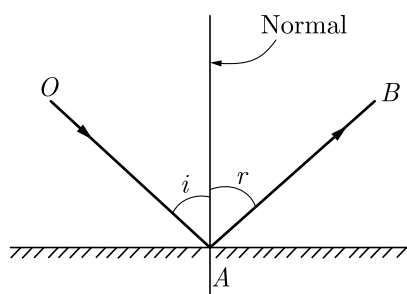
Ray Optics and Optical Instruments

SUMMARY

1. REFLECTION OF LIGHT

Reflection of light is the phenomenon of bouncing back of light in the same medium after striking a surface.

The reflection of light is shown in figure:



where, OA = incident ray

AB = reflected ray

$\angle i$ = incident angle

$\angle r$ = reflection angle

2. TOTAL INTERNAL REFLECTION

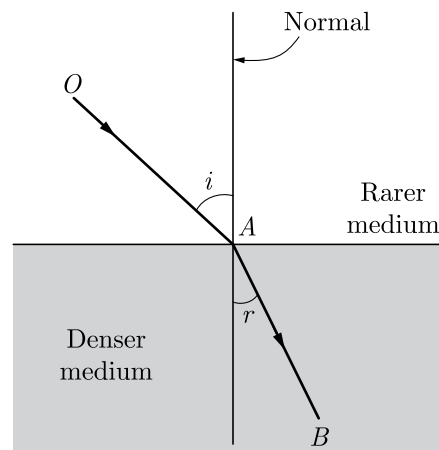
Critical angle is the angle of incidence in the denser medium for which angle of refraction in rarer medium is 90° .

When angle of incidence in the denser medium is greater than critical angle, the ray is reflected back in the denser medium. This phenomenon is called total internal reflection. In this case, angle of incidence is equal to the angle of reflection.

Practical applications of total internal reflection are mirage, optical fibre, brilliance of a diamond.

3. REFRACTION

Refraction of light is the change in path of a ray of light as it passes from one medium into another. Refraction of light follows two laws.



where, OA = incident ray

AB = refracted ray

NA = Normal

1st law

The incident ray, refracted ray and the normal at the point of incidence, all lie on the same plane.

2nd law

The ratio of sine of angle of incidence to the sine of angle of refraction is constant for any two given media.

$$\text{i.e.,} \quad \mu = \frac{\sin i}{\sin r}$$

$$\text{Also, } {}^a\mu_b \times {}^b\mu_c \times {}^c\mu_a = 1$$

$$\text{or } {}^b\mu_c = {}^b\mu_a \times {}^a\mu_c = \frac{{}^a\mu_c}{{}^a\mu_b}$$

$$\text{Again, } \mu = \frac{\text{Real Depth}}{\text{Apparent Depth}}$$

4. REFRACTION AT SPHERICAL SURFACES

When light travels from rarer to denser medium, then

$$\frac{\alpha_1}{-\alpha} + \frac{\alpha_2}{v} = \frac{\alpha_2 - \alpha}{R}$$

but when it travels from denser to rarer medium, then

$$\frac{\alpha_2}{-\alpha} + \frac{\alpha}{v} = \frac{\alpha_2 - \alpha}{R}$$

Mirror Formula

Mirror Formula is $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

where, f = focal length of mirror

u = object distance

v = image distance

Lens Formula

Lens formula is $\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$ and Lens maker's formula is the relation between the radii of curvatures R_1 , R_2 of the curved surfaces of lens, then if f is the focal length of the lens, then

$$\frac{1}{f} = (\mu - 1) \left\{ \frac{1}{R_1} - \frac{1}{R_2} \right\}$$

In our eye lens, we change its focal length by changing the radius of two sides of eye lens by the action of ciliary muscles and zinnule fibres.

5. LINEAR MAGNIFICATION OF THE IMAGE

The Ratio of the height of image and the Ratio of height of object is known as linear magnification. i.e.

$$m = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

6. POWER OF A LENS

Power of a lens is the reciprocal of its focal length. Power is measured in dioptre (D).

$$\text{Power (in dioptre)} = \frac{1}{f(\text{in metre})}$$

Power are added algebraically

$$P = P_1 + P_2 + P_3 + \dots + P_n$$

7. WHEN LENSES ARE COMBINED

When lenses are combined together side to side (touching) then combined focal lengths f is,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots + \frac{1}{f_n}$$

8. THE RESULTANT FOCAL LENGTH

The resultant focal length f of combination of two lens of focal lengths f_1 and f_2 placed distance x apart is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

and their combined power is

$$P = P_1 + P_2 - xP_1P_2$$

9. PRISM

Prism is a portion of transparent medium like glass, quartz, rock salt etc. bounded by three rectangular

and two triangular plane surfaces.

For a prism,

$$1. \quad A + \delta = i + e:$$

$$2. \quad \delta = A(\mu - 1) \text{ if angles are small}$$

$$3. \quad \mu = \frac{\sin \frac{A+\delta_m}{2}}{\sin \frac{A}{2}}$$

where, δ = deviation angle

i = incidence angle

e = emergent angle

μ = refractive index of prism

A = prism angle

10. DISPERSION

Dispersion is the phenomenon of splitting up of white light into constituent colours when passed through prism. Dispersion is due to different refractive index μ for different wavelengths.

According to Cauchy's law

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$

and

$$\delta = A(\mu - 1)$$

More the λ , lesser the μ and lesser deviation δ for ray as compared to violet ray.

Angle of dispersion,

$$\theta = \delta_v - \delta_r = (\alpha_v - \alpha_r) A$$

$$\begin{aligned} \text{Dispersion power, } \omega &= \frac{\theta}{\text{mean deviation } \delta} \\ &= \frac{\alpha_v - \alpha_r}{\alpha - 1} \end{aligned}$$

11. ANGULAR DISPERSIVE POWER

Angular dispersive power is the angle of dispersion per unit wavelength range. If $d\delta$ is the angle of dispersion between wavelength range λ and $\lambda + d\lambda$ (range of $d\lambda$), then angular dispersive power

$$\omega = \frac{d\delta}{d\lambda}$$

It can be proved that $\omega \propto \frac{1}{\lambda^3}$.

This shows that spectrum is more spread on the short wavelength side (violet) than on the long wavelength side (red).

Hence prism spectrum is not linear.

12. RAYLEIGH'S LAW OF SCATTERING

Rayleigh's law of scattering is that scattering of light by air etc. is inversely proportional to the fourth

power of wavelength i.e.

$$\text{Scattering} \propto \frac{1}{\lambda^4}$$

Blue colour of the sky, reddish colour of the sun at the time of sunrise and sunset can be explained on the basis of above said law.

$$\text{R.P. of microscope} = \frac{2\mu \sin \theta}{\lambda}$$

and

$$\text{R.P. of telescope} = \frac{D}{1.22\lambda}$$

13. OPTICAL INSTRUMENTS

Optical instruments are the instruments based upon laws of reflection and refraction.

14. MICROSCOPE

Microscope is an optical instrument used to observe minute particles in magnified form.

A simple microscope is an ordinary convex lens of small focal length placed at such a distance from object so that its clear image is formed at the least distance of distinct vision. It is used by biology students to observe dissected animal by watch repairer and by palmists. Its magnifying power,

$$M = \left(1 + \frac{D}{f_e}\right)$$

15. COMPOUND MICROSCOPE

Compound microscope consists of two lenses an objective of small aperture and small focal length.

16. MAGNIFYING POWER OF COMPOUND MICROSCOPE

1. When image formed at near point,

$$m = \frac{v_0}{u_0}(1 + Df_e)$$

2. When the final image is at infinity,

$$m = \frac{-C}{f_0} \cdot \frac{D}{f_e}$$

17. TELESCOPE

Telescope is used to study far off objects clearly. Astronomical telescope is used to observe heavenly bodies. It forms an inverted image. Its magnifying power in normal adjustment is,

$$M = -\frac{f_0}{f_e}$$

When the image of a distant object is formed at least distance of distinct vision, the magnifying power is,

$$M = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{D}\right)$$

18. RESOLVING POWER

Resolving power of an optical instrument is its ability to resolve two images of two point objects lying very close to each of these so that other images can be seen quite separately and distinctly.

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OBJECTIVE QUESTIONS

1. A ray of monochromatic light propagating in air is incident on the surface of water. Which of the following will be the same for the reflected and refracted rays?

(a) Energy carried (b) Speed
(c) Frequency (d) Wavelength

Ans :

OD 2023

The frequency of the light remains constant when it passes from one medium to another and it does not depend on the medium it passes through.

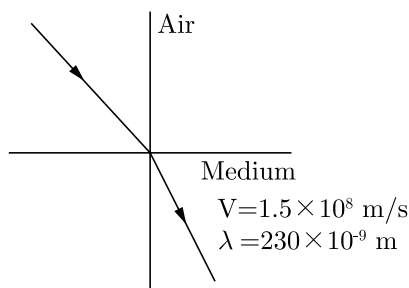
Thus (c) is correct option.

2. A beam of light travels from air into a medium. Its speed and wavelength in the medium are $1.5 \times 10^8 \text{ ms}^{-1}$ and 230 nm respectively. The wavelength of light in air will be

(a) 230 nm (b) 345 nm
(c) 460 nm (d) 690 nm

Ans :

OD 2023



Now,

$$\mu = \frac{\infty_2}{\infty_1}$$

here,

$$\mu_1 = 1 \quad (\text{it is of medium air})$$

Now,

$$\mu = \mu_2 = \frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}}$$

Now,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$$

$$\mu = \frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}}$$

$$2 = \frac{\lambda_{\text{air}}}{230 \times 10^{-9} \text{ m}}$$

$$\lambda_{\text{air}} = 460 \times 10^{-9} \text{ m} \\ = 460 \text{ nm}$$

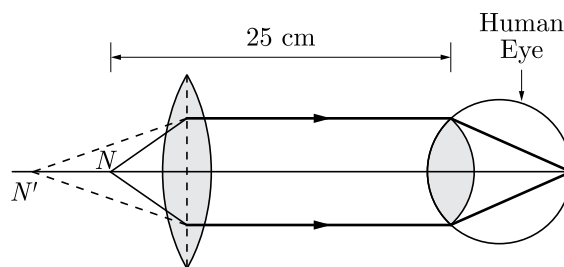
Thus (c) is correct option.

3. To remove hypermetropia lens used is
(a) Convex (b) Concave
(c) Cylindrical (d) Plano-convex

Ans :

OD 2018

A hypermetropia eye is corrected by using a convex lens of suitable focal length.



where, N = near point of normal eye

N' = near point of defective eye

Thus (a) is correct option.

4. The image formed by objective lens of a compound Microscope is
(a) Virtual and diminished
(b) Real and diminished
(c) Real and large
(d) Virtual and Large

Ans :

Delhi 2018, Comp 2002

The image formed by objective lens of a compound microscope is real and large.

Thus (c) is correct option.

5. A convex lens is dipped in a liquid, whose refractive index is equal to refractive index of material of lens. Then its focal length will
(a) become zero (b) become infinite
(c) decrease (d) increase

Ans :

SQP 2015

We have $\frac{1}{f} = (\mu_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\frac{1}{f} = \left(\frac{\infty_2}{\infty_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Here,

$$\mu_1 = \mu_2$$

Then,

$$\frac{1}{f} = 0$$

or

$$f = \infty$$

Thus (b) is correct option.

6. Critical angle for light moving from medium I to medium II is θ . The speed of light in medium I is v . Then speed in medium II is
(a) $v(1 - \cos \theta)$ (b) $\frac{v}{\sin \theta}$
(c) $\frac{v}{\cos \theta}$ (d) $v(1 - \sin \theta)$

Ans :

OD 2011

According to Snell's Law.

$$n_1 \sin i = n_2 \sin r \quad \dots(1)$$

where, n_1 = Refractive index of first medium

n_2 = Refractive index of second medium

i = Angle of incidence

r = Angle of refraction

For critical angle,

Angle of incidence, $i = \theta$

Angle of refraction, $r = 90^\circ$

Now, from Equation (1)

$$n_1 \sin \theta = n_2 \sin 90^\circ$$

$$\sin \theta = \frac{n_2}{n_1} \quad \dots(2)$$

Now $n = \frac{c}{v}$

where n is inversely proportional to v .

Hence, $\frac{n_2}{n_1} = \frac{v_1}{v_2}$

$$\sin \theta = \frac{v_1}{v_2}$$

$$v_2 = \frac{v}{\sin \theta}$$

Thus (b) is correct option.

7. Which of the following phenomena taken place when a monochromatic light is incident on a prism?

- (a) Dispersion (b) Deviation
(c) Interference (d) All of these

Ans :

Foreign 2014

Dispersion phenomena taken place when a monochromatic light is incident on a prism.

Thus (a) is correct option.

8. Optical fibre communication is based on which of the following phenomena

- (a) Total internal reflection
(b) Scattering
(c) Reflection
(d) Interference

Ans :

OD 2018

Optical fibre communication is based on total internal reflection. When light is incident on one end of the fibre at a small angle, it goes inside and suffers repeated total internal reflection because the angle of incidence is greater than the critical angle of the fibre material with respect to its outer coating.

Thus (a) is correct option.

9. The phase difference ϕ is related to path difference Δx by

- (a) $\frac{\lambda}{\pi} \phi$ (b) $\frac{\pi}{\lambda} \phi$
(c) $\frac{\lambda}{2\pi} \phi$ (d) $\frac{2\pi}{\lambda} \phi$

Ans :

Delhi 2018, 2016

The relation between phase difference (ϕ) and path difference (Δx) is given by,

$$\Delta x = \frac{\lambda}{2\pi} \times \phi$$

where, λ = wavelength of light

Thus (c) is correct option.

10. A short sighted person uses for clear vision

- (a) Convex Lens (b) Concave Lens
(c) Cylindrical Lens (d) Bi-focal Lens

Ans :

OD 2017, SQP 2004

A myopic or short sighted eye is corrected by using a concave lens of focal length equal to the distance of the far point F from the eye. This lens diverges the parallel ray from distant object as if they are coming from the far point f , finally the eye lens forms a clear image at the retina.

Thus (b) is correct option.

11. The angle of minimum deviation for thin prism of refractive index (μ) is

- (a) $(1 - \mu) A$ (b) $(\mu - 1) A$
(c) $(\mu + 1) A$ (d) $(\mu + 1) A^2$

Ans :

SQP 2017

Refractive index of the material of the prism,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}} \quad \dots(1)$$

If the prism angle is small than minimum.

Deviation (δ_m) is also small.

$$\text{Hence, } \sin\left(\frac{A + \delta_m}{2}\right) = \frac{A + \delta_m}{2}$$

$$\text{and } \sin \frac{A}{2} = \frac{A}{2}$$

From equation (1), we get

$$\mu = \frac{\frac{A + \delta_m}{2}}{\frac{A}{2}}$$

$$\mu = \frac{A + \delta_m}{A}$$

$$\mu A = A + \delta_m$$

$$\delta_m = \mu A - A = A(\mu - 1)$$

Thus (b) is correct option.

12. The focal length of a lens $\mu = 1.5$ in air is 20 cm. Its focal length in medium of refractive index 1.5 is

- (a) 20 cm (b) 40 cm
(c) 10 cm (d) ∞

Ans :

Foreign 2008

When lens is placed in medium, then

or reflection takes place on their surface of separation and the object becomes invisible.

Thus (d) is correct option.

27. The refractive indices of water and glass are 1.2 and 1.5 respectively. What will be the refractive index of glass with respect to water ?

- (a) 1.75 (b) 1.25
(c) 0.8 (d) 0.6

Ans :

OD 2008

Refractive index of water,

$$\mu_w = 1.2$$

and refractive index of glass,

$$\mu_g = 1.5$$

Refractive index of glass with respect to water,

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{1.5}{1.2} = 1.25$$

Thus (b) is correct option.

28. If ${}_m\mu_n$ represents refractive index when a light ray goes from medium m to medium n , then the value of the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to

- (a) ${}_3\mu_1$ (b) ${}_3\mu_2$
(c) ${}_4\mu_2$ (d) ${}_4\mu_1$

Ans :

SQP 2003

From the principle of reversibility that refractive index of medium,

$${}_m\mu_n = \frac{1}{{}_n\mu_m}$$

Therefore value of the product,

$$\begin{aligned} {}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3 &= \frac{1}{{}_1\mu_2} \times \frac{1}{{}_2\mu_3} \times \frac{1}{{}_3\mu_4} \\ &= \frac{1}{{}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4} \\ &= \frac{1}{{}_1\mu_4} = {}_4\mu_1 \\ &\dots (\text{where, } {}^i\mu_j \times {}^j\mu_k = {}^i\mu_k) \end{aligned}$$

Thus (d) is correct option.

29. An object is placed at a distance f in the front of a convex mirror. If focal length of the mirror is f , then distance of image from pole of the mirror is

- (a) f (b) $2f$
(c) $\frac{f}{2}$ (d) $\frac{f}{4}$

Ans :

Delhi 2016

Focal length of convex mirror = f .

and distance between object and mirror,

$$u = -f$$

(Minus sign due to object placed in front of mirror)

Relation for the distance of image from pole of the convex mirror v is:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} + \frac{1}{(-f)}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{f} = \frac{2}{f}$$

$$v = \frac{f}{2}$$

Thus (c) is correct option.

30. The velocity of light in a medium is $2 \times 10^8 \text{ m}(-)\text{s}^{-1}$. Refractive index of the medium is

- (a) 1 (b) 1.1
(c) 1.4 (d) 1.5

Ans :

Foreign 2009, Comp 2012

Velocity of light in medium,

$$v = 2 \times 10^8 \text{ m}(-)\text{s}^{-1}$$

Refractive index of the medium,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

(where, c = velocity of light equal to $3 \times 10^8 \text{ m} - \text{s}^{-1}$)

Thus (d) is correct option.

31. If the critical angle for a medium is 45° , then its refractive index will be

- (a) $\sqrt{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

Ans :

OD 2002

Critical angle of the medium,

$$C = 45^\circ$$

Refractive index of the medium,

$$\begin{aligned} \mu &= \frac{1}{\sin C} = \frac{1}{\sin 45^\circ} = \frac{1}{1/\sqrt{2}} \\ &= \sqrt{2} \end{aligned}$$

Thus (a) is correct option.

32. A ray of light strikes a glass plate at an angle θ . If angle between the reflected and refracted rays is a right angle, then refractive index of the glass plate is given by

- (a) $\mu = \sin \theta$ (b) $\mu = \cos \theta$
(c) $\mu = \tan \theta$ (d) $\mu = \cot \theta$

Ans :

Delhi 2011

Angle of incidence, $i = \theta$

Angle of reflection,

$$r' = \text{Angle of incidence } i = \theta$$

Since angle between reflected and refracted ray is 90° .

Therefore, $r' + r = 90^\circ$

$$f = \frac{1}{P} = \frac{1}{12} \text{ m} = \frac{100}{12} \text{ cm}$$

Magnifying lower of simple microscope is given by,

$$m = 1 + \frac{D}{f} \\ = 1 + \frac{25 \times 12}{100} = 1 + 3 = 4$$

Thus (a) is correct option.

- 20.** A path length t in a glass plate of refractive index n is equivalent to path length in vacuum.

- (a) $(n - 1)t$ (b) nt
(c) $\left[\frac{n}{t} - 1\right]$ (d) none of these

Ans :

Foreign 2011, Comp 2016

Path length = Refractive index of glass plate $\times$ thickness of plate.

Thus (b) is correct option.

- 21.** The radius of curvature of plane mirror is

- (a) infinite (b) zero
(c) +5 cm (d) -5 cm

Ans :

OD 2011

Since a plane mirror is a part of sphere of infinite focal length. So its radius of curvature is infinite [$\because r = 2f$].

Thus (a) is correct option.

- 22.** The critical angle of light passing from glass to air is minimum for

- (a) Red colour (b) Green colour
(c) Yellow colour (d) Violet colour

Ans :

Foreign 2017

The refractive index of glass,

$$\mu = \frac{c}{v} \quad \dots(1)$$

Here, c = velocity of light

v = velocity of light in medium

$$\text{Since, } \mu = \frac{1}{\sin C} \quad \dots(2)$$

Here, C = critical angle

From Eqn. (1) and (2), we get

$$\frac{c}{v} = \frac{1}{\sin C}$$

Hence, $v \propto \sin C$ [Since, $v = n\lambda$]

Thus critical angle is minimum for light with minimum v and that is violet.

Thus (d) is correct option.

- 23.** SI unit of power of a lens is

- (a) joule (b) dioptre
(c) candela (d) watt

Ans :

OD 2011

The reciprocal of focal length f is known as the power of the lens. i.e.,

$$\text{Power of lens, } P = \frac{1}{f(\text{focal length})}$$

The SI unit of power of lens is dioptre.

Thus (b) is correct option.

- 24.** A plane mirror produces a magnification of

- (a) 0 (b) -1
(c) +1 (d) between 0 and +1

Ans :

Delhi 2017

In a plane mirror, height of image formed is erect and of the same size as of the object i.e.,

$$h_2 = h_1 \text{ (where, } h_1 = \text{size of object)}$$

Therefore magnification of plane mirror,

$$m = +\frac{h_2}{h_1} = +\frac{h_1}{h_1} = +1$$

Thus (c) is correct option.

- 25.** If a ray of light is incident on a plane mirror at an angle of 30° , then deviation produced by the plane mirror is

- (a) 30° (b) 60°
(c) 90° (d) 120°

Ans :

SQP 2007

Given,

Angle of incidence $i = 30^\circ$

Angle of reflection,

$$r = \text{Angle of incidence } i = 30^\circ$$

We also know that deviation produced by plane mirror,

$$\begin{aligned} \delta &= 180^\circ - (i + r) \\ &= 180^\circ - (30^\circ + 30^\circ) \\ &= 180^\circ - 60^\circ \\ &= 120^\circ \end{aligned}$$

Thus (d) is correct option.

- 26.** An object is immersed in a fluid. In order that the object becomes invisible, it should

- (a) have refractive index one
(b) absorb all light falling on it
(c) behave as a perfect reflector
(d) have refractive index exactly matching with that of the surrounding fluid

Ans :

Foreign 2002

If refractive index of a material is exactly matching with that of the surrounding fluid, then no refraction

$$\frac{1}{f} = ({}^m\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{\infty_g}{\infty_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, $\mu_g = \mu_m = 1.5$

Hence, $\frac{1}{f} = 0$ or $f = \infty$

Thus (d) is correct option.

13. The final image in astronomical telescope is
 (a) real and erect (b) real and inverted
 (c) virtual and inverted (d) virtual and erect

Ans :

OD 2016

An astronomical telescope is an optical instrument which is used to see the magnified image of distant heavenly bodies like stars planets, satellites and galaxies etc. The final image formed by an astronomical telescope is always virtual, inverted and magnified.

Thus (d) is correct option.

14. When a ray of light enters a glass slab its wavelength
 (a) decreases
 (b) increases
 (c) remains unchanged
 (d) data are not complete

Ans :

SQP 2016, Foreign 2013

As the light enters from rarer medium to the denser medium, the speed of light decreases but frequency remains same. The relation between speed of light, V wavelength λ and the frequency n is given as,

$$V = n\lambda$$

In the denser medium frequency, n remains constant and Velocity V decreases therefore wavelength, λ also decreases to keep the frequency constant.

Thus (a) is correct option.

15. When the tube length of microscope is increased its magnifying power
 (a) increases (b) decreases
 (c) becomes zero (d) remains unchanged

Ans :

OD 2016

Tube length of microscope is directly proportional to magnifying power of microscope. When the tube length of microscope is increased, its magnifying power will be increases.

Thus (a) is correct option.

16. When two converging lenses of same focal f are placed in contact, the focal length of the combination is
 (a) f (b) $2f$
 (c) $\frac{f}{2}$ (d) $3f$

Ans :

Delhi 2015

Focal length of combination of lens is given by,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Here, $f_1 = f_2 = f$

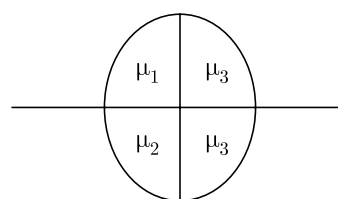
Then, $\frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f}$

$$\frac{1}{f_{eq}} = \frac{1+1}{f}$$

$$f_{eq} = \frac{f}{2}$$

Thus (c) is correct option.

17. How many images are formed by the lens if an object is kept on its axis?



- (a) 1 (b) 2
 (c) 3 (d) 7

Ans :

Foreign 2013

Mediums of refractive index μ_1 and μ_3 will form one lens and mediums of refractive Index μ_2 and μ_3 will form other lens both will have different focal length, so two images will be formed of an object.

Thus (b) is correct option.

18. If a glass rod is immersed in a liquid of the same refractive index, then it will
 (a) disappear (b) look bent
 (c) look longer (d) look shorter

Ans :

SQP 2015

If a glass rod is will immersed in a liquid of the same refractive index as that of the rod, then no refraction will take place.

Therefore, the glass rod will disappear.

Thus (a) is correct option.

19. The magnifying power of a magnifying glass of power 12 dioptre is
 (a) 4 (b) 1200
 (c) 3 (d) 25

Ans :

OD 2012

Given,

Magnifying power of glass = 12

Power of the lens, $P = \frac{1}{f}$

$$\theta + r = 90^\circ$$

$$r = 90^\circ - \theta$$

Refractive index of glass plate,

$$\begin{aligned}\mu &= \frac{\sin i}{\sin r} = \frac{\sin \theta}{\sin (90^\circ - \theta)} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta\end{aligned}$$

Thus (c) is correct option.

- 33.** A sound wave travels from air to water. The angle of incidence is α_1 and the angle of refraction is α_2 . If the Snell's law is valid, then

- (a) $\alpha_1 > \alpha_2$ (b) $\alpha_1 < \alpha_2$
(c) $\alpha_1 = \alpha_2$ (d) $\alpha_1 = 90^\circ$

Ans :

SQP 2010

Angle of incidence, $i = \alpha_1$

Angle of refractions, $r = \alpha_2$

Refractive index of water with respect to air,

$$\begin{aligned}{}^a\mu_w &= \frac{\text{Velocity of sound in air}}{\text{Velocity of sound in water}} \\ &= \frac{v_a}{v_w} \quad \dots(1)\end{aligned}$$

Refractive index of water with respect to air,

$${}^a\mu_w = \frac{\sin i}{\sin r} = \frac{\sin \alpha_1}{\sin \alpha_2} \quad \dots(2)$$

Equating equations (1) and (2),

$$\frac{v_a}{v_w} = \frac{\sin \alpha_1}{\sin \alpha_2}$$

Since velocity of sound in water, v_w is greater than that of air v_a ,

$$\text{Therefore, } \frac{v_a}{v_w} < 1$$

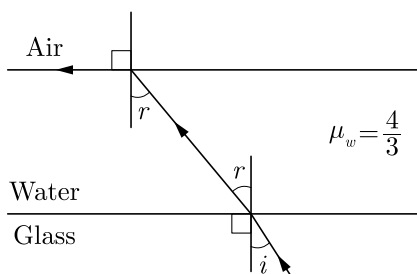
$$\frac{\sin \alpha_1}{\sin \alpha_2} < 1$$

$$\sin \alpha_1 < \sin \alpha_2$$

$$\alpha_1 < \alpha_2$$

Thus (b) is correct option.

- 34.** A ray of light is incident at the glass-water interface at an angle i . If it finally emerges parallel to the surface of water as shown in the figure, the value of μ_g would be



- (a) 1 (b) $\frac{4}{3}$
(c) $\frac{1}{\sin i}$ (d) $\frac{4 \sin i}{3}$

Ans :

OD 2004, SQP 2011

Angle of incidence at glass-water interface = i

Angle of refraction at glass-water interface = r

Angle of refraction at water-air interface = 90°

and refractive index of water,

$$\mu_w = \frac{4}{3}$$

For glass-water interface,

$$\frac{\mu_w}{\mu_g} = \frac{\sin i}{\sin r} \quad \dots(1)$$

Similarly, for water-air interface,

$$\frac{1}{\mu_w} = \frac{\sin r}{\sin 90^\circ}$$

$$\frac{1}{\mu_w} = \sin r$$

$$\mu_w = \frac{1}{\sin r}$$

Substituting the value of μ_w in equation (1),

$$\frac{1}{\sin r \times \mu_g} = \frac{\sin i}{\sin r}$$

Refractive index of glass,

$$\mu_g = \frac{1}{\sin i}$$

Thus (c) is correct option.

- 35.** Relation between critical angle of water (C_w) and that of the glass (C_g) is

- (a) $C_w > C_g$ (b) $C_w < C_g$
(c) $C_w = C_g$ (d) $C_w = C_g = 0$

Ans :

Foreign 2010

Critical angle of water = C_w

and Critical angle of glass = C_g

Refractive index of water,

$$\mu_w = \frac{4}{3}$$

and refractive index of glass,

$$\mu_g = 1.5$$

Critical angle of water,

$$\begin{aligned}C_w &= \sin^{-1} \left(\frac{1}{\mu_w} \right) \\ &= \sin^{-1} \left(\frac{1}{\frac{4}{3}} \right) \\ &= \sin^{-1} (0.75) = 48.6^\circ\end{aligned}$$

Similarly, critical angle of glass,

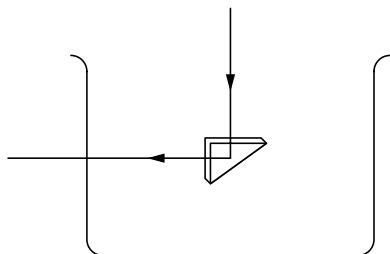
$$C_g = \sin^{-1} \left(\frac{1}{\mu_g} \right)$$

Ans :

Corpuscular theory fails to explain the velocity of light in air and water because it predicted light to have more velocity in denser medium where as the fact is just the opposite.

Thus (a) is correct option.

46. **Assertion :** The maximum refractive index of liquid for total internal reflection of the ray passing through the prism as shown in figure must be $\sqrt{2}$.



Reason : Here, critical angle is 45° .

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

In case of total internal reflection, the minimum refractive index is given by the relation.

$$\begin{aligned}\mu &= \frac{1}{\sin c} \\ &= \frac{1}{\sin 45^\circ} \\ &= \frac{1}{1/\sqrt{2}} = \sqrt{2} \\ \mu &= \sqrt{2}\end{aligned}$$

Thus (a) is correct option.

47. **Assertion :** A double convex lens ($\mu = 1.5$) has focal length 10 cm. When the lens is immersed in water ($\mu = 4/3$) its focal length becomes 40 cm.

Reason : $\frac{1}{f} = \frac{\mu_l - \mu_m}{\mu_m} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Using the given relation,

$$\begin{aligned}\frac{1}{10} &= \left(\frac{1.5 - 1}{1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ \frac{1}{10} &= 0.5 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(1)\end{aligned}$$

In the second case,

$$\begin{aligned}\mu &= \frac{4}{3} \\ \frac{1}{f} &= \left(\frac{1.5 - \frac{4}{3}}{\frac{4}{3}} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(2)\end{aligned}$$

Dividing (2) from (1),

$$\begin{aligned}\frac{f}{10} &= \frac{0.5}{0.5/4} = \frac{4 \times 0.5}{0.5} \\ f &= 4 \times 10 \\ &= 40 \text{ cm}\end{aligned}$$

Thus (a) is correct option.

48. **Assertion :** Different colours travel with different speed in vacuum.

Reason : Wavelength of light depends on refractive index of medium.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) Both the Assertion and Reason are incorrect.
 (d) The Assertion is incorrect but the Reason is correct.

Ans :

In vacuum all the colours have same velocity but their velocity changes when they enter in a medium. In the medium their velocity changes due to change in their wavelength.

$$v = n\lambda$$

If λ changes, v also changes for a particular colour therefore, different colours have different wavelength (λ).

Thus (d) is correct option.

49. **Assertion :** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will totally reflected when the angle of incidence is 30° .

Reason : $\mu = \frac{1}{\sin C}$, where μ is the refractive index of diamond with respect to liquid.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.

Since, $f_V < f_R$

Therefore, $\mu_R < \mu_V$

Thus (b) is correct option.

40. The angle of a prism is 60° . If green light of refractive index 1.5 passes through it, the angle of deviation will be

- (a) 30° (b) 40°
(c) 50° (d) 60°

Ans :

Foreign 2016

Angle of prism, $A = 60^\circ$

and refractive index for green light,

$$\mu = 1.5$$

Angle of deviation,

$$\begin{aligned}\delta &= (\mu - 1)A = (1.5 - 1) \times 60^\circ \\ &= 0.5 \times 60^\circ = 30^\circ\end{aligned}$$

Thus (a) is correct option.

41. If the critical angle for total internal reflection, from a medium to vacuum is 30° , then velocity of light in the medium is

- (a) $6 \times 10^8 \text{ m} - \text{s}^{-1}$ (b) $3 \times 10^8 \text{ m} - \text{s}^{-1}$
(c) $2 \times 10^8 \text{ m} - \text{s}^{-1}$ (d) $1.5 \times 10^8 \text{ m} - \text{s}^{-1}$

Ans :

SQP 2011

Critical angle, $C = 30^\circ$

Velocity of light in the medium,

$$\begin{aligned}v &= c \times \sin C = c \times \sin 30^\circ \\ &= (3 \times 10^8) \times 0.5 \\ &= 1.5 \times 10^8 \text{ m} - \text{s}^{-1}\end{aligned}$$

where, c = velocity of light equal to $3 \times 10^8 \text{ m} - \text{s}^{-1}$

Thus (d) is correct option.

42. A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out on the opposite parallel face, the red and green rays emerge from

- (a) one point propagating in the same direction
(b) one point propagating in two different directions
(c) two points propagating in two different parallel directions
(d) two points propagating in two different non-parallel directions

Ans :

OD 2007, Delhi 2001

When a beam of light is incident obliquely at a point on the face of a glass slab, it splits up into its constituents colours (or rays).

Therefore red and green rays emerge out from two

points propagating in two different parallel directions, because both the rays have different angles of refraction inside the glass slab.

Thus (c) is correct option.

ASSERTION AND REASON

43. **Assertion :** The setting sun appears to be red.

Reason : Scattering of light is directly proportional to the wavelength.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

Setting sun appears to be red because red light which has greatest wavelength is least scattered and reaches our eyes the most. Other wavelength are scattered to the longest extent. So, reason given is wrong.

Thus (c) is correct option.

44. **Assertion :** The colour of the green flower seen through red glass appears to be dark.

Reason : Red glass transmits only red light.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

A green flower absorbs all the light except green coloured light. So when red glass transmits only red light and falls on green flower, it absorbs all the light. Therefore, colour of the green flower becomes dark.

Thus (a) is correct option.

45. **Assertion :** Corpuscular theory fails in explaining the velocities of light in air and water.

Reason : According to corpuscular theory, light should travel faster in denser medium than in rarer medium.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

$$= \sin^{-1}\left(\frac{1}{1.5}\right) \\ = \sin^{-1}(0.6667) = 42^\circ$$

Therefore, $C_w > C_g$.

Thus (a) is correct option.

- 36.** If a ray of light in a denser medium strikes a rarer medium at an angle of incidence i , the angles of reflection and refraction are respectively r and r' . If the reflected and refracted rays are at 90° to each other, the critical angle for the given pair of media is

- (a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\tan r')$
(c) $\cot^{-1}(\tan i)$ (d) $\tan^{-1}(\sin i)$

Ans :

OD 2004

Angle of incidence = i

Angle of reflection = r

Angle of refraction = r'

and angle between reflected and refracted rays, = 90°

Angle of reflection is equal to the angle of incidence i.e.,

$$i = r$$

Relation for refractive index of denser medium with respect to rarer medium μ is,

$$\frac{1}{\mu} = \frac{\sin i}{\sin r'} = \frac{\sin r}{\sin r'} \\ = \frac{\sin r}{\sin(90^\circ - r)} = \frac{\sin r}{\cos r} \\ = \tan r$$

Critical angle for the given pair of media,

$$C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}(\tan r)$$

where, $r' = 90^\circ - r$

Thus (a) is correct option.

- 37.** Which should be the maximum acceptance angle at the air-core interface of an optical fibre, if μ_1 and μ_2 are the refractive indices of the core and the cladding respectively ?

- (a) $\sin^{-1}\left(\frac{\alpha_2}{\alpha_1}\right)$ (b) $\tan^{-1}\left(\frac{\alpha_1}{\alpha_2}\right)$
(c) $\tan^{-1}\left(\frac{\alpha_2}{\alpha_1}\right)$ (d) $\sin^{-1}(\sqrt{\alpha_1^2 - \alpha_2^2})$

Ans :

SQL 2017

Refractive index of core = μ_1

and, refractive index of cladding = μ_2

Critical angle for the optical fibre,

$$C = \sin^{-1}\left(\frac{\alpha_2}{\alpha_1}\right)$$

$$\sin C = \frac{\alpha_2}{\alpha_1}$$

And for maximum acceptance angle ($\theta_{\max}$) at air-core interface, the incidence angle at core-cladding interface is equal to the critical angle for the optical fibre (i.e., $i = C$)

Therefore maximum value of refracted angle at air-core interface,

$$r_{\max} = 90^\circ - i = 90^\circ - C$$

According to Snell's law that at air-core interface,

$$\mu_a \sin(\theta_{\max}) = \mu_1 \sin(r_{\max}) \\ \sin(\theta_{\max}) = \mu_1 \sin(90^\circ - C) \\ = \mu_1 \cos C \\ \sin(\theta_{\max}) = \mu_1 \sqrt{1 - \sin^2 C} \\ = \alpha_1 \sqrt{1 - \left(\frac{\alpha_2}{\alpha_1}\right)^2} \\ = \sqrt{\alpha_1^2 - \alpha_2^2} \\ \theta_{\max} = \sin^{-1}(\sqrt{\alpha_1^2 - \alpha_2^2})$$

where, μ_a = refractive index of air equal to 1

Thus (d) is correct option.

- 38.** A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is

- (a) 2 cm (b) 4 cm
(c) 6 cm (d) 12 cm

Ans :

Foreign 2005, Comp 2017

Radius of sphere, $R = 6$ cm

and, refractive index of glass,

$$\mu = 1.5$$

When rays from the object are incident normally on the surface of a sphere and emerge undeviating, then distance of the virtual image from the surface is equal to the radius of sphere i.e., 6 cm.

Thus (c) is correct option.

- 39.** If red and violet light rays are of focal lengths f_R and f_V , then which of the following is true relationship?

- (a) $\lambda_R < \lambda_V$ (b) $\alpha_R < \alpha_V$
(c) $\alpha_R > \alpha_V$ (d) $\lambda_R \leq \lambda_V$

Ans :

OD 2012

Focal length of red light = f_R

and focal length of violet light = f_V

Relation for focal length of the lens f is,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ f \propto \frac{1}{\mu}$$

- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) Both the Assertion and Reason are incorrect.
 (d) The Assertion is incorrect but the Reason is correct.

Ans :

$$\begin{aligned}
 {}^a\mu_d &= \sqrt{6} \\
 {}^a\mu_\ell &= \sqrt{3} \\
 {}^\ell\mu_d &= ? \\
 {}^a\alpha_d \times {}^d\alpha_\ell \times {}^\ell\alpha_a &= 1 \\
 \sqrt{6} \times {}^d\mu_\ell \times \frac{1}{\sqrt{3}} &= 1 \\
 {}^d\mu_\ell &= \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} \\
 {}^\ell\mu_d &= \sqrt{2}
 \end{aligned}$$

If C be the critical angle, then

$C = 45^\circ$. As angle of incidence $< 45^\circ$, it will not be internally reflected. So Assertion is incorrect Reason is correct.

Thus (d) is correct option.

50. **Assertion :** Newton's rings are formed in the reflected system when the space between the lens and the glass plate is filled with a liquid of refractive index greater than that of glass, the central spot of the pattern is bright.

Reason : This is because the reflection in these cases will be from a denser to rarer medium and the two interfering rays are reflected under similar conditions.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Newton's rings are formed in reflected system and if the refractive index of the first medium is more than the second medium, there is no reversal of phase in reflected ray so, central fringe remains bright.

Thus (a) is correct option.

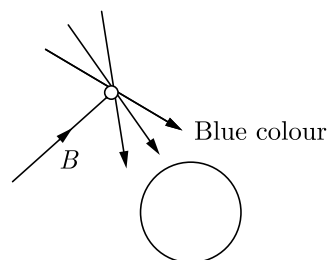
51. **Assertion :** Blue colour of sky appears due to scattering of blue colour.

Reason : Blue colour has shortest wave length in visible spectrum.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.

- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Blue colour of sky is due to scattering of blue colour to the maximum extent by dust particles. Blue colour appears to be coming from the sky. Blue colour has the least wavelength.



Thus (a) is correct option.

52. **Assertion :** The air bubble shines in water.

Reason : Air bubble in water shines due to refraction of light.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Air bubble shines in water due to total internal reflection from the surface of the bubble.

So, Assertion is correct and Reason is incorrect.

Thus (c) is correct option.

53. **Assertion :** The stars twinkle while the planets do not.

Reason : the stars are much bigger in size than the planets.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Stars twinkle because of changing refractive index of atmosphere. As the apparent size of stars are small, the effect of this change on the direction of rays coming from star is more pronounced.

Thus (b) is correct option.

- 54. Assertion :** In a movie, ordinarily 24 frames are projected per second from one end to the other of the complete film.

Reason : The image formed on retina of eye is sustained upto $\frac{1}{10}$ second after the removal of stimulus.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The image formed on retina of eye is sustained up-to $\frac{1}{16}$ th second after removal of stimulus. So, number of frames to be projected on the screen must be at most 16 per second.

Thus (c) is correct option.

- 55. Assertion :** A red object appears dark in the yellow light.

Reason : The red colour is scattered less.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

A red object appears dark in the yellow light because red object absorbs all the light falling on it except red. The Reason which is a fact can not be assigned to the Assertion.

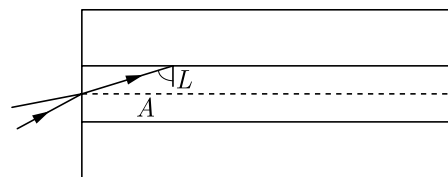
Thus (b) is correct option.

- 56. Assertion :** In optical fibre, the diameter of the core is kept small.

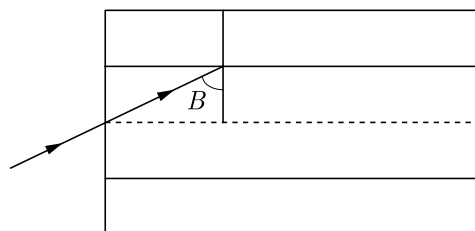
Reason : This smaller diameter of the core ensures that the fibre should have incident angle more than the critical angle required for total internal reflection.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :



Small diameter core, angle A is larger



Large diameter core, angle B is smaller. In the former case $\angle A$ is large so the possibility of this angle becoming greater than critical angle is more. Hence, the change of internal reflection is more.

Thus (a) is correct option.

- 57. Assertion :** Diamond glitters brilliantly.

Reason : Diamond does not absorb sunlight.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Diamond glitters brilliantly because of high refractive index and less critical angle.

Thus (b) is correct option.

- 58. Assertion :** By roughening the surface of a glass sheet its transparency can be reduced.

Reason : Glass sheet with rough surface absorbs more light.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The transparency of rough glass is reduced due to scattering of light.

Thus (c) is correct option.

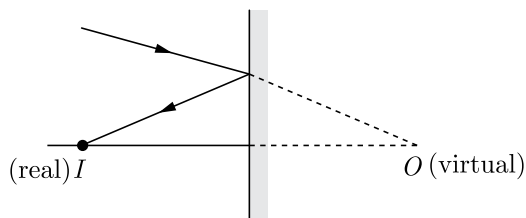
59. Assertion : Plane mirror may form real image.

Reason : Plane mirror forms virtual image, if object is real.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

Plane mirror may form real image, if object is virtual.



Thus (b) is correct option.

60. Assertion : A concave mirror and convex lens both have the same focal length in air. When they are submerged in water, they will have same focal length.

Reason : The refractive index of water is smaller than the refractive index of air.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

If a mirror is placed in a medium other than air, its focal length does not change as $f = R/2$. But for the lens,

$$\frac{1}{f_g} = ({}_a n_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

and
$$\frac{1}{f_w} = ({}_w n_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

As ${}_w n_g < {}_a n_g$, hence focal length of lens in water increase.

The refractive index of water is $4/3$ and that of air is 1. Hence, refractive index of water is greater than that of air.

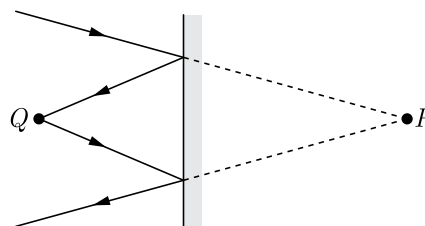
Thus (d) is correct option.

61. Assertion : The image of a virtual object due to a plane mirror is real.

Reason : If the rays seem to be converging at a point behind a plane mirror, they are reflected and they actually meet in front of the mirror.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :



The image of a real object is virtual while that of a virtual object (as shown) is real.

Thus (a) is correct option.

62. Assertion : Position of image approaches focus of a lens, only when object approaches infinity.

Reason : Par-axial rays incident parallel to principal axis intersect at the focus after refraction from lens.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

Thus (b) is correct option.

63. Assertion : There exists two angles of incidence for the same magnitude of deviation (except minimum deviation) by a prism kept in air.

Reason : In a prism kept in air, a ray is incident on first surface and emerges out of second surface. Now if another ray is incident on second surface (of prism) along the previous emergent ray, then this ray emerges out of first surface along the previous incident ray. This principle is called principle of reversibility of light.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.

- (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Thus (a) is correct option.

- 64. Assertion :** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

Reason : Laws of reflection are strictly valid for plane surfaces, but not for large spherical surface.

- (a) Both Assertion and Reason are correct and Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Laws of reflection can be applied to any type of surface.

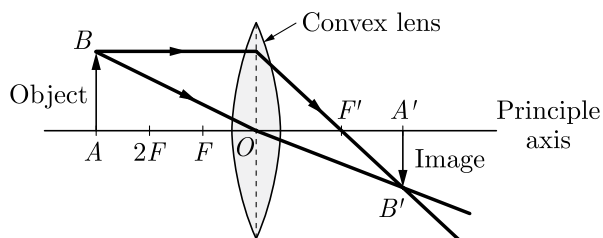
Thus (c) is correct option.

VERY SHORT ANSWER QUESTIONS

- 65.** A point object is placed above principal axis at a distance three times focal length of a convex lens. Draw ray diagram showing position of image.

Ans :

Comp 2020



- 66.** For the same angle of incidence, the angle of refraction in two media A and B are 25° and 35° respectively. In which one the two media is the speed of light lesser?

Ans :

Delhi 2021

$$\text{Medium, } A; \mu = \frac{\sin i}{\sin r}$$

$$= \frac{\text{Speed of light on air}}{\text{Speed of light in medium}}$$

μ is higher therefore speed is lesser.

- 67.** For the same value of angle of incidence, the angles of refraction in three media A, B and C are $15^\circ, 25^\circ$ and 35° respectively. In which medium, would the velocity of light be minimum?

Ans :

OD 2021

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$$

$v \propto \sin r$ for given value of i

Smaller angle of refraction, smaller the velocity of light in medium.

Velocity of light is minimum in medium A as angle of refraction is minimum, i.e., 15° .

- 68.** How does focal length of a lens change when red light incident on it is replaced by violet light? Give reason for your answer.

Ans :

Foreign 2010, OD 2014

The refractive index of the material of a lens increased with the decrease in wavelength of the incident light. So, focal length will decrease with decrease in wavelength according to the formula.

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Thus, when we replace red light with violet light then due to increase in wavelength the focal length of the lens will decrease.

- 69.** State the criteria for the phenomenon of total internal reflection of light to take place.

Ans :

Comp 2020

Following are the criteria for total internal reflection

- (i) Light must pass from a denser to a rarer medium.
 (ii) Angle of incidence must be greater than critical angle.
- 70.** (i) What is the relation between critical angle and refractive index of a material ?
 (ii) Does critical angle depend on the colour of light ?

Ans :

Delhi 2020

- (i) Refractive index $(\mu) = \frac{1}{\sin C}$ where, C is the critical angle.
 (ii) Since, refraction index depends upon the wavelength of light, the critical angle for a given pair of media is different for different wavelengths (colours) of light.

- 71.** Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid?

Ans :

OD 2019

When, $\mu_l = \mu_g$

where μ_l = Refractive index of liquid and μ_g = Refractive index of glass.

- 72.** A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65 what is the nature of the lens?

Ans :**OD 2016**

A concave lens is made up of certain material behaves as a diverging lens, when it is placed in a medium of refractive index less than the refractive index of the material of the lens and behaves as a converging lens, when it is placed in a medium of refractive index greater than the refractive index of the material of the lens.

In the given case, concave lens is immersed in medium having refractive index greater than the refractive index of the material of the lens ($1.65 > 1.5$). Therefore, it will behave as a converging lens.

- 73.** The refractive index of diamond is much greater than that of glass. How does a diamond cutter make use of that fact?

Ans :**OD 2017**

The refractive index of diamond is much higher than that of glass. Due to high refractive index, the critical angle for diamond air interface is low. The diamond is cut suitably so that the light entering the diamond from any face suffers multiple total internal reflections at the various surfaces.

- 74.** A lens behaves as a converging lens in air and a diverging lens in water ($\mu = \frac{4}{3}$). What will be the condition on the value of refractive index (μ) of the material of the lens?

Ans :**Delhi 2015**

Refractive index of the material of lens is less than the refractive index of water.

- 75.** When light travels from an optically denser medium to a rarer medium, why does the critical angle of incidence depend on the colour of light?

Ans :**Comp 2018**

The refractive index is different for different colour wavelength as $\mu = a + \frac{b}{\lambda^2}$. Hence, critical angle $\sin i_c = \frac{1}{\mu}$ would also be different colour of light.

- 76.** A biconvex lens made of a transparent material of refractive index 1.5 is immersed in a water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason.

Ans :**Foreign 2016**

A biconvex lens acts as a converging lens in air because the refractive index of air is less than that of the material of the lens. The refractive index of water is less than the refractive index of the material of the lens (1.5). So, its nature will not change, it behaves as a converging lens.

- 77.** Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid?

Ans :**SQP 2012**

A biconvex lens will act like a plane sheet of glass if than is immersed in a liquid the same index of refraction as itself, In this case, the focal length $\frac{1}{f} = 0$ or $f \rightarrow \infty$.

- 78.** A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging lens? Give reason.

Ans :**OD 2014, Foreign 2008**

No, it will behave as a diverging lens.

Using thin lens maker formula

$$\frac{1}{f_w} = \left(\frac{n_g}{n_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

On using convention $R_1 = +Ve$, $R_2 = -Ve$ and $n_g = 1.25$ and $n_m = 1.33$

$$\frac{1}{f_w} = \left(\frac{1.25}{1.33} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$\left(\frac{1}{R_1} + \frac{1}{R_2} \right) = +Ve$ value and $\left(\frac{1.25}{1.33} - 1 \right) = -Ve$ value

Hence $f_w = -Ve$ So it behaves as a diverging lens.

- 79.** Write the relationship between angle of incidence i , angle of prism A and angle of minimum deviation for triangular prism.

Ans :**Delhi 2017**

$$i = \frac{A + \delta_m}{2}$$

Where δ_m = angle of minimum deviation.

- 80.** How does the angle of minimum depiction of a glass prism vary, if the incident violet light is replaced by red light? Give reason.

Ans :**Delhi 2010**

$$\mu = \sin \frac{\left(\frac{\delta_m + A}{2} \right)}{\sin \frac{A}{2}}$$

for red light $\mu = \text{least}$, $\delta_m \rightarrow$ will reduce

- 81.** Does the critical angle depend on the wavelength of light?

Ans :**Foreign 2013**

Yes, Since critical angle depends upon the refractive index of the medium and refractive index of the medium depends upon the wavelength of light. So critical angel also depends upon the wavelength of light.

- 82.** Why does a convex lens of glass $\mu = 1.5$ behave as a diverging lens when immersed in carbon disulphide of $\mu = 1.65$?

Ans :**OD 2016**

The lens formula is given by,

$$\frac{1}{f} = \left(\frac{\alpha}{\alpha_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

98. What is total internal reflection? What are the conditions for it?

Ans :

OD 2020

The phenomenon in which a ray of light travelling at an angle of incidence greater than the critical angle from denser to rarer medium is totally reflected back into the denser medium is called total internal reflection.

Necessary Conditions for Total Internal Reflection :

1. Light must travel from an optically denser to an optically rarer medium.
 2. The angle of incidence in the denser medium must be greater than the critical angle for the two media.
99. What do you mean by refraction of light?

Ans :

OD 2019

The phenomenon of bending of light rays as they pass from one medium to another is called refraction of light. The bending of light rays is due to the fact that the speed of light changes as it passes from one medium to another. If the light goes from rarer medium to denser medium, it bends towards the normal and if it goes from denser to rarer medium it bends away from the normal. The refraction of light obeys two laws (a) the incident ray, refracted ray and the normal at the point of incidence all lie on the same plane and (b) the ratio of sine of angle of incidence and sine of angle of refraction is constant for a given pair of media.

100. When observed from under water, all the objects above the surface can be seen within a cone of 97° . Why? Explain

Ans :

Comp 2011, OD 2006

Since the critical angle C and refractive index μ is given by

$$\sin C = \frac{1}{\mu}$$

For water, $\mu = 1.33$

Hence, $\sin C = \frac{1}{1.33} = 0.7519$

or $C = 48.75^\circ$

Hence, Angle of the cone $= 48.75^\circ \times 2 = 97^\circ$

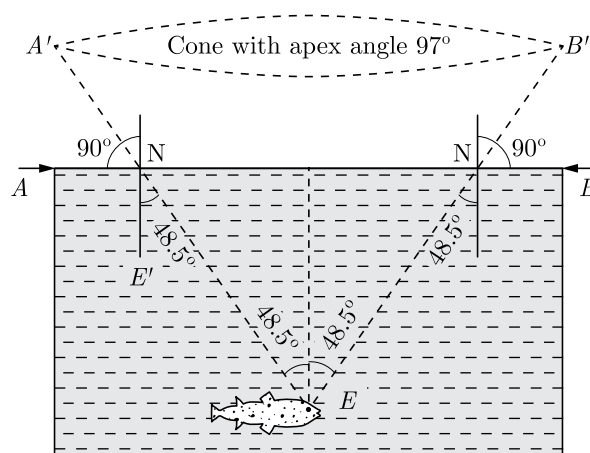
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101. How does the surface of the earth appear to a fish or a person sitting inside clear water?

Ans :

Delhi 2015

Whole of the earth will appear to lie within 97° instead of actual 180° . A ray of light from A incident at angle of 90° at N will go towards NE at an angle of 48.5° . [Refractive index of water is 1.33 and critical angle is 48.5°]. It shall appear to fish to come from A' instead of A'. Similarly, another ray from B will appear to come from B'. Whole of the earth will appear to lie within a cone of apex angle $48.5^\circ + 48.5^\circ = 97^\circ$.

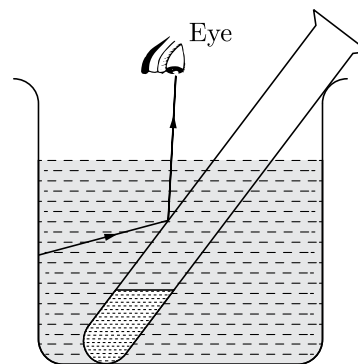


102. An empty test tube is placed slanting in the water and viewed from above. What will you observe?

Ans :

OD 2012

The portion of the tube within water will shine like a mirror due to total internal reflection. If the tube is partially filled with water, the portion of tube containing water will no longer show this effect.



103. How will you explain twinkling of stars?

Ans :

Comp 2018

Unless a star is directly above and the rays from it are normal to the atmosphere, atmospheric refraction causes an angular displacement of nearly $\frac{1^\circ}{2}$ and a star is not where it appears to be (Figure). This effect has to be taken into account by astronomers and navigators.

is 90° . The value of critical angle depends on the nature of two media in contact.

Condition for Critical Angle :

1. Ray of light should be goes from denser of rarer medium.
 2. Refraction angle is always 90° .
92. A concave mirror and a convex lens are held separately in water. What changes, if any, do you expect in the focal length of either?

Ans :

SQP 2013

The focal length of concave mirror will remain constant because its focal length $f = \frac{R}{2}$, so f does not depend upon μ .

The focal length of convex lens will increase, because, in this case, focal length is given by,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

As, μ of lens decreases with reference to water.

93. When does Snell's law fail in refraction?

Ans :

Delhi 2016

Snell's law fails, if a ray of light in a denser medium incident on a rarer medium at an angle more than the critical angle for that pair of media, then the ray of light is totally reflected back into the denser medium instead of going to the rarer medium as predicted by Snell's law. Hence in this situation, Snell's law fails in refraction.

94. What are the advantages of total reflecting prism over a plane mirror?

Ans :

OD 2017

1. It reflects whole light i.e. its reflection is 100%.
2. No silvering is required for reflection of light through total reflecting prisms.
3. In reflecting prism, multiple reflection does not take place and the single image formed is bright.

SHORT ANSWER QUESTIONS

95. Define critical angle for a given pair of media and total internal reflection. Obtain the relation between the critical angle and refractive index of the medium.

Ans :

OD 2023

The critical angle for a pair of refracting media can be defined as that angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° .

Total internal reflection when a ray of light travelling from an optically denser medium to an optically

rarer medium is incident at an angle greater than the critical angle for the pair of media in contact, the ray is totally reflected back into the denser medium. This phenomenon is referred as total internal reflection.

Derivation : Let the angle of incidence i be the critical angle C . Let, the angle of refraction, $r = 90^\circ$.

Refractive index of rarer medium be μ_a .

Refractive index of denser medium be μ_d .

On applying Snell's law

$$\frac{\sin i}{\sin r} = \frac{\mu_d}{\mu_a}$$

$$\mu_a \sin C = \mu_d \sin 90^\circ$$

$$\frac{\mu_a}{\mu_d} = \frac{1}{\sin C}$$

$$\mu_a = \frac{1}{\sin C}$$

96. Explain the brilliance of a diamond.

Ans :

Foreign 2014, SQP 2010

The refractive index of diamond is 2.42 and the critical angle for diamond is given by,

$$\sin C = \frac{1}{\mu} = \frac{1}{2.42} = 0.4132$$

or

$$C = 24.4^\circ$$

As the value of critical angle is very small so the diamond is cut in such a way that it has multiple faces at suitable angles. A ray of light on entering into the diamond suffers total internal reflection from other faces and comes out of the diamond as an intense beam from selected directions and the faces shine when viewed from outside.

97. What is the difference between Magnification and Magnifying power?

Ans :

Delhi 2019

The difference between Magnification and Magnifying power are as follows :

	Magnification	Magnifying Power
1.	Magnification is the ratio of image size to object size.	Magnifying power is the ratio of visual angle subtended by the image (β) and visual angle subtended by the object (α).
2.	The range of magnification is ∞ to $+\infty$	The value of magnifying power lies between $\frac{D}{f}$ and $\left(1 + \frac{D}{f}\right)$

Here, $\frac{\alpha_o}{\alpha_i} = \frac{\alpha_g}{\alpha_c} = \frac{1.5}{1.65} < 1$

Hence, f becomes negative, so the lens behaves like a diverging lens.

83. When monochromatic light travels from one medium to another, its wavelength changes but frequency remains the same. Explain

Ans :

Foreign 2011

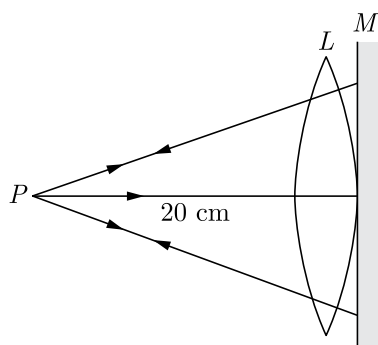
Because refractive index for a given pair of media depends on the ratio of wavelengths and velocity of light in two medium and not on a frequency.

84. A convex lens is placed in contact with a plane mirror. A point object at a distance of 20 cm on the axis of is combination has its image coinciding with itself. That is the length of the lens?

Ans :

SQP 2014, Comp 2009

The figure shows a convex lens L in contact with a plane mirror M . P is the point object kept in front of this combination at a distance of 20 cm from it.



Since, the image is coinciding with the object itself, the rays always from the object after refraction from the lens fall on the mirror M and from an image coinciding with the object itself.

The image is formed at the focus of the lens, So, focal length of the lens is 20 cm.

85. What are the assumptions followed to study refraction from spherical surfaces?

Ans :

Delhi 2019

1. The object is taken a point object on the principle axis.
2. The aperture of the spherical refracting surface is small.
3. Incident and refracted rays make small angles with principal axis.

86. What is the cause of dispersion of light?

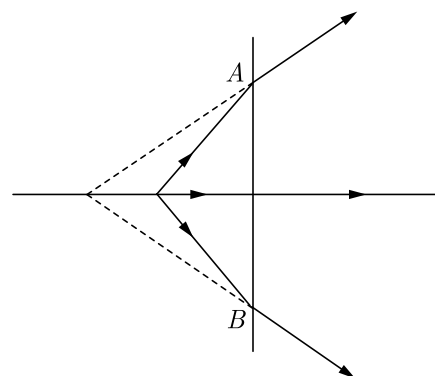
Ans :

Comp 2009

The refractive index of a material depends upon the wavelength as the velocity of light in a material depends upon wavelength, since white light is composed of

different colours, so different colours are deviated through different angles and we get dispersion of light.

87. The line AB in the ray diagram represents a lens. State whether the lens is convex or concave.



Ans :

Comp 2015

It is a convex or converging lens.

Reason The refracted ray is bending towards the principal axis.

88. Why does the Sun look reddish at sunset or sunrise?

Ans :

OD 2013

During sunset or sunrise, the sun is just above the horizon, the blue colour gets scattered most by the atmospheric molecules while red light gets scattered least, hence sun appears red.

Reason : Scattering intensity $I \propto \frac{1}{\lambda^4}$ and $\lambda_B < \lambda_R$.

89. Define absolute refractive index of a medium.

Ans :

Comp 2019

The absolute refractive index of a medium (μ) is defined as the ratio of velocity of light in vacuum (c) to the velocity of light in the medium (v)

$$\text{i.e.,} \quad \mu = \frac{c}{v}$$

$$\text{As,} \quad c > v \quad (\text{Hence } \mu > 1)$$

90. How is chromatic aberration reduced?

Ans :

Comp 2020

To remove chromatic aberration, we use two or more lenses in contact in place of single lens.

Mathematically condition of achromatic is

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

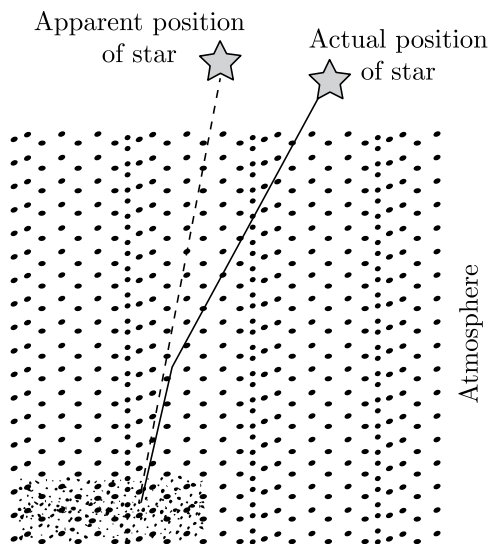
$$\omega_1 f_2 = -\omega_2 f_1$$

91. Define critical angle and write its conditions.

Ans :

Delhi 2020

Critical angle for a pair of given media in contact can be defined as, "the angle of incidence in denser medium for which angle of refraction in rarer medium



The atmosphere consists of several moving masses of air, some hot and some cold, some dense and some rarer. When light from a distant star enters the Earth's atmosphere, it is bent as it passes from one mass of air to another. The masses themselves are in motion. This gives rise to the twinkle in stars.

- 104.** How does a combination of lenses affect the size, position and nature of the image?

Ans :

Comp 2006

Two or more lenses may be combined by placing them in contact or at a finite distance apart. We can find size, nature and position of the final image by considering refraction through individual lenses. The image formed by first lens acts as an object for second lens. Similarly, the image formed by second lens acts as an object for third lens and so on. The standard formula can be applied in case of individual lenses to obtain the final size, nature and position of the image. If $m_1, m_2, m_3, \dots$ are the magnifications produced by the individual lenses, then total magnification produced by the combination is,

$$M = m_1 \times m_2 \times m_3 \times \dots$$

- 105.** A prism can produce spectrum of white light but a glass slab having same material as prism cannot produce any spectrum. Why?

Ans :

Delhi 2008

In case of a prism, when white light is incident on it, the white light break up into constituent colours. And the emergent light is not parallel to incident light, so the different colours for deviation at different angles and we get a spectrum of white light.

In case of glass slab, when white light is incident on it, it also break up into its constituent colours, after

refraction in the glass slab, but the emergent ray is parallel to incident ray, so all the different colours are deviated through same angle and image formed is white in colour and no spectrum of white light is observed.

- 106.** Show diagrammatically the various types of convex and concave lenses.

Ans :

Delhi 2011, Foreign 2009

Convex Lenses



Double convex lens

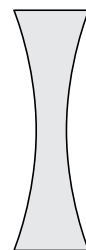


Plano-convex lens

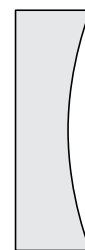


Concavo-convex lens

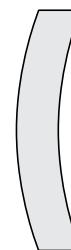
Concave Lenses



Double concave lens



Plano-concave lens



Concavo-concave lens

- 107.** Violet light is incident on a converging lens of focal length f . State with reasons how focal length of the lens will change, if the violet light is replaced by red light.

Ans :

OD 2017

The focal length of a lens is given by,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Since, $\alpha_r > \alpha_v$.

Hence, $\frac{1}{f_r} < \frac{1}{f_v}$

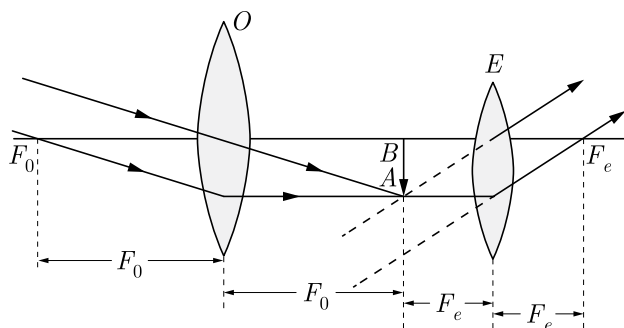
or $f_r > f_v$

Thus, the focal length will increase.

- 108.** Draw path of rays for astronomical telescope in normal adjustment.

Ans :

OD 2010



In an astronomical telescope, the objective is a convex lens of large focal length. The eye lens is also a convex lens of small focal length. The position of the eye - piece is adjusted so that final image is seen at infinity. Then distance between two lenses,

$$d = f_o + f_e$$

- 109.** Two convex lenses of same focal length but of apertures A_1 and A_2 ($A_2 < A_1$), are used as the objective lenses in two astronomical telescopes having identical eyepieces. What is the ratio of their resolving power? Which telescope will you prefer and why? Give reason.

Ans :

Comp 2010

Resolving power of telescope,

$$R_p = \frac{A}{1.22\lambda}$$

where, A = aperture or diameter of the objective telescope and λ = wavelength of the objective.

$$R \propto A$$

Hence, Ratio of resolving powers of two telescopes

$$\frac{R_1}{R_2} = \frac{A_1}{A_2}$$

Since, $A_2 > A_1$

Hence, $R_2 > R_1$

The larger the aperture of objective, higher the resolving power of telescope. As well more gathering of light to form the image and hence, brighter image would be obtained.

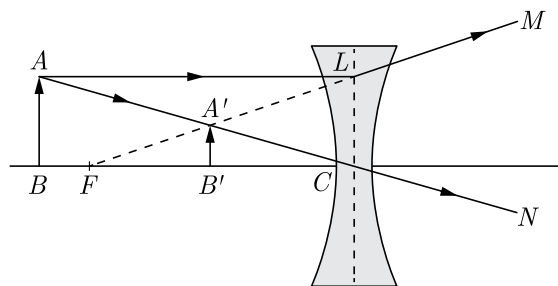
- 110.** Describe the formation of image by a concave lens.

Ans :

SQP 2009

Let, us consider a concave mirror of focal length F and optical centre C as shown in figure. Let, a ray of light AL be incident parallel to the principal axis of the concave lens and is refracted along LM . When this ray is produced in backward direction it meets at F , i.e. the ray LM appears to be coming from the focus F . Other ray AN which passes through the optical centre C of the concave lens, goes undeviated and these two refracted rays meet at A' . So $A'B'$ is the virtual image of object AB .

Here, the image is always virtual (independent of position of object), erect, diminished and is formed in front of the lens. If the object is moved closer to the lens, the image moves towards the optical centre is always formed in front of the lens.



- 111.** Use mirror equation to show that convex mirror always produces a virtual image independent of the location of the object.

Ans :

OD 2015, Delhi 2011

From equation, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$u < 0$, i.e., always negative for the object position in front of convex/concave mirror, $f > 0$, i.e., focal length is always positive for a convex mirror.

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$v = \frac{fu}{u-f}$$

On using sign convention,

$$v = \frac{(+f)(-u)}{(-u) - (+f)}$$

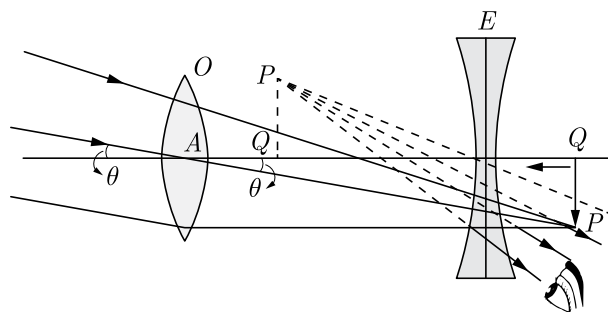
$$v = +ve$$

$$v > 0$$

- 112.** Draw a labelled diagram of a telescope which gives erect images of distant objects.

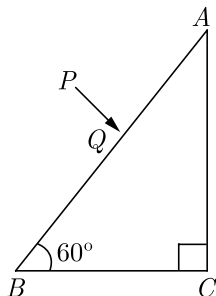
Ans :

Delhi 2015



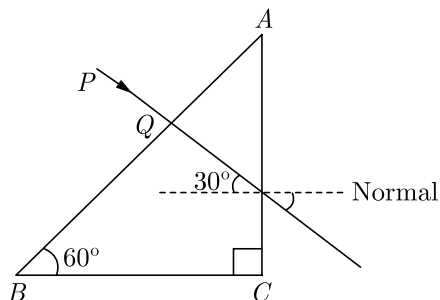
We use a Galileo telescope to get an erect image of objects. The objective is a convex lens while the eye - piece is a concave.

113. A ray PQ incident normally on the refracting face BA is refracted in the prism BAC made of material of refractive index 1.5. Complete the path of ray through the prism. From which face will the ray emerge? Justify your.



Ans :

Delhi 2017



For face, $AB, \angle i = 0^\circ$

i.e., $\angle r = 0^\circ$

They ray will pass through AB

Now, at face AC

$$\begin{aligned} \text{Here, } i_c &= \sin^{-1}\left(\frac{1}{\mu}\right) \\ &= \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}(0.66) \end{aligned}$$

$\angle i$ on face AC is 30° which is less than $\angle i_c$. Hence, they ray get refracted.

and applying Snell's law at face AC

$$\begin{aligned} \sin 30^\circ \times \frac{3}{2} \sin r &= 1 \\ \sin r &= \frac{1}{2} \times \frac{3}{2} \\ r \sin^{-1}\left(\frac{3}{4}\right) &= \sin^{-1}(0.75) \end{aligned}$$

and clearly $r > i$, as ray passes from denser to rarer medium.

114. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2 . Find the focal length of the combination.

Ans :

SQP 2013, 10

For convex lens,

Focal length, $f = f_1$

and for concave lens,

Focal length, $f = -f_2$

The equivalent focal length of the combination of lenses

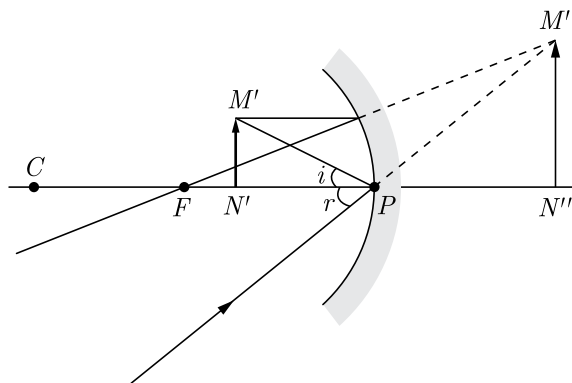
$$\begin{aligned} \frac{1}{F} &= \frac{1}{f_1} + \frac{1}{-f_2} \\ F &= \frac{f_1 f_2}{f_2 - f_1} \end{aligned}$$

115. Draw a ray diagram to show the image formation by a concave mirror. When the object is kept between its focus and the pole. Using this diagram. Derive the magnification formula for the image formed.

Ans :

Comp 2011

Ray diagram of image formation by a concave mirror.



In $\triangle M'N'P$ and $M''N''P$

$$\frac{M''N''}{M'N'} = \frac{N''P}{N'P}$$

By sign convention,

$$PN' = -u$$

$$PN'' = +v$$

$$M'N' = h_1$$

116. Draw a labelled ray diagram to show the image formation by an astronomical telescope in the near point adjustment.

or

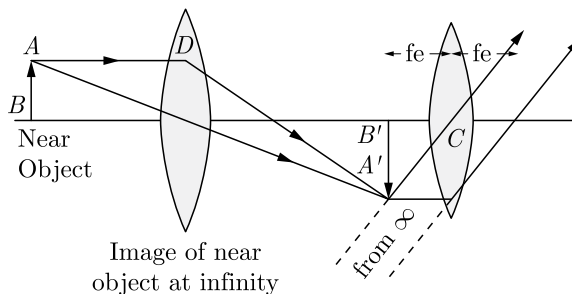
Draw a ray diagram to illustrate image formation by an astronomical telescope used in near point position.

Ans :

Foreign 2014, SQP 2002

Image of the near object can be formed at infinity or at D .

Image of near object at infinity,

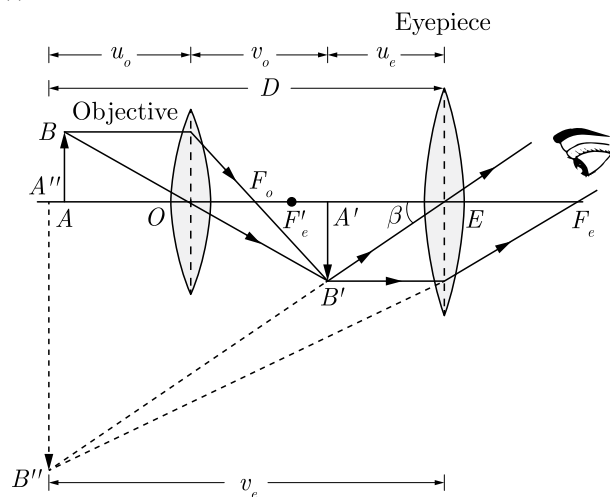


127. (i) Draw a schematic ray diagram of a compound microscope when image is formed at distance of distinct vision.
- (ii) Write the expression for resolving power of a compound microscope. How can the resolving power of a microscope be increased ?

Ans :

SQP 2007

(i)



- (ii) Resolving power of compound microscope

$$= \frac{2\mu \sin \beta}{1.22\lambda}$$

Where μ = Refractive index of the medium between the object and the objective lens.

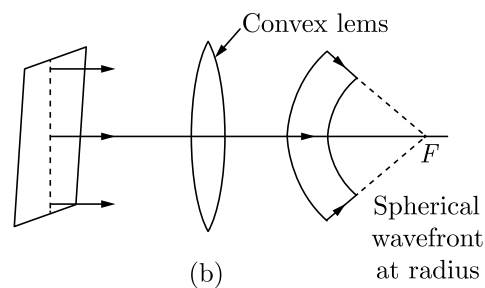
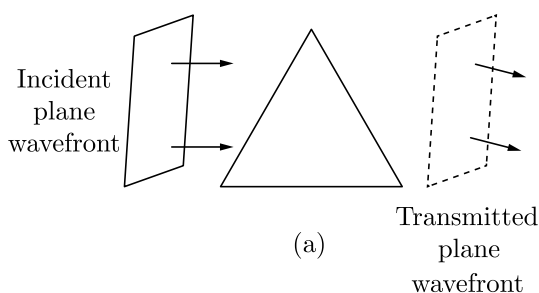
Resolving power can be increased by decreasing wavelength and by increasing refractive index of medium.

128. Draw the diagrams to show behaviour of plane wavefronts as they (a) pass through a thin prism, and (b) pass through a thin convex lens and (c) reflect by a concave mirror.

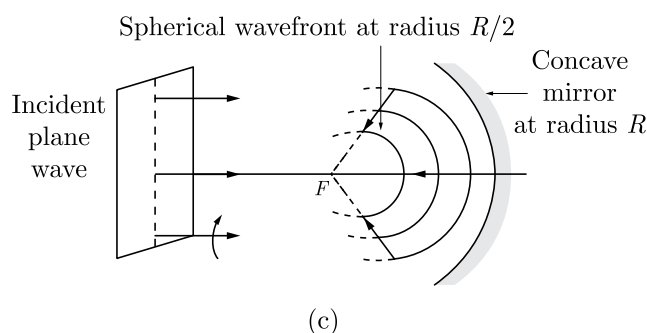
Ans :

Delhi 2014

The behaviour of a thin prism and a thin convex lens are shown in figs (a) and (b) respectively.



A plane wave front becomes spherical convergent after refraction

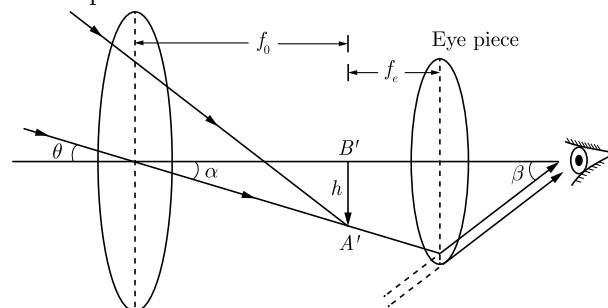


129. Draw a labelled ray diagram showing image formation by a refracting telescope. Define its magnifying power. Write two limitations of a refracting telescope over a reflecting telescope.

Ans :

OD 2023, Comp 2010

Ray diagram showing image formation by a refracting telescope :

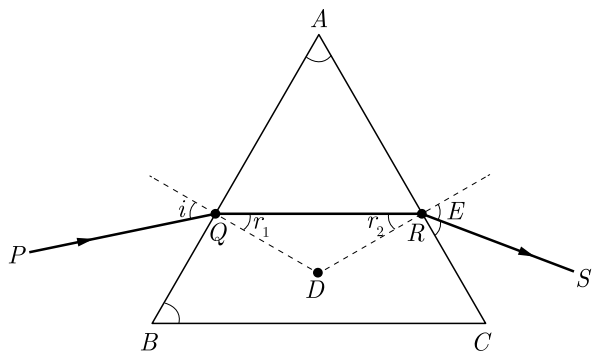


Magnifying power,

$$m = \frac{\tan \beta}{\tan \alpha} \approx \frac{\beta}{\alpha}$$

Limitations of refracting telescope reflecting telescope.

- (A) Refracting telescope suffers chromatic observation as it uses large lenses.
- (B) The image formed by refracting telescope is less brighter than the image formed by the reflecting type telescope due to some loss of light by reflection at the lens and by absorption.
- (C) The resolving power of refracting telescope is less than the resolving Power of reflecting type



$$n_1 = n_2 = r \quad (\text{let}) \dots (1)$$

We know that, $n_1 + n_2 = A$

From eq. (1), we get

$$2r = A, r = \frac{A}{2} \quad \dots (2)$$

Hence, $n_1 = n_2 = \frac{A}{2}$

(ii) Also, we have

$$A + D = i + e$$

Substituting, $D = D_m$ and $e = i$

$$A + D_m = i + i$$

Hence, $D_m = 2i - A$

- 120.** How is the working of a telescope different from that of a microscope?

Ans :

OD 2017

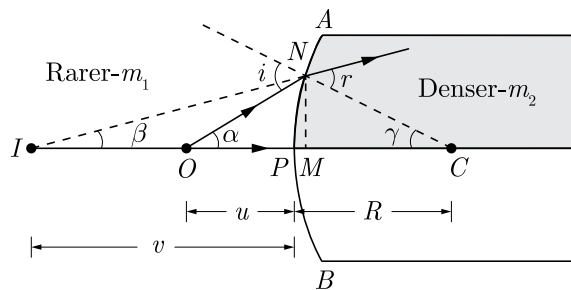
Difference in working of telescope and microscope

- Objective of telescope forms the image of a very far off object at or within the focus of its eyepiece. The microscope does the same for a small object kept just beyond the focus of its objective.
 - The final image formed by a telescope is magnified relative to its the seen by the unaided eye while the final image formed by a microscope is magnified relative to its absolute size.
 - The objective of a telescope has large focal length and large aperture while the corresponding parameters for a microscope have very small values.
- 121.** With the help of a suitable ray diagram, derive a relation between the object distance (u), image distance (v) and radius of curvature R for the convex spherical surface when a ray of light travels from a rarer to denser medium.

Ans :

Foreign 2012, SQP 2004

Let an object O is placed at a distance u from convex S spherical refracting surface whose virtual image formed at I at a distance v from surface. Let R is the radius of curvature of surface



$$\text{In } \triangle ONC, \quad i = \alpha + \gamma \quad \dots (1)$$

$$\text{In } \triangle INC, \quad r = \beta + \gamma \quad \dots (2)$$

Also, for small angle α , β and γ

$$1\alpha \simeq \tan \alpha = \frac{NM}{OM} \simeq \frac{NM}{PO} = \frac{h}{-u} \quad \dots (3)$$

[Minimum close to P]

Where, $h = NM$

$$\beta \simeq \tan \beta = \frac{NM}{IM} \simeq \frac{NM}{PI} = \frac{h}{-v} \quad \dots (4)$$

$$\text{Also, } \gamma \simeq \tan \gamma = \frac{NM}{MC} \simeq \frac{NM}{PC} = \frac{h}{+R} \quad \dots (5)$$

But, by Snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

where n_2, n_1 are the refractive indices of denser medium and rarer medium, respectively.

Since, Angles i and r are small.

$$\text{Hence, } \sin i \simeq i, \sin r \simeq r$$

$$\frac{i}{r} = \frac{n_2}{n_1}$$

$$n_1 i = n_2 r$$

$$\text{Hence, } n_1(\alpha + \gamma) = n_2(\beta + \gamma)$$

[from eqs.(1) and (2)]

$$n_1\alpha - n_2\beta = \gamma(n_2 - n_1)$$

$$n_1\left(\frac{h}{-u}\right) - n_2\left(\frac{h}{-v}\right) = \left(\frac{h}{+R}\right)(n_2 - n_1)$$

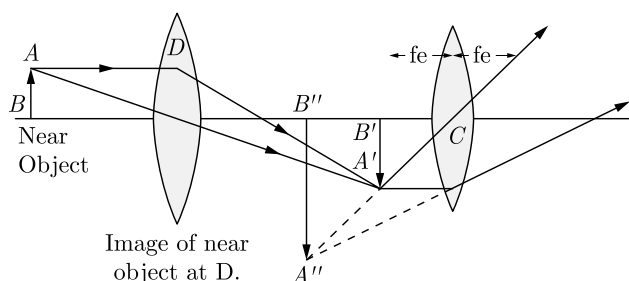
[from eqs.(4) and (5)]

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

This is the required expression.

- 122.** (i) A mobile phone lies along the principal axis of a concave mirror. Show with the help of suitable diagram the formation of its image. Explain why magnification is not a uniform.
- (ii) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain.

Image formed at least distance of distinct vision.

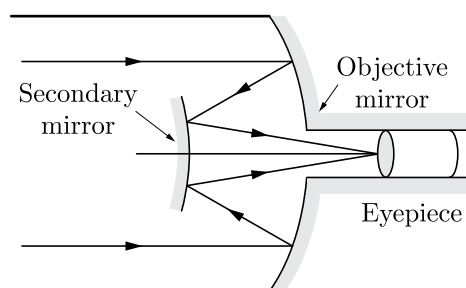


117. Draw a schematic arrangement of a reflecting telescope (Cassegrain) showing how rays coming from a distant object are received at the eyepiece. Write its two important advantages over a refracting telescope.

Ans :

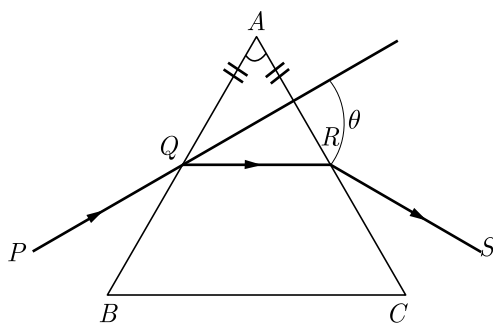
OD 2018, Comp 2015

Diagram of a reflecting telescope (Cassegrain) is shown as below



Advantages of reflecting telescope over a refracting telescope are given as below

- A reflecting telescope reflects all wavelengths of light at the same angle, so there are no colour halos.
 - A mirror has only one surface to be figured, so it is easier to control the shape.
 - A mirror reflects the light, so the material that is made from does not have to be transparent and infrared and ultraviolet light reflects equally well.
118. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that $AQ = AR$. If angle of prism $A = 60^\circ$ and refractive index of material of prism is $\sqrt{3}$, calculate angle θ .



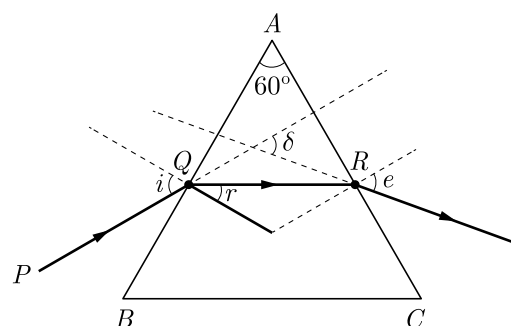
Ans :

Given,

$$AQ = AR$$

We have,

$$QR \parallel BC$$



At the minimum deviation, the refracted ray inside the prism becomes parallel to its base.

Hence, θ the angle of minimum deviation

$$\mu = \frac{\sin\left(\frac{A+\theta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sqrt{3} = \frac{\sin\left(\frac{60^\circ+\theta}{2}\right)}{\sin 30^\circ}$$

$$\sin\left(\frac{60^\circ+\theta}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{60^\circ+\theta}{2}\right) = \sin 60^\circ$$

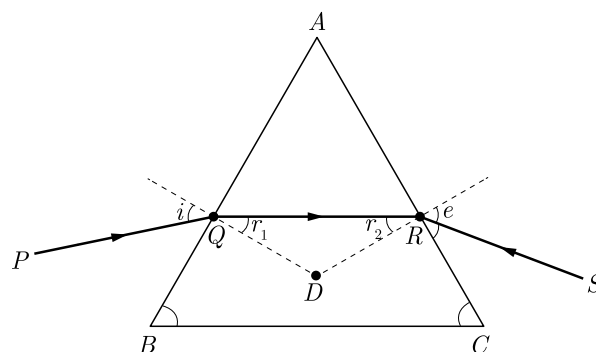
$$\frac{60^\circ+\theta}{2} = 60^\circ$$

$$\theta = 60^\circ$$

119. Figure shows a ray of light passing through a prism. If the refracted ray QR is parallel to the base BC , show that

(i) $r_1 = r_2 = \frac{A}{2}$ and

(ii) Angle of minimum deviation, $D_m = 2i - A$



Ans :

Foreign 2014

- (i) When QR is parallel to the base BC , we have $i = e$

Consider two lenses A and B of focal lengths, f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the lens A .

The first lens produces an image (real image) at I_1 , which serves as a virtual object for the second lens B producing final image at I .

Since, the lenses are thin, we assume the optical centres P of the lenses to be coincident. For the image formed by the first lens A , we obtain

$$\frac{1}{u_1} - \frac{1}{u} = \frac{1}{f_1} \quad \dots(1)$$

For the image formed by the second lens B , we obtain

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \dots(2)$$

Adding eqs. (1) and (2), we obtain

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2} + \frac{1}{f_1} \quad \dots(3)$$

If the two lenses system is regarded as equivalent to a single lens of focal length f , we have

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{f} \quad \dots(4)$$

From eqs. (3) and (4), we obtain

$$\frac{1}{f} + \frac{1}{f_2} = \frac{1}{f}$$

- 125.** State the necessary conditions for producing total internal reflection of light. Draw ray diagrams to show specially designed prism make use of total internal reflection to obtain inverted image of the object by deviation of rays

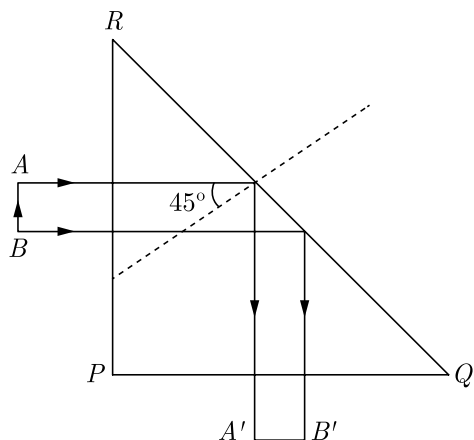
(i) thought 90° and (ii) thought 180° .

Ans :

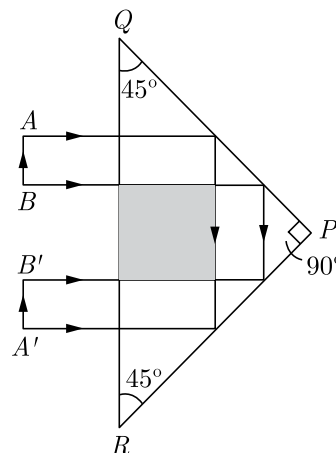
OD 2010

Following are the criteria for total internal refraction

- Light must pass from a denser to a rarer medium.
 - Angle of incidence must be greater than critical angle.
- (i) Deviation of light rays through 90° .



- (ii) Deviation of light rays through 180° .

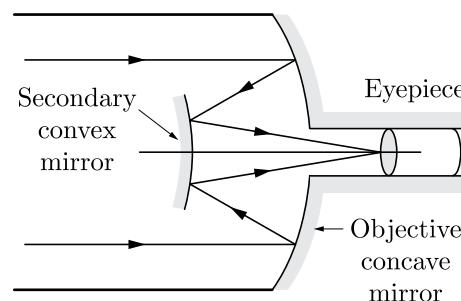


- 126.** (a) Draw a schematic labelled ray diagram of a reflecting type telescope (cassegrain).
 (b) The objective of telescope is a larger focal length and of larger aperture (compared to the eyepiece.) Why ? Given reasons.
 (c) State the advantages of reflecting telescope over refracting telescope.

Ans :

Delhi 2014, Comp 2016

- (a)



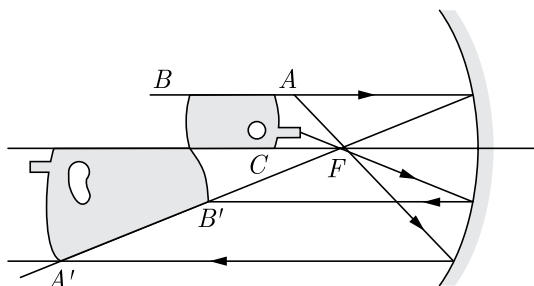
- (b) In normal adjustment, magnifying power of the telescope, $M = \frac{f_o}{f_e}$.
 (i) If focal length of the objective lens is large in comparison to the eyepiece, magnifying power increases.
 (ii) Resolving power of the telescope $RP = \frac{D}{1.22\lambda}$
 D being the diameter of the objective.
 To increase the resolving power of the telescope, large aperture of the objective lens is required.
 (c) Advantages
 (i) There is no chromatic aberration in a mirror.
 (ii) Brighter image.
 (iii) High resolving power.
 (iv) Large magnifying power.

Ans :

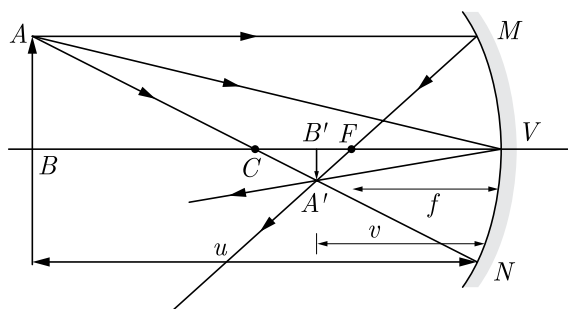
Comp 2007

- (i) The ray diagram for the formation of the image of the phone is shown as below. The image of the part which is on the plane perpendicular to the principal axis will be on the same plane. It will be of the same size,

i.e., $B'C = BC$



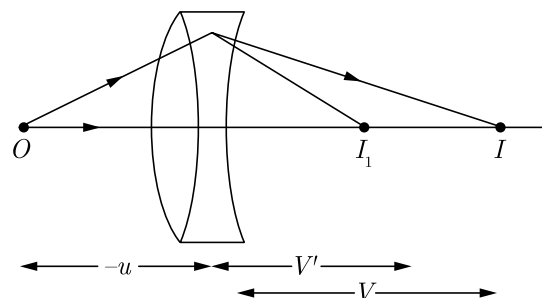
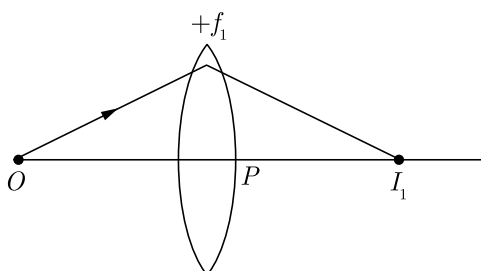
- (ii) We may think that the image will now show only half of the object, but considering the laws of reflection to be true for all points of the remaining part of the mirror, the image will be the of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low, i.e., half.



123. A convex lens of focal length f_1 is kept in contact with a concave lens of focal length f_2 . Find the focal length of the combination.

Ans :

SQP 2006



For convex lens of focal length $+(f_1)$

$$+\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} \quad \dots(1)$$

For convex lens of focal length $(-f_2)$

$$-\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v'} \quad \dots(2)$$

Adding equation (1) and (2)

$$\frac{1}{f_1} - \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \quad \dots(3)$$

For an equivalent lens (Using lens formula)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \dots(4)$$

where, f is the focal length of combination

From equation (3) and (4),

$$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2}$$

124. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2}$ for two thin lenses kept in contact coaxially.

or

Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal length of the lenses.

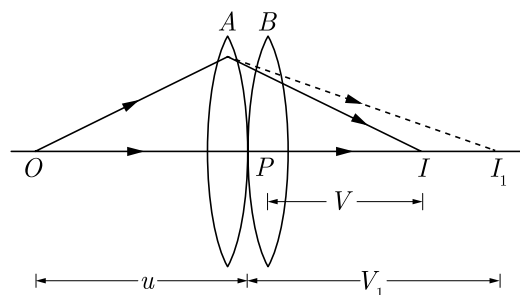
Ans :

Comp 2017, OD 2012

The power of a lens is equal to the reciprocal of its focal length when it is measured in metre. Power of a lens,

$$P = \frac{1}{f(\text{metre})}$$

It's SI unit is dioptre (D),



telescope as the mirror of reflecting type telescope has large diameter.

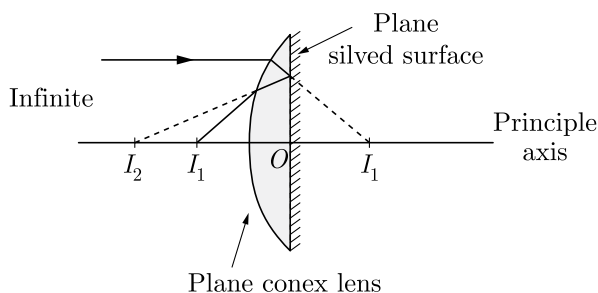
LONG ANSWER QUESTIONS

- 130.** An object is at infinity from a planoconvex lens whose plane surface is silvered. Explain information of its image and hence find its focal length.

Ans :

Comp 2021

According to the question,



Let the focal length of lens and mirror are f_l and f_m respectively.

For lens,

$$\frac{1}{f_l} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Here, $R_1 = \infty$ (for plane surface)

$$R_2 = -R$$

μ = Refractive index of lens

$$\begin{aligned} \frac{1}{f_l} &= (\mu - 1) \left[\frac{1}{\infty} - \frac{1}{(-R)} \right] \\ &= (\mu - 1) \left(\frac{1}{R} \right) \end{aligned}$$

$$P_l = \frac{1}{f_l} = \frac{\mu - 1}{R} \quad \dots(1)$$

For Mirror, $f_m = \frac{-R}{2}$

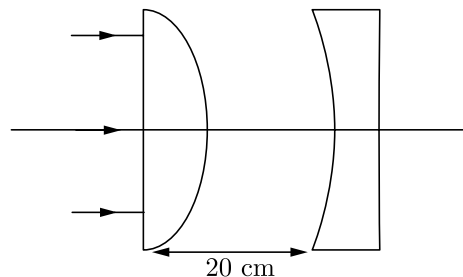
Since, $P_m = \frac{-1}{f_m} = \frac{-2}{R} \quad \dots(2)$

$$\begin{aligned} \text{Power of lens, } P &= P_l + P_m + P_l \\ &= \frac{\mu - 1}{R} + \frac{2}{R} + \frac{\mu - 1}{R} \\ &= \frac{2(\mu - 1)}{R} + \frac{2}{R} \\ &= \frac{2\mu}{R} - \frac{2}{R} + \frac{2}{R} = \frac{2\mu}{R} \end{aligned}$$

Focal length, $f = \frac{1}{P} = \frac{R}{2\mu}$

The lens will be equivalent to a converging mirror of focal length $\left(\frac{R}{2\mu} \right)$.

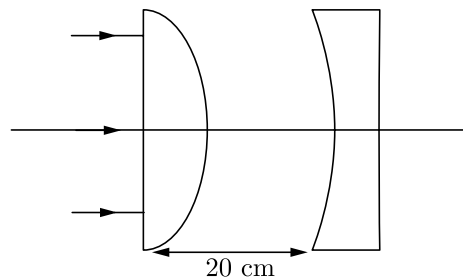
- 131.** In the given figure the radius of curvature of curved face in the plano-convex and the plano-concave lens is 15 cm each. The refractive index of the material of the lenses is 1.5. Find the final position of the image formed.



Ans :

OD 2023, SQP 2004

Focal length of plano-convex lens is :



$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{15} - \frac{1}{\infty} \right)$$

$$= \frac{0.5}{15} = \frac{1}{30}$$

$$f = 30 \text{ cm}$$

Focal length of piano-concave lens is,

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{15} \right)$$

$$= \frac{-0.5}{15} = -\frac{1}{30}$$

$$f = -30 \text{ cm}$$

Since, parallel beams are incident on the lens, its image from piano-concave lens will be formed at +20 cm from it (at the focus) and will act as an object for piano-concave lens. Since, the two lens are at a distance of 20 cm from each other, therefore, for the next lens.

$$u = +20 \text{ cm}$$

$$v = \frac{uf}{u + f} = \frac{20 \times -30}{20 - 30}$$

$$= \frac{-600}{-10} = 60 \text{ cm}$$

- 132.** Explain with a neat diagram, the working of a compound microscope. Obtain an expression for its magnifying power.

Using mirror formula,

$$\frac{1}{2u} + \frac{1}{u} = -\frac{1}{10}$$

$$\frac{3}{2u} = -\frac{1}{10}$$

$$u = -15 \text{ cm}$$

Hence, $v = 2 \times (-15) = -30 \text{ cm}$

(b) Using sign convention for convex mirror, we get

$$f > 0, u < 0$$

Hence, from the formula

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

As f is positive and u negative, v is always positive, hence image is always virtual.

- 175.** A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index 1.6. It is immersed in a liquid of refractive index 1.3. Calculate its new focal length.

Ans :

Delhi 2011

Given, $f_1 = +20 \text{ cm}$

$${}_a n_g = 1.6, {}_a n_w = 1.3$$

$${}_w n_g = \frac{{}_a n_g}{{}_a n_w} = \frac{1.6}{1.3}$$

Using lens maker's formula (in water) for converging lens,

$$\frac{1}{f_2} = ({}_w n_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \dots(1)$$

$$\text{In air,} \quad \frac{1}{f_1} = ({}_a n_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \dots(2)$$

Dividing eq. (2) by eq. (1), we get

$$\begin{aligned} \frac{f_2}{f_1} &= \frac{({}_a n_g - 1)}{({}_w n_g - 1)} \\ &= \frac{(1.6 - 1)}{\left(\frac{1.6}{1.3} - 1\right)} = \frac{0.6 \times 1.3}{0.3} \end{aligned}$$

$$\frac{f_2}{f_1} = 2.6$$

New focal length, $f_2 = 2.6 \times f_1 = 2.6 \times 20$

$$f_2 = 52 \text{ cm}$$

- 176.** A convex lens made up of a glass of refractive index 1.5 is dipped in turn in :

(i) a medium of refractive index 1.65.

(ii) a medium of refractive index 1.33.

(a) Will it behave as a converging lens or a diverging lens in the two cases?

(b) How will its focal length change in the two media?

Ans :

Comp 2006, OD 2010

(i) From lens maker's formula,

$$\begin{aligned} \frac{1}{f} &= ({}_w n_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= \left(\frac{{}_a n_g}{{}_a n_w} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(1) \end{aligned}$$

As, ${}_a n_g < {}_a n_m$ (1.65) for the first medium with refractive index 1.65.

(ii) And ${}_a n_g > {}_a n_m$ (1.33) for the second medium with refractive index 1.33.

Hence, the value of focal length f will be negative in the first medium.

(a) So, the convex lens will behave as the diverging lens for first medium and will behave as the converging lens for the second medium as the sign of the focal length will not change in second case.

Given, ${}_a n_g = 1.5, {}_a n_w = \frac{4}{3}$

$${}_w n_g = \frac{{}_a n_g}{{}_a n_w} = \frac{1.5}{\frac{4}{3}} = \frac{4.5}{4}$$

$$\text{As,} \quad \frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\begin{aligned} \text{(b) Hence,} \quad \frac{f_2}{f_1} &= \frac{({}_a n_g - 1)}{({}_w n_g - 1)} \\ &= \frac{(1.5 - 1)}{\left(\frac{4.5}{4} - 1\right)} = \frac{0.5}{\frac{0.5}{4}} = 4 \end{aligned}$$

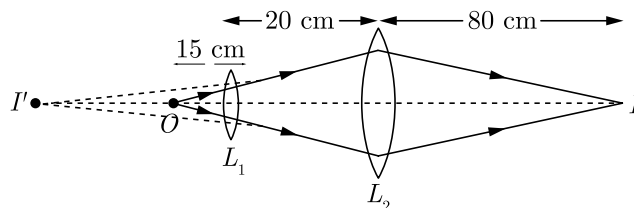
$$\frac{f_2}{f_1} = 4$$

$$f_2 = 4f_1$$

Change in focal length $= 4f_1 - f_1 = 3f_1$.

Change in focal length is equal to thrice of its original focal length.

- 177.** In the following diagram, an object O is placed 15 cm in front of a convex lens L_1 of focal length 20 cm and the final image is formed at I at a distance of 80 cm from the second lens L_2 . Find the focal length of the L_2 .



Ans :

OD 2010

Let focal length of lens L_2 be x cm.

Now, for lens L_1 ,

$$u = -15; f = +20 \text{ cm}, v = ?$$

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} + \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to the eyepiece then

$$m = \frac{-L}{f_o} \cdot \frac{D}{f_e}$$

where, L = Length of the tube of microscope in this case the microscope is said to be in normal adjustment.

133. Use the mirror equation to show that

- an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.
- a convex mirror always produces a virtual image independent of the location of the object.
- an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Ans :

Delhi 2015, OD 2011

(a) Mirror equation is,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

For a concave mirror, f is negative, i.e. $f < 0$.

For a real object (on the left of mirror).

For u between f and $2f$ implies $\frac{1}{f}$ and $\frac{1}{2f}$, i.e., $\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$ (as, u, f are negative)

$$-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < 0$$

$$\frac{1}{2f} < \frac{1}{v} < 0$$

i.e., $\frac{1}{v}$ is negative

This implies that v is negative and greater than $2f$. This means that the image lies beyond $2f$ and it is real.

- (b) For a convex mirror, f is positive i.e., $f > 0$. For a real object on the left u is negative

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \text{ implies } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

As u is negative and f is positive, $\frac{1}{v}$ must be positive, so v must be positive, i.e., image lies behind the mirror. Hence, image is virtual whatever the value of u may be.

- (c) For a mirror,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \dots(1)$$

For a concave mirror, f is negative $f < 0$

As u is also negative, so $f < u < 0$

This implies, $\frac{1}{f} - \frac{1}{u} > 0$

Then from eq. (1), $\frac{1}{v} > 0$ or v is positive.

i.e., image is on the right and hence virtual.

$$\text{Magnification, } m = -\frac{1}{u} = -\frac{f}{u-f}$$

As u is negative and f is positive, magnification

$$m = \frac{|f|}{|f| - |u|} > 1$$

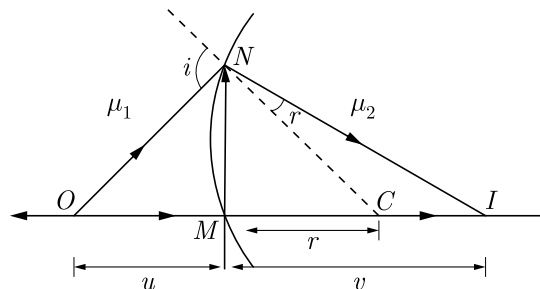
i.e., image is enlarged.

134. Establish the formula $\frac{\alpha_o}{v} - \frac{\alpha}{v} = \frac{\alpha_o - \alpha}{r}$

Ans :

Delhi 2021

A refracting surface which from a part of a sphere of transparent refracting material is called a spherical refracting surface.



In the figure, the geometry of formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R .

Assumptions

- The aperture of the surface is small as compared to other distances involved.
- NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan \angle NOM = \frac{MN}{OM},$$

$$\tan \angle NCM = \frac{MN}{MC},$$

$$\tan \angle NIM = \frac{MN}{MI}$$

For small angles, $\tan \theta \approx \sin \theta \approx \theta$.

$$\text{So, } \angle NOM = \frac{MN}{OM}$$

$$\angle NCM = \frac{MN}{MC}$$

$$\angle NIM = \frac{MN}{MI}$$

For $\triangle NOC$, i is the exterior angle.

Hence, $i = \angle NOM + \angle NCM$

$$= \frac{MN}{OM} + \frac{MN}{MC} \quad \dots(1)$$

Ans :

OD 2020

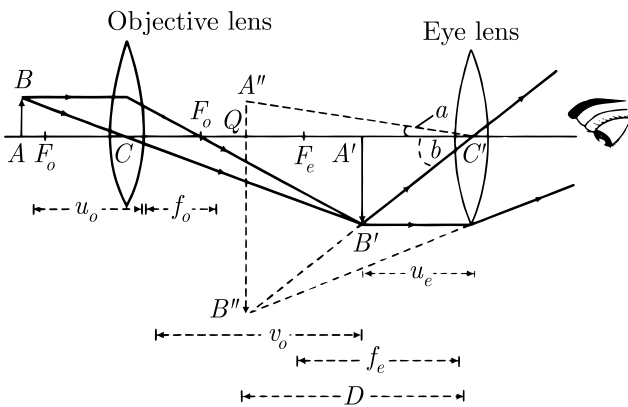
Compound Microscope

A compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

Working

The objective of compound microscope forms the real, inverted and magnified image of the object. This image serves as the object for the second lens, i.e., eyepiece which produces the final image, which is enlarged and virtual.

The first inverted image is thus near the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity or a little closer for image formation at the near point. The final image is inverted with respect to the original object.



Angular magnification or magnifying power of compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at least distance of distinct vision.

Hence, Angular magnification,

$$m = \frac{\beta}{\alpha}$$

Since, the angles are small, then

$$\alpha \approx \tan \alpha \text{ or } \beta \approx \tan \beta$$

$$\text{Hence, } m = \frac{\tan \beta}{\tan \alpha} \quad \dots(1)$$

From right angles $\Delta C'QB''$, we have

$$\tan \beta = \frac{B''Q}{C'Q} = \frac{B''Q}{D} = \frac{A'B''}{D}$$

Also, from right angled $\Delta C'A''Q$, we have

$$\begin{aligned} \tan \alpha &= \frac{A''Q}{C'Q} \\ &= \frac{AB}{D} \quad [\text{Since, } A''Q = AB] \end{aligned}$$

Substituting the values of $\tan \alpha$ and $\tan \beta$ in equation (1), we have

$$\begin{aligned} m &= \frac{B''Q}{D} \times \frac{D}{AB} = \frac{B''Q}{AB} \\ &= \frac{B''Q}{A'B'} \times \frac{A'B'}{AB} \end{aligned}$$

Thus, the magnification produced by the compound microscope is the product of the magnification produced by the eyepiece and objective.

$$\text{Hence, } m = m_e \times m_o \quad \dots(2)$$

Where, m_e and m_o are the magnifying powers of the eyepiece and objective, respectively.

The linear magnification of the real inverted image produced by the eyepiece is $\frac{A'B''}{A'B'}$.

Case I When the Final Image is Formed at Near Point

Linear magnification is given by,

$$m_e = 1 + \frac{D}{f_e} \quad \dots(3)$$

where, f_e is focal length of the eyepiece.

$\frac{A'B'}{AB}$ is the linear magnification of the object produced by the objective.

$$m_o = \frac{v_o}{u_o} \quad \dots(4)$$

From equation. (2), (3) and (4), we have

$$m = \frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right) \quad \dots(5)$$

$$\text{Now } \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

Multiplying both sides by v_o , we have

$$\begin{aligned} \frac{v_o}{v_o} - \frac{v_o}{u_o} &= \frac{v_o}{f_o} \\ -\frac{v_o}{u_o} &= -1 + \frac{v_o}{f_o} \\ \frac{v_o}{u_o} &= 1 - \frac{v_o}{f_o} \end{aligned}$$

Substituting the value of $\frac{v_o}{u_o}$ in equation (5), we have

$$m = \left(1 - \frac{v_o}{f_o} \right) \left(1 + \frac{D}{f_e} \right)$$

Case II When the Final Image is at Infinity

If u_o is the distance of the object from the objective and v_o is the distance of the image from the objective, then the magnifying power of the objective is,

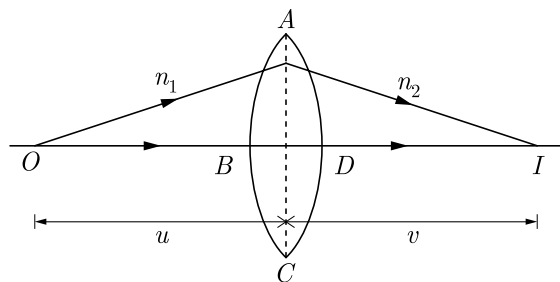
$$m_o = \frac{v_o}{u_o}$$

When the final image is at infinity, then angular magnification is given by,

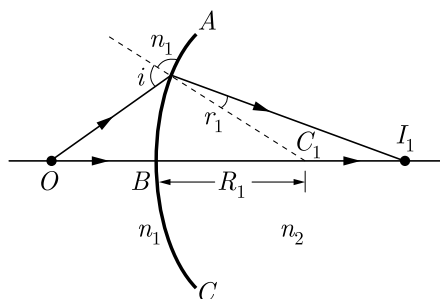
$$m_e = \frac{D}{f_e}$$

The total magnification when image is at infinity is given by,

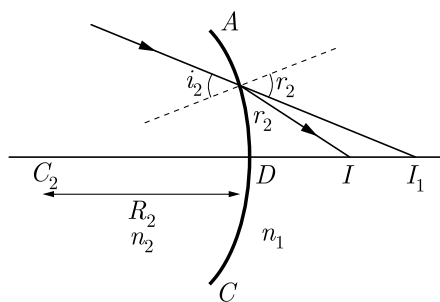
$$m = m_e \times m_o = \left(\frac{v_o}{u_o} \times \frac{D}{f_e} \right)$$



(a)



(b)



(c)

The above figures show the image formation by a convex lens.

Assumptions Made in the Derivation

Some assumptions made from the derivation are as :

1. The lens is thin, so that distances measured from the poles of its surfaces can be taken as equal to the distance from the optical centre of the lens.
2. The aperture of the lens is small.
3. The object considered as a point lying on the principal axis of the lens.
4. The incident ray and refracted ray make small angles with the principal axis of the lens.
5. A convex lens is made up of two convex spherical refracting surfaces.
6. The first refracting surface forms image I_1 of the object O [Fig. (b)].
7. Image I_1 acts as virtual object for the second surface that forms the image at I [Fig. (c)].

Applying the equation for spherical refracting surface to the first interface ABC , we get

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \quad \dots(1)$$

A similar procedure applied to the second interface ADC , we get

$$-\frac{n_2}{DI_1} + \frac{n_1}{DI} = \frac{n_2 - n_1}{DC_2} \quad \dots(2)$$

For a thin lens,

Adding Eqs. (1) and (2), we get

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1) \left(\frac{1}{BC_1} + \frac{1}{DC_2} \right) \quad \dots(3)$$

Suppose the object is at infinity, i.e.,

$OB \rightarrow \infty$ and $DI \rightarrow f$

So, Eq. (3) can be written as,

$$\frac{n_1}{f} = (n_2 - n_1) \left(\frac{1}{BC_1} + \frac{1}{DC_2} \right) \quad \dots(4)$$

The point where image of an object placed at infinity is formed is called the focus (f) of the lens and the distance f gives its focal length. A lens has two foci, F and F' on either side of it by sign convention.

$$BC_1 = R_1$$

or

$$DC_2 = -R_2$$

Therefore, Eq. (4) can be written as,

$$\frac{1}{f} = (n_2 - n_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \left[\text{Since, } \frac{n_2}{n_1} = \frac{n_2}{n_1} \right] \quad \dots(5)$$

Eq. (5) is known as the lens Maker's formula.

Putting $\frac{n_2}{n_1} = n$, refractive index of material of lens w.r.t. its surroundings, we get

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

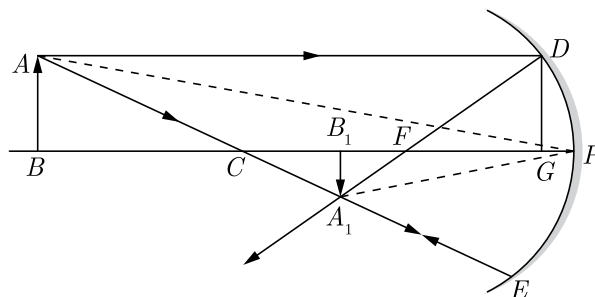
137. What is meant by linear magnification of a spherical mirror? Derive expression for it.

Ans :

OD 2018

Linear magnification or simply magnification of a spherical mirror is the ratio of the size of the image formed by the mirror to the size of the object.

It is represented by m ,



$$\frac{n_1}{v_1} - \frac{n_1}{v_1} = \frac{n_2 - n_1}{-R} \quad \dots(2)$$

On adding equation (1) and (2), we get

$$\frac{n_2}{v} - \frac{1}{u} = \frac{2n_1 - n_2 - 1}{R}$$

If rays are coming from infinity, i.e., $u = -\infty$ then $v = f$

$$\frac{n_2}{f} + \frac{1}{\infty} = \frac{2n_1 - n_2 - 1}{R}$$

$$f = \frac{n_2 R}{2n_1 - n_2 - 1}$$

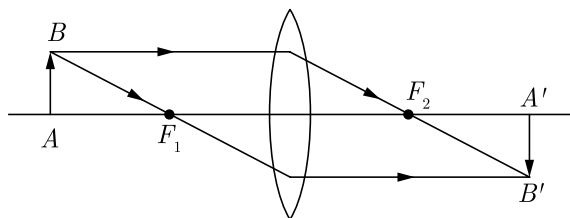
- (b) If the combination behave as a diverging system then $f < 0$. This is possible only when,

$$2n_1 - n_2 - 1 < 0$$

$$2n_1 < n_2 + 1$$

$$n_1 < \frac{(n_2 + 1)}{2}$$

(c)



If the combination behaves as a converging lens then $f > 0$. It is possible only when,

$$2n_1 - n_2 - 1 > 0$$

$$2n_1 > n_2 + 1$$

$$n_1 > \frac{(n_2 + 1)}{2}$$

Nature of the image formed is real.

NUMERICAL QUESTIONS

150. A concave mirror of focal length 12 cm forms a three times magnified virtual image of an object. Find the distance of the object from the mirror.

Ans :

OD 2023

Here $m = +3$ (virtual image)

For mirror, $m = \frac{-v}{u}$

$$\frac{-v}{u} = 3$$

$$v = -3u$$

By mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-12} = \frac{1}{-3u} + \frac{1}{u}$$

...(i)

$$\frac{-1}{12} = \frac{2}{3u}$$

$$u = -8 \text{ cm}$$

Hence, object is placed at a distance of 8 cm from the mirror.

151. The focal lengths of the objective the eye-piece of a compound microscop are 1.0 cm and 2.5 cm respectively. Find the tube length of the microscope obtaining a magnification of 300.

Ans :

OD 2023, Comp 2016

Magnification of compound microscope

$$m = m_o \times m_e = \left(\frac{L}{f_o} \times \frac{D}{f_e} \right)$$

Here, $L = ?$, $D = 25 \text{ cm}$

$$f_o = 1 \text{ cm and } f_e = 2.5 \text{ cm, } m = 300$$

$$300 = \left(\frac{L}{1} \times \frac{25}{2.5} \right)$$

$$L = \frac{300 \times 2.5}{25}$$

$$= \frac{300}{100} = 30 \text{ cm}$$

152. The focal length of a equiconvex lens is equal to the radius of curvature of either face. What is the value of refractive index of the material of the lens ?

Ans :

Delhi 2018

As we know that,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{2}{f} \right)$$

$$(\text{Since } f = R_1 = R_2 = R)$$

$$\frac{1}{2} = (\mu - 1) \Rightarrow \mu = 1.5$$

153. Calculate the speed of light in a medium, whose critical angle is 45° .

Ans :

OD 2017

Critical angle $i_c = 45^\circ$

$$\text{Now } \mu = \frac{c}{v} = \frac{1}{\sin i_c}$$

Hence,

$$v = c \sin i_c = 3 \times 10^8 \times \sin 45^\circ$$

$$= 3 \times 10^8 \times 0.707$$

$$= 2.121 \times 10^8 \text{ ms}^{-1}$$

154. A ray of light falls on a transparent sphere with centre C as shown in figure. The ray emerges from the sphere parallel to the line AB Find the angle of refraction at A if refractive index of the material of the sphere is $\sqrt{3}$.

Suppose a ray OA traversing parallel to the principal axis is incident on lens L_1 . It is refracted along AF , F being the second principal focus of L_1 . The deviation produced by L_1 is,

$$\delta_1 = \tan \delta_1 = \frac{h_1}{f}$$

The emergent ray is further refracted by second lens L_2 along BF' . Since the incident ray OA is parallel to the principle axis, F' should be second principal focus of the combination. The deviation produced by the second lens L_2 is,

$$\delta_2 \simeq \tan \delta_2 = \frac{h_1}{f}$$

The final emergent ray BF' , when produced backwards, meets the incident ray at point D . Obviously, δ is the final deviation produced. A single thin lens placed at C will produce the same deviation as by the combination of two lenses. Thus distance CF' is the second focal length of the combination. If f is the focal length of the combination, then

$$\delta = \frac{h_1}{f}$$

It is obvious from figure, that

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

As $\Delta AC_1F \sim \Delta BC_2F$, therefore, we have

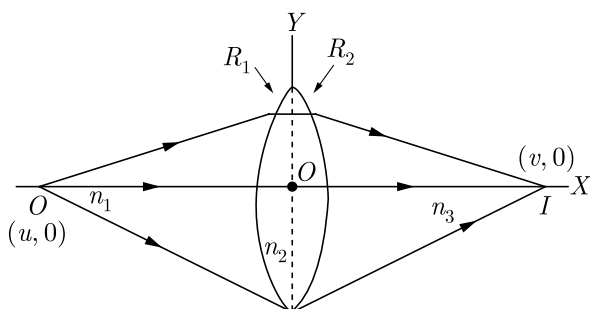
$$\frac{AC_1}{C_1F} = \frac{BC_2}{C_2F} \text{ or } \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

$$h_2 = \frac{f_1 - d}{f_1} \cdot h_1$$

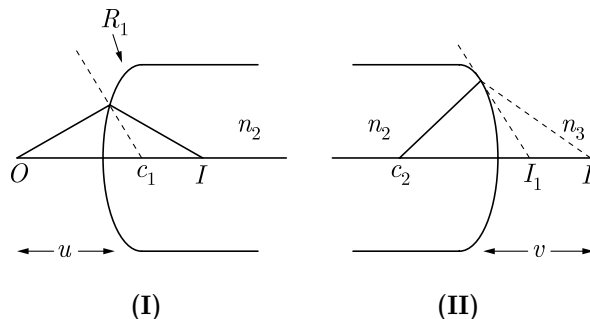
Hence,
$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{f_1 - d}{f_1 f_2} \cdot h_1$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

2.



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For first figure,

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \dots(1)$$

For second figure,

$$\frac{n_3}{v} - \frac{n_2}{v'} = \frac{n_3 - n_2}{R_2} \quad \dots(2)$$

After adding equation 1 and 2, we get

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

- 139.** Draw a ray diagram to show image formation by a compound Microscope. Derive an expression for magnifying power of your instrument. Explain the applications of your instrument.

Ans :

Foreign 2014

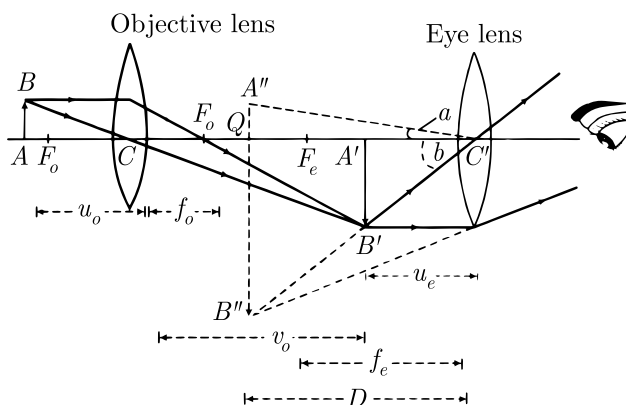
Compound Microscope

A compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

Working

The objective of compound microscope form the real, inverted and magnified image of the object. This image serves as the object for the second lens, i.e., eyepiece which produces the final image, which is enlarged and virtual.

The first inverted image is thus near the focal plane of the eyepiece at a distance appropriate for final image formation at infinity or a little closer for image information at the near point. The final image is inverted with respect to the original object.



For $\triangle NIC$, $\angle NCM$ is the exterior angle.

Hence, $\angle NCM = r + \angle NIM$

$$r = \angle NCM - \angle NIM$$

$$\text{i.e., } r = \frac{MN}{MC} - \frac{MN}{MI} \quad \dots(2)$$

By Snell's law,

$$\mu_1 \sin i = \mu_2 \sin r$$

For small angles, $\mu_1 = \mu_2$

Substituting the value of i and r from Equation. (1) and (2), we get

$$\mu_1 \left(\frac{MN}{OM} + \frac{MN}{MC} \right) = \mu_2 \left(\frac{MN}{MC} - \frac{MN}{MI} \right)$$

$$\text{or } \frac{\alpha_1}{OM} + \frac{\alpha}{MI} = \frac{\alpha_2 - \alpha}{MC} \quad \dots(3)$$

Applying new Cartesian sign convention,

$$OM = -u$$

$$MI = +v$$

$$MC = +r$$

Substituting these values in equation (3), we get

$$\frac{\alpha_2}{v} - \frac{\alpha}{u} = \frac{\alpha_2 - \alpha}{r}$$

This equation holds for any curved spherical surface.

135. Show for a prism that the refractive index n is given by,

$$\eta = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where, symbol have their usual meaning.

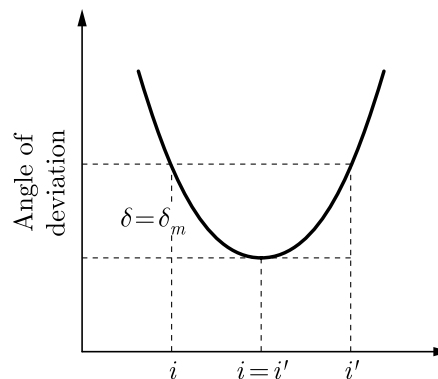
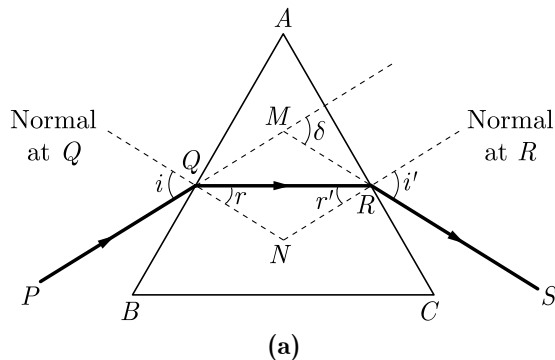
or

Draw a ray diagram to show the refraction of light through a glass prism. Derive the formula for the determination of refractive index of the material of the prism.

Ans :

Delhi 2020

Figure (a) shows the path of a ray of light suffering refraction through a prism of refracting angle A . Figure (b) shows the variation of angle of



(b)

deviation δ with the angle of incidence i . For a given prism and for a given colour of light, the angle δ depends on i only. As i increases, the angle of δ first decreases and reaches a minimum value δ_m and then increase. Clearly, any given value of δ corresponds to two angles of incidence i and i' in the equation : $\delta = i + i' - A$ i.e., δ remains the same as i and i' are interchanged. Physically, it means that the path of the ray in Figure (a) can be traced back, resulting in the same angle of deviation.

The minimum value of the angle of deviation suffered by a ray on passing through a prism is called the angle of minimum deviation and is denoted by δ_m .

Relation between Refractive Index and Angle of Minimum Deviation

When a prism is in the position of minimum deviation, a ray of light passes symmetrically (parallel to the base) through the prism so that,

$$i = i', r = r', \delta = \delta_m$$

$$\text{As, } A + \delta = i + i'$$

$$A + \delta_m = i + i \text{ or } i = \frac{A + \delta_m}{2}$$

$$\text{Also, } A = r + r' = r + r = 2r$$

$$\text{Hence, } r = \frac{A}{2}$$

From Snell's law, the refractive index of the material of the prism will be.

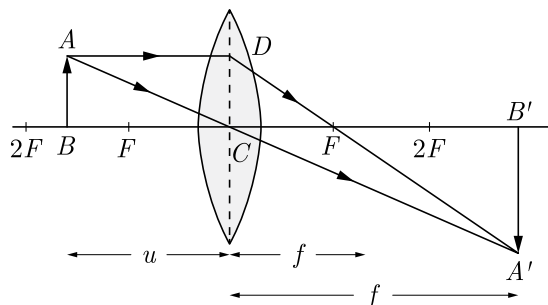
$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{A+\delta_m}{2}}{\sin \frac{A}{2}}$$

By measuring the value of A and δ_m , with the help of a spectrometer, the refractive index μ of the prism glass can be determined accurately.

136. Establish the formula $\frac{1}{f} = (\mu - 1) \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$.

Ans :

Comp 2019, OD 2013



$\triangle ABC$ and $\triangle A'B'C$ are similar, so we have

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \dots(1)$$

$\triangle DCF$ and $\triangle A'B'F$ are similar, so we have

$$\frac{A'B'}{DC} = \frac{FB'}{CF}$$

But, $DC = AB$

$$\text{Hence, } \frac{A'B'}{AB} = \frac{FB'}{CF} \quad \dots(2)$$

From equations (1) and (2), we have

$$\frac{CB'}{CB} = \frac{FB'}{CF}$$

$$\text{or } \frac{CB'}{CB} = \frac{CB' - CF}{CF} \quad \dots(3)$$

Using sign conventions,

Let, $CB' = v$, $CB = -u$, $CF = f$

So equation (3) becomes,

$$\frac{v}{-u} = \frac{v-f}{f}$$

$$\text{or } vf = -uv + uf$$

Dividing both sides by uvf , we get

$$\frac{uf}{uvf} = \frac{-uv}{uvf} + \frac{uf}{uvf}$$

$$\frac{1}{u} = -\frac{1}{f} + \frac{1}{v}$$

$$\text{or } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\text{or } -\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This is lens formula.

141. Draw a ray diagram to show image formation by a Astronomical Telescope. Derive an expression for magnifying power of your instrument. Explain the applications of your instrument.

Ans :

Foreign 2014

Astronomical (Refracting) Telescope

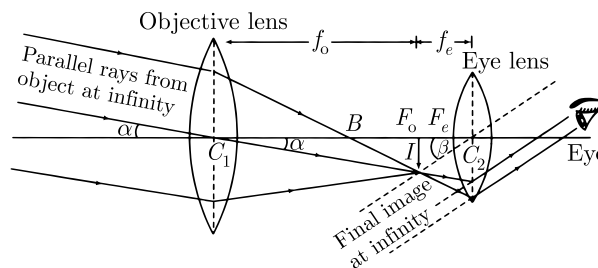
An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like stars, planets, etc., when the final image is formed at infinity.

Astronomical telescope has two convex lenses coaxially separated by some distance. The lens towards the object is called objective and has much larger aperture than the eyepiece of the lens towards the eye.

Working

Light from the distant object enters the objective and real image is formed at second focal point of objective. The eyepiece magnifies this image producing a final inverted image.

Case I When the Final Image is Formed at Infinity



Angular magnification is given by,

$$m = \frac{\beta}{\alpha}$$

Since, β and α are very small.

$$\text{Hence, } \beta \approx \tan \beta$$

$$\text{or } \alpha \approx \tan \alpha$$

$$m = \frac{\tan \beta}{\tan \alpha}$$

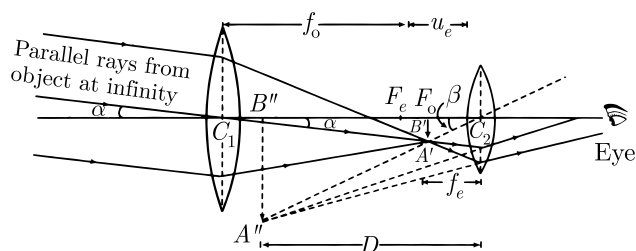
$$\text{Now, } \tan \alpha = \frac{I}{f_0}$$

$$\text{and } \tan \beta = \frac{I}{f_e}$$

Where, I is the image formed by the objective, F_0 and f_e are the focal length of objective and eyepiece, respectively. Substituting the values of $\tan \alpha$ and $\tan \beta$ in equation (1), we get

$$m = \frac{-\frac{I}{f_e}}{-\frac{I}{f_0}} \text{ or } m = -\frac{f_0}{f_e}$$

Case II When Final Image is Formed at Near Point



Angular magnification,

$$m = \frac{\beta}{\alpha} = \frac{\tan \beta}{\tan \alpha} \quad [\text{Since, } \beta \text{ and } \alpha \text{ are small}]$$

This in figure.

$$m = \frac{\text{size of image (I)}}{\text{size of object (O)}} = \frac{A_1 B_1}{AB}$$

$\Delta s ABP$ and $A_1 B_1 P$ are similar.

$$\frac{A_1 B_1}{AB} = \frac{PB_1}{PB}$$

In case of concave mirror, using new Cartesian sign conventions.

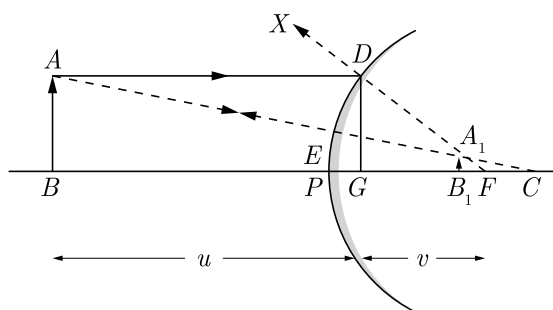
$$A_1 B_1 = -I, AB = +O$$

$$PB_1 = -v, PB = -u$$

Hence,
$$\frac{-I}{O} = \frac{-v}{-u} = \frac{v}{u}$$

Hence,
$$m = \frac{I}{O} = -\frac{v}{u}$$

In case of a convex mirror, using new Cartesian sign conventions, [Figure].



$$A_1 B_1 = +I, AB = +O$$

$$PB_1 = +v, PB = -u$$

Hence,
$$\frac{I}{O} = \frac{v}{-u}$$

Hence,
$$m = \frac{I}{O} = -\frac{v}{u}$$

When $m > 1$, image formed is enlarged.

When $m < 1$, image formed is diminished.

Again, when m is +ve, image must be erect (i.e. virtual)

When m is -ve, image must be inverted (i.e. real)

Other formula for magnification.

From mirror formula,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \dots(1)$$

Multiplying both side of equation (1) by v ,

$$\frac{v}{u} + 1 = \frac{u}{f}$$

$$\frac{v}{u} = \frac{v}{f} - 1 = \frac{u-f}{f}$$

As,
$$m = -\frac{v}{u}$$

Hence,
$$m = \frac{f-v}{f}$$

Again, multiplying both sides of mirror formula in equation (1) by u ,

$$1 + \frac{u}{v} = \frac{u}{f}$$

$$\frac{u}{v} = \frac{u}{f} - 1 = \frac{u-f}{f}$$

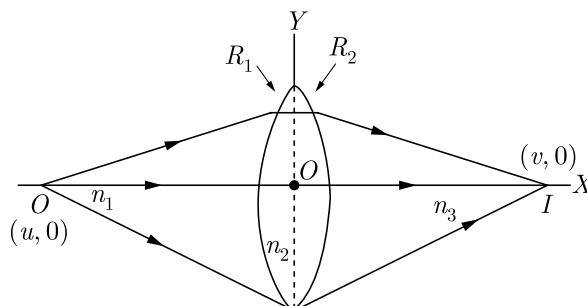
$$\frac{v}{u} = \frac{f}{u-f}$$

As,
$$m = -\frac{v}{u}$$

$$m = \frac{f}{f-u}$$

Hence,
$$m = -\frac{v}{u} = \frac{f-v}{f} = \frac{f}{f-u}$$

138. 1. What do you mean by the equivalent lens? Derive an expression for the equivalent focal length of a system of two lenses placed at a certain separation.
2. A lens having two curved surfaces of radii of curvature R_1 and R_2 is placed as shown in the figure.



Prove that,

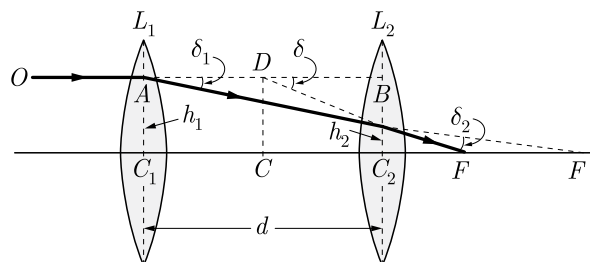
$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

Ans :

Delhi 2019, Comp 2005

1. **Equivalent lens** : A single lens which forms the image of an object at the same position as is formed by a combination of lenses is called an equivalent lens.

Thin lenses separated by a small distance : As shown in Figure consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2 , respectively. The two lenses are placed coaxially, distance ' d ' apart.



$$-\alpha_1 \frac{AM_1}{M_1O} + \alpha_2 \frac{AM_1}{M_1I_1} = (\alpha_2 - \alpha_1) \frac{AM_1}{M_1C_1}$$

$$-\frac{\alpha_1}{M_1O} + \frac{\alpha_2}{M_1I_1} = \frac{(\alpha_2 - \alpha_1)}{M_1C_1}$$

Since point M_1 is very close to C . So,

$$M_1O = CO, M_1I_1 = CI_1 \text{ and } M_1C_1 = CC_1$$

$$\text{Hence, } -\frac{\alpha_1}{CO} + \frac{\alpha_2}{CI_1} = \frac{\alpha_2 - \alpha_1}{CC_1} \quad \dots(2)$$

Refraction at X_2Y_2

Actually the lens material is not continuous and the ray AB suffers refraction at B and bend away from the normal along BQ . When BQ is produced in backward direction, it meets at I on the principal axis and I is the virtual image of virtual object I_1 .

From Snell's law for small i_2 and r_2 , we have

$$\frac{\sin i_2}{\sin r_2} = \frac{i_2}{r_2} = \frac{\alpha_1}{\alpha_2}$$

$$\text{or } \alpha_1 \alpha_2 = \mu_2 i_2 \quad \dots(3)$$

$$\text{In } \triangle BC_2I \quad i_2 = \beta_1 + \gamma_2$$

Putting the value of i_2 and r_2 in equation (3), we get

$$\mu_1(\beta_2 + \gamma_2) = \mu_2(\beta_1 + \gamma_2)$$

$$\text{or } \mu_1\beta_2 + \mu_1\gamma_2 = \mu_2\beta_1 + \mu_2\gamma_2$$

$$\text{or } \mu_1\beta_2 - \mu_2\beta_1 = (\mu_2 - \mu_1)\gamma_2$$

Since, β_2 , β_1 and γ_2 are small, so they can be replaced by their tangents. So,

$$\mu_1 \tan \beta_2 - \mu_2 \tan \beta_1 = (\mu_2 - \mu_1) \tan \gamma_2$$

$$\alpha_1 \frac{BM_2}{M_2I} - \alpha_2 \frac{BM_2}{M_2I_1} = (\alpha_2 - \alpha_1) \frac{BM_2}{M_2C_2}$$

$$\frac{\alpha_1}{M_2I} - \frac{\alpha_2}{M_2I_1} = \frac{\alpha_2 - \alpha_1}{M_2C_2}$$

Since, point M_2 is every close to C , so

$$\frac{\alpha_1}{CI} - \frac{\alpha_2}{CI_1} = \frac{\alpha_2 - \alpha_1}{CC_2} \quad \dots(4)$$

Adding equations (2) and (4), we get

$$-\frac{\alpha_1}{CO} + \frac{\alpha_2}{CI} = (\alpha_2 - \alpha_1) \left[\frac{1}{CC_1} + \frac{1}{CC_2} \right]$$

Using sign conventions, $CO = -u$, $CI = v$

$$CC_1 = -R \text{ and } CC_2 = -R_2$$

$$\frac{-\alpha_1}{-u} + \frac{\alpha_2}{-v} = (\alpha_2 - \alpha_1) \left(\frac{1}{-R_1} + \frac{1}{R_2} \right)$$

$$\frac{-\alpha_1}{u} + \frac{\alpha_2}{v} = (\alpha_2 - \alpha_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Dividing both sides by μ_1 , we get

$$-\frac{1}{u} + \frac{1}{v} = \left(\frac{\alpha_2}{\alpha_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Writing, $\frac{\alpha_2}{\alpha_1} = \mu$, we get

$$-\frac{1}{u} + \frac{1}{v} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{Since, } -\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Where f = focal length of the lens

$$\text{So, } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

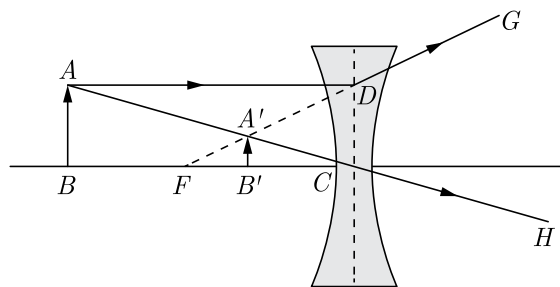
This is required lens maker's formula for this concave lens.

144. Derive lens formula for a concave lens.

Ans :

SQP 2011, Comp 2009

Consider a concave lens of focal length f and optical centre C . Let an object AB be placed perpendicular to the principal axis at any suitable distance. The image formed by this lens is always virtual, erect and on the same side of the lens as shown in figure



$\triangle ABC$ and $\triangle A'B'C$ are similar, so

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \dots(1)$$

$\triangle DCF$ and $\triangle A'B'F$ are similar, so

$$\frac{A'B'}{DC} = \frac{B'F}{CF}$$

$$\text{But, } DC = AB$$

$$\text{Hence, } \frac{A'B'}{AB} = \frac{B'F}{CF} \quad \dots(2)$$

From equations (1) and (2), we have

$$\frac{CB'}{CB} = \frac{B'F}{CF}$$

$$\text{or } \frac{CB'}{CB} = \frac{CF' - CB'}{CF} \quad \dots(3)$$

Using sign conventions,

$$\text{Let } CB' = -v, CB = -u, CF = -f$$

So equation (3) becomes,

$$\frac{-v}{-u} = \frac{-f + v}{-f}$$

$$\text{or } -vf = -uf + uv$$

Dividing both sides by uvf , we get

$$\frac{-uf}{uvf} = \frac{-uf}{uvf} + \frac{uv}{uvf}$$

$$-\frac{1}{u} = -\frac{1}{v} + \frac{1}{f}$$

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at least distance of distinct vision.

Hence, Angular magnification,

$$m = \frac{\beta}{\alpha}$$

Since, the angles are small, then

$$\alpha \approx \tan \alpha$$

or

$$\beta \approx \tan \beta$$

Hence,
$$m = \frac{\tan \beta}{\tan \alpha} \quad \dots(1)$$

From right angles $\Delta C'QB''$, we have

$$\begin{aligned} \tan \beta &= \frac{B''Q}{C'Q} \\ &= \frac{AB}{D} \quad [\text{Since, } A''Q = AB] \end{aligned}$$

Substituting the values of $\tan \alpha$ and $\tan \beta$ in equation (1), we have

$$\begin{aligned} m &= \frac{B''Q}{D} \times \frac{D}{AB} = \frac{B''Q}{AB} \\ m &= \frac{B''Q}{A'B'} \times \frac{A'B'}{AB} \end{aligned}$$

Thus, the magnification produced by the compound microscope is the product of the magnification produced by the eyepiece and objective.

Hence,
$$m = m_e \times m_0 \quad \dots(2)$$

Where, m_e and m_0 are the magnifying power of the eyepiece and objective, respectively.

The linear magnification of the real inverted image produced by the eyepiece is $\frac{A'B''}{A'B'}$.

Case I When the Final Image is Formed at Near Point

Linear magnification is given by,

$$m_e = 1 + \frac{D}{f_e} \quad \dots(3)$$

where, f_e is focal length of the eyepiece $\frac{A'B''}{AB}$ is the linear magnification of the object produced by the objective.

$$m_0 = \frac{v_0}{u_0} \quad \dots(4)$$

From equation (2), (3) and (4), we have

$$m = \frac{v_0}{u_0} \left(1 + \frac{D}{f_e} \right) \quad \dots(5)$$

Now
$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

Multiplying both sides by v_0 , we have

$$\frac{v_0}{v_0} - \frac{v_0}{u_0} = \frac{v_0}{f_0}$$

$$-\frac{v_0}{u_0} = -1 + \frac{v_0}{f_0}$$

$$\frac{v_0}{u_0} = 1 - \frac{v_0}{f_0}$$

Substituting the value of $\frac{v_0}{u_0}$ in equation (5), we have

$$m = \left(1 - \frac{v_0}{f_0} \right) \left(1 + \frac{D}{f_e} \right)$$

Case II When the Final Image is at Infinity

If u_0 is the distance of the object from the objective and v_0 is the distance of the image from the objective, then the magnifying power of the objective is,

$$m_0 = \frac{v_0}{u_0}$$

When the final image is at infinity, then angular magnification is given by,

$$m_e = \frac{D}{f_e}$$

The total magnification when image is at infinity is given by,

$$m = m_0 \times m_e = \left(\frac{v_0}{u_0} \times \frac{D}{f_e} \right)$$

If the object is very close to the principle focus of the objective and the image formed by the objective is very close to the eyepiece, then

$$m = \frac{-L}{f_0} \cdot \frac{D}{f_e}$$

where, L = length of the tube of microscope in this case, the microscope is said to be in normal adjustment.

- 140.** What is a lens formula? Derive an expression for lens formula for a convex lens forming a real image?

Ans :

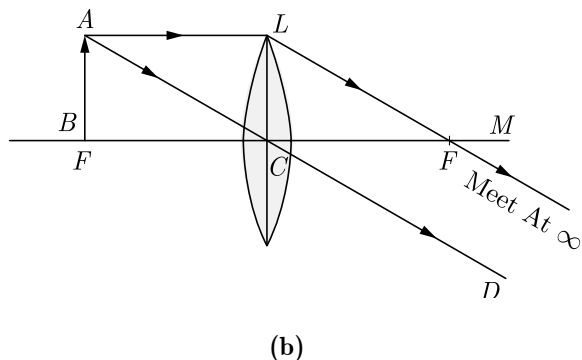
Delhi 2016, Foreign 2003

Lens Formula: Lens formula is an equation relating the distance of object, distance of image formed and the focal length of the lens, and is given by,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

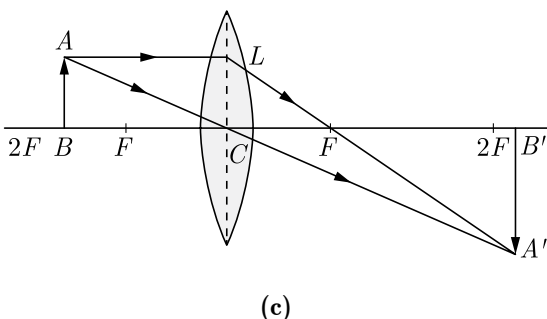
Where u is the distance of object from optical centre of the lens, v is the distance of image from the optical centre of the lens and f is the focal length of the lens.

When Image Formed is Real : Consider a convex lens of focal length f and optical centre C . Let an object AB be placed perpendicular to the principal axis at a point between F and $2F$. The real image of this object $A'B'$ is formed beyond $2F$ on the other side of the lens. The image is real, inverted and magnified and magnified as shown in figure.



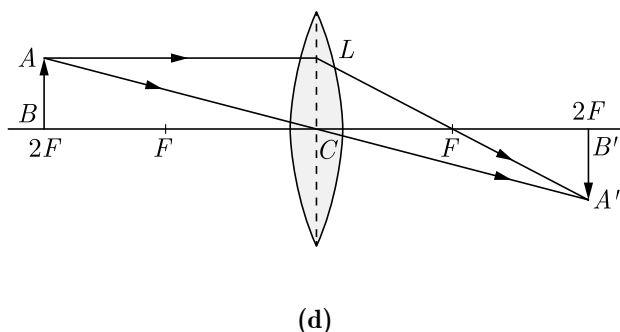
Here the image is real, inverted, highly enlarged and is formed at infinity.

3. **When Object is Placed between F and 2F :** The incident ray AL , which is parallel to the principal axis, passes through the focus F after refraction and ray AA' , which passes through the optical centre C , goes undeviated. These two rays meet at point A' . So $A'B'$ is the real image of object AB .

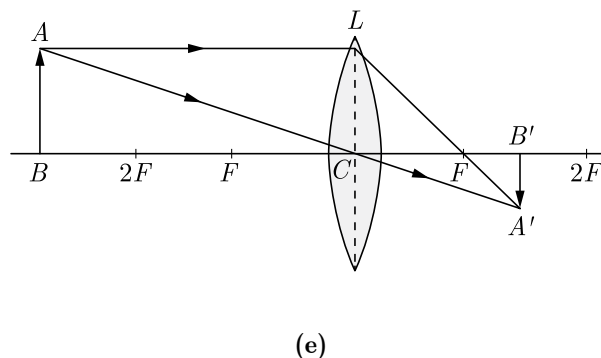


Here the image is real, inverted, magnified and is formed beyond $2F$. Figure (c)

4. **When Object is Placed at $2F$:** The incident ray AL , which is parallel to the principal axis, passes through the focus F after refraction and ray AA' , which passes through the optical centre C goes undeviated. These two rays meet at point A' . So $A'B'$ is the real image of object AB Figure (d). Here the image is real, inverted, equal to the size of object and is formed at $2F$ on other side of the lens.

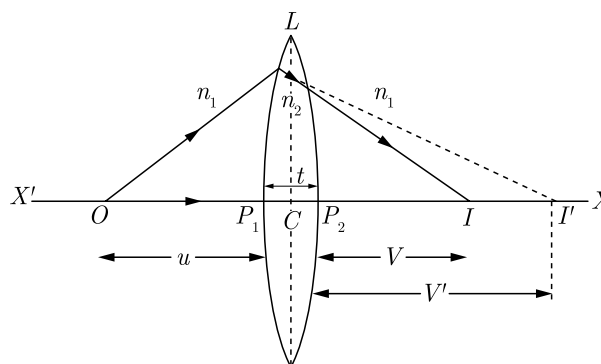


5. **When the Object is Placed beyond $2F$:** The incident ray AL , which is parallel to the principal axis, passes through the principal focus F after refraction, and the ray AA' which passes through the optical centre C goes undeviated. These two rays meet at point A' . So $A'B'$ is the real image of object AB . Figure (e).



Here, the image is real, inverted, smaller in size and is formed between F and $2F$ on the other side of the lens.

147. (a) Draw a ray diagram for formation of image of a point object by a thin double convex lens having radii of curvature R_1 and R_2 . Hence, derive lens maker's formula for a double convex lens. State the assumptions made and sign convention used.



- (b) A convex lens is placed over a plane mirror. A pin is positioned so that there is no parallax between the pin and its image formed by this lens-mirror combination. How will you use this observation to find focal length of the lens? Explain briefly.

Ans :

OD 2013, SQP 2015

- (a) **Lens Maker's Formula :** Suppose L is a thin lens. The refractive index of the material of lens is n_2 and it is placed in a medium of refractive index n_1 . The optical centre of lens is C and $X'OX$ is principal axis. The radii of curvature of the surfaces of the lens are R_1 and R_2 and their poles are P_1 and P_2 . The thickness of lens is t ,

$$\begin{aligned}
 &= \frac{\frac{A'B'}{C_2B'}}{\frac{A'B'}{C_1B'}} = \frac{C_1B'}{C_2B'} \\
 &= \frac{f_0}{-u_e} \quad \dots(1)
 \end{aligned}$$

Using lens formula $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ for the eyepiece, we have

$$\begin{aligned}
 \frac{1}{-D} - \frac{1}{-u_e} &= \frac{1}{f_e} \\
 \frac{1}{u_e} &= \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e} \left(1 + \frac{f_e}{D}\right)
 \end{aligned}$$

Putting the value of $\frac{1}{u_e}$ in equation (1), we have

$$m = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{D}\right)$$

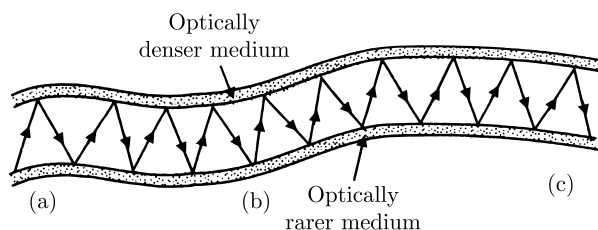
- 142.** What are Fibre Optics? Mention some of their applications.

Ans :

Delhi 2015, SQP 2001

Optics fibres too make use of the total internal reflection principle. Optical fibres consist of many thousands of very long fine quality, glass/quartz fibres. They are coated with a thin layer of a material of low refractive index. For example, the strand can be 0.0001 cm in diameter and the refractive index of the main fibre 1.7 and that of coating 1.5 (Figure).

When the light is incident on one end of the fibre at a small angle, the light passes inside. It undergoes repeated total internal reflections along the fibre and finally comes out. The angle of incidence here is larger than the critical angle of the fibre material with respect of its coating. Even if the fibre is bent, the light can easily travel along the fibre.



Uses of Optical Fibre :

1. A bundle of optical fibres can be used as a 'light pipe' in medical and optical examination called Endoscopy.
2. It can also be used for optical signal transmission.
3. Optical fibres have also been used for transmission and receiving electrical signals which are converted to light by suitable transducers. The main requirement is that there be very little absorption of light as it travels for long distances inside the optical fibre. This has been achieved by purification and special preparation of materials such as quartz.

- 143.** Derive Lens maker's formula for a thin concave lens.

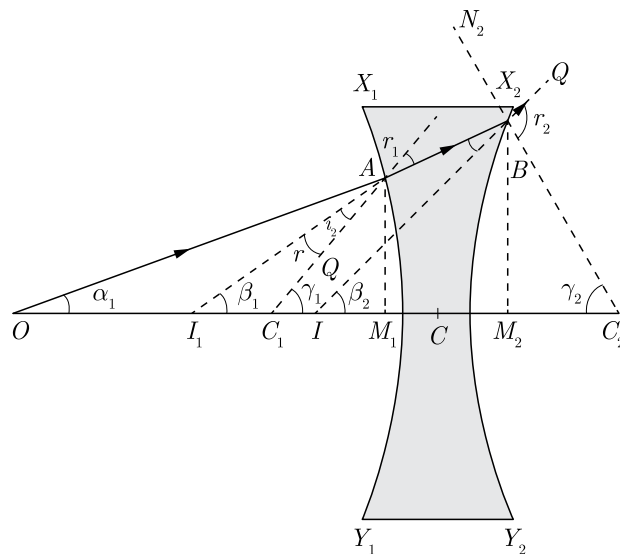
Ans :

Comp 2016

Let C_1 and C_2 be centres of curvature of two spherical surfaces of the concave lens having optical centre C as shown in the figure. Let μ_2 be the refractive index of the material of the lens and μ_1 that of medium surrounding the lens ($\mu_2 > \mu_1$).

Refraction at Surface X_1Y_1

Let a ray OA starts from the point object O which is placed on the principal axis of the concave lens and it refracts into the denser medium at point A . The refracted ray AB goes undeviated if the lens material were continuous. When AB is produced back it meets at I_1 . So I_1 would be the virtual image of point object O .



From Snell's law for small i_1 and r_1 , we have

$$\frac{\sin i_1}{\sin r_1} = \frac{i_1}{r_1} = \frac{\mu_2}{\mu_1} \quad \dots(1)$$

$$\begin{aligned}
 \text{In } \triangle AOC_1, \quad \gamma_1 &= i_1 + \alpha_1 \\
 i_1 &= \gamma_1 - \alpha_1
 \end{aligned}$$

$$\begin{aligned}
 \text{In } \triangle AI_1C_1, \quad \gamma_1 &= \beta_1 + r_1 \\
 r_1 &= \gamma_1 - \beta_1
 \end{aligned}$$

Putting the values of i_1 and r_1 in equation (1), we get

$$\mu_1(\gamma_1 - \alpha_1) = \mu_2(\gamma_1 - \beta_1)$$

$$\mu_1\gamma_1 - \mu_1\alpha_1 = \mu_2\gamma_1 - \mu_2\beta_1$$

$$-\mu_1\alpha_1 + \mu_2\beta_1 = (\mu_2 - \mu_1)\gamma_1$$

Since, α_1 , β_1 and γ_1 are small, so they can be replaced by their tangents.

$$\text{So,} \quad -\mu_1 \tan \alpha_1 + \mu_2 \tan \beta_1 = (\mu_2 - \mu_1) \tan \gamma_1$$

$$\text{or} \quad r = \gamma - \beta \quad \dots(2)$$

$$\text{From Snell's law,} \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad \dots(3)$$

If point A is very close to P , then angles i, r, α, β and γ will be very small, therefore

$$\sin i = i$$

and

$$\sin r = r$$

$$\text{From eq. (3),} \quad \frac{i}{r} = \frac{n_2}{n_1}$$

Substituting values of i and r from (1) and (2), we get

$$\frac{\gamma + \alpha}{\gamma - \beta} = \frac{n_2}{n_1}$$

$$\text{or} \quad n_1(\gamma + \alpha) = n_2(\gamma - \beta) \quad \dots(4)$$

Let h be the height of perpendicular drawn from A on principal axis i.e., $AM = h$. As α, β and γ are very small angles.

$$\text{Hence,} \quad \tan \alpha = \alpha, \quad \tan \beta = \beta$$

$$\text{and} \quad \tan \gamma = \gamma$$

Substituting these value in eq. (4),

$$n_1(\tan \gamma + \tan \alpha) = n_2(\tan \gamma - \tan \beta) \dots(5)$$

As point A is very close to point P , point M is coincident with P .

From figure,

$$\tan \alpha = \frac{AM}{OM} = \frac{h}{OP}$$

$$\tan \beta = \frac{AM}{MI} = \frac{h}{PI}$$

$$\tan \gamma = \frac{AM}{MC} = \frac{h}{PC}$$

Substituting these value in eq. (5), we get

$$n_1 \left(\frac{h}{PC} + \frac{h}{OP} \right) = n_2 \left(\frac{h}{PC} - \frac{h}{PI} \right)$$

$$\text{or} \quad n_1 \left(\frac{1}{PC} + \frac{1}{OP} \right) = n_2 \left(\frac{1}{PC} - \frac{1}{PI} \right) \quad \dots(6)$$

If the distances of object O , image I , centre of curvature C from the pole be u , v and R respectively. Then by sign convention PO is negative while PC and PI are positive.

Thus,

$$u = -PO, \quad v = +PI, \quad R = +PC$$

Substituting these value in eq. (6), we get

$$n_1 \left(\frac{1}{R} - \frac{1}{u} \right) = n_2 \left(\frac{1}{R} - \frac{1}{v} \right)$$

$$\frac{n_1}{R} - \frac{n_1}{u} = \frac{n_2}{R} - \frac{n_2}{v}$$

$$\text{or} \quad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The focal length of a convex lens is given by

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

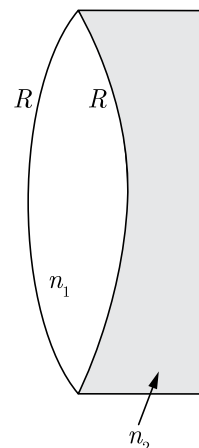
According to Cauchy's formula,

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \dots$$

n is inversely proportional to λ .

When wavelength increase, the refractive index n decreases; so focal length of lens increase with increase of wavelength.

149. A biconvex lens with its two faces of equal radius of curvature R is made of a transparent medium of refractive index n_1 . It is kept in contact with a medium of refractive index n_2 as shown in the figure.



- Find the equivalent focal length of the combination.
- Obtain the condition when this combination acts as diverging lens.
- Draw the ray diagram for the case $\mu_1 > \frac{(\mu_2 + 1)}{2}$. When the equal the object is kept far away from the lens. Point out the nature of the image formed by the system.

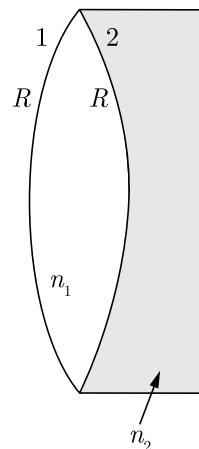
Ans :

Comp 2015

- If refraction occurs at first surface,

$$\frac{n_1}{v_1} - \frac{1}{u} = \frac{n_1 - 1}{R} \quad \dots(1)$$

If refraction occurs at second surface, and the image of the first surface acts as an object



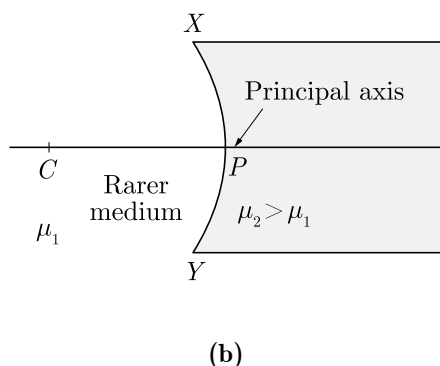
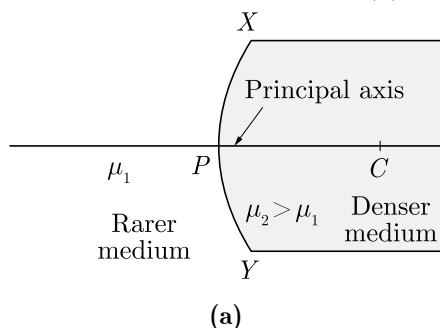
145. What is spherical refracting surface ? What do you mean by pole, centre of curvature, radius of curvature, aperture and principal axis.

Ans :

SQP 2012

A spherical refracting surface is formed when the refracting surface forms a part of a sphere. It is of two types.

1. **Convex spherical surface** : If the convex surface is towards the rarer medium, it is called convex spherical refracting surface, figure (a).
2. **Concave spherical surface** : If the concave surface is towards the rarer medium, it is called concave spherical refracting surface, figure (b).



1. **Pole (P)** : The centre of spherical refracting surface is called its pole.
 2. **Centre of Curvature (C)** : It is the centre of the sphere from which the curved surface is taken a part.
 3. **Radius of Curvature (PC)** : It is the radius of the sphere from which the curved surface is taken a part.
 4. **Aperture (XY)** : The diameter of the curved surface is called its aperture.
 5. **Principal Axis** : The principal axis is the line joining the pole and centre of curvature of refracting surface.
146. Distinguish between a real image and a virtual image. Describe the formation of real and virtual images by a convex lens with the help of diagrams at different positions of the object.

Ans :

Delhi 2013

Real Image

Real image is the image formed due to actual intersection of refracted light rays, and real image can be formed on a screen . Figure (c), (d), (e)

Virtual Image

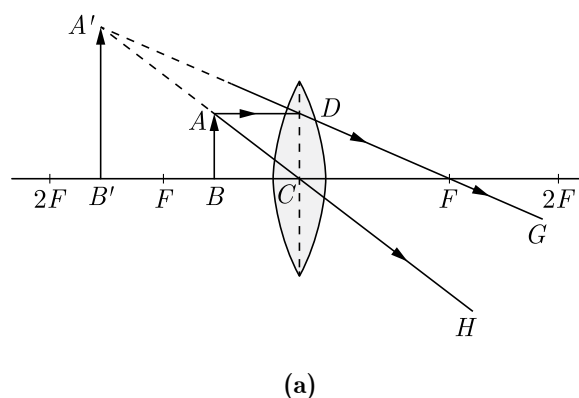
Virtual image is the image formed due to apparent interaction of refracted light rays when they are produced in backward directions, and virtual image cannot be obtained on the screen Figure (a).

Rules for forming the images by convex lens :

1. A ray parallel to the principal axis, passes through focus after refraction.
2. A ray passing through the principal focus, becomes parallel after refraction.
3. A ray passing through the optical centre of the lens goes undeviated after refraction.

Let us discuss following cases :

1. **When Object is Placed between Optical Centre and the Principal Focus of the Convex Lens** : The incident ray AD which is parallel to the principal axis, passes through the principal focus F after refraction and ray AH which is passing through the optical centre goes undeviated. These two rays when produced in backward direction, meet at A' , so $A'B'$ is the virtual image of real object AB Figure (a).

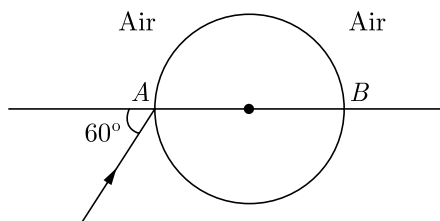


Here the image is virtual, erect, magnified and is formed behind the object.

2. **When Object is Placed at the Principal Focus of the Convex Lens** : The incident ray AL which is parallel to the principal axis, passes through the focus F after refraction and ray AD , which passes through the optical centre C , goes undeviated. These two rays are parallel and they meet at infinity. Thus real image of object AB is formed at infinity. Figure (b).

Ans :

Delhi 2019

Refractive index, $\mu = \frac{\sin i}{\sin r}$

$$\sqrt{3} = \frac{\sin 60^\circ}{\sin r}$$

$$\sin r = \frac{\sqrt{3}}{2} \times \frac{\sqrt{1}}{3} = \frac{1}{2}$$

$$\sin r = \sin 30^\circ$$

$$r = 30^\circ$$

Angle of refraction = 30°

155. Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

Ans :

OD 2017

Wavelength in air, $\lambda_a = 589 \text{ nm} = 5.89 \times 10^{-7} \text{ m}$ Refractive index of water, $\mu_w = 1.33$

Since, Speed of light in Vacuum,

$$c = 3 \times 10^8 \text{ m/s}$$

$$\text{Hence, frequency, } v = \frac{c}{\lambda_a} = \frac{3 \times 10^8 \text{ m/s}}{5.89 \times 10^{-7} \text{ m}} = 5.093 \times 10^{14} \text{ Hz}$$

Now, Speed of light in water,

$$v = \frac{c}{\mu_w} = \frac{3 \times 10^8 \text{ m/s}}{1.33} = 2.2605 \times 10^8 \text{ m/s}$$

Hence, Wavelength in water,

$$\lambda_w = \frac{v_w}{v_a} = \frac{c/\mu_w}{c/\lambda_a} = \frac{\lambda_a}{\mu_w}$$

(Since, frequency remains the same in all media)

$$= 4.43 \times 10^{-7} \text{ m}$$

156. A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm.

Ans :

OD 2017, Delhi 2006

Given, $n = 1.55$, $f = 20 \text{ cm}$

$$|R_1| = |R_2| = R \text{ (let)}$$

Since, For double convex lens as $R_1 > 0$ and $R_2 < 0$.So, $R_1 = R$ and $R_2 = -R$

Using lens maker's formula,

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{20} = (1.55-1) \cdot \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$R = 0.55 \times 2 \times 20$$

$$= 22 \text{ cm}$$

Hence, the radius of curvature is 22 cm.

157. A compound microscope uses an objective lens of focal length 4 cm and eyepiece lens of focal length 10 cm. An object is placed at 6 cm from the objective lens microscope. Also calculate the length of the microscope.

Ans :

Foreign 2013

Given, $f_o = 4 \text{ cm}$, $f_e = 10 \text{ cm}$, $u_o = -6 \text{ cm}$

Magnifying power of microscope,

$$M = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right)$$

From lens formula,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12}$$

$$v_o = 12 \text{ cm}$$

Hence,

$$M = -\frac{12}{6} \left(1 + \frac{25}{10} \right)$$

$$= -2 \times 3.5 = -7$$

Negative sign shows that the image is inverted.

Hence, Length of microscope,

$$L = |v_o| + |u_e|$$

$$= 12 + 7.14 = 19.14 \text{ cm}$$

158. A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm. The two are kept 15 cm apart. A point object is placed 40 cm in front of the convex lens. Find the position of the image formed by this combination. Draw the ray diagram showing the image formation.

Ans :

OD 2016

For convex lens,

$$u = -40 \text{ cm}, f = 20 \text{ cm}$$

which is very small. O from pole P_1 is u . The first refracting surface forms the image of O at I' at distance v' from P_1 . From the refraction formula at spherical surface,

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \dots(1)$$

The image I' acts as a virtual object for second surface and after refraction at second surface, the final image is formed at I . The distance of I from pole P_2 of second surface is v . The distance of virtual object (I') from pole P_2 is $(v' - t)$.

For refraction at second surface, the ray is going from second medium (refractive index n_2) to first medium (refractive index n_1), therefore, from refraction formula at spherical surface,

$$\frac{n_1}{v} - \frac{n_2}{(v' - t)} = \frac{n_1 - n_2}{R_2} \quad \dots(2)$$

For a thin lens t is negligible as compared to v' , therefore from eq. (2)

$$\frac{n_1}{v} - \frac{n_2}{v'} = -\frac{n_2 - n_1}{R_2} \quad \dots(3)$$

Adding equations (1) and (3), we get

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{or} \quad \frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{i.e.,} \quad \frac{1}{v} - \frac{1}{u} = ({}_1n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(4)$$

where, ${}_1n_2 = \frac{n_2}{n_1}$ is refractive index of second medium i.e., medium of lens with respect to first medium. If the object O is at infinity will be formed at second focus i.e.,

$$\text{if} \quad u = \infty \quad v = f_2 = f$$

Therefore, from equation (4),

$$\frac{1}{f} - \frac{1}{\infty} = ({}_1n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{i.e.,} \quad \frac{1}{f} = ({}_1n_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(5)$$

This formula is called Lens-Maker's formula.

If first medium is air and refractive index of material of lens be n , then ${}_1n_2 = n$, therefore, the modified equation (5) may be written as

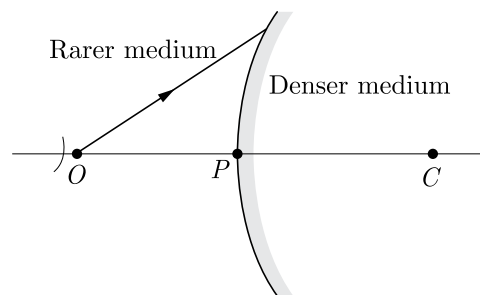
$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(6)$$

- (b) Focal length = distance of the pin from the mirror.

The rays from the object after refraction from lens should fall normally on the plane mirror. So, they retrace their path. Hence, rays must be originating from focus and thus distance of the pin from the plane mirror gives focal length of the lens.

148. A spherical surface of radius of curvature R , separates a rarer and a denser medium as shown in the figure.

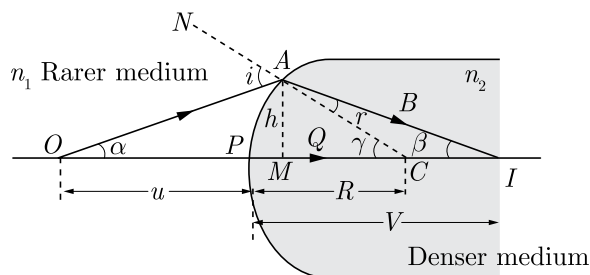
Complete the path of the incident ray of light. Showing the formation of a real image. Hence derive the relation connecting object distance u , image distance v radius of curvature R and the refractive indices n_1 and n_2 of two media. Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light.



Ans :

Delhi 2017

Relation of object and image distance of a convex spherical surface : Let SPS' be the convex spherical refracting surface, separating the two media of refractive indices n_1 and n_2 respectively ($n_1 < n_2$) i.e., medium 1 is rarer and medium 2 is denser. Let P be the pole, C the centre of curvature and PC the principal axis of convex refracting surface. O is a distant point object on the principal axis. The ray OA starting from O incident on point A of the spherical surface, CAN is normal at point A of the surface. Due to going from rarer to denser medium the ray OA deviates along the normal CAN and is refracted along the direction AB . The another ray OP starting from O is incident normally on the spherical surface and passes undeviated after refraction along PQ . Both the rays AB and PQ meet at point I on the principal axis. i.e., I is the real image of point object O .



Let i be the angle of incidence of ray OA and r be the angle of refraction in the denser medium i.e., $\angle OAN = i$ and $\angle CAI = r$, Let $\angle AOP = \alpha$, $\angle AIP = \beta$ and $\angle ACP = \gamma$

$$\text{In triangle } OAC, \quad i = \gamma + \alpha \quad \dots(1)$$

$$\text{In triangle } AIC, \quad \gamma = \beta + r$$

Hence,
$$\mu = \frac{\sin A}{\sin \frac{A}{2}} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}}$$

$$= 2 \cos \frac{A}{2}$$

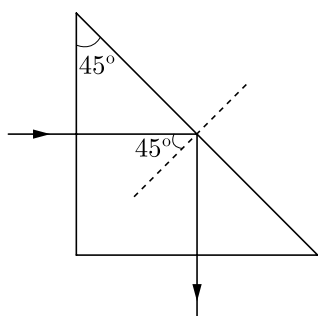
$$\cos \frac{A}{2} = \frac{\sqrt{3}}{2}$$

or
$$\frac{A}{2} = 30$$

$$A = 60^\circ$$

(ii)
$$\mu = \sqrt{3} = \frac{1}{\sin i_c}$$

$$\sin i_c = \frac{1}{\sqrt{3}}$$



Alternatively,

Which is less than $\frac{1}{\sqrt{2}}$.

Hence, Angle of incidence $> i_c$

Total internal reflection takes place.

- 161.** The radii of curvature of both the surfaces of a lens are equal. If one of the surfaces is made plane by grinding then will the focal length of lens change? Will the power change?

Ans :

Comp 2015

Focal length of lens,
$$\frac{1}{f} = (n-1) \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$f = \frac{R}{2(n-1)}$$

When one surface is made plane,

$$\frac{1}{f} = (n-1) \left(\frac{1}{R} + \frac{1}{\infty} \right)$$

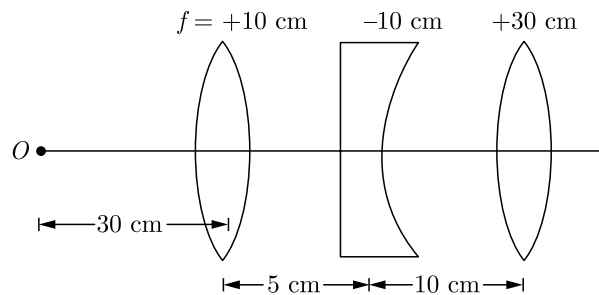
Hence,
$$f' = \frac{R}{(n-1)} = 2f$$

That is, the focal length will be doubled.

As $P = \frac{1}{f}$, so power will be halved.

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- 162.** Find the position of the image formed of an object O by the lens combination given in the figure.



Ans :

Delhi 2014, SQP 2003

For first lens, $u_1 = -30$ cm, $f_1 = +10$ cm

Hence lens formula,

$$\frac{f}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$$

$$v_1 = 15$$
 cm

This means that the image formed by first is at a distance of 15 cm to the right of first lens. This image serves as a virtual object for second lens.

For second lens, $f_2 = -10$ cm,

$$u_2 = 15 - 5 = +10$$
 cm

Hence,
$$\frac{1}{v_2} = \frac{f}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10}$$

$$v_2 = \infty$$

This means that the real image is formed by second lens at infinite distance. This acts as an object for third lens.

For third lens, $f_3 = +30$ cm, $u_3 = \infty$

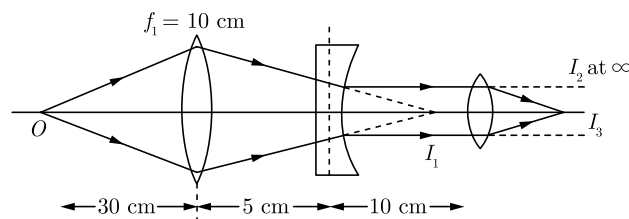
From lens formula,

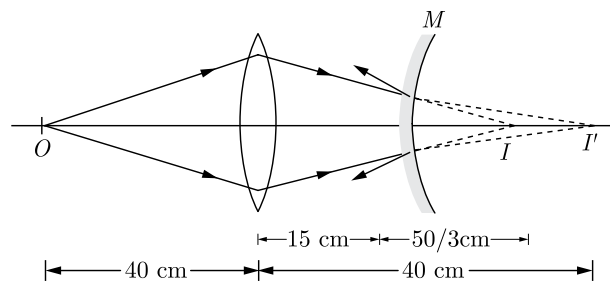
$$\frac{1}{v_3} = \frac{1}{f_3} + \frac{1}{u_3} = \frac{1}{30} + \frac{1}{\infty}$$

$$v_3 = 30$$
 cm

i.e., final image is formed at a distance 30 cm to the right of third lens.

The ray diagram of formation of image is shown in figure.





$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$v = 40 \text{ cm}$$

This image acts as a virtual object for the convex mirror.

$$u = 40 - 15 = 25 \text{ cm}$$

$$f = \frac{20}{2} = +10 \text{ cm}$$

Using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{25}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{25}$$

$$v = \frac{15}{3} \text{ cm} = 16.67 \text{ cm}$$

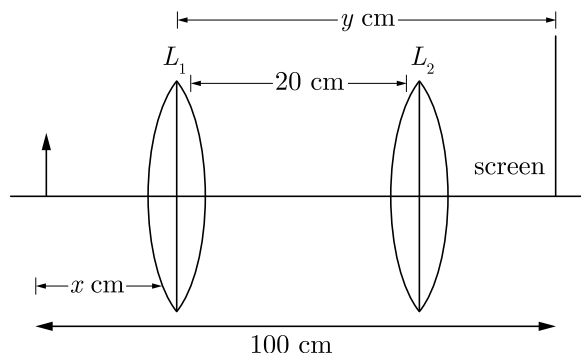
Hence, the final image is a virtual image formed at a distance of 16.67 cm.

159. (i) A screen is placed at a distance of 100 cm from of object. The image of the object is formed on the screen by a convex lens for two different locations of the lens separated by 20 cm. Calculate the focal length of the used.
- (ii) A converging lens is kept coaxially in contact with a diverging lens both the lenses being of equal focal length. What is the focal length of the combination?

Ans :

Comp 2012, OD 2009

- (i) For first position of the lens, we have



$$\frac{1}{f} = \frac{1}{y} - \frac{1}{(-x)}$$

$$\frac{1}{f} = \frac{1}{y} + \frac{1}{x} \quad \dots(1)$$

For second position of lens, we have

$$\frac{1}{f} = \frac{1}{y-20} - \frac{1}{[-(x+20)]}$$

$$\frac{1}{f} = \frac{1}{y-20} + \frac{1}{x+20} \quad \dots(2)$$

From eq (1) and eq (2), we have

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{(y-20)} + \frac{1}{(x+20)}$$

$$\frac{x+y}{xy} = \frac{(x+20) + (y-20)}{(y-20)(x+20)}$$

$$\frac{x+y}{xy} = \frac{x+y}{(y-20)(x+20)}$$

Hence,

$$xy = (y-20)(x+20)$$

$$xy = xy - 20x + 20y - 400$$

$$20x - 20y = -400$$

Hence,

$$x - y = -20$$

Also,

$$x + y = 100$$

On solving, we have,

$$x = 40 \text{ cm}$$

and

$$y = 60 \text{ cm}$$

Hence,

$$\frac{1}{f} = \frac{1}{60} - \frac{1}{-40} = \frac{5}{120}$$

Hence,

$$f = 24 \text{ cm}$$

(ii) Let focal length of the combination be f .

Hence,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{f} + \left(-\frac{1}{f}\right)$$

$$\frac{1}{f} = 0$$

$$f = \frac{1}{0} = \text{infinity}$$

160. (i) For a glass prism ($\mu = \sqrt{3}$) the angle of minimum deviation is equal to the angle of the prism. Calculate the angle of the prism.
- (ii) Draw ray diagram when incident ray normally on one of the two equal sides of a right angled isosceles prism having refractive index $\mu = \sqrt{3}$.

Ans :

Delhi 2010

(i) At minimum deviation,

$$\mu = \frac{\sin\left[\frac{(A + \delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

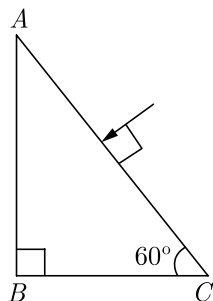
Given,

$$\delta_m = A$$

Since, $A = 60^\circ$ (being an equilateral glass prism)

So,
$$D = \frac{60^\circ}{2} = 30^\circ$$

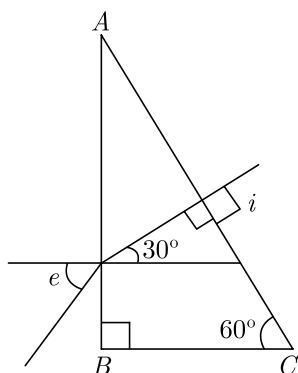
- 167.** Trace the path of a ray of light passing through a glass prism ABC as shown in the figure. If the refractive index of glass is $\sqrt{3}$, then find out the value of the angle of emergence from the prism.



Ans :

Refractive index of glass, $\mu_g = \sqrt{3}$

Since, $i = 0$



At the interface AC , we have (according to Snell's law)

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$$

But, $\sin i = \sin 0^\circ = 0$

Thus, $\sin r = \frac{\mu_a \sin i}{\mu_g} = 0$

Hence, $r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. Here, we can see angle of incidence becomes 30° . Thus, applying Snell's law,

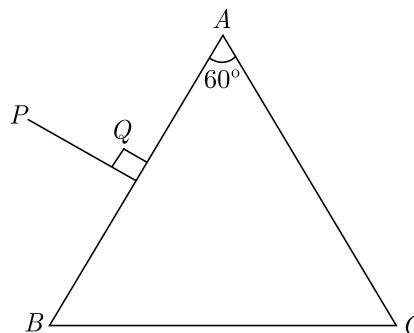
$$\frac{\sin 30^\circ}{\sin e} = \frac{\mu_g}{\mu_a} = \frac{1}{\sqrt{3}}$$

$$\sin e = \sqrt{3} \times \sin 30^\circ = \frac{\sqrt{3}}{2}$$

Thus, $e = 60^\circ$

Hence, angle of emergence is 60° .

- 168.** A ray PQ is incident normally on the face AB of a triangular prism of refracting angle of 60° made of a transparent material of refracting index $\frac{2}{\sqrt{3}}$ as shown in the figure. Trace the path of the ray as it passes through the prism. Also, calculate the angle of emergence and angle of deviation



Ans :

Delhi 2014, SQP 2002

If the i_c is the critical angle for the prism.

Now,
$$\mu = \frac{1}{\sin i_c}$$

Hence,
$$\sin i_c = \frac{1}{\mu} = \frac{\frac{1}{2}}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

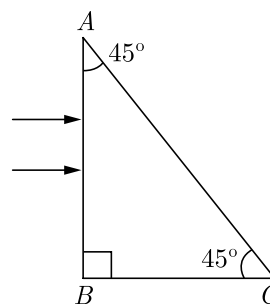
$$i_c = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow i_c = 60^\circ$$

Angle of incidence at face AC of the prism is 60° . Hence, refracted ray grazes the surface AC .

Angle of emergence = 90°

Angle of deviation = 30° .

- 169.** Two monochromatic rays of light are incident normally on the face AB of an isosceles right-angled prism ABC . The refractive indices of the glass prism for the two rays 1 and 2 are respectively 1.35 and 1.45. Trace the path of these rays after entering through the prism.



Ans :

SQP 2010

The paths are shown as below

- 163.** An object is placed 40 cm from a convex lens of focal length 30 cm. If a concave lens of focal length 50 cm is introduced between the convex lens and the image formed such that it is 20 cm from the convex lens, find the change in the position of the image.

Ans :

OD 2016

For the convex lens, $f_1 = +30$ cm and object distance $u_1 = -40$ cm, therefore,

$$\begin{aligned}\frac{1}{f_1} &= \frac{1}{v_1} - \frac{1}{u_1} \\ \frac{1}{+30} &= \frac{1}{v_1} - \frac{1}{-40} \\ \frac{1}{v_1} &= \frac{1}{30} - \frac{1}{40} = \frac{1}{120} \\ v_1 &= +120 \text{ cm}\end{aligned}$$

Real image is formed.

On introducing a concave lens,

$$f_2 = 50 \text{ cm}$$

and $u_2 = 120 - 20$

$= +100$ cm from the concave lens

$$\begin{aligned}\frac{1}{f_2} &= \frac{1}{v_2} - \frac{1}{u_2} \\ \frac{1}{-50} &= \frac{1}{v_2} - \frac{1}{+100} = -\frac{1}{100} \\ v_2 &= -100 \text{ cm}\end{aligned}$$

A virtual image is formed at the distance of 100 cm from the concave lens.

The change in position between the real image and the virtual image is $100 \text{ cm} + 100 \text{ cm} = +200 \text{ cm}$ to the left of its original position.

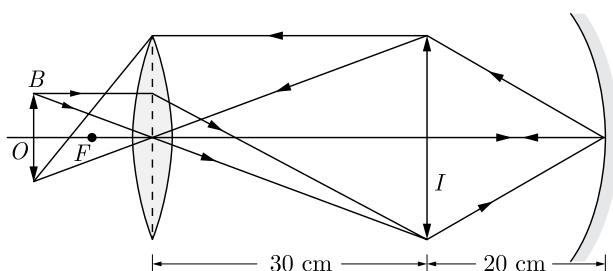
- 164.** An object is placed 15 cm in front of a convex lens of focal length 10 cm. Find the nature and position of the image formed. Where should a concave mirror of radius of curvature 20 cm be placed so that the final image is formed at the position of the object itself?

Ans :

Delhi 2013

For lens, $u = -15$ cm,

$$f = +10 \text{ cm}$$



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} + \frac{1}{u} \\ &= \frac{1}{10} - \frac{1}{15} \\ v &= 30 \text{ cm}\end{aligned}$$

For image to be formed at O , the rays incident on mirror should form the image at centre of curvature. It will be so if the image I formed by the lens lies at the centre of curvature of the mirror.

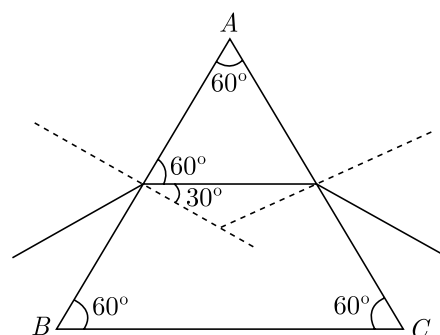
Hence, $d = |v| + |R| = 30 + 20 = 50 \text{ cm}$

- 165.** A ray of light incident on an equilateral glass prism ($\mu_g = \sqrt{3}$) moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray.

Ans :

Foreign 2012, OD 2007

The reflection of light through prism is shown as below



By geometry Angle of refraction, $r = 30^\circ$

Given, refractive index, $\mu = \sqrt{3}$

Using Snell's law, $\mu = \frac{\sin i}{\sin r}$

$$\sin i = \mu \sin r$$

$$= (\sqrt{3}) \sin(30^\circ) = \frac{\sqrt{3}}{2}$$

Angle of incidence, $i = 60^\circ = \frac{\pi}{3}$

Hence, $i = \frac{\pi}{3}$

The image is inverted.

- 166.** A ray of light passes through an equilateral glass prism such that the angle of incidence is equal angle of emergence and each of these angles is equal to $\frac{3}{4}$ of angle of prism. What is the value of deviation?

Ans :

OD 2021

In prism, $i + e = A + D$

and $i = e = \frac{3}{4}A$ (given)

So, $A + D = \frac{3}{4}A + \frac{3}{4}A$

$$D = \frac{3A}{2} - A = \frac{A}{2}$$

$$= \frac{1}{0.25} + \frac{1}{-0.20} = \frac{100}{25} + \frac{100}{-20}$$

$$= \frac{400 - 500}{100} = \frac{-100}{100} = -1\text{D}$$

Here, the focal length of the combination = 100 cm = -1m

Since, the focal length is in negative, so the system will be diverging in nature.

- 172.** A small telescope has an objective lens of focal length 105 cm and eyepiece of focal length 5 cm. What is the magnifying power of the telescope for viewing distance objects in normal adjustments.

In this telescope is used to view a 100 km tall tower 3 km away, than what is the height of the tower formed by the objective lens.

Ans :

Delhi 2012

When final image is at D , then magnifying power,

$$m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

In normal adjustment, $m = \frac{-f_o}{f_e}$

For telescope,

Focal length of objective lens, $f_o = 150\text{ cm}$

Focal length of eye lens, $f_e = 5\text{ cm}$

When final image forms at, $D = 25\text{ cm}$

As we know that,

$$\text{Magnification, } m = \frac{-f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

$$= \frac{-150}{5} \left(1 + \frac{5}{25} \right)$$

$$= \frac{-150}{5} \times \frac{6}{5}$$

$$m = -36$$

Let height of final image be $h\text{ cm}$.

$$\tan \beta = \frac{h}{25}$$

β = visual angle formed by final image at eye.

α = visual angle subtended by object at objective.

$$\tan \alpha = \frac{100\text{ m}}{3000\text{ m}} = \frac{1}{30}$$

But, $m = \frac{\tan \beta}{\tan \alpha}$

$$-36 = \frac{\left(\frac{h}{25} \right)}{\left(\frac{1}{30} \right)}$$

$$-36 = \frac{h}{25} \times 30$$

$$-36 = \frac{6h}{5}$$

$$h = \frac{-36 \times 5}{6}$$

$$h = -30\text{ cm}$$

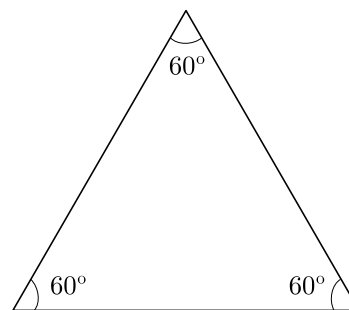
- 173.** A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $3/4$ th of the angle of prism. Calculate the speed of light in the prism.

Ans :

OD 2018, Delhi 2013

For equilateral prism, $A = 60^\circ$

For minimum angle of deviation



$$i + e = A + \delta_m$$

$$2i = A + \delta_m$$

$$2 \times \frac{3A}{4} = A + \delta_m$$

$$\delta_m = \frac{3A}{2} - A = \frac{A}{2}$$

$$\mu = \frac{\sin\left(\frac{A + \frac{A}{2}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\mu = \frac{\sin \frac{3A}{4}}{\sin \frac{A}{2}} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

$$= \frac{0.7071}{0.5} = 1.414$$

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.414}$$

$$= 2.121 \times 10^8 \text{ m/s}$$

- 174.** (a) Calculate the distance of an object of height h from a concave mirror of radius of curvature 20 cm, so as to obtain a real image of magnification 2. Find the location of image also.
(b) Using mirror formula, explain why does a convex mirror always produce a virtual image.

Ans :

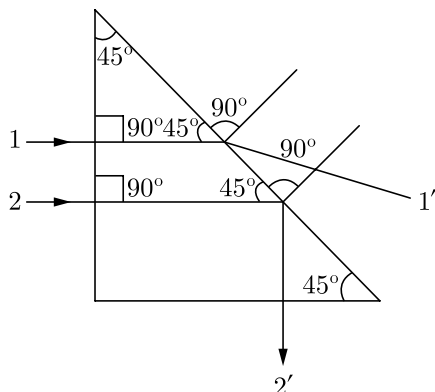
Foreign 2007

(a) $R = -20\text{ cm}$ and $M = -2$

Focal length, $f = \frac{R}{2} = -10\text{ cm}$

Magnification, $M = -\frac{v}{u} = -2$ (given)

Hence, $v = 2u$



From the figure, it is clear that angle of incidence for ray 1 is 45° .

For ray 1, $\sin i = \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{1}{1.414}$

For ray 1, refractive index of the prism is $\mu = 1.35$

$$\mu = \frac{1}{\sin C}$$

$$\sin C = \frac{1}{\mu} = \frac{1}{1.35}$$

Here, $\frac{1}{1.414} < \frac{1}{1.35}$

i.e., $\sin i < \sin C$

or $i < C$

So, ray 1 will be refracted by the prism.

For ray 2, angle of incidence,

$$i = 45^\circ$$

$$\sin i = \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{1}{1.414}$$

For ray 2, the refractive index, $\mu = 1.45$

$$\mu = \frac{1}{\sin C}$$

$$\sin C = \frac{1}{\mu} = \frac{1}{1.45}$$

Here, $\frac{1}{1.414} > \frac{1}{1.45}$

i.e., $\sin i > \sin C$

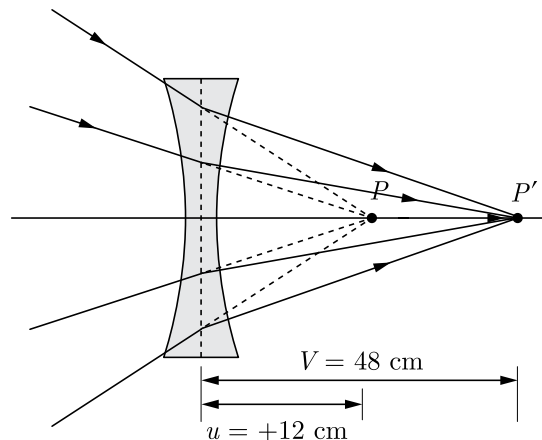
or $i > C$

So, ray 2 will get total internally reflected.

170. A beam of light converges at a point P . A concave lens of focal length 16 cm is placed in the path of this beam 12 cm from P . Draw a ray diagram and find the location of the point at which the beam would now converge.

Ans :

Delhi 2013



Given, $u = +12$ cm, $f = -16$ cm $v = ?$

Using lens formula,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{v} - \frac{1}{u} \\ -\frac{1}{16} &= \frac{1}{v} - \frac{1}{12} \\ \frac{1}{v} &= \frac{1}{12} - \frac{1}{16} = \frac{4-3}{48} = \frac{1}{48} \\ v &= +48 \text{ cm} \end{aligned}$$

The image of virtual object at P from at P' at a distance 48 cm from the lens.

171. A convex lens of focal length 25 cm is placed coaxially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

Ans :

OD 2016, Comp 2000

Given, focal length of convex lens,

$$f_1 = +25 \text{ cm} = +0.25 \text{ m}$$

Focal length of concave lens,

$$f_2 = -20 \text{ cm} = -0.20 \text{ m}$$

Equivalent focal length of convex and concave lens,

$$\begin{aligned} F &= \frac{1}{f_1} + \frac{1}{f_2} \\ &= \frac{1}{25} + \frac{1}{-20} = -\frac{1}{100} \end{aligned}$$

Hence, $F = -100 \text{ cm} = -1 \text{ m}$

Now, the power of lens,

$$P = \frac{1}{f}$$

For convex lens, $P_1 = \frac{1}{f_1} = \frac{1}{0.25}$

For concave lens, $P_2 = \frac{1}{f_2} = \frac{1}{-0.20}$

Hence, power of the combination

$$P = P_1 + P_2$$

$$= \frac{1}{20} + \frac{1}{-15} = \frac{15-20}{300}$$

$$= -\frac{5}{300} = -\frac{1}{60}$$

$$v = -60 \text{ cm}$$

i.e., -60 cm from lens in the direction of object.

Now, for lens L_2 ,

The image formed by lens L_1 , will act as object for lens L_2 ,

$$u = -60 + (-2) = -80 \text{ cm}$$

$$v = +80 \text{ cm} \quad (\text{given})$$

and

$$f = x \text{ cm}$$

Applying lens formula for lens L_2 ,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{x} = \frac{1}{80} - \frac{1}{(-80)} = \frac{1}{80} + \frac{1}{80}$$

$$\frac{1}{x} = \frac{2}{80}$$

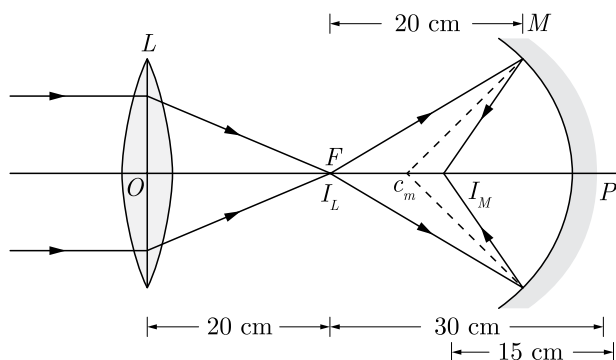
$$x = 40$$

Hence, focal length of lens L_2 is 40 cm .

- 178.** A convex lens of focal length 20 cm is placed coaxially with a concave mirror of focal length 10 cm at a distance of 50 cm apart from each other. A beam of light coming parallel to the principal axis is incident on the convex lens. Find the position of the final image formed by this combination. Draw the ray diagram showing the formation of the image.

Ans :

SQP 2014



For the convex lens,

$$u = \infty, f = 20 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Hence,

$$v = 20 \text{ cm}$$

For the convex mirror, the image formed by the lens acts as the object.

$$\text{Hence, } u = -(50 - 20) \text{ cm}$$

$$= -30 \text{ cm and } f = -10 \text{ cm}$$

Using mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{u} + \frac{1}{-30} = \frac{1}{-10}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{10} = -\frac{1}{15}$$

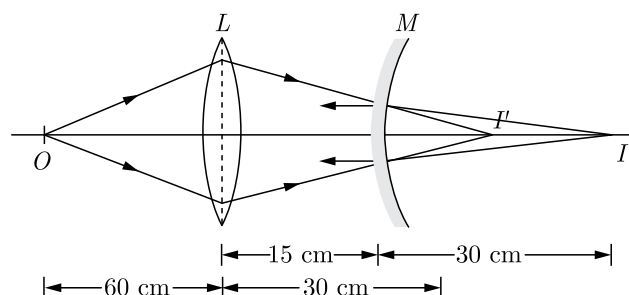
$$v = -15 \text{ cm}$$

The lens mirror combination, therefore, forms a real image I_2 at a distance of 15 cm to the concave mirror or at a distance of 35 cm to the right of convex lens.

- 179.** A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm . The two are kept at 15 cm from each other. A point object lies 60 cm in front of the convex lens. Draw a ray diagram to show the formation of the image by the combination. Determine the nature and position of the image formed.

Ans :

OD 2017, Comp 2014



$$\text{For convex lens, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$f = +20 \text{ cm}$$

$$u = -60 \text{ cm}$$

$$\frac{1}{+20} = \frac{1}{v} - \frac{1}{-60}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{60} = \frac{1}{30}$$

$$v = +30 \text{ cm}$$

On the absence of the mirror, the lens would formed be image of I' Which acts as a virtual object for the convex mirror.

f_m = distance of virtual object I' from mirror

$$= +30 \text{ cm} - 15 \text{ cm} = +15 \text{ cm}$$

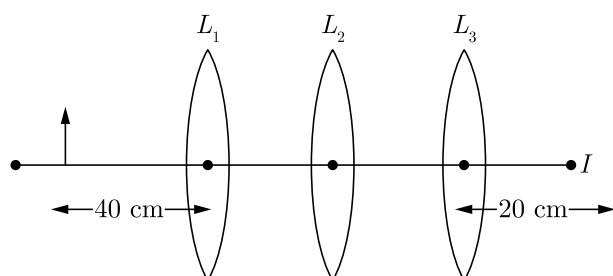
$$f_m = \frac{R}{2} = \frac{+20}{2} = +10 \text{ cm}$$

From mirror formula,

$$\begin{aligned}\frac{1}{f_m} &= \frac{1}{v_2} + \frac{1}{u_2} \\ \frac{1}{f_m} &= \frac{1}{v_2} + \frac{1}{15} \\ \frac{1}{v_2} &= \frac{1}{10} - \frac{1}{15} = \frac{1}{30} \\ v_2 &= +30 \text{ cm}\end{aligned}$$

Hence, the final image I is a virtual image formed at a distance of 30 cm to the right of convex mirror or 45 cm from the convex lens.

180. You are given three lenses L_1 , L_2 and L_3 each of focal length 20 cm. An object is kept at 40 cm in front of L_1 , as shown. The final real image is formed at the focus I of L_3 . Find the separations between L_1, L_2 and L_3 .



Ans :

Delhi 2012

Given, $f_1 = f_2 = f_3 = 20 \text{ cm}$

For lens L_1 , $u_1 = -40 \text{ cm}$

By lens formula,

$$\begin{aligned}\frac{1}{v_1} - \frac{1}{u_1} &= \frac{1}{f} \\ \frac{1}{v_1} &= \frac{1}{20} - \frac{1}{-40} \\ v_1 &= 40 \text{ cm}\end{aligned}$$

For lens L_1 , $f_3 = 20 \text{ cm}$, $v_3 = 20 \text{ cm}$, $u_3 = ?$

By lens formula,

$$\begin{aligned}\frac{1}{v_3} - \frac{1}{u_3} &= \frac{1}{f_3} \\ \frac{1}{20} - \frac{1}{u_3} &= \frac{1}{20} \\ \frac{1}{u_3} &= 0 \Rightarrow u_3 = \infty\end{aligned}$$

Thus lens L_2 should produce image at infinity.

Hence, for L_2 , its objective should be at focus. The image formed by lens L_1 is at 40 cm on the right side of lens L_1 . Which lies at 20 cm left of lens L_2 , i.e., focus of lens L_2 .

Hence, the distance by lens L_2 lies at infinity, then the distance between lens L_2 and L_3 does not matter.

Hence, the distance between L_2 and L_3 can have any value.

181. Two convex lenses of focal length 20 cm and 1 cm constitute a telescope is focused on a point which is 1 m away from the objective. Calculate the magnification produced and the length of the tube if the final image is formed at a distance 25 cm from the eyepiece.

Ans :

Delhi 2011, OD 2003

Given, $f_o = 20 \text{ cm}$, $f_e = 1 \text{ cm}$, $v_e = -25 \text{ cm}$

and

$$u_0 = 1 \text{ m} = 100 \text{ cm}$$

For objective

$$u_0 = 1 \text{ m} = 100 \text{ cm}$$

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{20}$$

$$= \frac{1}{v_o} - \frac{1}{(-100)}$$

$$\frac{1}{v_o} = \frac{1}{20} - \frac{1}{100} = \frac{5-1}{100} = \frac{4}{100}$$

$$v_o = 25 \text{ cm}$$

For eye piece

$$f_e = 1 \text{ cm}, u_e = ?, v_e = -25 \text{ cm}$$

$$\frac{1}{f_e} = \frac{1}{u_e} = \frac{25}{-25} = -\frac{1}{u_e}$$

$$1 + \frac{1}{25} = -\frac{1}{u_e} = \frac{25}{26} = -\frac{1}{u_e}$$

$$u_e = -\frac{25}{26}$$

$$|u_e| = -0.96 \text{ cm}$$

$$\begin{aligned}\text{Magnification, } m &= -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right) \\ &= -\left(\frac{25}{100}\right) \left(1 + \frac{25}{1}\right) \\ m &= -\frac{1}{4} \times 26 = -6.5\end{aligned}$$

Length of telescope,

$$L = v_o + u_e = 25 + 0.96$$

$$L = 25.96 \text{ cm}$$

182. You are given two converging lenses of focal length 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, find out the separation between the objective and the eyepiece.

Ans :

Comp 2015, OD 2010

The magnification due to the objective lens,

$$m_o = \frac{v_o}{u_o}$$

If the object is close to focus of the objective lens then,

$$u_o = f_o \text{ and } v = L$$

(where, L = distance between two lenses)

$$m_o = \frac{L}{f_o}$$

If the final image is at the near point, then magnification

due to eye lens's

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

$$M = m_o \times m_e = \frac{L}{f_o} \left(1 + \frac{D}{f_e}\right)$$

Use, $f_o = 1.25 \text{ cm}$ and $f_e = 5 \text{ cm}$

$$30 = \frac{L}{1.25} \left(1 + \frac{25}{5}\right)$$

$$30 = \frac{L}{1.25} \times 6$$

$$L = \frac{30 \times 1.25}{6} = 6.25 \text{ cm}$$

Separation between two lenses is 6.25 cm.

- 183.** The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. The microscope is focussed on a certain object. The distance between the objective and eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the of objective and the eyepiece.

Ans :

Foreign 2008

Here, $M = -20$, $m_e = 5$, $v_e = -20 \text{ cm}$

$$\begin{aligned} \text{For eyepiece, } m_e &= \frac{v_e}{u_e} \\ 5 &= \frac{-20}{u_e} \\ u_e &= \frac{-20}{5} = -4 \text{ cm} \end{aligned}$$

Using lens formula,

$$\begin{aligned} \frac{1}{v_e} - \frac{1}{u_e} &= \frac{1}{f_e} \\ -\frac{1}{20} + \frac{1}{4} &= \frac{1}{f_e} \\ \frac{-1+5}{20} &= \frac{1}{f_e} \\ f_e &= 5 \text{ cm} \end{aligned}$$

Now, total magnification,

$$\begin{aligned} M &= m_e \times m_o \\ -20 &= 5 \times m_o \\ m_o &= -4 \end{aligned}$$

$$\begin{aligned} \text{Also, } |v_o| + |u_e| &= 14 \\ |v_o| + |-4| &= 14 \\ v_o &= 14 - 4 = 10 \text{ cm} \end{aligned}$$

$$\begin{aligned} m_o &= 1 - \frac{v_o}{f_o} = -4 = 1 - \frac{10}{f_o} \\ -5 &= -\frac{10}{f_o} \\ f_o &= 2 \text{ cm} \end{aligned}$$

- 184.** An object is kept in front of a concave mirror of focal length 15 cm. The image formed is three times the size of the object. Calculate the two possible distances of the object from the mirror.

Ans :

OD 2016, Foreign 2004

As the mirror is concave, so

$$f = -15 \text{ cm}$$

When the image formed is real,

$$m = \frac{h_2}{h_1} = -\frac{v}{u} = -3$$

or

$$v = +3u$$

As,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Hence,

$$\frac{1}{u} + \frac{1}{3u} = -\frac{1}{15}$$

$$\frac{4}{3u} = -\frac{1}{15}$$

$$u = -\frac{15 \times 4}{3} = -20 \text{ cm}$$

When the image formed is virtual,

$$m = \frac{h_2}{h_1} = -\frac{v}{u} = +3$$

$$v = -3u$$

As

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} - \frac{1}{3u} = -\frac{1}{15}$$

$$\frac{2}{3u} = -\frac{1}{15}$$

$$\text{Hence, } u = -\frac{15 \times 2}{3} = -10 \text{ cm}$$

- 185.** A ray of light of frequency $5 \times 10^{14} \text{ Hz}$ is passed through a liquid. The wavelength of light measured inside the liquid is found to be $450 \times 10^{-9} \text{ m}$. Calculate the refractive index of the liquid.

Ans :

Comp 2018

Given,

$$\nu = 5 \times 10^{14} \text{ Hz}$$

$$\lambda = 450 \times 10^{-9} \text{ m}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

Refractive index of the liquid,

$$\begin{aligned} \mu &= \frac{c}{v} = \frac{c}{\nu \lambda} \\ &= \frac{3 \times 10^8}{5 \times 10^{14} \times 450 \times 10^{-9}} \\ &= 1.33 \end{aligned}$$

- 186.** An object is placed (1) 10 cm, (2) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature and magnification of the image in each case.

Ans :

Delhi 2014, OD 2012

Given,

Angular magnification, $M = 10$ Length of the tube, $L = 44$ cmAngular magnification, $M = \frac{f_o}{f_e}$

$$10 = \frac{f_o}{f_e}$$

$$f_o = 10f_e \quad \dots(1)$$

According to the question

Length of tube, $L = f_o + f_e = 44 \quad \dots(2)$

From equation (1), we get

$$10f_e + f_e = 44$$

$$f_e = \frac{44}{11} = 4 \text{ cm}$$

Now, from equation (2), we get

$$f_o + 4 = 44$$

$$f_o = 44 - 4$$

$$= 40 \text{ cm}$$

- 196.** Double convex lenses are to be manufacture from a glass of refractive index 1.55 with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Ans :

SQP 2006

Given,

Refractive index of glass, $\mu = 1.55$ Focal length of lens, $f = 20$ cmLet the radius of curvature of lens are R_1 and R_2 .

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Let,

$$R_1 = R_2, \text{ then}$$

$$R_1 = R$$

$$R_2 = -R \quad (\text{Since, Lens is convex})$$

$$\frac{1}{20} = (1.55 - 1) \left[\frac{1}{R} - \frac{1}{(-R)} \right]$$

$$\frac{1}{20} = 0.55 \left[\frac{1}{R} + \frac{1}{R} \right]$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$R = 0.55 \times 2 \times 20 = 22 \text{ cm}$$

- 197.** The power of two lenses are +12D and -2D. They are placed in contact on the same axes. What will be the focal length of the combination?

Ans :

Delhi 2012

Given,

Power of first lens, $P_1 = +12$ DPower of second lens, $P_2 = -2$ D

Now, Power of equivalent lens,

$$P = P_1 + P_2$$

$$= 12 \text{ D} - 2 \text{ D} = 10 \text{ D}$$

$$\text{Focal length, } f = \frac{1}{P} = \frac{1}{10} \text{ meter}$$

$$= \frac{1}{10} \times 100 = 10 \text{ cm}$$

- 198.** 1. Two thin convex lenses of focal length 15 cm and 30 cm are placed in contact. What will be power of the combination?
2. A glass Prism ($\mu_g = 1.66$) of angles of Prism 72° , is placed in Water ($\mu_w = 1.33$). Calculate the value of minimum deviation produced by prism for parallel incident ray.

Ans :

OD 2015

1. Given,

Focal length of first convex lens,

$$f_1 = 15 \text{ cm}$$

Focal length of second convex lens,

$$f_2 = 30 \text{ cm}$$

Focal Length of combination,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{15} + \frac{1}{30} = \frac{2+1}{30}$$

$$f_{eq} = 10 \text{ cm or } 0.1 \text{ m}$$

Power of the lens combination,

$$P = \frac{1}{f_{eq} \text{ (in meter)}} = \frac{1}{0.1} = 10 \text{ D}$$

2. Angle of deviation,

$$\delta_m = \left(\frac{n_2}{n_1} - 1 \right) A$$

Here,

$$n_1 = 1.33$$

$$n_2 = 1.66$$

Angle of prism, $A = 72$

$$\text{Hence, } \delta_m = \left(\frac{1.66}{1.33} - 1 \right) 72$$

$$= (1.24 - 1) \times 72$$

$$= 0.24 \times 72 = 17.8^\circ$$

- 199.** At what distance an object be placed from a convex lens of focal length 15 cm so that its three times magnified image is formed?

Ans :

Delhi 2017

Given,

Focal length of the lens,

$$f = +15 \text{ cm}$$

- 191.** The velocity of light in a liquid is $1.5 \times 10^8 \text{ ms}^{-1}$ and in air, it is $3 \times 10^8 \text{ ms}^{-1}$. If a ray of light passes from this liquid into air, calculate the value of critical angle.

Ans :

Delhi 2012, Comp 2017

Given, $v = 1.5 \times 10^8 \text{ ms}^{-1}$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

Refractive index of the liquid,

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$$

or $\sin i_c = \frac{1}{\mu} = \frac{1}{2}$

Hence, Critical angle, $i_c = 30^\circ$

- 192.** The radius of curvature of each face of biconcave lens, made of glass of refractive index 1.5 is 30 cm. Calculate the focal length of the lens in air.

Ans :

OD 2007

Given, $\mu = 1.5$

$$R_1 = -30 \text{ cm}$$

$$R_2 = +30 \text{ cm}$$

Using lens maker's formula,

$$\begin{aligned} \frac{1}{f} &= (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \\ &= (1.5 - 1) \left[\frac{1}{-30} - \frac{1}{30} \right] \\ &= -0.5 \times \frac{2}{30} = -\frac{1}{30} \end{aligned}$$

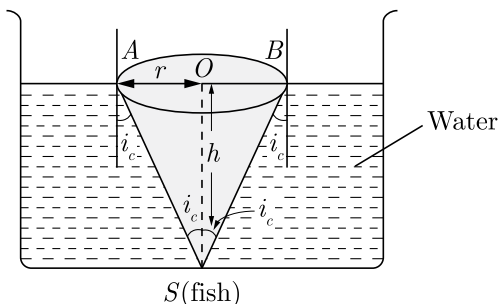
Hence, $f = -30 \text{ cm}$

- 193.** A fish is at a depth of $\sqrt{7}$ cm and it can see only through a circular section of water surface. What will be radii of this circular surface? The refractive index of water with respect to air is $\frac{4}{3}$. Sketch the diagram also.

Ans :

Delhi 2016

Suppose the height $OS = h$ is such that $\angle OSA = i_c$. Then any other ray from S (fish) will be totally internally reflected because then the angle of incidence would be greater than i_c .



In $\triangle OSA$,

$$\sin i_c = \frac{OA}{AS} = \frac{r}{\sqrt{r^2 + h^2}}$$

Here,

$$\mu = \frac{4}{3}$$

$$h = \sqrt{7} \text{ cm}$$

Hence,

$$\frac{3}{4} = \frac{r}{\sqrt{r^2 + (\sqrt{7})^2}}$$

On taking square both side, we get

$$\frac{9}{16} = \frac{r^2}{r^2 + 7}$$

$$9r^2 + 63 = 16r^2$$

$$16r^2 - 9r^2 = 63$$

$$7r^2 = 63$$

$$r = \sqrt{9} = 3 \text{ cm}$$

- 194.** Two thin lenses of powers 5 D and 2 D are placed coaxially at a distance 20 cm apart. Find the focal length and power of the combination of lenses.

Ans :

Delhi 2013

Given,

Power of first lens, $P_1 = 5 \text{ D}$

Power of second lens, $P_2 = 2 \text{ D}$

Distance between the lens,

$$d = 20 \text{ cm}$$

Focal length of first lens, $f_1 = \frac{1}{P_1} \times 100 \text{ cm}$

$$= \frac{1}{5} \times 100 = 20 \text{ cm}$$

Similarly,

$$f_2 = \frac{1}{P_2} \times 100$$

$$= \frac{1}{2} \times 100 = 50 \text{ cm}$$

Focal length of combination of lens,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{50} - \frac{20}{20 \times 50}$$

$$\frac{1}{f} = \frac{1}{20}$$

$$f = 20 \text{ cm}$$

Power of lens combination,

$$P = \frac{1}{f(\text{meter})}$$

$$= \frac{1}{20} \times 100 = 5 \text{ D}$$

- 195.** The minimum angular magnification of an astronomical telescope is 10. The length of its tube is 44 cm. Determine the focal length of the objective lens.

Ans :As R is negative for a concave mirror, so

$$f = \frac{R}{2} = -\frac{15}{2} = -7.5 \text{ cm}$$

1. Here object distance,

$$u = -10 \text{ cm}$$

By mirror formula,

$$\begin{aligned} \frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = \frac{1}{-7.5} - \frac{1}{-10} \\ &= \frac{-2.5}{7.5 \times 10} = -\frac{1}{30} \\ v &= -30 \text{ cm} \end{aligned}$$

As v is negative a real image is formed 30 cm from the mirror on the same side as the object.

Magnification,

$$m = -\frac{v}{u} = -\frac{-30}{-10} = -3$$

The image is magnified, real and inverted.

2. Here object distance,

$$u = -5 \text{ cm}$$

By mirror formula,

$$\begin{aligned} \frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = \frac{1}{-7.5} - \frac{1}{-5} \\ &= \frac{-5 + 7.5}{7.5 \times 5} = \frac{1}{15} \\ v &= +15 \text{ cm} \end{aligned}$$

As v is positive a virtual image is formed 15 cm behind the mirror.

Magnification,

$$m = -\frac{v}{u} = -\frac{15}{-5} = 3$$

The image is magnified, virtual and erect.

- 187.** The refractive index of glass is 1.5 and that of water is 1.3. If the speed of light in water is $2.25 \times 10^8 \text{ ms}^{-1}$, what is the speed of light in glass?

Ans :

As we know that,

$${}^a\mu_g = \frac{c}{v_g} = 1.5$$

and

$${}^a\mu_w = \frac{c}{v_w} = 1.3$$

$$\text{Hence, } \frac{c}{v_w} \times \frac{v_g}{c} = \frac{1.3}{1.5}$$

or

$$\begin{aligned} v_g &= \frac{1.3}{1.5} \times v_w \\ &= \frac{1.3}{1.5} \times 2.25 \times 10^8 \\ &= 1.95 \times 10^8 \text{ ms}^{-1} \end{aligned}$$

Delhi 2012

- 188.** Light from a point source in air falls on a convex spherical glass surface ($\mu = 1.5$, radius of curvature = 20 cm). The distance of light source from the glass surface is 100 cm. At what position is the image formed?

Ans :

Foreign 2010, OD 2007

Given,

$$\mu_1 = 1$$

$$\mu_2 = 1.5$$

$$u = -100 \text{ cm}$$

$$R = +20 \text{ cm}$$

[R is positive for a convex refracting surface]

$$\text{As, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\text{Hence, } \frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20} = \frac{1}{40}$$

$$\text{or } \frac{3}{2v} = \frac{1}{40} - \frac{1}{100} = \frac{5 - 2}{200} = \frac{3}{200}$$

Hence,

$$v = +100 \text{ cm}$$

Thus the image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

- 189.** The velocity of light in glass is $2 \times 10^8 \text{ ms}^{-1}$ and that in air is $3 \times 10^8 \text{ ms}^{-1}$. By how much would an ink dot appear to be raised, when covered by a glass plate 6.0 cm thick?

Ans :

OD 2011

Given,

$$v = 2 \times 10^8 \text{ ms}^{-1}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\text{Refractive index of glass, } \mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

$$\text{Real depth} = 6.0 \text{ cm}$$

$$\begin{aligned} \text{Hence, Apparent depth} &= \frac{\text{Real depth}}{\mu} \\ &= \frac{6.0}{1.5} = 4.0 \text{ cm} \end{aligned}$$

Distance through which the ink dot appears to be raised

$$= 6.0 - 4.0 = 2.0 \text{ cm}$$

- 190.** Find the value of critical angle for a material of refractive index $\sqrt{3}$.

Ans :

Foreign 2014

Given,

$$\mu = \sqrt{3}$$

$$\begin{aligned} \sin i_c &= \frac{1}{\mu} = \frac{1}{\sqrt{3}} \\ &= \frac{\sqrt{3}}{3} = 0.5773 \end{aligned}$$

$$\text{Hence, Critical angle, } i_c \simeq 35.3^\circ$$

Magnification, $m = \pm 3$

$$m = \frac{v}{u}$$

$$v = mu = \pm 4u$$

For real image, $v = -3u$

For virtual image, $v = 3u$

1. For virtual image,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ (lens formula)}$$

$$\frac{1}{15} = \frac{1}{3u} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1-3}{3u}$$

$$\frac{1}{15} = \frac{-2}{3u}$$

$$u = \frac{-2}{3} \times 15 = -10 \text{ cm}$$

2. For real image,

$$v = -3u$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{1}{-3u} - \frac{1}{u}$$

$$\frac{1}{15} = \frac{-1-3}{3u}$$

$$\frac{1}{15} = \frac{-4}{3u}$$

$$u = \frac{-4}{3} \times 15 = -20 \text{ cm}$$

- 200.** A compound microscope has $f_0 = 2 \text{ m}$ and $f_e = 5 \text{ cm}$, the distance between the two lenses $d = 15 \text{ cm}$. Locate the position of placement of object in front of objective so that final image formed by the compound microscope is (i) at least distance of distinct vision $D = 25 \text{ cm}$ (ii) at infinity. Also evaluate the magnification in each case.

Ans :

Foreign 2011, Delhi 2016

Focal length of objective,

$$f_0 = 2 \text{ m} = 200 \text{ cm}$$

Focal length of eyepiece,

$$f_e = 5 \text{ cm}$$

The distance between the lens,

$$d = 15 \text{ cm}$$

(i) at least distance of distinct vision $D = 25 \text{ cm}$

When the final image formed at least distance of distant vision.

$$m = \frac{v_0}{u_0} \left(1 + \frac{D}{f_e} \right)$$

Image distance for objective,

$v_0 =$ Tube length or distance between lens

$$v_0 = 15 - 5 = 10 \text{ cm}$$

Using Cartesian sign convention for the objective

$$v_0 = 10 \text{ cm}$$

$$f_0 = 2 \text{ cm}$$

$$u_0 = ?$$

$$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{10} - \frac{1}{2}$$

$$= \frac{1-5}{10} = \frac{-4}{10}$$

$$u_0 = \frac{-10}{4} = \frac{-5}{2} \text{ cm}$$

$$\text{Now, } m = \frac{10}{-5} \times 2 \left(1 + \frac{25}{5} \right)$$

$$m = -4(1+5)$$

$$m = -24 \text{ or } 24 \text{ (Numerically)}$$

(ii) ∞ .

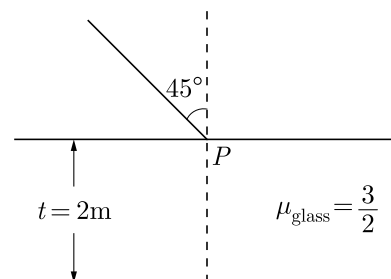
When the final image is formed at infinity. The magnifying power of the microscope will be,

$$m = \frac{v_0}{u_0} \times \frac{D}{f_e}$$

$$= \frac{10}{-5} \times 2 \times \frac{25}{5}$$

$$= -20 \text{ or } 20 \text{ (Numerically)}$$

- 201.** The speed of light in vacuum is $c = 3 \times 10^8 \text{ ms}^{-1}$. In the given figure $\theta = 45^\circ$ and $\mu_{\text{glass}} = \frac{3}{2}$. Light enters the air-glass interface at P . How much time will light ray take to cross the glass slab?



Ans :

SQP 2009

Refractive index of glass,

$$\mu_{\text{glass}} = \frac{3}{2} = 1.5$$

Velocity of light, $c = 3 \times 10^8 \text{ m/sec}$

Velocity of light in glass,

$$c_g = \frac{\text{Velocity of light in air}}{\text{Refractive index of glass}} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

- 219.** A ray of light suffers minimum deviation when incident on a prism of refractive index $\sqrt{2}$. What is the angle of incidence?

Ans :

Foreign 2008

Refractive index of prism,

$$\mu = \sqrt{2}$$

Refractive index of prism,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\begin{aligned}\sin\left(\frac{A+\delta_m}{2}\right) &= \mu \sin\left(\frac{A}{2}\right) = \sqrt{2} \sin\left(\frac{60^\circ}{2}\right) \\ &= \sqrt{2} \sin 30^\circ = \sqrt{2} \times \frac{1}{2} \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\frac{A+\delta_m}{2} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Angle of incidence,

$$i = \frac{A+\delta_m}{2} = 45^\circ$$

- 220.** For a prism of refractive index 1.732, the angle of minimum deviation is equal to the angle of prism. What is the angle of the prism?

Ans :

Delhi 2012

Refractive index of prism,

$$\mu = 1.732 = \sqrt{3}$$

and angle of minimum deviation,

$$\delta_m = A$$

(where, A = Angle of prism)

Refractive index of the prism,

$$\begin{aligned}\mu &= \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \\ &= \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \\ &= \frac{\sin A}{\sin\left(\frac{A}{2}\right)} \\ &= \frac{2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \\ &= 2 \cos\left(\frac{A}{2}\right) \\ \sqrt{3} &= 2 \cos\left(\frac{A}{2}\right) \\ \frac{\sqrt{3}}{2} &= \cos\left(\frac{A}{2}\right) \\ \frac{A}{2} &= 30^\circ \\ A &= 60^\circ\end{aligned}$$

- 221.** The refractive index of a flint glass for red and blue colours are 1.648 and 1.668 respectively. What is the dispersive power of the lens material ?

Ans :

OD 2014, Comp 2015

Refractive index of flint glass for red colour,

$$\mu_R = 1.648$$

and refractive index of flint glass for blue colour,

$$\mu_B = 1.668$$

Mean refractive index of two colours,

$$\begin{aligned}\mu &= \frac{\mu_B + \mu_R}{2} \\ &= \frac{1.668 + 1.648}{2} = 1.658\end{aligned}$$

Therefore dispersive power of the lens material,

$$\begin{aligned}\omega &= \frac{\mu_B - \mu_R}{\mu - 1} \\ &= \frac{1.668 - 1.648}{1.658 - 1} \\ &= \frac{0.02}{0.658} = 0.03\end{aligned}$$

- 222.** A small-angled prism of refractive index 1.7 gives a deviation of 4.9° . What is the angle of prism?

Ans :

OD 2007

Refractive index of prism,

$$\mu = 1.7$$

and angle of deviation,

$$\delta = 4.9^\circ$$

Angle of deviation produced by the small-angled prism,

$$\delta = (\mu - 1) A$$

Angle of prism, $A = \frac{\delta}{\mu - 1}$

$$= \frac{4.9^\circ}{1.7 - 1} = \frac{4.9^\circ}{0.7} = 7^\circ$$

- 223.** If a thin prism of 6° gives a deviation of 3° , what is the refractive index of the prism material?

Ans :

Delhi 2016

Angle of prism, $A = 6^\circ$

Angle of deviation, $\delta = 3^\circ$

Angle of deviation produced by small-angled prism,

$$\delta = (\mu - 1) A$$

$$\mu - 1 = \frac{\delta}{A} = \frac{3^\circ}{6^\circ} = 0.5$$

$$\mu = 0.5 + 1 = 1.5$$

where, μ = refractive index of the prism material

and $R_2 = -20$ cm

$$\frac{1}{f} = (1.5 - 1) \left[\frac{1}{20} - \left(\frac{1}{-20} \right) \right]$$

$$\frac{1}{f} = (1.5 - 1) \left[\frac{1}{20} + \frac{1}{20} \right]$$

$$\frac{1}{f} = 0.5 \times \frac{1}{10}$$

$$f = \frac{10}{0.5} = 20 \text{ cm}$$

Now, Power of lens

$$P = \frac{1}{f(\text{in meter})}$$

$$= \frac{1}{20} \times 100 = 5 \text{ D}$$

- 207.** Light is incident on a glass plate at an angle of 60° . If reflected and refracted rays are mutually perpendicular, What is the refractive index of the glass plate?

Ans :

Delhi 2007

Angle of incidence, $i = 60^\circ$

Angle of reflection,

$$r' = \text{Angle of incidence } i = 60^\circ$$

Since refracted rays and reflected rays are mutually perpendicular, therefore,

$$r' + r = 90^\circ$$

$$i + r = 90^\circ$$

$$r = 90^\circ - i = 90^\circ - 60^\circ = 30^\circ$$

Refractive index of glass plate,

$$\mu = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 60^\circ}{\sin 30^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

- 208.** A double convex lens is manufactured from a glass of refractive index 1.55, with both faces of same radius of curvature. If focal length of lens is to be 20 cm. What is the radius of the curvature of the lens?

Ans :

OD 2010

Refractive index of lens,

$$\mu = 1.55$$

Radius of curvature of first face of convex lens,

$$R_1 = R$$

Radius of curvature of second face of convex lens,

$$R_2 = -R$$

Focal length, $f = 20$ cm

Relation for the focal length of a double convex lens f is,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{20} = (1.55 - 1) \left(\frac{1}{R} - \frac{1}{(-R)} \right)$$

$$= 0.55 \times \frac{2}{R} = \frac{1.1}{R}$$

Radius of curvature,

$$R = 20 \times 1.1$$

$$= 22 \text{ cm}$$

- 209.** A planoconvex lens is made of refractive index 1.6. The radius of curvature of the curved surface is 60 cm. What is the focal length of the lens?

Ans :

Foreign 2015, OD 2001

Refractive index, $\mu = 1.6$

Radius of curvature of curved surface,

$$R_1 = 60 \text{ cm}$$

and radius of curvature of plane side,

$$R_2 = \infty \quad (\text{due to plane surface})$$

Relation for the focal length of a planoconvex lens f is,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= (1.6 - 1) \left(\frac{1}{60} - \frac{1}{\infty} \right)$$

$$= \frac{0.6}{60} = 0.01$$

$$f = \frac{1}{0.01} = 100 \text{ cm}$$

- 210.** In a planoconvex lens the radius of curvature of the convex lens is 10 cm. If the plane side is polished, What is the focal length of the lens? (Refractive index $\mu = 1.5$)

Ans :

Delhi 2013

Radius of curvature of curved side,

$$R = 10 \text{ cm}$$

Radius of curvature of plane side,

$$R_2 = \infty \quad (\text{due to plane surface})$$

Refractive index, $\mu = 1.5$

Relation for the focal length of a planoconvex lens f is,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= (1.5 - 1) \left(\frac{1}{10} - \frac{1}{\infty} \right) = \frac{0.5}{10}$$

$$f = \frac{10}{0.5} = 20 \text{ cm}$$

Time, $t = \frac{t}{c_g}$

$$= \frac{2}{2 \times 10^8} = 10^{-8} \text{ sec}$$

- 202.** Green light of wavelength $5460 \text{ \AA}$ is incident on an air and glass interface. If refractive index of glass is 1.5, What is the wavelength of light in glass?

Ans :

Delhi 2011

Actual wavelength of light,

$$\lambda = 5460 \text{ \AA}$$

and refractive index of glass,

$$\mu = 1.5$$

Wavelength of light in glass,

$$\lambda = \frac{\lambda}{\mu} = \frac{5460}{1.5} = 3640 \text{ \AA}$$

- 203.** A magnifying glass is to be used at the fixed object distance of 1 inch. If it is to produce an erect image 5 times magnified. What is the focal length of magnifying glass ?

Ans :

OD 2018

Given, $u = 1 \text{ inch}$

Magnification, $m = 5$

Magnification, $m = \frac{v}{u}$

where, $u = \text{Object distance}$

$v = \text{Image distance}$

$$5 = \frac{v}{u}$$

$$v = 5u \quad (\text{Here, } u = 1 \text{ inch})$$

$$v = 5 \times 1 = 5 \text{ inch}$$

Now, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Using sign convention.

Here, $u = -1 \text{ inch}$

$$v = -5 \text{ inch}$$

Hence, $\frac{1}{f} = \frac{1}{-5} - \left(\frac{1}{-1}\right)$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

$$\frac{1}{f} = 0.8 \text{ inch}$$

$$f = \frac{1}{0.8} = 1.25''$$

- 204.** A monochromatic light of wavelength $6000 \text{ \AA}$ in vacuum enters a medium of refractive index 1.5. Calculate the wavelength of light in this medium?

Ans :

OD 2016, Comp 2011

Wavelength of monochromatic light,

$$\lambda_0 = 6000 \text{ \AA}$$

Refractive index of medium,

$$\mu = 1.5$$

As we know $n \propto \frac{1}{\lambda}$ that,

$$n = \text{Refractive index}$$

and $\lambda = \text{Wavelength}$

So, $\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$

For Air, $n_1 = 1$

$$\lambda_1 = 6000 \text{ \AA}$$

For Glass, $n_2 = 1.5$

$$\lambda_2 = ?$$

$$\frac{1}{1.5} = \frac{\lambda_2}{6000}$$

$$\lambda_2 = 4000 \text{ \AA}$$

- 205.** In a pond of water, a flame is held 2 m above the surface of water. A fish is at depth of 4 m from water surface. Refractive index of water is $\frac{4}{3}$. What is the apparent height of the flame from the eyes of fish ?

Ans :

Comp 2008

Given, Height of flame above water surface,

$$h = 2 \text{ m}$$

Depth of fish, $d = 4 \text{ m}$

and refractive index of water,

$$\mu = \frac{4}{3}$$

Apparent height of flame above water surface,

$$h' = \mu \times h = \frac{4}{3} \times 2 = \frac{8}{3} \text{ m}$$

Therefore, apparent height of the flame from the eyes of fish

$$= d + h' = 4 + \frac{8}{3} = \frac{20}{3} \text{ m}$$

- 206.** A biconvex lens ($\mu = 1.5$) has equal curvature each of 20 cm. What is the power of the lens?

Ans :

Delhi 2012

Refractive index of lens,

$$\mu = 1.5$$

Relation for the focal length of a double convex lens f is,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where, $R_1 = 20 \text{ cm}$

Ans :**Delhi 2012**

Radius of curvature of convex air lens,

$$R_1 = 10 \text{ cm}$$

Radius of curvature of concave air lens,

$$R_2 = -10 \text{ cm}$$

(Minus sign due to concave)

and refractive index of glass to air,

$${}^a\mu_g = 1.5$$

$${}^g\mu_a = \frac{1}{1.5}$$

Relation for the focal length of the lens f is,

$$\begin{aligned}\frac{1}{f} &= ({}^g\mu_a - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= \left(\frac{1}{1.5} - 1 \right) \left(\frac{1}{10} - \frac{1}{-10} \right) \\ &= -\frac{0.5}{1.5} \times \frac{1}{5} = -\frac{1}{15}\end{aligned}$$

$$f = -15 \text{ cm}$$

Minus sign of focal length shows the air lens is a concave type.

Therefore the focal length of air lens is 15 cm and it is a concave lens.

- 216.** A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. What is the power of the combination?

Ans :**Delhi 2017**

Focal length of convex lens,

$$f_1 = 40 \text{ cm} = 0.4 \text{ m}$$

and focal length of concave lens,

$$f_2 = -25 \text{ cm} = -0.25 \text{ m}$$

Minus sign due to concave lens.

Power of the combination when the lenses are in contact,

$$\begin{aligned}P &= P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2} \\ &= \frac{1}{0.4} + \frac{1}{-0.25} = -1.5 \text{ D}\end{aligned}$$

- 217.** The radius of curvature of convex surface of a thin planoconvex lens is 15 cm and refractive index of its material is 1.6. What is the power of lens?

Ans :**OD 2015**

Radius of curvature the convex surface,

$$R_1 = 15 \text{ cm}$$

Radius of plane surface,

$$R_2 = \infty \quad (\text{due to plane surface})$$

and refractive index of material,

$$\mu = 1.6$$

Relation for the focal length of a thin planoconvex lens f is,

$$\begin{aligned}\frac{1}{f} &= (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= (1.6 - 1) \left(\frac{1}{15} - \frac{1}{\infty} \right) \\ &= \frac{0.6}{15} = \frac{1}{25} \\ f &= 25 \text{ cm} = 0.25 \text{ m}\end{aligned}$$

Therefore power of lens,

$$P = \frac{1}{f} = \frac{1}{0.25} = 4 \text{ D}$$

- 218.** A convex lens of glass ($\mu = 1.5$) has focal length of 20 cm. The lens is immersed in water of refractive index 1.33. What is the change in the power of convex lens?

Ans :**OD 2011, Comp 2005**

Refractive index of glass with respect to air,

$${}^a\mu_g = 1.5$$

Refractive index of water with respect to air,

$${}^a\mu_w = 1.33$$

and focal length of convex lens in air,

$$f_a = 20 \text{ cm} = 0.2 \text{ m}$$

Refractive index of glass with respect to water,

$${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.5}{1.33} = 1.128$$

Relation for the focal length of convex lens in air f_a is,

$$\begin{aligned}\frac{1}{f_a} &= ({}^a\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ \frac{1}{0.2} &= (1.5 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ \frac{1}{R_1} - \frac{1}{R_2} &= 10\end{aligned}$$

Similarly, relation for the focal length of convex lens when immersed in water f_w is,

$$\begin{aligned}\frac{1}{f_w} &= ({}^w\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= (1.128 - 1) \times 10 = 1.28 \\ f_w &= \frac{1}{1.28} = 0.78 \text{ m}\end{aligned}$$

Therefore power of convex lens in air,

$$P_a = \frac{1}{f_a} = \frac{1}{0.2} = 5 \text{ D}$$

and power of convex lens when immersed in water,

$$\begin{aligned}P_w &= \frac{1}{f_w} = \frac{1}{0.78} \\ &= 1.28 \text{ D}\end{aligned}$$

Therefore change in power of convex lens

$$\begin{aligned}&= P_a - P_w \\ &= 5 \text{ D} - 1.28 \text{ D} = 3.72 \text{ D}\end{aligned}$$

Therefore, focal length of lens when its plane surface is polished

$$= \frac{f}{2} = \frac{20}{2} = 10 \text{ cm}$$

- 211.** The projection lens of a projector has focal length 5 cm. It is desired to get an image with a magnification 30. What is the distance of the screen from the lens?

Ans :

OD 2011

Focal length of lens, $f = 5 \text{ cm}$

Magnification, $m = 30$

Magnification $m = -\frac{v}{u}$

$$30 = -\frac{v}{u}$$

$$u = -\frac{v}{30}$$

Relation for the distance of screen from the lens v is,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{v} + \frac{30}{v} = \frac{31}{v}$$

$$\frac{1}{5} = \frac{31}{v}$$

$$v = 5 \times 31$$

$$= 155 \text{ cm} = 1.55 \text{ m}$$

- 212.** A convex lens of focal length 0.1 m and made of glass ($\mu = 1.5$) is immersed in water ($\mu = 1.33$). What is the change in its focal length?

Ans :

SGP 2008

Focal length of lens in air,

$$f_a = 0.1 \text{ m}$$

Refractive index of glass,

$$\mu_g = 1.5$$

and refractive index of water,

$$\mu_w = 1.33$$

Focal length of convex lens when immersed in water,

$$\begin{aligned} f_w &= \left[\frac{\mu_g - 1}{\left(\frac{\mu_g}{\mu_w} - 1\right)} \right] \times f_a \\ &= \left[\frac{1.5 - 1}{\left(\frac{1.5}{1.33} - 1\right)} \right] \times 0.1 \\ &= \left[\frac{0.5}{1.13 - 1} \right] \times 0.1 \\ &= \frac{0.5}{0.13} \times 0.1 = 0.4 \text{ m} \end{aligned}$$

Therefore, change in focal length of the lens

$$= f_w - f_a = 0.4 - 0.1$$

$$= 0.3 \text{ m}$$

- 213.** A thin symmetrical convex lens of refractive index $\frac{3}{2}$ and radius of curvature 0.3 m is immersed in water of

refractive index $\frac{4}{3}$. What is the focal length in water?

Ans :

Comp 2016, OD 2008

Refractive index of glass,

$$\mu_g = \frac{3}{2}$$

and refractive index of water,

$$\mu_w = \frac{4}{3}$$

Since the lens is symmetrical, therefore radius of curvature of two surfaces of convex lens are,

$$R_1 = 0.3 \text{ m}$$

and

$$R_2 = -0.3 \text{ m}$$

Refractive index of glass with respect to water,

$${}^w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8}$$

Relation for focal length of convex lens when immersed in water f_w is,

$$\begin{aligned} \frac{1}{f_w} &= ({}^w\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ &= \left(\frac{9}{8} - 1 \right) \left(\frac{1}{0.3} + \frac{1}{0.3} \right) \\ &= \frac{1}{8} \times \frac{2}{0.3} = \frac{20}{24} = \frac{10}{12} \\ f_w &= \frac{12}{10} = 1.2 \text{ m} \end{aligned}$$

- 214.** What is the power of combination of two lenses of powers +1.5 D and -2.5 D ?

Ans :

Delhi 2014

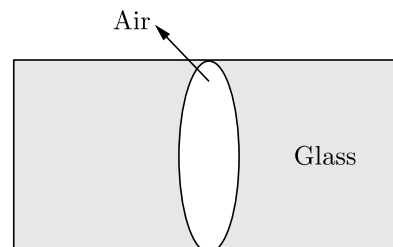
Power of first lens, $P_1 = +1.5 \text{ D}$

Power of second lens, $P_2 = -2.5 \text{ D}$

Power of the combination of two lenses when placed in contact with each other,

$$\begin{aligned} P &= P_1 + P_2 \\ &= +1.5 + (-2.5) \\ &= -1 \text{ D} \end{aligned}$$

- 215.** In the figure, an air lens of radii of curvature ($R_1 = R_2 = 10 \text{ cm}$) is cut in a cylinder of glass ($\mu_g = 1.5$). What is the focal length and the nature of lens?



- 224.** The refractive index of material of prism for blue and red colours are 1.532 and 1.514 respectively. If angle of prism is 8° , what is the angular dispersion produced by the prism?

Ans :

SQP 2009

Refractive index of lens for blue colour,

$$\mu_B = 1.532$$

Refractive index of lens for red colour,

$$\mu_R = 1.514$$

and angle of prism,

$$A = 8^\circ$$

Angular dispersion produced by small-angled prism,

$$\begin{aligned} d\delta &= (\alpha_B - \alpha_R)A \\ &= (1.532 - 1.514) \times 8^\circ \\ &= 0.018 \times 8^\circ = 0.144^\circ \end{aligned}$$

CASE BASED QUESTIONS

- 225.** Ankit goes to visit a museum. A special mirror is lying there. When he stands in front of the mirror he finds his image having a very small head, a fat body and legs of normal size. He becomes frightened and comes to his friend Ramesh to know the facts. Being a science student Ramesh explains the reason behind it.

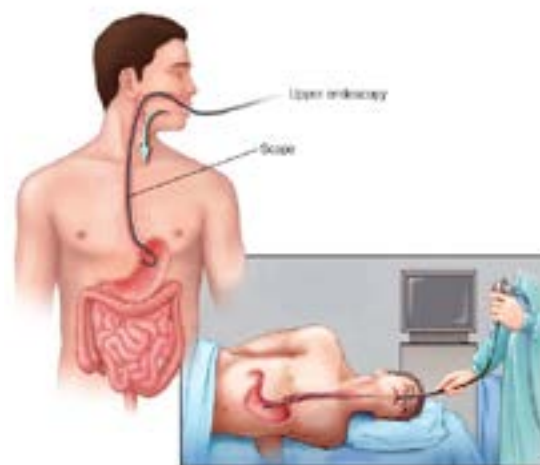


- Name the kinds of mirror.
- How do you explain such a thing properly?

Ans :

- Plane, concave and convex mirrors.
 - The special mirror is the combination of three mirrors which show their characteristics properly.
 - convex mirror is the upper portion of the mirror.
 - convex mirror is the middle portion of the mirror.
 - Plane mirror is the lower portion of then mirror.
- 226.** One day Chetan's mother developed a severe Stomach ache all of a sudden. She was rushed to the doctor who suggested for an immediate endoscopy test and gave an estimate of expenditure for the same. Chetan immediate contacted his class teacher and shared the

information with her. The class teacher arranged for the money and rushed to the hospital. On tantalising that Chetan belonged to a below average income group family, even the doctor offered concession for the test. The test was conducted successfully.



- Which principal in optics is used in endoscopy? Explain this principal briefly.
- Briefly explain the values reflected in the action asked by the teacher.

Ans :

- Total internal reflection. If a light ray enters at one end of an optic fibre coated with a material of low refractive index, it is refracted and strikes the walls at an angle greater than the critical angle.
- The teacher knows that Chetan belongs to a below Average income group family, So he/she immediately Arranged the money required to be paid as test fee. His/her caring and helping attitude towards others resulted in timely help to Chetan's mother.

Such helping attitude on the part of the person living in the society makes it a better society to live in.

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- 227.** A glass window has been broken into tiny particles of glass in a robbery case. Some of there tiny Particles are found at the scene of crime and some in the robbers clothing If the police can both particles found from the places are similar. They have a strong case. Being a responsible person Deepak helped the police to prove such case.

spectacles for his uncle from his pocket money. By using spectacles he should now read with great ease, For this, he expressed his gratitude to his nephew. Based on the above paragraph.



- (i) Why does the least distance of distinct vision increase with age ?
- (ii) What type of lens is required to correct this defect ?

Ans :

- (i) The least distance of distinct vision increases with age because of the increase in the minimum possible focal length of the eye lens or contraction of the eye ball.
 - (ii) This defect can be corrected by placing a convex lens in front of the eye.
- 233.** Ravi goes to visit a museum. A special mirror is lying there. When he stands in front of the mirror he finds his image having a very small head, a fat body and legs of normal size. He becomes frightened and comes to his friend Shiva to know the facts. Being a science student Shiva explains the reason behind it.
- (i) Name the kinds of mirror.
 - (ii) How do you explain such a thing properly ?

Ans :

- (i) Plane, concave and convex mirrors.
 - (ii) The special mirror is the combination of three mirrors which show their characteristics property.
 - (a) Convex mirror is the upper portion of the mirror.
 - (b) Concave mirror is the middle portion of the mirror.
 - (c) Plane mirror is the lower portion of the mirror.
- 234.** School arrange an educational trip to Kavalur in Tamil Nadu where an observatory having India largest telescope. On hearing this, the whole class got excited and left for their way to kavalur. In the observatory, the physics teacher was explaining about the type of telescope, with diameter of objective (2.34 m) and several other details. Everyone was ready to look through the telescope.



- (i) What type of telescope is the teacher referring to ?
- (ii) Mention any two advantages of this telescope.

Ans :

- (i) Reflecting telescope.
- (ii) No chromatic aberration, mirrors are relatively lighter and cheaper compared to the lens.

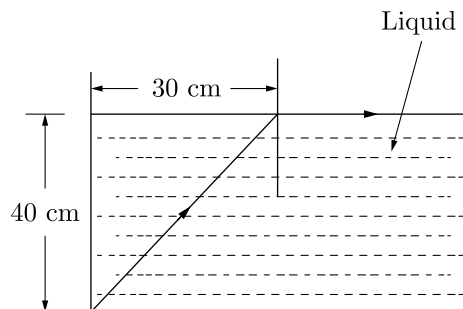
- 235.** Nisha was working on a physics project. In this she had to construct a good astronomical telescope, while three lenses of power 0.5 D, 4 D, and 10 D are available for her project. She was not sure as to which lenses would she use for constructing such telescope. So she consulted her friend Rani for her problem. Rani explained details and significance of a telescope, which helped Nisha to choose the lenses for her project.

- (i) What do you understand by normal adjustment of a telescope ?
- (ii) Which lenses will be used as objective and which one as eyepiece from given above lenses ?

Ans :

- (i) In normal adjustment of telescope, the final image is formed at infinity.
- (ii) Out of these three lenses, lens of power 0.5 D will be used for objective while lens of power 10 D as eye piece for construction of such telescope.

- 236.** Mr. RP Kothari, a retired professor of physics was walking with his grandson. It was last week of December and so it was dark around 5.30 pm. The street lights were on and the yellow light flooded the area around. The boy asked professor why yellow lights were used when white light were brighter. The professor answered that during foggy days the tiny droplets act as prisms splitting the white light into its constituent colours and thus reducing the clarity.
- (i) What phenomenon was the professor referring to ? Why does it happen ?
 - (ii) Give one application of prism.

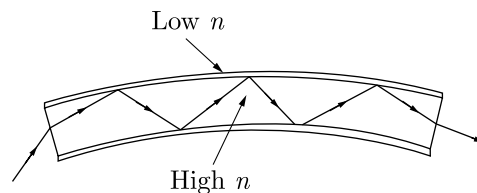
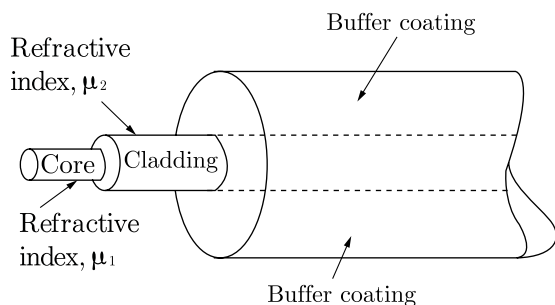


- What is refractive index of a medium? (in terms of speed of light)
- In the above diagram, calculate the speed of light in the liquid of unknown refractive index?
- What is refractive index of a medium (in terms of real and apparent depth)?
- What is the relation between refractive index and critical angle for a medium?

Ans :

- Speed of light in vacuum/speed of light in medium.
- 1.8×10^8 m/s.
- Real depth/Apparent depth.
- $n = 1/\sin i_c$

230. Now-a-days optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres too make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end. Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal. Optical fibres are lubricated such that light reflected at one side of inner surface strikes the other at an angle larger than the critical angle. Even if the fibre is bent, light can easily travel along its length. Thus, an optical fibre can be used to act as an optical pipe.



- What is the condition for total internal reflection to occur?
- Which signal is transmitted by optical fibres?
- What is the internal reflection of light??

Ans :

- Angle of incidence must be greater than the critical angle.
- Optical Signal.
- The phenomenon which occurs when the light rays travel from a more optically denser medium to a less optically denser medium.

231. Ravi is a student of mechanical engineering studying in one of the engineering colleges. The other day he saw an old man who suddenly collapsed as he walked out of the house in his neighbourhood. Ravi rushed towards him immediately made a call to the nearby hospital, asked for the ambulance and took him to the emergency ward of the hospital. On getting the medical aid, the old man soon got recovered. he did not forget to thank Ravi for the timely help he rendered. He was wondering that in his times to get the telephone connection, one had to wait for years whereas these days it takes no time to get the connection. Ravi told him it was all because of the technological progress/development due to which the simple phenomenon in physics could be easily used.

- To which phenomenon in physics was Ravi referring to, which made the land line links so easily accessible ?
- What are the essential conditions required to observe this phenomenon ?

Ans :

- Ravi was referring to the phenomenon of total Internal Reflection
- Conditions required for the occurrence of total internal reflection :
 - Angle of incidence $>$ Critical Angle
 - Light should travel from denser to rarer medium

232. Amit's uncle was finding great difficulty in reading a book placed at normal position. He was not going to the doctor because he could not afford the cost. When Amit came to know of it, he took his uncle to the doctor. After thoroughly checking his eyes, the doctor prescribed the proper lenses for him. Amit bought the



- (i) Which phenomenon is responsible for proving Such case?
- (ii) How do you prove such case?
- (iii) What do you mean by refraction of light?

Ans :

- (i) Refraction
 - (ii) Suspend the particles of glass in a special liquid and particles are viewed through a microscope. Change the temperature or liquid slowly. At a certain temperature particles of glass disappear. Then they probably came from the same broken pane of glass. In this case refractive index of liquid becomes equal to the refractive index of particles of glass.
 - (iii) Refraction of light is the change in the direction of light as it passes from one medium to another.
- 228.** While driving his car, Rahul suddenly heard loud sound coming from behind. He happened to look through his rear view mirror and saw that an ambulance is approaching. He recollect that mostly emergency vehicles have their names written in mirror script. He quickly give the way for the ambulance murmuring a prayer for fast recovery of patient who is inside the ambulance.



- (i) What type of mirror is as a rear-view mirror and why ?
- (ii) What values did Rahul exhibit ?

Ans :

- (i) Convex mirror, to get a wide view of traffic behind.
- (ii) Compassion, courtesy, concern for others, and knowledgeable.

- 229.** Total internal reflection (TIR) is the optical phenomenon in which waves arriving at the interface (boundary) from one medium to another (e.g., from water to air) are not refracted into the second ("external") medium, but completely reflected back into the first ("internal") medium. It occurs when the second medium has a higher wave speed (i.e., lower refractive index) than the first, and the waves are incident at a sufficiently oblique angle on the interface. For example, the water-to-air surface in a typical fish tank, when viewed obliquely from below, reflects the underwater scene like a mirror with no loss of brightness.

TIR occurs not only with electromagnetic waves such as light and microwaves, but also with other types of waves, including sound and water waves. If the waves are capable of forming a narrow beam, the reflection tends to be described in terms of "rays" rather than waves; in a medium whose properties are independent of direction, such as air, water or glass, the "rays" are perpendicular to the associated wave fronts.

Repeated total internal reflection of a 405nm laser beam between the front and back surfaces of a glass pane. The colour of the laser light itself is deep violet; but its wavelength is short enough to cause fluorescence in the glass, which re-radiates greenish light in all directions, rendering the zigzag beam visible.

Refraction is generally accompanied by partial reflection. When waves are refracted from a medium of lower propagation speed (higher refractive index) to a medium of higher speed e.g., from water to air the angle of refraction (between the outgoing ray and the surface normal) is greater than the angle of incidence (between the incoming ray and the normal). As the angle of incidence approaches a certain threshold, called the critical angle, the angle of refraction approaches 90° , at which the refracted ray becomes parallel to the boundary surface. As the angle of incidence increases beyond the critical angle, the conditions of refraction can no longer be satisfied, so there is no refracted ray, and the partial reflection becomes total. For visible light, the critical angle is about 49° for incidence from water to air, and about 42° for incidence from common glass to air.

Ans :

- (i) Dispersion. Because the speed of each colour is different when the colours enter the glass.
- (ii) Studying and analysing the spectrum of distant light sources.

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CHAPTER 10

Wave Optics

SUMMARY

1. HUYGEN'S PRINCIPLE

Huygen's principle is essentially a geometrical construction, which gives the shape of the wavefront at any time, allows us to determine the shape of the wavefront at a later time, According to Huygen's principle:

1. Every point on a wave-front behaves like a light source and emits secondary wavelets.
2. The secondary wavelets spread in all directions in space (vacuum) with the velocity of light.
3. The envelope of wave-front of secondary wavelets, after a given time, along forward direction gives the new position of wave-front.

2. COHERENT SOURCES

Coherent sources of light are the sources which emit light waves of same frequency, same wavelength and have a constant initial phase difference.

Coherent source of light can be obtained by deriving two sources from a primary (or initial) source by,

1. Division of wave-front
2. Division of amplitude

Two such sources of light, which do not emit light waves with a constant phase difference are called incoherent sources.

3. INTERFERENCE OF LIGHT

Interference is the phenomenon of superposition of two light waves of same frequency and constant phase difference travelling in same direction. The position of maximum intensity are called maxima while those of minimum intensity are called minima.

4. CONDITION OF MAXIMA AND MINIMA

If a_1 and a_2 are amplitudes of interfering waves and θ is the phase difference at a point under consideration, then resultant intensity at a point in the region of superposition

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

where,

$$I_1 = a_1^2 = \text{intensity of one wave}$$

$$I_2 = a_2^2 = \text{intensity of other wave}$$

Condition of Maxima

Phase difference, $\phi = 2n\pi$

Path difference, $\Delta = n\lambda$, n being integer

Maximum amplitude, $A_{\max} = a_1 + a_2$

Maximum intensity, $I_{\max} = A_{\max}^2 = (a_1 + a_2)^2$

$$= a_1^2 + a_2^2 + 2a_1 a_2$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Condition of Minima

Phase difference, $\phi = (2n - 1)\pi$

Path difference, $\Delta = (2n - 1)\frac{\lambda}{2}$

$$n = 1, 2, 3, \dots$$

Minimum amplitude, $A_{\min} = (a_1 - a_2)$

Minimum intensity, $I_{\min} = (a_1 - a_2)^2$

$$= a_1^2 + a_2^2 - 2a_1 a_2$$
$$= I_1 + I_2 - 2\sqrt{I_1 I_2}$$

5. YOUNG'S DOUBLE SLIT EXPERIMENT

1. Fringe width of bright and dark fringe,

$$\beta = \frac{D\lambda}{d}$$

where, λ = wavelength of light

D = distance between slit and screen

and d = distance between two slits

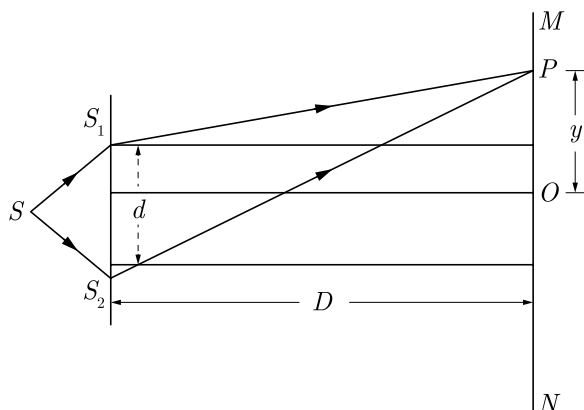
Angular fringe width,

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

2. Separation of n th order bright fringe from central fringe,

$$Y_n = \frac{Dn\lambda}{d}$$

where, $n = 1, 2, 3, \dots$



3. Separation of n th order dark fringe from central fringe,

$$Y_n = (2n - 1) \frac{D\lambda}{2d}$$

where, $n = 1, 2, 3, \dots$

4. Angular position of n th order,

$$\text{Bright fringe} = \frac{Y_n}{D} = \frac{n\lambda}{d}$$

$$\text{Dark fringe} = \frac{Y_n}{D} = (2n - 1) \frac{\lambda}{d}$$

where, $n = 1, 2, 3, \dots$

6. INTENSITY OF LIGHT

Intensity of light I is proportional to the width d of slit and ratio of slit-width a .

$$\frac{d_1}{d_2} = \frac{I_2}{I_1} = \frac{a_1^2}{a_2^2}$$

Ratio of maximum and minimum intensity of light,

$$\begin{aligned} \frac{I_{\max}}{I_{\min}} &= \left(\frac{a_1 + a_2}{a_1 - a_2} \right)^2 \\ &= \left(\frac{r + 1}{r - 1} \right)^2 \end{aligned}$$

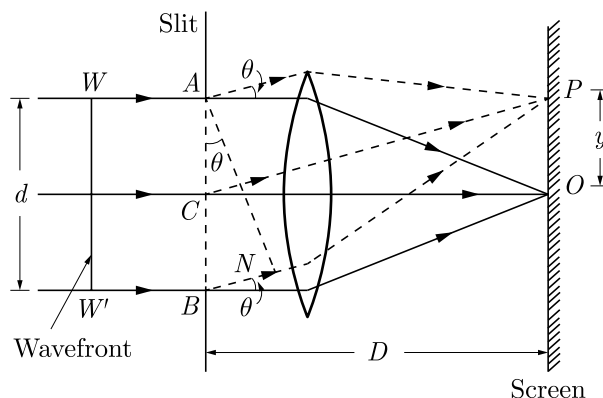
where, $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$

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7. DIFFRACTION DUE TO A SINGLE SLIT

A parallel beam of light with a plane wave-front WW' is made to fall on a single slit AB . Width of the slit is of the order of wavelength of light, therefore, diffraction occurs on passing through the slit.

As shown in the diagram below,



1. n th order secondary minima is obtained when,

$$d \sin \theta = n\lambda$$

where, $r = 1, 2, 3, \dots$

2. n th order secondary maximum is obtained when,

$$d \sin \theta = (2n + 1) \frac{\lambda}{2}$$

where, $n = 0, 1, 2, 3, \dots$

3. Angular separation for n th minima,

$$\theta_n = \frac{n\lambda}{d}$$

where, $n = 0, 1, 2, 3, \dots$

4. Linear separation of n th secondary minima,

$$Y_n = \frac{Dn\lambda}{d}$$

5. Angular position of n th secondary maxima,

$$\theta_n = (2n + 1) \frac{\lambda}{2d}$$

6. Angular width of central maxima $= \frac{2\lambda}{d}$.

7. Linear width of central maxima $= \frac{2D\lambda}{d}$.

8. Angular width of secondary maxima or minima $= \frac{\lambda}{d}$.

9. Linear width of secondary maxima or minima $= \frac{D\lambda}{d}$.

8. MALUS LAW

It states that if completely plane polarised light is passed through an analyser, the intensity of light transmitted $\propto \cos^2 \theta$, where θ is angle between planes of transmission of polariser and analyser i.e.,

$$I = I_0 \cos^2 \theta \quad (\text{Malus Law})$$

If incident light is unpolarised, then

$$I = \frac{I_0}{2}$$

Since $(\cos^2 \theta)_{\text{average}}$ for all directions $= \frac{1}{2}$.

This rule is called cosine squared rule.

where, I_0 is intensity of plane polarised light.

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OBJECTIVE QUESTIONS

- Transverse nature is shown by
 - Interference
 - Reflection
 - Polarisation
 - Dispersion

Ans :

OD 2017, SQP 2015

The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called polarisation of waves. Only transverse waves show the property of polarisation.

Thus (c) is correct option.

- The colour of a thin film is due to
 - scattering
 - interference
 - dispersion
 - diffraction

Ans :

SQP 2015

The colour band in thin films are a result of wavelength dependent interference caused by the close spacing of the surfaces of the film.

Thus (b) is correct option.

- Brewster's law is
 - $\mu = \sin i_p$
 - $\mu = \cos i_p$
 - $\mu = \tan i_p$
 - $\mu = \tan^2 i_p$

Ans :

Delhi 2015

According to the Brewster's law, when unpolarised light is incident at polarising angle, i_p on a interface separating air from a medium of refractive index μ , then the reflected light is fully polarised.

$$\mu = \tan i_p$$

Thus (c) is correct option.

- In the Young's double slit experiment, the central bright fringe can be identified
 - as it is wider than bright fringes
 - as it is narrower than other bright fringes
 - by using white light instead of monochromatic light
 - as it has a greater intensity of light than that of other bright fringes

Ans :

Foreign 2017

In Young's double slit experiment, the central bright fringe can be identified as it has a greater intensity of light than that of the other bright fringes.

And width of central bright fringe is equal to other bright and dark fringes.

Thus (d) is correct option.

5. Huygen's wave theory of light can not explain

- (a) diffraction (b) interference
(c) polarisation (d) photoelectric effect

Ans :

SQP 2016

Huygen's wave theory of light can explain reflection, refraction, interference, polarisation and diffraction, while quantum theory can explain the photoelectric effect.

Thus (d) is correct option.

6. Which nature of the wavefront is associated with a parallel beam of light?

- (a) plane (b) spherical
(c) cylindrical (d) all of these

Ans :

OD 2011

We know from the Huygen's wave theory that the locus of all points oscillating in the same phase in a parallel beam of light forms plane wavefront.

Thus (a) is correct option.

7. The inverse square law of intensity (illuminance) is valid for

- (a) plane wavefront (b) conical wavefront
(c) spherical wavefront (d) cylindrical wavefront

Ans :

Foreign 2009, Comp 2014

Inverse square law of intensity (illuminance) is valid for a point source, which produces spherical wavefront.

Thus (c) is correct option.

8. In Huygen's wave theory, the locus of all points oscillating in the same phase is called a

- (a) ray (b) vibrator
(c) wavefront (d) half period zone

Ans :

OD 2014

We know from the Huygen's wave theory that the locus of all points oscillating in the same phase is called a wavefront.

In other words, a wavefront is defined as a surface of constant phase.

Thus (c) is correct option.

9. Bright colours exhibited by spider's web exposed to sun light are due to

- (a) diffraction (b) polarisation
(c) interference (d) resolution

Ans :

SQP 2011

The phenomenon of non-uniform distribution of energy in a medium, due to superposition of light waves is called interference of light.

And the bright colours exhibited by the spider's web exposed to sun light is due to this phenomenon.

Thus (c) is correct option.

10. When exposed to sunlight, thin films of oil on water often exhibit brilliant colours due to the phenomenon of

- (a) interference (b) diffraction
(c) dispersion (d) polarisation

Ans :

Foreign 2007

When a thin films of oil on water is exposed to sunlight, then light waves, reflected from the upper and lower surfaces of the thin film of oil interfere with each other.

And the brilliant colours due to the constructive and destructive interference, depends upon wavelength the light.

As a result of this, thin films of oil on water often exhibit brilliant colours due to the phenomenon of interference.

Thus (a) is correct option.

11. In Young's double slit experiment, if one slit is covered with red filter and the other slit is covered by green filter, then their interference pattern will be

- (a) red (b) green
(c) yellow (d) invisible

Ans :

OD 2008

Red and green colours are complementary to each other. Therefore if one slit is covered with red filter and another slit is covered by green filter, then their interference pattern will be invisible.

Thus (d) is correct option.

12. In Young's double slit experiment, interference was observed, when air was present in the interference chamber. If the chamber is evacuated and the same light is used, then we will see

- (a) no interference
(b) interference with dark bands
(c) interference with bright bands
(d) interference with increased fringe width

Ans :

SQP 2013

Fringe width, $\beta = \frac{\lambda D}{d} \propto \lambda$

Since, refractive index of air is more than that of vacuum, therefore when air is evacuated, then wavelength λ of light increases.

As a result of this, fringe width also increases.

Thus (d) is correct option.

13. If light waves emitted by two coherent sources have wavelengths λ_1 and λ_2 , then

- (a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) $\lambda_1 \lambda_2 = 1$

Ans :

SQP 2005

The light waves emitted from two coherent sources have same wavelength, same frequency and are in same phase or have a constant phase difference

Therefore, $\lambda_1 = \lambda_2$

Thus (a) is correct option.

14. Laser light is considered to be coherent as it consists of
- (a) divergent beam
(b) different wavelengths
(c) uncoordinated wavelength
(d) coordinated waves of same wavelength

Ans :

OD 2009

Laser light is highly coherent, monochromatic and completely parallel. It is considered to be coherent as it consists of coordinated waves of exactly the same wavelength.

Thus (d) is correct option.

15. Interference takes place due to change in
- (a) velocity (b) intensity
(c) amplitude (d) phase difference

Ans :

Foreign 2006

When two waves of nearly same frequency and same amplitude with zero initial phase difference or constant phase difference superimpose over each other, then non-uniform distribution of energy in the medium is called interference. Therefore interference takes place due to change in phase difference.

Thus (d) is correct option.

16. What happens if a monochromatic light used in the Young's double slit experiment is replaced by white light?
- (a) no fringes are observed
(b) all bright fringes become white
(c) all bright fringes are coloured between violet and red
(d) only central fringe is white and all other fringes are coloured

Ans :

Delhi 2001

If white light is used in Young's double slit experiment, we get a central white fringe, because all wavelengths of white light have zero path difference. And all other points on both sides of central white fringe have different path differences for different wavelengths of

white light. Therefore no condition of brightness or darkness is satisfied.

As a result of this, coloured fringes are observed on both sides of central white fringe.

Thus (d) is correct option.

17. In Young's double slit experiment, for which colour, fringe width is the least?
- (a) green (b) blue
(c) red (d) yellow

Ans :

OD 2001

Fringe width, $\beta = \frac{\lambda D}{d} \propto \lambda$

Since, wavelength λ of blue colour is least among the given colours, therefore fringe width is least for blue colour.

Thus (b) is correct option.

18. In Young's experiment, the monochromatic light is used to illuminate two slits. The interference fringes are observed on a screen placed in front of the slits. Now, if a thin transparent glass plate is introduced normally in one of the beam, then
- (a) fringe will disappear
(b) fringe width will increase
(c) fringe width will decrease
(d) fringe width remains the same

Ans :

SQP 2013, Comp 2004

If a thin transparent sheet of some refractive index is introduced normally in the path of one of the interfering waves (or beam), then the entire interference fringe pattern is shifted through some distance without any change in the fringe width. Therefore if a thin glass plate which is transparent is introduced normally in one of the beam, then fringe width remains the same.

Thus (d) is correct option.

19. The interference occurs in which of the following waves?
- (a) transverse (b) longitudinal
(c) electromagnetic (d) all of these

Ans :

OD 2015

The phenomenon of non-uniform distribution of energy in a medium, due to superposition of light waves, is called interference of light. It occurs in transverse, longitudinal and electromagnetic waves.

Thus (d) is correct option.

20. In Young's double slit experiment, if separation between the slits is halved and distance between the slits and screen is doubled, then fringe width will

ASSERTION AND REASON

34. Assertion (A) : In Young's double slit experiment all fringes are of equal width.

Reason (R) : The fringe width depends upon wavelength of light (λ) used distance of screen from plane of slits (D) and slits separation (d).

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is NOT the correct explanation of Assertion (A).
- (c) Assertion (A) is true and Reason (R) is false.
- (d) Assertion (A) is false and Reason (R) is also false.

Ans :

The fringe width is given as :

$$\beta = \frac{\lambda_0}{d}$$

So, if λ , D and d remains constant, then fringe width remains constant.

Thus (a) is correct option.

35. Assertion : In Young's experiment, the fringe width for dark fringes is different from that for white fringes.

Reason : In Young's double slit experiment the fringes are performed with a source of white light, then only black and bright fringes are observed.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

In Young's experiment, fringe width of dark and white fringes are equal. If white light is used as source, coloured fringes are observed representing bright band of different colours.

Thus (d) is correct option.

36. Assertion : Goggles have zero power.

Reason : Radius of curvature of both sides of lens is same.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.

(c) The Assertion is correct but Reason is incorrect.

(d) Both the Assertion and Reason are incorrect.

Ans :

Goggles protect from harmful UV light of sun rays and do not correct sight defects hence have zero power.

Both lens of goggles are identical hence have same curvature.

Thus (b) is correct option.

37. Assertion : The clouds in sky generally appear to be whitish.

Reason : Diffraction due to clouds is efficient in equal measure at all wavelengths.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

The clouds in the sky appear white as the size of cloud particle is not small enough to permit diffraction. So all the wavelength gets reflected and it appears white.

Thus (c) is correct option.

38. Assertion : When tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.

Reason : Destructive interference occurs at the centre of the shadow.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

A bright spot is found at the centre of circular fringe patterns formed due to diffraction of light at the edge of circular obstacles. This bright spot is due to constructive interference there by secondary wavelets. Thus (c) is correct option.

39. Assertion : A famous painting was painted by not using brush strokes in the usual manner, but rather a myriad of small colour dots. In this painting the colour you see at any given place on the painting changes as you move away.

Reason : The angular separation of adjacent dots changes with the distance from the painting.

- (c) diffraction bands become broader and farther apart
 (d) diffraction bands become narrower and crowded together

Ans :**Foreign 2005**

Width of a diffraction pattern for central band,

$$\beta = \frac{\lambda D}{d} \propto \lambda$$

Since, wavelength of blue light is less than that of red light, therefore width of the bands is reduced or bands becomes narrower and crowded together.

Thus (d) is correct option.

- 28.** Which of the following waves can be polarised?

- (a) light waves (b) sound waves
 (c) ultra violet rays (d) longitudinal waves

Ans :**OD 2011**

Light waves can be polarised because these waves are transverse in nature.

Since sound waves, ultraviolet rays are longitudinal waves, can not be polarised.

Thus (a) is correct option.

- 29.** The diffraction of sound waves is easier than light waves, because the sound waves are

- (a) longitudinal
 (b) mechanical waves
 (c) of shorter wavelength
 (d) of longer wavelength

Ans :**SQP 2000, Delhi 2015**

A wave is diffracted only when size of the obstacle is comparable with the wavelength of the wave.

Since wavelength of sound waves is longer than light waves, therefore their diffraction is easier than light waves around big obstacles.

Thus (d) is correct option.

- 30.** Which of the following can not be polarised?

- (a) radio waves
 (b) ultrasonic waves
 (c) ultra-violet-rays
 (d) X-rays and infra-red rays

Ans :**Delhi 2013**

Ultrasonic waves can not be polarised, because these waves are longitudinal in nature.

Since radio-waves ultra-violet rays X-rays, infra-red rays are transverse in nature, therefore they can be polarised.

Thus (b) is correct option.

- 31.** 2 ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle ϕ . If μ represents the refractive index of glass with respect to air, then the angle between reflected and refracted rays is

- (a) 90° (b) $90^\circ + \phi$
 (c) $\sin^{-1}(\mu \cos \phi)$ (d) $\sin^{-1}(\mu \tan \phi)$

Ans :**Foreign 2003**

Angle of polarisation, $i_p = \text{Brewster's angle} = \phi$
 and refractive index of glass with respect to air $= \mu$.

When the light is incident at Brewster's angle, the angle between reflected and refracted rays is 90° irrespective of the value of refractive index.

Thus (a) is correct option.

- 32.** A polaroid is used to

- (a) reduce intensity of light
 (b) produce polarised light
 (c) increase intensity of light
 (d) produce unpolarised light

Ans :**OD 2017**

A polaroid is a device which is used to produce polarised light, when unpolarised light (i.e., ordinary light) is incident on it.

Thus (b) is correct option.

- 33.** A single slit of width 0.5 mm is illuminated by parallel light of wavelength $5000 \text{ \AA}$ and diffraction pattern is observed on a screen 1 m from the slit. The width of the central maxima is

- (a) 500 mm (b) 600 mm
 (c) 700 mm (d) 800 mm

Ans :**Delhi 20011**Width of slit, $a = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$ Wavelength of light, $\lambda = 5000 \text{ \AA} = 5000 \times 10^{-10} \text{ m}$

and distance between slit and screen,

$$D = 1 \text{ m}$$

Width of central maxima,

$$\begin{aligned} \beta &= \frac{2\lambda D}{a} \\ &= \frac{2 \times (5000 \times 10^{-10}) \times 1}{0.5 \times 10^{-3}} \\ &= 500 \times 10^{-3} \text{ m} = 500 \text{ mm} \end{aligned}$$

Thus (d) is correct option.

- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

When d is negligibly small, fringe width β which is proportional to $1/d$ may become too large. Even a single fringe may occupy the whole screen. Hence the pattern cannot be detected.

Thus (a) is correct option.

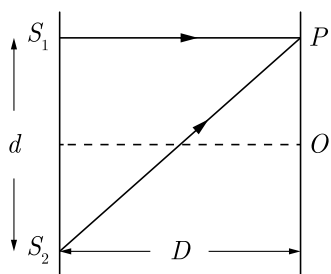
- 44. Assertion :** In Young's double slit experiment the two slits are at distance d apart. Interference pattern is observed on a screen at distance D from the slits. At a point on the screen when it is directly opposite to one of the slits, a dark fringe is observed. Then, the wavelength of wave is proportional to square of distance of two slits.

Reason : For a dark fringe intensity is zero.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

When dark fringe is obtained at the point opposite to one of the slits then



$$S_1P = D$$

and

$$\begin{aligned} S_2P &= \sqrt{D^2 + d^2} = D\left(1 + \frac{d^2}{D^2}\right)^{1/2} \\ &= D\left(1 + \frac{d^2}{2D^2}\right) \quad (\text{By binomial theorem}) \end{aligned}$$

$$\begin{aligned} \text{Path difference} &= S_2P - S_1P \\ &= D\left(1 + \frac{d^2}{2D^2}\right) - D \\ &= \frac{d^2}{2D} = \frac{\lambda}{2} \end{aligned}$$

$$\begin{aligned} \text{or} \quad \lambda &= \frac{d^2}{D} \\ \lambda &\propto d^2 \end{aligned}$$

Now, intensity of a dark fringe is zero.

Thus (d) is correct option.

- 45. Assertion :** A white source of light during interference forms only white and black fringes.

Reason : Width of fringe is inversely proportional to the wavelength of the light used.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

A white source of light during interference will form coloured fringes.

Fringe width is given by $\beta = \frac{\lambda D}{d}$ i.e., it is directly proportional to wavelength.

Thus (d) is correct option.

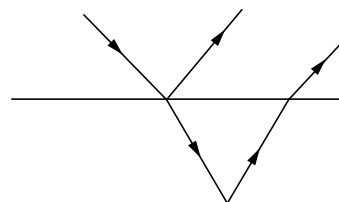
- 46. Assertion :** Thin films such a soap bubble or a thin layer of oil on water show beautiful colours when illuminated by white light.

Reason : It happens due to the interference of light reflected from the upper surface of the thin film.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Interference in between two rays, one is reflected from the upper surface and second from the lower surface.



Thus (c) is correct option.

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- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

We see two closely situated very small dots separate when their angular separation for the viewer is more than that required by Rayleigh's criterion.

$$\theta_R = 1.22 \frac{\lambda}{d}$$

Here, d is diameter of eye and λ is wavelength of light seen.

If distance between spot is D and L be the distance of observer from the painting then,

$$\theta = \frac{D}{L}$$

So, $1.22 \frac{\lambda}{d} = \frac{D}{L}$

$$L = \frac{Dd}{1.22\lambda}$$

In for red light having greater λ , the value of L will be smaller and for violet colour, this distance is large. If we move away from the painting, adjacent red dots become indistinguishable before adjacent blue dots do. So, to make two dots distinguishable we shall have to give distance of the observer (L) a minimum value. At a greater distance colours of adjacent dots blend together. In this way we can have different view of dots by looking at it from different distance (or changing L). So, both assertion and reason are correct.

Thus (a) is correct option.

- 40. Assertion :** The resolving power of a telescope is more if the diameter of the objective lens is more.

Reason : Objective lens of large diameter collects more light.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

The resolving power of a telescope increases as diameter of objective lens increases.

$$\text{Resolving Power} = \frac{D}{1.22\lambda}$$

where D is diameter of objective and λ is wavelength

of light used.

Thus (a) is correct option.

- 41. Assertion :** Standard optical diffraction gratings can not be used for discriminating between X-ray wavelength.

Reason : The grating spacing is not of the order of X-ray wavelengths.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

For diffraction purpose, target should have size of the order of wavelength of light used. So, X-ray cannot be used in standard diffraction between the lines in different grating is large as compared with very short wavelength of X-rays.

Thus (a) is correct option.

- 42. Assertion :** At the first glance, the top surface of the Morpho butterfly's wing appears a beautiful blue-green. If the wind moves the colour changes.

Reason : Different pigments in the wing reflect light at different angles.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

When wind moves the colour of the wing changes. The visible colour of the wing is different from its original colour due to interference of light. Interference occurs between incident ray and reflected ray. Due to wind the reflectivity of the upper surface of the wing changes (as it is covered by tiny hairs) which changes the reflected wave. The resultant colour due to interference also changes. This is the principle behind the change of colour of wings of Morpho Butterfly.

Thus (c) is correct option.

- 43. Assertion :** No interference pattern is detected when two coherent sources are infinitely close to each other.

Reason : The fringe width is inversely proportional to the distance between the two slits.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.

- (a) be halved (b) be two times
(c) be four times (d) remains constant

Ans :

Delhi 2007

Final distance between the slits,

$$d_2 = \frac{d_1}{2}$$

where, d_1 = initial distance between the slits and
final distance between slit and screen

$$D_2 = 2D_1$$

where, D_1 = initial distance between slit and screen

Fringe width, $\beta = \frac{\lambda D}{d} \propto \frac{D}{d}$

Therefore $\frac{\beta_1}{\beta_2} = \frac{D_1}{D_2} \times \frac{d_2}{d_1} = \frac{D_1}{2D_1} \times \frac{d_1}{\frac{d_1}{2}} = \frac{1}{4}$

or $\beta_2 = 4\beta_1$

Thus the fringe width will be quadrupled.

Thus (b) is correct option.

21. The maximum number of possible interference maxima for slit separation equal to twice the wavelength in Young's double-slit experiment is
(a) zero (b) three
(c) five (d) infinite

Ans :

OD 2005

Slit separation, $d = 2\lambda$

We know from the Young's double slit experiment that interference of light occurs when slit separation

d = less than the wavelength of light used

Since, $d = 2\lambda$

i.e., $d > \lambda$

Therefore no interference occurs.

Thus (a) is correct option.

22. The bending of beam of light around the corners of an obstacle is called
(a) reflection (b) refraction
(c) diffraction (d) interference

Ans :

SQP 2006

The phenomenon of bending of light around the corners of an obstacle or aperture in the path of light is called diffraction.

Thus (c) is correct option.

23. When light is incident on a diffraction grating, then zero order principal maxima will be
(a) white
(b) absent

- (c) spectrum of the colours
(d) one of the component colours

Ans :

Delhi 2017

When light is incident on a diffraction grating, then diffraction bands obtained are coloured. But the zero order principal maxima will be white.

Thus (a) is correct option.

24. When a compact disc is illuminated by a source of white light, coloured lanes are observed. This is due to
(a) dispersion (b) diffraction
(c) interference (d) refraction

Ans :

SQP 2003

When a compact disc is illuminated by a source of white light, then the small ripples on the surface of disc, break up white light into different colours due to diffraction.

As a result of this, coloured lanes are observed.

Thus (b) is correct option.

25. The waves that can be polarised are
(a) light waves
(b) transverse waves
(c) longitudinal waves
(d) electromagnetic waves

Ans :

Foreign 2016, OD 2009

Longitudinal waves can not be polarised. As the light waves and electromagnetic waves are transverse in nature, therefore they can be polarised.

Thus (c) is correct option.

26. The phenomenon of diffraction can be treated as the phenomenon of interference, if the number of coherent sources are
(a) one (b) two
(c) zero (d) infinite

Ans :

OD 2007

Diffraction at a single slit is equivalent to the interference of light from infinite number of coherent sources contained in the slit. Therefore the phenomenon of diffraction can be treated as interference phenomenon, if the number of coherent sources are infinite.

Thus (d) is correct option.

27. A diffraction pattern is obtained by using a beam of red light. What happens, if the red light is replaced by blue light?
(a) no change
(b) diffraction bands disappear

VERY SHORT ANSWER QUESTIONS

47. In what way is the diffraction from each slit related to interference pattern in double slit experiment ?

Ans :

Delhi 2020

The intensity of interference fringes in a double slit arrangement is modulated by the diffraction pattern of each slit. Alternatively, in double slit experiment the interference pattern on the screen is actually superposition of single slit diffraction for each slit.

48. What will be the effect of interference fringes if red light is replaced by blue light ?

Ans :

OD 2019

$\beta = \frac{\Delta\lambda}{d}$, i.e., $\beta \propto \lambda$; the wavelength of blue light is less than that of red light is less than that of red light, hence if red light is replaced by blue light, the fringe width decrease, i.e., fringes come closer.

49. Define the term 'coherent sources' which are required to produce interference pattern in Young's double slit experiment.

Ans :

OD 2018

Two monochromatic sources, which produce light waves, having a constant phase difference are known as coherent sources.

50. Define the term 'wavefront'.

Ans :

Delhi 2021

It is defined as the locus of all points in a medium vibrating in the same phase.

51. State Huygen's principle of diffraction of light ?

Ans :

Comp 2021, SQP 2017

When a wavefront strikes to the corner of an obstacle, light wave bends around the corner because every point on wavefront again behaves like a light source and emit secondary wavelets in all directions (Huygen's wave theory) including the region of geometrical shadow. This explains diffraction.

52. In a single-slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ?

Ans :

Foreign 2012

In single slit diffraction experiment fringe width is

$$\beta = \frac{2\lambda D}{d}$$

If d is doubled, the width of central maxima is halved. Thus, size of central maxima is reduced to half. Intensity of diffraction pattern varies square of slit width. So, when the slit gets double, it makes the intensity four times.

53. Why a longitudinal wave cannot be polarized?

Ans :

Foreign 2012

Longitudinal wave like sound waves oscillate only along the direction of its propagation so longitudinal waves cannot be polarised.

54. Which of the following waves can be polarized (i) Heat waves (ii) Sound waves ? Give reason to support your answer.

Ans :

OD 2013, 16

Heat wave are transverse or electromagnetic in nature whereas sound wave are not. Polarisation is possible only for transverse waves.

55. Write down the expression of fringe width in interference.

Ans :

SQP 2014

Fringe width, $\beta = \frac{\lambda D}{d}$

where,

λ = wavelength of light

D = Distance of screen from slit

d = Distance between slit

56. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled ?

Ans :

Comp 2020

Angular separation is, $\theta = \frac{\beta}{D} = \frac{\frac{\Delta\lambda}{d}}{D} = \frac{\lambda}{d}$

Since θ is independent of D , angular separation would remain same.

57. Can white light produce interference? What is its nature?

Ans :

Comp 2016

Yes, white light can produce interference. In this case, the central fringe will be white and first few colored fringes will be formed and then the fringes of different colors overlap to produce general illumination, white in color.

58. Why two independent sources cannot produce sustained interference?

Ans :

Comp 2017

The two independent sources cannot act as coherent sources because the phase difference changes 10^8 times in one second and due to this fast variation of phase, the position of maxima and minima also varies, so the interference pattern but undetected i.e. the two independent sources can not produce sustained interference.

59. What happens to the interference pattern when two coherent sources are
- infinitely close, and
 - far apart from each other.

Ans :

OD 2023

- No interference pattern is detected when two coherent sources are infinitely close to each other.
 - If the sources are far apart, d is large, so 'fringe width Δx ' will be so small that the fringes are not resolved and they do not appear separate. That is why the interference pattern is not detected for large separation of coherent sources.
60. Can two electric bulbs of same power and having filaments of the same material placed close to each other produce interference on the screen?

Ans :

Comp 2018

No, the interference pattern will not be obtained on the screen because the two electric bulbs are not coherent sources and the path difference from two incoherent sources will change randomly. Thus sustained interference pattern will not be observed but there will be general illumination on the screen.

61. Radio-waves diffract pronouncedly around buildings, while light waves, which are electromagnetic waves, do not. Why?

Ans :

OD 2010

Since phenomenon of diffraction is observed only when size of the obstacle is comparable to the wavelength of the wave. The wavelength of radio waves (particularly short waves) have wavelength comparable to general objects like buildings and hence easily get diffracted. But wavelength of light is very small ($3900\text{\AA}$ to $7800\text{\AA}$). So they are not diffracted.

62. Why light can travel in vacuum whereas sound cannot do so?

Ans :

Foreign 2009

Light wave are electromagnetic waves and hence do not require any material medium for propagation hence light can travel in vacuum. The sound waves are mechanical waves and a material medium is must for them to travel, hence they cannot travel in vacuum.

63. What are polaroid ? Mention its uses.

Ans :

Delhi 2009, OD 2018

Polaroid is a material which can polarise light. Tourmaline is a natural polarising material. These are now artificially made. They are also used for the identification of a given light i.e. whether the light is polarised or unpolarised.

Polaroids are used :

- In sunglasses
- To prepare filters.

64. Light from two coherent sources is reaching the screen. If the path difference at a point on the screen for the yellow light be $\frac{3\lambda}{2}$, then what will be the colour of the fringe at that point?

Ans :

Delhi 2019

If the light is white, the fringes will be colored, however, the yellow and neighbouring colors shall be absent. If the source is emitting yellow light (i.e. monochromatic) there will be dark fringe at that point.

65. When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of shadow of the obstacle. Explain.

Ans :

Foreign 2008

The light waves diffracted from the edges of the circular obstacle interfere destructively at the center of the shadow producing bright spot at the center of the shadow.

66. Explain why the intensity of light coming out of a polaroid does not change irrespective of the orientation of the pass axis of the polaroid.

Ans :

Comp 2016

When unpolarised light passes through a polariser, vibrations perpendicular to the axis of the polaroid are blocked.

Hence, if the polariser is rotated, the unblocked vibrations remain same with reference to the axis of polariser.

Hence, for all positions of polaroid, half of the incident light always get transmitted. Hence, the intensity of the light does not change.

67. State the principle of superposition of waves.

Ans :

Comp 2021

This principle state that when a number of waves travelling through a medium superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of the displacement due to the individual waves at that point.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \dots, \vec{y}_n$ are the displacements due to the different waves acting separately, then according to the principle of superposition, the resultant displacement when all the waves act together is given by the vector sum:

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \dots + \vec{y}_n$$

68. What happens to fringe width, when the separation between the slits as well as distance of the screen from the slit are halved?

Ans :

OD 2012

Since fringe width, $\beta = \frac{\lambda D}{d}$

When, $D' = \frac{D}{2}$ and $d' = \frac{d}{2}$ then,

The new fringe width,

$$\begin{aligned}\beta' &= \frac{\lambda D'}{d'} \\ &= \frac{\lambda D}{2} \times \frac{2}{d} = \frac{\lambda D}{d} = \beta\end{aligned}$$

i.e. the fringe width remains the same.

69. What will happen if Young's double slit experiment is performed in water?

Ans :

Foreign 2010

On immersing Young's double slit apparatus in water having index μ , then the wavelength of light λ becomes,

$$\lambda' = \frac{\lambda}{\mu}$$

Since, fringe width β in air,

$$\beta = \frac{\lambda D}{d}$$

So in water, the fringe width β' will be

$$\beta' = \frac{\lambda D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$$

i.e. the fringe will decrease by a factor $\frac{1}{\mu}$.

SHORT ANSWER QUESTIONS

70. Two independent sources of light cannot be coherent. Explain why.

or

Can two different bulbs, similar in all respects, act as coherent sources? Give reasons for your answer.

Ans :

Delhi 2020, Foreign 2013

This is because of the following reasons:

1. Light is emitted by individual atoms and not by the bulk of matter acting as a whole.
2. Even a tiniest source consists of millions of atoms and emission of light by them takes place independently.
3. Even an atom emits an unbroken wave of about 10^{-8} second due to its transition from a higher energy state to a lower energy state.

The millions of atoms of a source cannot emit waves in the same phase. The light emitted by the commonly used monochromatic source (a sodium lamp) remains coherent for about 10^{-8} s. After this time, the atoms responsible for emission of light get changed. The phase difference and hence the interference pattern

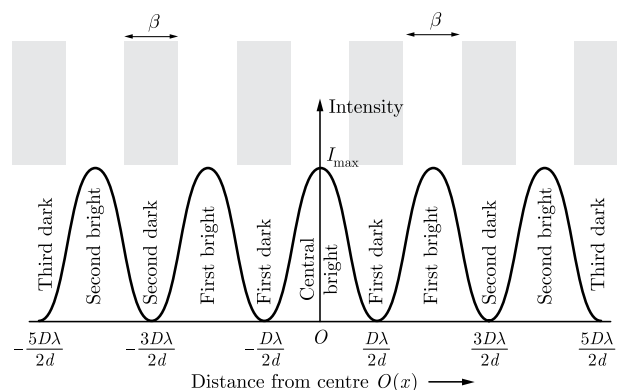
changes 10^8 times in one second. Our eyes cannot see such rapid changes and a uniform illumination is seen on the screen. So two independent light sources cannot produce a sustained interference.

71. Draw a graph showing the variation of intensity against the position x on the screen in Young double slit experiment.

Ans :

OD 2020

On plotting the intensities of bright and dark fringes against distance x from O , we get a curve as shown in figure. The intensity is maximum at the central point O . Then it becomes zero and maximum alternately on either side of O , depending on x is odd multiple of $\frac{D\lambda}{2d}$ and integral multiple of $\frac{D\lambda}{2d}$, respectively.



72. Two sources of intensity I_1 and I_2 undergo interference in Young's double slit experiment. Show that $\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$, where a_1 and a_2 are the amplitudes of disturbance of two sources S_1 and S_2 .

Ans :

Comp 2020

Let, a_1 and a_2 be the amplitudes and I_1 and I_2 be the intensities of light waves from two different sources.

As, Intensity $\propto$ Amplitude²

Hence, $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

Amplitude at a maximum in interference pattern = $a_1 + a_2$.

Amplitude at a minimum in interference pattern = $a_1 - a_2$.

Therefore, the ratio of intensities at maxima and minima is,

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

or

$$\frac{I_{\max}}{I_{\min}} = \left[\frac{r+1}{r-1}\right]^2$$

where, $r = \frac{a_1}{a_2} \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of the two waves

This means width of beam due to diffraction remain equal to the size of the width of the slit (3×10^{-3}) i.e. diffraction effect can be neglected (upto 18 m) and ray optics is valid.

82. Give two differences between interference and diffraction of light.

or

State three characteristic features which distinguish between interference pattern due to two coherently illuminated sources as compared to that observed in a diffraction pattern due to a single slit.

Ans :

Foreign 2014, OD 2017

Difference between interference and diffraction are as follows :

	Interference	Diffraction
(i)	It is due to the superposition of two waves coming from two coherent sources.	It is due to the superposition of secondary wavelets originating from different parts of the same wavefront.
(ii)	The width of the interference bands is equal.	The width of the diffraction bands is not the same.
(iii)	The intensity of all maxima (fringes) is same.	The intensity of central maximum is maximum and goes on decreasing rapidly with increase in order of maxima.

83. What is difference between diffraction and interference?

Ans :

OD 2013

Differences between interference and diffraction are as follows.

	Interference	Diffraction
1.	Interference is the result of superposition of light from two different wave fronts produced by two coherent sources.	Diffraction is the superposition of secondary waves emitted from various points of the same wave front.
2.	Interference fringes are of the same intensity.	Intensity of diffraction fringes fall rapidly.
3.	Interference fringes are equally spaced.	Diffraction fringes are unequally spaced.
4.	Minimas of interference fringes are of zero intensity.	In case of diffraction fringes, minimas are never perfectly dark.

84. Two nearby narrow slits are illuminated by a single monochromatic source. Name the pattern obtained on the screen. One of the slits is now covered. What is the name of the pattern now obtained on the screen? Write two differences between the patterns obtained in the two cases.

Ans :

Comp 2019

When two nearby narrow slits are illuminated by a single monochromatic source of light, we get interference fringes on the screen i.e., we have alternative bright and dark bands.

When one of the slits is covered, we have no interference pattern but have a general illumination on the screen. However, there may be a little diffraction pattern at the edge due to Fraunhofer's diffraction.

Differences

In first case interference bands are alternative bright and dark bands. All bands are of equal brightness.

In second case diffraction pattern at the edges are not of uniform brightness.

85. Explain, polarisation affords a convincing evidence of transverse nature of light.

Ans :

Delhi 2018, OD 2010

The phenomenon of interference and diffraction establishes the wave nature of light. Both these phenomena are exhibited by both the types of wave motion i.e., longitudinal and transverse.

In transverse wave motion the vibrations of particles take place in a direction perpendicular to the direction of propagation of the wave. Hence the vibration in ordinary light are distributed symmetrically in all directions in a plane perpendicular to the direction of propagation. Such light is called unpolarised light.

Light waves which exhibit different properties in different directions are called polarised light waves.

This can be demonstrated by using a pair of tourmaline crystals in the path of light waves.

86. Describe two commonly used devices which use polarised light.

Ans :

OD 2009

The two commonly used devices using polarised light are:

- Liquid Crystal Display :** They are found in many watches and calculators. Liquid crystals have long molecules whose direction can be controlled by applying electric fields. This is used to modify the light produced by polariser to make its polarisation perpendicular to the axis of an analyser. The analyser cuts these lights and produces dark regions. These dark regions are controlled and are used to form numbers and letters.

For two different frequencies, the interference term becomes zero.

Hence, the intensities of two bulbs get up.

$$\text{i.e., } I = I_1 + I_2.$$

77. In Young's double slit experiment, the separation of slits is doubled and the distance of the slits and screen is halved. How will it affect the fringe width?

Ans :

SQP 2013

In interference pattern, the fringe width β is given,

$$\beta = \frac{\lambda D}{d} \quad \dots(1)$$

Let β' be new fringe width, when

$$d' = 2d$$

$$\text{and } D' = \frac{D}{2}$$

$$\text{Hence, } \beta' = \frac{\lambda D'}{d'} = \frac{\lambda D}{2 \cdot 2d} = \frac{\lambda D}{4d}$$

Using Eq. (1), we get

$$\beta' = \frac{\beta}{4}$$

So the fringe width is reduced to one-fourth of its previous value.

78. Show that maximum intensity in interference pattern is four times the intensity due to each slit. Hence show that interference involves only redistribution of energy.

Ans :

Delhi 2019, OD 2005

Let, a be amplitude of waves from each slit, hence, a^2 is the intensity due to each slit.

Hence, Before interference,

$$\text{Average intensity} = a^2 + a^2 = 2a^2$$

$$\begin{aligned} \text{So, Maximum intensity} &= (a + a)^2 \\ &= (2a)^2 = 4a^2 \end{aligned}$$

i.e. maximum intensity is four times the intensity due to each slit.

$$\text{and Minimum intensity} = (a - a)^2 = 0$$

Also average intensity in interference pattern

$$= \frac{0 + 4a^2}{2} = 2a^2$$

$$= \text{Average intensity before interference}$$

Hence the interference involves only redistribution of energy keeping the total energy constant.

79. What will be the effect on interference fringes obtained in Young's double slit experiment if (1) one slit is covered, (2) a source of light of higher wavelength is used, (3) distance between two slits be increased to 1 cm and (4) distance between screen and double slit is increased?

Ans :

Delhi 2015

The fringe width β is given by,

$$\beta = \frac{\lambda D}{d}$$

where, λ is the wavelength of light incident, D is distance between the screen and double slit and d is the distance between the two slits.

1. On covering one slit, the interference pattern will be replaced by diffraction pattern due to single slit.
 2. On increasing the wave length of incident light λ , the fringe width will increase.
 3. On increasing d by 1 cm, the fringe will disappear and there will be uniform illumination.
 4. On increasing D , the fringe width will increase.
80. Compare single slit diffraction pattern due to monochromatic light and white light.

Ans :

OD 2016

When source of light is monochromatic, the diffraction pattern consists of alternate bright and dark bands of unequal width. The central bright fringe has maximum intensity. The intensity of secondary maxima falls off rapidly.

When source is emitting white light, the diffraction pattern is colored. The central maxima is white, but other bands are colored. As band width is directly proportional to the $\propto \lambda$, therefore, red band with higher wavelength is wider than violet with smaller wavelength.

81. A slit or an aperture diffracts light. Even then we say light travels in a straight line and ray optic is valid. Comment.

Ans :

Delhi 2014

A slit of width a diffracts a beam of light. Half angular width of central maximum is:

$$\theta \cong \sin \theta = \frac{\lambda}{a}$$

In travelling distance z , half angular width of beam would become $\left(\frac{z\lambda}{a}\right)$.

This width will become more than width of slit, i.e.

$$\frac{z\lambda}{a} > a$$

$$\text{when } z > \frac{a^2}{\lambda}$$

We call $\frac{a^2}{\lambda} = z_F$, Fresnel distance.

$$\text{Now, } a = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$\lambda = 5000 \text{ \AA} = 5 \times 10^{-7} \text{ m}$$

$$z_F = \frac{a^2}{\lambda} = \frac{(3 \times 10^{-3})^2}{5 \times 10^{-7}} = 18 \text{ m}$$

73. State Malus law. Draw a graph showing the dependence of intensity of transmitted light on the angle between polariser and analyser.

Ans :

Foreign 2012

When a beam of completely plane polarised light is passed through analyser, the intensity I of transmitted light varies directly as the square of the cosine of the angle θ between the transmission directions of polariser and analyser. This statement is known as the law of Malus.

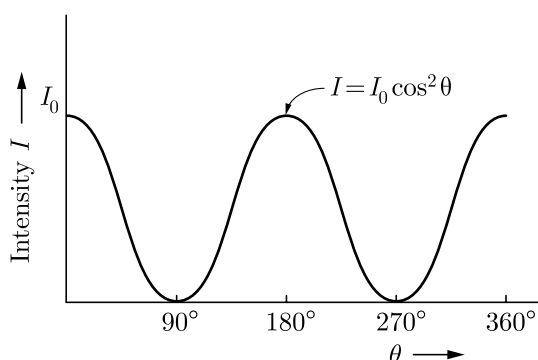
Mathematically, $I \propto \cos^2 \theta$

or $I = I_0 \cos^2 \theta$

Here, I_0 is the maximum intensity of transmitted light.

Intensity Curve

As the angle θ between the transmission direction of polariser and analyser is varied, the intensity I of the light transmitted by the analyser varies as a function of $\cos^2 \theta$, as shown in figure.



74. Explain why do we need coherent sources to produce interference of light.

Ans :

Delhi 2021

When two monochromatic waves of intensity I_1 , I_2 and phase difference ϕ meet at a point, the resultant intensity is given by,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

The last term $2\sqrt{I_1 I_2} \cos \phi$ is called interference term. There are two possibilities:

1. If $\cos \phi$ remains constant with time, the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points where $\cos \phi$ is $+1$ and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points where $\cos \phi = -1$. The sources in this case are coherent.
2. If $\cos \phi$ varies continuously with time, then the interference term averages to zero. There will be same intensity, $I = I_1 + I_2$ at every point i.e., there

will be general illumination on the observation screen. The two sources in the case are incoherent.

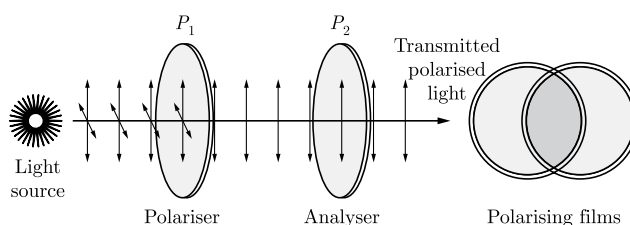
Hence to observe interference, we need to have two sources with the same frequency and with a stable phase difference. Such a pair of sources are called coherent sources.

75. How does one demonstrate, using a suitable diagram, that unpolarised light when passed through a polaroid gets polarised?

Ans :

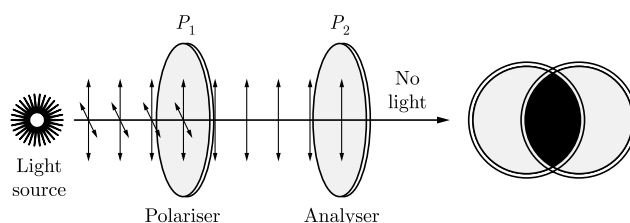
OD 2016, Comp 2012

As shown in figure, when a beam of unpolarised light falls on a polaroid P_1 , it transmits only those vibrations which are parallel to its polaroid axis. It absorbs the vibrations in the perpendicular direction. Thus the transmitted light is plane-polarised. This can be examined by using a second polaroid P_2 . When the polaroid axes of the two polaroids are parallel to each other [Figure (a) and (b)], the plane-polarised light transmitted by P_1 is also transmitted by P_2 , when the second polaroid is rotated through 90° (cross polaroids), no light is transmitted by P_2 [Figure (c) and (d)]. This experiment demonstrates that light has transverse wave nature.



(a)

(b)



(c)

(d)

76. Why are two bulbs lighting the same walls considered as incoherent sources? How do their intensities add up?

Ans :

OD 2015

The bulbs are incoherent source at any frequency, the radiation from one has no stable phase relation with the other. This is because independent atoms and electrons are responsible for the radiation for each source.

2. **Sun Glasses (Dark Glasses) :** Sun glasses have polaroids. Glare from sunlight reflection from water can be reduced, if the polaroids cut out horizontally polarised light.

87. How will the interference pattern in Young's double slit experiment get affected, when
(i) distance between the slits S_1 and S_2 reduced and
(ii) the entire set up is immersed in water ? Justify your answer in each case.

Ans :

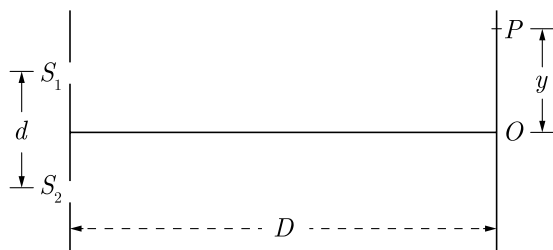
OD 2010

- (i) The fringe width of interference pattern increases with the decrease in separation between S_1S_2 as

$$\beta \propto \frac{1}{d}$$

- (ii) The fringe width decrease as wavelength gets reduced when interference set up is taken from air to water.

88. The intensity at the central maxima (O) in a Young's double slit experiment is I_0 . If the distance OP equals one-third of the fringe width of the pattern, show that the intensity at point P would be $\frac{I_0}{4}$.



Ans :

Foreign 2011, 12

Fringe width, $\beta = \frac{\lambda D}{d}$

$$y = \frac{\beta}{3} = \frac{\lambda D}{3d}$$

Path difference, $\Delta p = \frac{yd}{D}$

$$\Delta p = \frac{\lambda D}{3d} \cdot \frac{d}{D} = \frac{\lambda}{3}$$

$$\begin{aligned} \Delta \phi &= \frac{2\pi}{\lambda} \cdot \Delta p \\ &= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3} = \frac{2\pi}{3} \end{aligned}$$

Intensity at point, $P = I_0 \cos^2 \Delta \phi$

$$= I_0 \left[\cos \frac{2\pi}{3} \right]^2 = I_0 \left(\frac{1}{2} \right)^2 = \frac{I_0}{4}$$

89. State two conditions required for obtaining coherent sources.

In Young's arrangement to produce interference pattern, show that dark and bright fringes appearing on the screen are equally spaced.

Ans :

Delhi 2009

Conditions for obtaining coherent sources :

- (i) Coherent sources of light should be obtained from a single source by same device.
(ii) The two sources should give monochromatic light. The separation between the centres or two consecutive bright fringes is the width of a dark fringe.

$$\beta = x_n - x_{n-1}$$

$$= n \frac{\lambda D}{2d} - (n-1) \frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$

Similarly for dark fringe, fringe width

$$\beta' = x'_n - x'_{n-1}$$

$$= (2n-1) \frac{\lambda D}{2d} - [2(n-1)-1] \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{2d}$$

Hence, all bright and dark fringes are equally spaced on screen.

90. For a single slit of width a , the first minimum of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of $\frac{\lambda}{a}$. At the same angle of $\frac{\lambda}{a}$, we get a maximum for two narrow slits separated by a distance a . Explain.

Ans :

Delhi 2014, Comp 2002

In diffraction, angular position $\theta = \frac{\Delta x}{a}$ for first minima, $\Delta x = \lambda$

$$\theta = \frac{\lambda}{a}$$

In interference, $d = a$ (given) and angular position, $\theta = \frac{\Delta x}{d}$

Angular position of first maxima

$$(\Delta x = \lambda) \quad \theta = \frac{\lambda}{a}$$

91. State briefly two features which can distinguish the characteristic features of an interference pattern from those observed in the diffraction pattern due to a single slit.

Ans :

Delhi 2018

Features of interference pattern are given below.

- (i) The interference pattern has a number of equally a central bright maximum which is twice as wide as the other maxima. The intensity in diffraction pattern falls as we go to successive maxima away from the centre on either side.
(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

92. (a) Why are coherent sources necessary to produce a sustained interference pattern ?
- (b) In Young's double slit experiment using monochromatic light of wavelength λ , the intensity of light at a point on the screen where path difference is λ is K units. Find out the intensity of light at a point where path difference is $\frac{\lambda}{3}$.

Ans :

Delhi 2014

- (a) This is because coherent sources are needed to ensure that the positions of maxima and minima do not change with time.

If the phase difference between wave, reaching at a point change with time, intensity will change and sustained interference will not be obtained.

- (b) We know,

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

For path difference λ , phase difference $\phi = 2\pi$

Intensity of light = K

Hence, $K = 4I_0 \cos^2 \pi = 4I_0$

For path difference $\frac{\lambda}{3}$, phase difference $\phi = \frac{2\pi}{3}$

$$\begin{aligned} \text{Intensity of light, } I' &= 4I_0 \cos^2 \frac{\phi}{2} \\ &= 4I_0 \cos^2 \frac{\pi}{3} = I_0 \end{aligned}$$

$$I' = \frac{K}{4}$$

93. Distinguish between polarized and unpolarized light. Does the intensity of polarized light emitted by a polaroid depend on its orientation ? Explain briefly. The vibrations in a beam of polarized light make an angle of 60° with the axis of the polaroid sheet. What percentage of light is transmitted through the sheet?

Ans :

Comp 2016

A light which has vibrations in all directions in a plane perpendicular to the direction of propagation is said to be unpolarised light. The light from the sun, an incandescent bulb or a candle is unpolarised.

If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then it is said to be polarised or linearly polarised light.

Yes, the intensity of polarised light emitted by a polaroid depends on orientation of polaroid. When polarised light is incident on a polaroid, the resultant intensity of transmitted light varies directly as the square of the cosine of the angle between polarisation direction of light and the axis of the polaroid.

$$I \propto \cos^2 \theta$$

$$\text{or } I = I_0 \cos^2 \theta$$

where I_0 = maximum intensity of transmitted light;
 θ = angle between vibrations in light and axis of polaroid sheet.

$$\text{or } I = I_0 \cos^2 60^\circ = \frac{I_0}{4}$$

Percentage of light transmitted

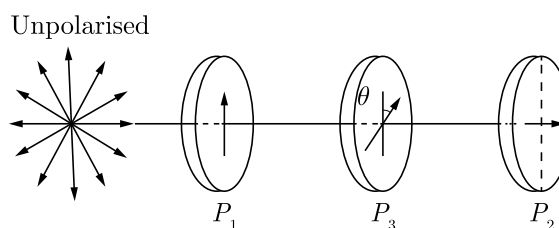
$$\begin{aligned} &= \frac{I}{I_0} \times 100 \\ &= \frac{1}{4} \times 100 = 25\% \end{aligned}$$

94. Find an expression for intensity of transmitted light, when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum ?

Ans :

SQP 2009, OD 2002

Let us consider two crossed polarisers, P_1 and P_2 with a polaroid sheet P_3 placed between them.



Let I_0 be the intensity of polarised light after passing through the first polariser P_1 . If θ is the angle between the axes of P_1 and P_3 , then the intensity of the polarised light after passing through P_3 will be $I = I_0 \cos^2 \theta$. As, P_1 and P_2 are crossed, the angle between the axes of P_1 and P_2 is 90° .

Angle between the axes of P_2 and P_3 is $(90^\circ - \theta)$.

The intensity of light emerging from P_2 will be given by

$$\begin{aligned} I &= [I_0 \cos^2 \theta] \cos^2 (90^\circ - \theta) \\ &= [I_0 \cos^2 \theta] \sin^2 \theta \\ &= \frac{I_0}{4} (4 \cos^2 \theta \sin^2 \theta) \\ &= \frac{I_0}{4} (2 \sin \theta \cos \theta)^2 \\ &= \frac{I_0}{4} \sin^2 (2\theta) \end{aligned}$$

The intensity of polarised light transmitted from P_2 will be maximum, when

$$\sin 2\theta = \text{maximum} = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

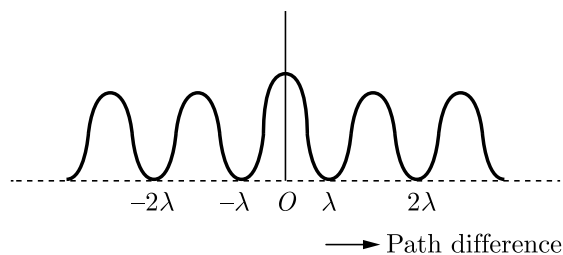
Also, the maximum transmitted intensity will be given by $I = \frac{I_0}{4}$.

95. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two difference between interference and diffraction patterns.

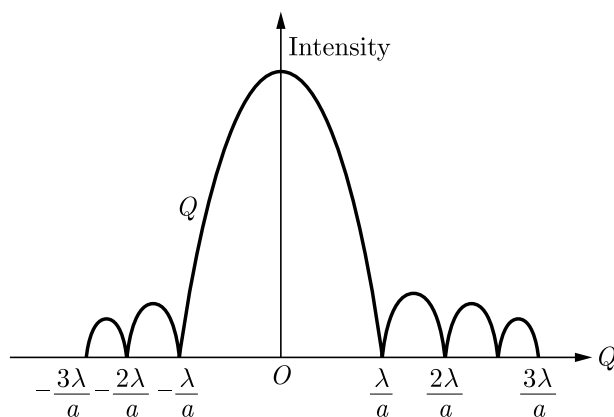
Ans :

OD 2013

Intensity pattern for single slit diffraction



Intensity pattern for double slit interference



Difference between interference and diffraction

	Interference	Diffraction
1.	Fringe widths are same.	Fringe widths are different.
2.	Generally fringe pattern is hyperbolic.	Generally fringe pattern is circular.

96. A parallel beam of monochromatic light falls normally on a narrow slit of width d to produce a diffraction pattern on the screen placed parallel to the plane of the slit. Use Huygen's principle to explain that
- the central bright maxima is twice as wide as the other maxima.
 - the intensity falls as we move to successive maxima away from the centre on either side.

Ans :

Delhi 2012

- (i) In diffraction pattern, intensity will be minimum at an angle $\theta = \frac{n\lambda}{d}$

Where, d = width of slit and $n = 1, 2, 3, \dots$

There will be a first minimum at an angle

$$\theta = \frac{\lambda}{d}$$

On either side of central maximum.

Since, Width of central maxima = $\frac{2\lambda}{d}$

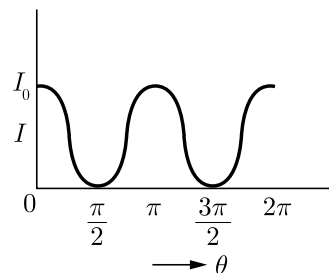
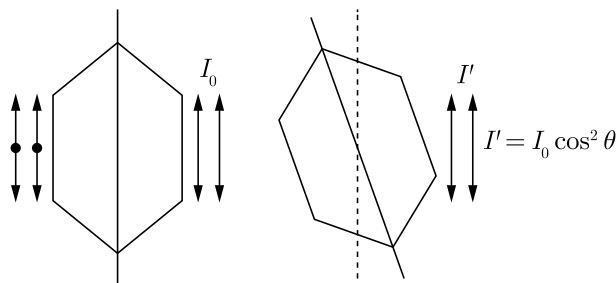
Whereas the width of other minimum/maximum = $\frac{\lambda}{d}$

- (ii) The intensity of maxima decreases as the order (n) or diffraction maxima increases. This is because, on dividing the slit into odd number of parts, the contributions of the corresponding (outermost) pairs cancel each other, the contributions of the corresponding pair cancel each other, leaving behind the contribution of only the innermost segment e.g., for maxima, slit is dividing into three parts, out of these three parts of the slit, the contribution from first two parts cancel each other, only $\frac{1}{3}$ rd position of slit contributes of the maxima of intensity. Similarly, for second maxima, slit is dividing into five parts, contribution of first four parts will be zero. The remaining $\frac{1}{5}$ th portion will only contribute for maxima and so on.

97. Unpolarised light is pass through a polarised P_1 , when this polarised beam pass through another polarised P_2 and if the pass axis of P_2 makes angle θ with the pass axis of P_1 , when write the expression for the polarised beam passing through P_2 . Draw a plot showing the variation of intensity when θ varies from 0 to 2π .

Ans :

OD 2017, Comp 2009



$$\begin{aligned}
 P_2 &= \frac{I_0}{2} \cos^2 \beta \cos^2 (90 - \beta) \\
 &= \frac{I_0}{2} \cdot \cos^2 \beta \sin^2 \beta \\
 &= \frac{I_0}{2} \left[\frac{(2 \cos \beta \cdot \sin \beta)^2}{(2)^2} \right] \\
 I &= \frac{I_0}{8} (\sin 2\beta)^2
 \end{aligned}$$

But it is given that intensity transmitted from P_2 is

$$I = \frac{I_0}{8}$$

So,

$$\frac{I_0}{8} = \frac{I_0}{8} (\sin 2\beta)^2$$

or

$$(\sin 2\beta)^2 = 1$$

$$\sin 2\beta = \sin \frac{\pi}{2}$$

$$\beta = \frac{\pi}{4}$$

103. (a) Describe briefly, with the help of suitable diagram, how the transverse nature of light can be demonstrated by the phenomenon of polarization of light.
- (b) When unpolarized light passes from air to a transparent medium, under what condition does the reflected light get polarised ?

Ans :

Delhi 2016, Comp 2011

- (a) Light from a source S is allowed to fall normally on the flat surface of a thin plate of a tourmaline crystal, cut parallel to its axis. Only a part of this light is transmitted through A .

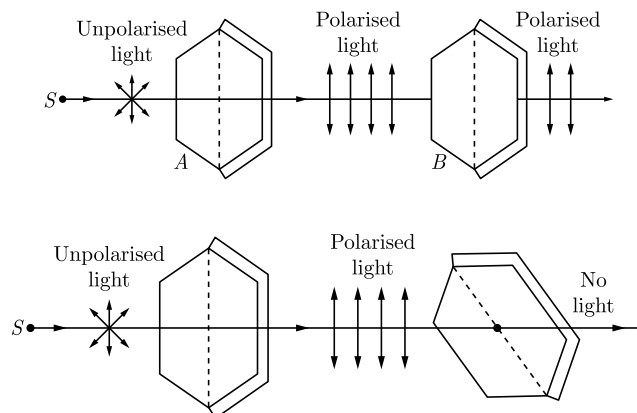
If now the plate A is rotated, the character of transmitted light remains unchanged. Now another similar plate B is placed at some distance from A such that the axis of B is parallel to that of A . If the light transmitted through A is passed through B the light is almost completely transmitted through B and no change is observed in the light coming out of B .

If now the crystal A is kept fixed and B is gradually rotated in its own plane, the intensity of light emerging out of B decreases and becomes zero when the axis of B is perpendicular to that of A . If B is further rotated, the intensity begins to increase and becomes maximum when the axes of A and B are again parallel.

Thus, we see that the intensity of light transmitted through B is maximum when axes of A and B are parallel and minimum when they are at right angles.

From this experiment, it is obvious that light waves are transverse and not longitudinal because,

if they were longitudinal, the rotation of crystal B would not produce any change in the intensity of light.



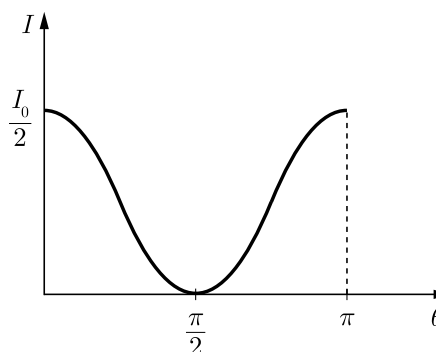
- (b) The reflected ray is totally plane polarised, when reflected and refracted rays are perpendicular to each other.

104. (i) Unpolarised light of intensity I_0 passes through two polaroids P_1 and P_2 such that pass axis of P_2 makes an angle θ with the pass axis of P_1 . Plot a graph showing the variation of intensity of light transmitted through P_2 as the angle θ varies from 0° to 180° .
- (ii) A third polaroid P_3 is placed between P_1 and P_2 with pass axis of P_3 making an angle β with that of P_1 and the angle between P_1 and P_2 is θ . If I_1 , I_2 and I_3 , represents the intensities of light transmitted by P_1 , P_2 and P_3 , then determine the values of angle θ and β for which $I_1 = I_2 = I_3$.

Ans :

SQP 2010

- (i) The required graph would have the form as shown in figure below



Using, $I_2 = I_1 \cos^2 \theta$

- (ii) Given,

$$I_1 = \text{light transmitted by } P_1$$

$$I_2 = \text{light transmitted by } P_2$$

$$I_3 = \text{light transmitted by } P_3$$

When polariser and analyser are perpendicular to each other

$$\theta = 90^\circ$$

$$\cos \theta = \cos 90^\circ = 0$$

$$I = 0$$

In unpolarised light, vibrations are probable in all the direction in a plane perpendicular to the direction of propagation.

Therefore, θ can have any value from 0 and 2π .

$$\begin{aligned} \text{Hence, } [\cos^2 \theta]_{av} &= \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta d\theta \\ &= \frac{1}{2\pi} \int_0^{2\pi} \frac{(1 + \cos 2\theta)}{2} d\theta \\ &= \frac{1}{2\pi \times 2} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \frac{1}{2} \end{aligned}$$

Using law of Malus,

$$I = I_0 \cos^2 \theta$$

$$I = I_0 \times \frac{1}{2} = \frac{1}{2} I_0$$

As per the question, I_0 is the intensity of incident unpolarised light and I_1 and I_2 are the intensities of polaroids P_1 and P_2 respectively, then we can say that when unpolarised light of intensity I_0 get polarised on passing through a polaroid P_1 its intensity become half, i.e., $I_1 = \frac{I_0}{2}$.

- (ii) When this polarised light of intensity I_1 passes through polaroid P_2 , then its intensity will be given by $I_2 = I_1 \cos^2 \theta$

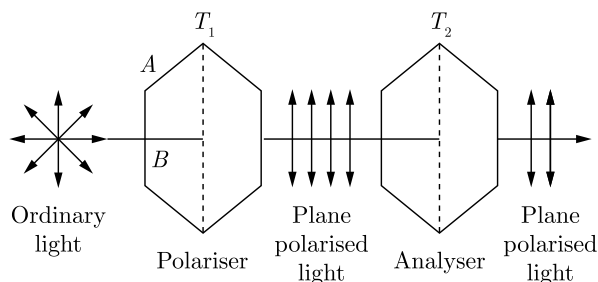
This is a required relation between intensities I_1 and I_2 .

- 101.** (i) Using the phenomenon of polarisation, show how transverse nature of light can be demonstrated.
 (ii) Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I_0 is incident on P_1 . A third polaroid P_3 is kept in between P_1 and P_2 such that its pass axis makes an angle of 30° with that of P_1 . Determine the intensity of light transmitted through P_1 , P_2 and P_3 .

Ans :

OD 2013, Comp 2011

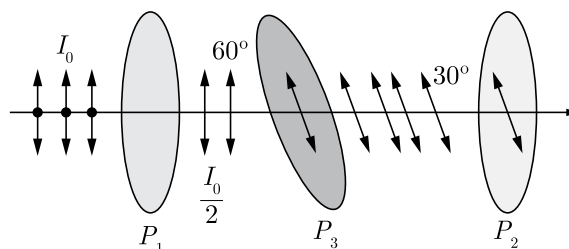
- (i) To show how the transverse nature of light waves



We observe that intensity and character of light transmitted by T_1 and T_2 remain unaffected only when T_1 and T_2 are set with their axes parallel. If both T_1 and T_2 are rotated with same angular velocity in the same direction, no change in intensity of transmitted light is observed.

The phenomenon can be explained only when we assume that light waves are transverse.

(ii)



$$\text{Intensity through } P_1 = \frac{I_0}{2}$$

(After polarisation intensity of light becomes half)

$$\begin{aligned} \text{Intensity through } P_3 &= \frac{I_0}{2} \cos^2 60^\circ \\ &= \frac{I_0}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{I_0}{8} \end{aligned}$$

(Since, From Brewster's law)

$$\begin{aligned} \text{Intensity through } P_2 &= \frac{I_0}{8} \cos^2 30^\circ \\ &= \frac{I_0}{8} \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{3I_0}{32} \end{aligned}$$

- 102.** (i) Light passes through two polaroids P_1 and P_2 with axis of P_2 making an angle θ with the pass axis of P_1 for what value of θ is the intensity of emergent light zero?
 (ii) A third polaroid is placed between P_1 and P_2 with its pass axis making an angle β with the pass axis of P_1 . Find a value of β for which the intensity of light emerging from P_2 is $\frac{I_0}{8}$, where I_0 is the intensity of light on the polaroid P_1 .

Ans :

Foreign 2007

- (i) At $\theta = 90^\circ$, the intensity of emergent light is zero.
 (ii) Intensity of light coming out through polariser

$$P_1 = \frac{I_0}{2}$$

Intensity of light coming out from polariser,

$$P_3 = \left(\frac{I_0}{2}\right) \cos^2 \beta$$

Intensity of light coming out from polariser P_2 ,

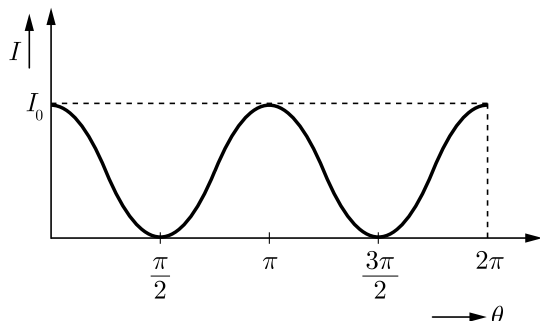
98. (i) State law of Malus.
 (ii) Draw a graph showing the variation of intensity (I) of polarised light transmitted by an analyser with angle (θ) between polariser and analyser.

Ans :

SQP 2008

- (i) Malus law states that when the pass axis of a polaroid makes an angle θ with the plane of polarisation of polarised light of intensity I_0 incident on it, then the intensity of the transmitted emergent light is given by $I = I_0 \cos^2 \theta$.

(ii)



99. (a) Write the necessary conditions to obtain sustained interference fringes. Also write the expression for the fringe width.
 (b) In Young's double slit experiment, plot a graph showing the variation of fringe width versus the distance of the screen from the plane of the slits keeping other parameters same. What information can one obtain from the slope of the curve?
 (c) What is the effect on the fringe width if the distance between the slits is reduced keeping other parameters same?

Ans :

Comp 2019

- (a) Conditions for sustained interference
 (i) The interfering sources must be coherent i.e., source must have same frequency and constant initial phase.
 (ii) Interfering waves must have same or nearly same amplitude, so that there may be contrast between maxima and minima.

$$\text{Fringe width } \beta = \frac{D\lambda}{d}$$

where D = distance between slits and screen

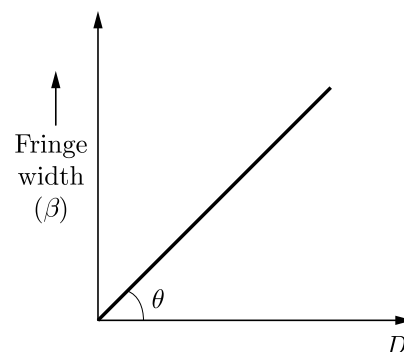
d = separation between slits

λ = wavelength of light

- (b) Information from the slope :
 Wavelength, $\lambda = \text{slope} \times d = d \cdot \tan \theta$
 (c) Effect From relation,

$$\beta = \frac{\lambda D}{d}$$

$$\text{Fringe width, } \beta \propto \frac{1}{d}$$



If distance d between the slits is reduced, the size of fringe width will increase.

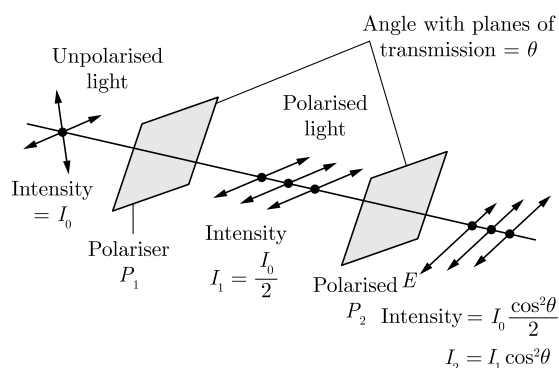
100. State clearly how an unpolarised light get linearly polarised, when passed through a polaroid.

- (i) Unpolarised light of intensity I_0 is incident on a polaroid P_1 which is kept near polaroid P_2 whose pass axis is parallel to that of P_1 . How will the intensities of light, I_1 and I_2 transmitted by the polaroids P_1 and P_2 respectively, change on rotating P_1 without disturbing P_2 ?
 (ii) Write the relation between the intensities I_1 and I_2 .

Ans :

Delhi 2015, OD 2010

(i)



According to law of Malus, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light I transmitted from the analyser varies directly as the square of the cosine of angle θ between the plane of transmission of analyser and polariser.

$$\text{i.e., } I \propto \cos^2 \theta$$

$$I = I_0 \cos^2 \theta$$

When polariser and analyser are parallel, $\theta = 0^\circ$ or 180° .

$$\text{So, that } \cos \theta = \pm 1$$

$$I = I_0$$

According to Malus law

$$I_3 = I_1 \cos^2 \beta \quad \dots(1)$$

$$I_2 = I_3 \cos^2(\theta - \beta) \quad \dots(2)$$

According to question,

$$I_2 = I_3$$

Substituting the value of I_2 and I_3 from Equation (1) and (2), we get

$$I_3 \cos^2(\theta - \beta) = I_1 \cos^2 \beta$$

Substituting the value of I_3 from equation (1),

$$I_1 \cos^2 \beta \cos^2(\theta - \beta) = I_1 \cos^2 \beta$$

$$\cos^2(\theta - \beta) = 1$$

$$(\theta - \beta) = \cos^{-1}(1)$$

$$(\theta - \beta) = 0$$

$$\theta = \beta \quad \dots(3)$$

According to question $I_1 = I_2$

Substituting the value of I_2 from equation (2),

$$I_1 = I_3 \cos^2(\theta - \beta)$$

Substituting the value of I_3 from equation (1),

$$I_1 = I_1 \cos^2 \beta \cos^2(\theta - \beta)$$

or $\cos^2 \beta = 1$ [From eq. (3), $\theta = \beta$]

$$\beta = 0^\circ \text{ or } \pi$$

c_1 = speed of light in rarer medium

c_2 = speed of light in denser medium

The point P on incident wavefront is reached at Q on refracted wavefront in time t as taken by point B to reach at C and point A to reach at D .

i.e., $t = \frac{PO}{c_1} + \frac{OQ}{c_2}$

In $\triangle APO$, $\sin i = \frac{PO}{AO}$

$$PO = AO \sin i$$

In $\triangle CQO$, $\sin r = \frac{OQ}{OC}$

$$OQ = OC \sin r$$

Hence,
$$t = \frac{AO \sin i}{c_1} + \frac{OC \sin r}{c_2}$$

$$= \frac{AO \sin i}{c_1} + \frac{(AC - AO) \sin r}{c_2}$$

$$= AO \left(\frac{\sin i}{c_1} - \frac{\sin r}{c_2} \right) + \frac{AC \sin r}{c_2}$$

AO and AC independent on time, hence we get

$$\frac{\sin i}{c_1} - \frac{\sin r}{c_2} = 0$$

$$\frac{\sin i}{c_1} = \frac{\sin r}{c_2}$$

$$\frac{c_1}{c_2} = \frac{\sin i}{\sin r}$$

The ratio of speed of light in rarer medium to denser medium is called refractive index of optical medium.

i.e., $\mu = \frac{c_1}{c_2}$

Hence, $\mu = \frac{\sin i}{\sin r}$

This is Snell's law.

106. Two harmonic waves of monochromatic light

$y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$ are superimposed on each other. Show that maximum intensity in interference pattern is four times the intensity due to each slit. Hence write the conditions for constructive and destructive interference in terms of the phase angle ϕ .

Ans :

Delhi 2016, Comp 2008

The resultant displacement will be given by

$$y = y_1 + y_2$$

$$= a \cos \omega t + a \cos(\omega t + \phi)$$

$$= a[\cos \omega t + \cos(\omega t + \phi)]$$

$$= 2a \cos\left(\frac{\phi}{2}\right) \cos\left(\omega t + \frac{\phi}{2}\right)$$

The amplitude of the resultant displacement is $2a \cos\left(\frac{\phi}{2}\right)$.

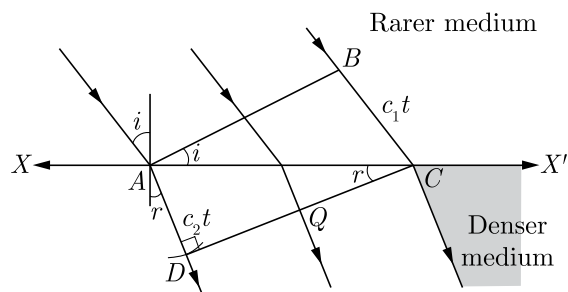
LONG ANSWER QUESTIONS

105. How are wavefront and secondary wavelets defined? Verify laws of reflection or laws of refraction on the basis of Huygen's wave theory.

Ans :

OD 2021

Laws of Refracting using Huygen's Principle



where, XX' = two media separating surface

AB = incident wavefront

CD = refracted wavefront

$\angle i$ = incident angle

$\angle r$ = refracted angle

Hence,

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

$$= 4I_0 \cos^2 \pi = 4I_0 = k$$

1. When $p = \frac{\lambda}{4}$ and $\phi = \frac{\pi}{2}$, then

$$I = 4I_0 \cos^2 \frac{\pi}{4}$$

$$= 4I_0 \times \frac{1}{2} = 2I_0 = \frac{k}{2}$$

2. When $p = \frac{\lambda}{3}$ and $\phi = \frac{2\pi}{3}$, then

$$I = 4I_0 \cos^2 \frac{\pi}{3}$$

$$= 4I_0 \times \frac{1}{4} = I_0 = \frac{k}{4}$$

3. When $p = \frac{\lambda}{2}$ and $\phi = \pi$, then

$$I = 4I_0 \cos^2 \frac{\pi}{2} = 0$$

147. (i) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1° . Find the spacing between the two slits.
- (ii) Light of wavelength $5000 \text{ \AA}$ propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

Ans :

Comp 2019

- (i) Angular width (θ) of the fringe in double slit experiment is given by

$$\theta = \frac{\lambda}{d}$$

Where, d = spacing between the slits

Given, wavelength of light,

$$\lambda = 600 \text{ nm}$$

Angular width of fringe,

$$\theta = 0.1^\circ = \frac{\pi}{1800}$$

$$= 0.0018 \text{ rad}$$

Hence,

$$d = \frac{\lambda}{\theta}$$

$$d = \frac{600 \times 10^{-9}}{18 \times 10^{-4}}$$

$$= 0.33 \times 10^{-3} \text{ m}$$

- (ii) The frequency and wavelength of reflected wave will not change. The refracted wave will have same frequency. The velocity of light in water is given by $\nu = \lambda f$

where, ν = velocity of light

f = frequency of light

λ = wavelength of light

If velocity will decrease, then wavelength (λ) will also decrease.

148. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture $2 \times 10^{-4} \text{ m}$. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

Ans :

SQP 2013, OD 2000

For maxima other than central maxima.

$$a \cdot \theta = \left(n + \frac{1}{2}\right) \lambda$$

and

$$\theta = \frac{y}{D}$$

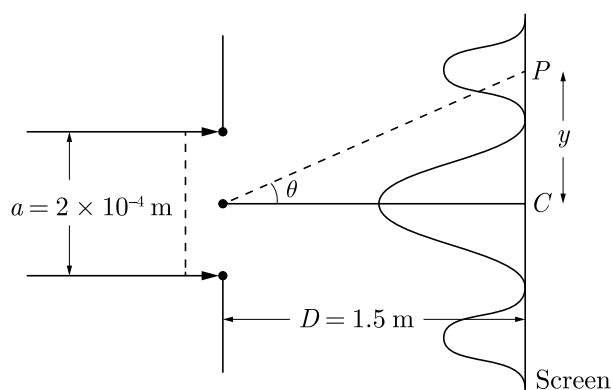
$$a \cdot \frac{y}{D} = \left(n + \frac{1}{2}\right) \lambda$$

For light of wavelength $\lambda_1 = 590 \text{ nm}$

$$2 \times 10^{-4} \times \frac{y_1}{1.5} = \left(1 + \frac{1}{2}\right) \times 590 \times 10^{-9}$$

$$y_1 = \frac{3}{2} \times \frac{590 \times 10^{-9} \times 1.5}{2 \times 10^{-4}}$$

$$= 6.64 \text{ mm}$$



For light of wavelength $\lambda_2 = 596 \text{ nm}$

$$2 \times 10^{-4} \times \frac{y_2}{1.5} = \left(1 + \frac{1}{2}\right) \times 596 \times 10^{-9}$$

$$y_2 = \frac{3}{2} \times \frac{596 \times 10^{-9} \times 1.5}{2 \times 10^{-4}}$$

$$= 6.705 \text{ mm}$$

Separation between two positions of first maxima

$$\Delta y = y_2 - y_1$$

$$= 6.705 - 6.64 = 0.065 \text{ mm}$$

149. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Calculate the width of the slit.

or

State Huygen's principle. With the help of a diagram, show how a plane wave reflected from a surface. Hence verify law of reflection.

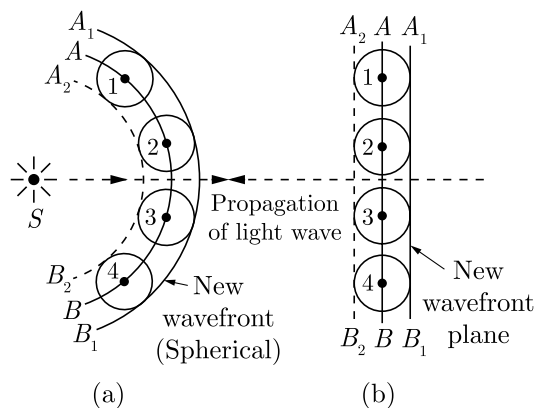
Ans :

Foreign 2023, OD 2018

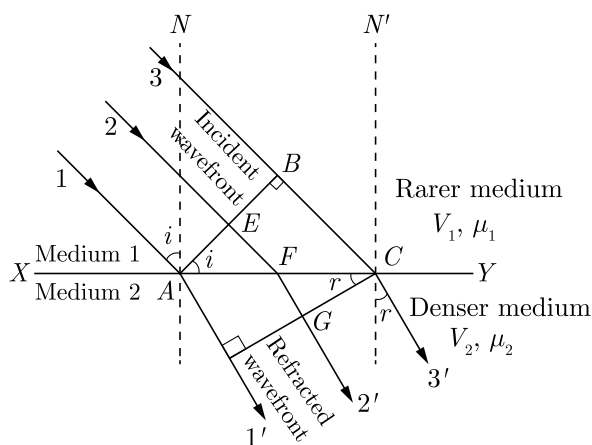
According to Huygen's principle

- (i) Each point on the given wavefront (called primary wavefront) is the source of a secondary disturbance (called secondary wavelets) and the wavelets emanating from these point spread out in all the directions with the speed of the wave.
- (ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant.

This is called secondary wavefront.



Propagation of wave front



If v_1, v_2 are the speed of light into two mediums and t is the time taken by light to go from B to C or A to D or E to G through F , then

$$t = \frac{EF}{v_1} + \frac{FG}{v_2}$$

$$\text{In } \triangle AFE, \quad \sin i = \frac{EF}{AF}$$

$$\text{In } \triangle FGC, \quad \sin r = \frac{FG}{FC}$$

$$t = \frac{AF \sin i}{v_1} + \frac{FC \sin r}{v_2}$$

$$t = \frac{AC \sin r}{v_2} + AF \left(\frac{\sin i}{v_1} - \frac{\sin r}{v_2} \right)$$

For rays of light from the different part on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront. So, t should not depend on F . This is possible only, if

$$\frac{\sin i}{v_1} - \frac{\sin r}{v_2} = 0$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \mu$$

Now, if c represents the speed of light in vacuum, then $\mu_1 = \frac{c}{v_1}$ and $\mu_2 = \frac{c}{v_2}$ are known as the refractive index of medium 1 and medium 2 respectively.

$$\text{Then,} \quad \mu_1 \sin i = \mu_2 \sin r$$

$$\mu = \frac{\sin i}{\sin r}$$

This is known as Snell's law of refraction.

109. What are coherent sources of light? Draw the variation of intensity with position in the interference of Young's double slit experiment.

Ans :

Comp 2020

Coherent sources are those sources which emit continuously light of same wavelength and magnitude either in the same phase or with a constant phase difference.

Let the waves of two coherent sources be,

$$y_1 = a \sin \omega t$$

and

$$y_2 = b \sin(\omega t + \phi)$$

where, a and b are the respective amplitudes of the two waves and ϕ is the constant phase angle by which the second wave leads the first wave. [Figure (a)]

According to superposition principle, the displacement y of resultant wave is

$$\begin{aligned} y &= y_1 + y_2 = a \sin \omega t + b \sin(\omega t + \phi) \\ &= a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi \\ &= \sin \omega t (a + b \cos \phi) + \cos \omega t \cdot b \sin \phi \end{aligned}$$

$$\text{Substituting,} \quad a + b \cos \phi = A \cos \theta \quad \dots(1)$$

$$\text{and} \quad b \sin \phi = A \sin \theta \quad \dots(2)$$

$$\begin{aligned} \text{We have, } y &= \sin \omega t \cdot A \cos \theta + \cos \omega t \cdot A \sin \theta \\ &= A \sin(\omega t + \theta) \end{aligned}$$

where, A is the amplitude of the resultant wave.

Squaring and adding (1) and (2), we have

$$a^2 + b^2 \cos^2 \phi + 2ab \cos \phi + b^2 \sin^2 \phi = A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

The intensity of light is directly proportional to the square of amplitude of the wave. The resultant intensity will be given by

$$I = 4a^2 \cos^2 \frac{\phi}{2}$$

Hence, Intensity $= 4I_0 \cos^2 \left(\frac{\phi}{2} \right)$

where $I_0 = a^2$ is the intensity of each harmonic wave at the maxima,

$$\phi = \pm 2n\pi$$

$$\cos^2 \frac{\phi}{2} = 1$$

At the maxima, $I = 4I_0 = 4 \times \text{intensity due to one slit}$

$$I = 4I_0 \cos^2 \left(\frac{\phi}{2} \right)$$

For constructive interference, I is maximum.

It is possible when $\cos^2 \left(\frac{\phi}{2} \right) = 1$; $\frac{\phi}{2} = n\pi$, $\phi = 2n\pi$

For destructive interference, I is minimum, i.e., $I = 0$

It is possible when

$$\cos^2 \left(\frac{\phi}{2} \right) = 0$$

$$\frac{\phi}{2} = \frac{(2n-1)}{2}$$

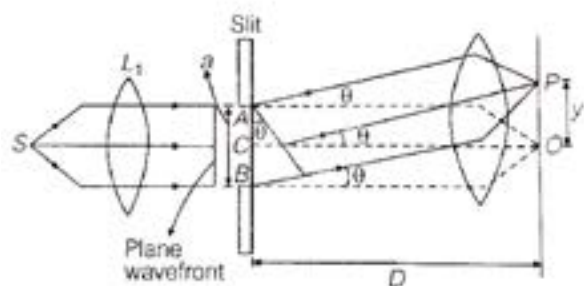
$$\phi = (2n \pm 1) \frac{\pi}{2}$$

107. (i) Using Huygen's construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
- (ii) Show that the angular width of the first diffraction fringe is half that of the central fringe.
- (iii) Explain why the maxima at $\theta = \left(n + \frac{1}{2} \right) \frac{\lambda}{a}$ become weaker and weaker with increasing n .

Ans :

Foreign 2015

(i)

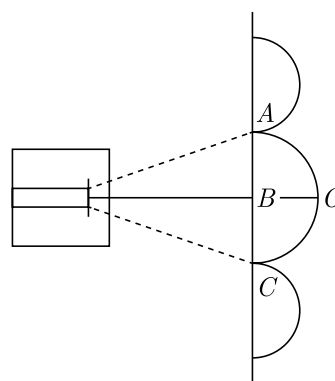


Consider a parallel beam of light from a lens falling on a slit AB . As, diffraction occurs, the pattern is focused on the screen with the help of lens L_2 . We will obtain a diffraction pattern that is central maximum at the centre O , flanked by a number of dark and bright fringes called secondary maxima and minima.

Each point on the plane wavefront AB sends out the secondary wavelets in all directions. The waves from points equidistant from the centre C , lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at O .

- (ii) Let, A and a be the wavelength and slit width of diffracting system respectively.

Let O be the position of central maximum.



Condition for the first minimum is given by

$$d \sin \theta = m\lambda \quad \dots(1)$$

Let θ be the angle of diffraction.

As, diffraction angle is small.

$$\sin \theta = \theta$$

For first diffraction minimum,

$$\theta = \theta_1 \quad (\text{let})$$

For the first minimum, take $m = 1$

$$a\theta_1 = \lambda$$

$$\theta_1 = \frac{\lambda}{a}$$

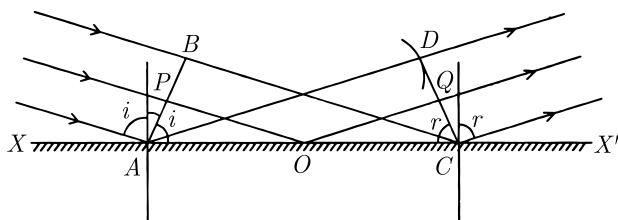
Now, angular width, $AB = \theta_1$

Angular width, $BC = \theta_1$

Angular width, $AC = 2\theta_1$

- (iii) On increasing the value of n , the part of slit contributing to the maximum decrease. Hence, the maximum becomes weaker.

108. Use Huygen's principle to show how a plane wavefront propagates from a denser to rarer medium. Hence, verify Snell's law of refraction.

Laws of Reflection using Huygen's Principle

where, XX' = reflecting surface
 AB = incident wavefront
 $\angle i$ = incident angle
 CD = reflected wavefront
 $\angle r$ = reflected angle
 c = speed of light

The point P on incident wavefront is reached at Q on reflected wavefront in time t , as taken by point A to reach at D and point B to reach at C .

$$\text{i.e., } t = \frac{PO}{c} + \frac{OQ}{c}$$

In $\triangle APO$,

$$\sin i = \frac{PO}{AO}$$

$$PO = AO \sin i$$

In $\triangle CQO$,

$$\sin r = \frac{OQ}{OC}$$

$$OQ = OC \sin r$$

$$\begin{aligned} \text{Hence, } t &= \frac{AO \sin i + OC \sin r}{c} \\ &= \frac{AO \sin i + AC \sin r - AO \sin r}{c} \\ &= \frac{AO(\sin i - \sin r) + AC \sin r}{c} \end{aligned}$$

AO and AC independent on time, hence we get

$$\sin i - \sin r = 0$$

$$\sin i = \sin r$$

$$\angle i = \angle r$$

1. The incident angle is equal to reflected angle.
 2. The incident wavefront, reflected wavefront and normal all lie on same plane.
- iii.** In the experiment of diffraction due to a single slit, show that
- (i) the intensity of diffraction fringes decreases as the order (n) increases.
 - (ii) angular width of the central maximum is twice that of the first order secondary maximum.

Ans :

Delhi 2011, SQP 2001

- (i) In diffraction due to a single slit the path difference is given by :

$$\Delta x = a \sin \theta$$

where, a is the width of the slit

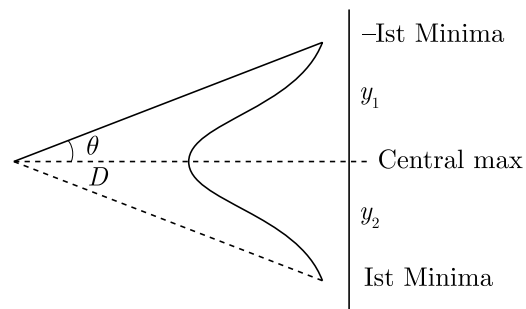
$$\text{For maxima, } \Delta x = (2x+1) \frac{\lambda}{2}$$

$$\Delta x = a \sin \theta = (2x+1) \frac{\lambda}{2}$$

$$\text{For } x=1, \quad \sin \theta = \frac{3 \cdot \lambda}{2a}$$

Let us divide the slit into three equal parts. If we take first two parts of slit, the path difference between rays diffracted from the extreme ends of first two parts.

$$\frac{2}{3} a \sin \theta = \frac{2}{3} a \times \frac{3\lambda}{2a} = \lambda$$



Then the first two parts will have a path difference of $\frac{\lambda}{2}$ and cancel the effect of each other. The remaining third part will contribute to the intensity at a point between two minima. This is called first secondary maxima. In similar manner we can show that the intensity of the other secondary maxima will go on decreasing.

- (ii) For first minima,

$$a \sin \theta = \lambda$$

$$a \theta = \lambda$$

Since, for small angle $\sin \theta \simeq \theta$

$$\tan \theta = \frac{y_1}{D}$$

$$\theta = \frac{y_1}{D}$$

$$\frac{\lambda}{a} D = y_1 = y_2$$

Hence, whole width on secondary maxima on one side is $\frac{\lambda D}{a}$.

The angular width of the central maxima = $\frac{2\lambda D}{a}$

So, angular width of the central maxima is twice that of the first order secondary maximum.

112. 1. What is diffraction and explain diffraction at a single slit.
 2. Show that the central maximum in the single slit diffraction is twice as wide as the second maxima and the pattern becomes narrow as the width of slit is increased.

or

What is single slit diffraction, explain it with diagram?

Ans :

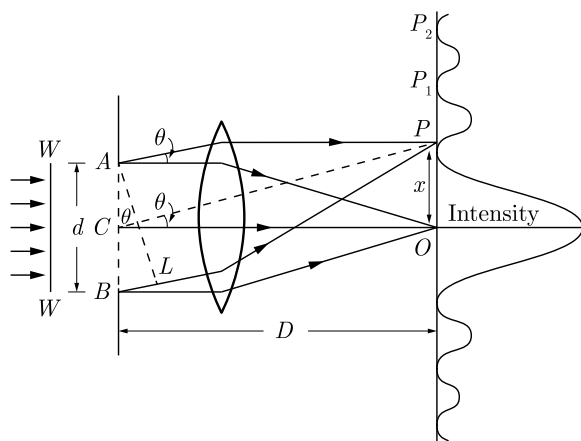
OD 2019

1. **Diffraction** : Is the phenomenon of bending of light around the corner of an obstacle or aperture.

Diffraction of Light by Narrow Slit

Consider a narrow slit AB perpendicular to plane of paper on which a monochromatic light be incident normally. Due to diffraction pattern consists of a central band much wider than the slit width just opposite to the slit and bordered on either side by dark and bright fringes of decreasing intensity. It is found that,

- The intensity of the secondary maxima goes on decreasing.
- The width of central maximum is double that of a secondary maximum.



Consider a plane wave-front WW incident on the slit AB as shown in the Figure. Imagine the slit to be divided into a large number of very narrow strips of equal width parallel to the slit. When the wave-front reaches the slit, each narrow strip parallel to the slit can be considered to be a source of Huygen's secondary wavelets.

Central Maxima

Let us first consider the effect of all the wavelets at the point O just opposite to the middle of the slit. Since distance between the slit and the screen is very large with the width of the slit, the wavelets covers the same distance and reach O in the same phase. Thus the wavelets reinforce each other's effect to give maximum intensity at O .

Let us now consider the intensity at a point P above O produced by the rays travelling at an $\angle \theta$ with the original direction. Since the width of the slit is very small, all the rays reaching P proceed almost parallel to each other. The wavelets from all the points in the slit start in the same phase but as they travel different distances, they reach P in different phases. Draw AL normal to BP .

The path difference between the extreme rays is,

$$BP - AP = BL$$

Let, C be the mid-point of slit or path difference between AP and $CP = \frac{\lambda}{2}$

Hence, Wavelets starting from A and C shall reach the point P in opposite phases and cancel each other effect. Similarly, the wavelets from a strip below A are cancelled by the wavelets from a corresponding strip below C .

$$\text{In } \triangle ABL, \quad \frac{BL}{AB} = \sin \theta$$

$$\text{Hence,} \quad BL = AB \sin \theta$$

$$BL = d \sin \theta$$

For first minima, $BL = \lambda$;

$$\text{Hence,} \quad d \sin \theta = \lambda$$

$$\sin \theta = \frac{\lambda}{d}$$

Since θ is small.

$$\text{Hence,} \quad \sin \theta = \theta = \frac{x}{D}$$

$$\frac{x}{D} = \theta = \frac{\lambda}{d}$$

$$\text{or} \quad x = \frac{D}{d} \lambda$$

At a point slightly above P i.e. P_1 , the path difference between the two rays is BL_1 .

$$\text{If,} \quad BL_1 = \frac{3}{2} \lambda$$

Then slit AB can be divided into three equal parts as shown in the figure. The waves from corresponding points AC_1 and C_1C_2 cancel each other. The waves from C_2B produce secondary maximum at an angle θ_1 given by,

$$d \sin \theta_1 = \frac{3}{2} \lambda$$

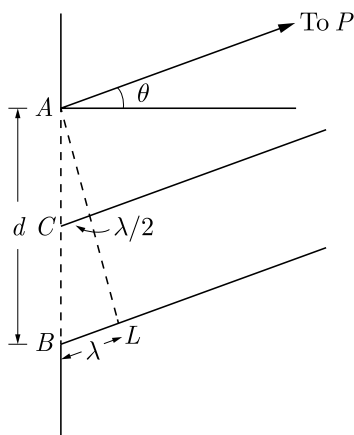
$$\sin \theta_1 = \frac{3}{2} \frac{\lambda}{d}$$

$$\text{If } \theta_1 \text{ is small then} \quad \sin \theta_1 = \theta_1$$

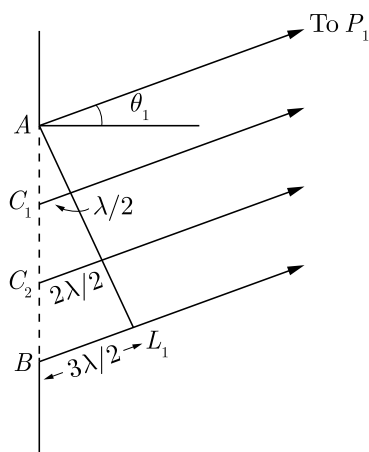
$$\theta_1 = \frac{3}{2} \frac{\lambda}{d}$$

At another point P_2 , (figure (c)), the path difference between two rays is $BL_2 = 2\lambda$, then, the slit AB can be supposed to be divided into four equal parts. The

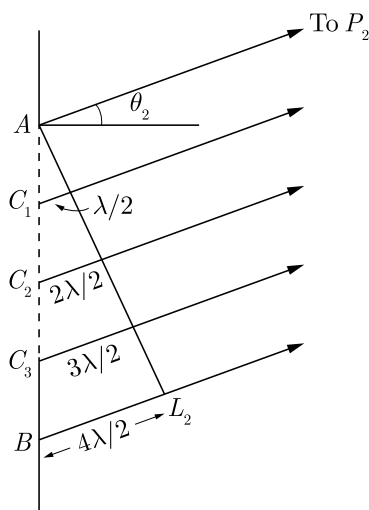
waves from corresponding points of the parts AC_1 and C_1C_3 cancel each other. Also the waves from the corresponding points C_2C_3 and C_3B cancel each other producing second minima at angle θ_2 given by,



(a)



(b)



(c)

$$BL_2 = d \sin \theta_2 = 2\lambda$$

$$\sin \theta_2 = \frac{2\lambda}{d}$$

$$\theta_2 = \frac{2\lambda}{d} \text{ provided } \theta_2 \text{ is small}$$

In general, the various minima occur if the path difference between extreme rays is even multiple of $\frac{\lambda}{2}$.

i.e.,
$$d \sin \theta_n = \frac{2n\lambda}{2} = n\lambda$$

$$\sin \theta_n = \frac{n\lambda}{d}$$

$$\frac{x}{D} = \theta_n = \frac{n\lambda}{d}$$

$$(\text{As, } \theta_n \text{ is very small, } \sin \theta_n \approx \theta_n)$$

The intensity of central fringes is maximum whereas that of other fringes falls off rapidly in either direction from the centre of the fringe pattern.

- Width of central maxima is the distance between first secondary minimum on either side of central point in front of slit.

Hence, $d \sin \theta = \lambda$ for first secondary minima

or
$$\sin \theta = \frac{\lambda}{d}$$

If f is the focal length of the focussing lens (on screen side), held close to the slit, then D , the distance of slit from the screen is equal to f i.e.,

$$D = f$$

Hence,
$$\sin \theta \approx \theta = \frac{y}{f} = \frac{y}{D}$$

$$\frac{y}{D} = \frac{\lambda}{d}$$

$$y = \frac{D\lambda}{d}$$

Since θ is half the angular width of central maxima of single slit,

Hence, Width of central maxima

$$= 2y = \frac{2D\lambda}{d} = \frac{2f\lambda}{d} \quad \dots(1)$$

Width of secondary maxima is the distance between n th and $(n+1)$ th maxima.

We know for a n th minima

$$d \sin \theta_n = n\lambda$$

$$\sin \theta_n = \frac{n\lambda}{d}$$

and

$$\sin \theta_{n+1} = (n+1) \frac{\lambda}{d}$$

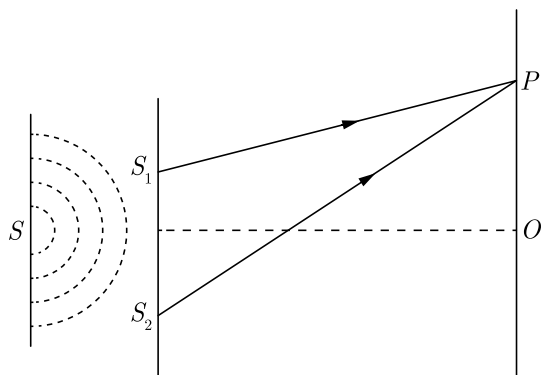
Hence,
$$\sin \theta_{n+1} - \sin \theta_n = [n+1 - n] \frac{\lambda}{d} = \frac{\lambda}{d}$$

Since angles are small,

$$\sin \theta \approx \theta$$

$$a^2 + b^2(\cos^2 \phi + \sin^2 \phi) + 2ab \cos \phi = A^2(\cos^2 \theta + \sin^2 \theta)$$

$$A^2 = a^2 + b^2 + 2ab \cos \phi$$



(a)

Since resultant intensity is proportional to square of amplitude,

Hence, $I \propto A^2$

or $I \propto (a^2 + b^2 + 2ab \cos \phi)$

or $I = a^2 + b^2 + 2ab \cos \phi \quad \dots(3)$

(For convenience, we assume that intensity of light is equal to square of amplitude)

1. For constructive interference (or for maximum).

I should be maximum

i.e., $\cos \phi = 1$

and $I_{\max} = (a + b)^2$

$$\phi = 0, 2\pi, 4\pi$$

$$\phi = 2\pi n$$

$$\text{Path diff. } \frac{\lambda}{2\pi} \phi = \frac{\lambda}{2\pi} 2\pi n = n\lambda \quad \dots(4)$$

So, for constructive interference (or for maximum intensity), path difference should be integral multiple of λ .

2. For destructive interference (or for minima)

I should be minima

i.e. $\cos \phi = -1$

and $I_{\min} = (a - b)^2$

$$\phi = \pi, 3\pi, 5\pi, \dots$$

$$\phi = (2n + 1)\pi \text{ (where, } n = 0, 1, 2, \dots)$$

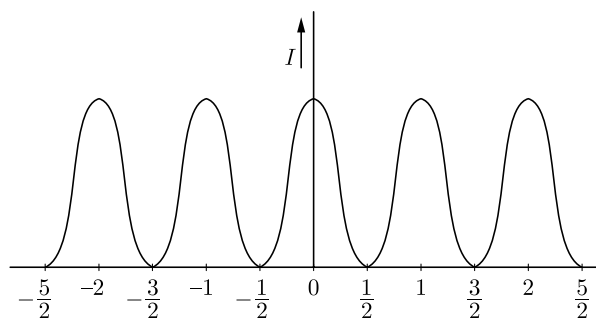
$$\begin{aligned} \text{Path difference} &= \frac{\lambda}{2\pi} (2n + 1)\pi \\ &= (2n + 1) \frac{\lambda}{2} \quad \dots(5) \end{aligned}$$

So, for destructive interference (or for minimum intensity), path difference should be an odd multiple of $\frac{\lambda}{2}$.

Variation of Intensity with Position

From above discussion, we find that intensity of light is maximum when path difference is $n\lambda$ i.e. $\pm \lambda, 2\lambda, 3\lambda \dots$ and minimum when path difference is $(2n + 1)\frac{\lambda}{2}$ i.e. $\pm \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \dots$

The variation in intensity of light with position in interference is as shown in Figure (b).



(b)

110. Explain Huygens principle of secondary wavelets and on the basis of this principle establish the law of Refraction 'OR' reflection.

Ans :

Delhi 2021, OD 2017

Wavefront

A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant.

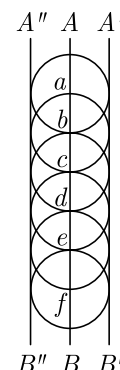
Huygen's Principle

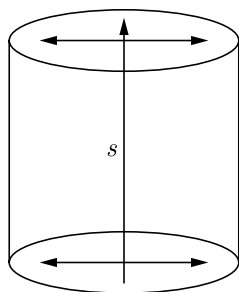
1. Every point of primary wavefront acts as source of secondary wavelets.
2. The common envelope over secondary wavelets gives the formation of secondary wavefront.

AB - Primary wavefront

$A'B'$ - Forward secondary wavefront

$A''B''$ - Backward secondary wavefront

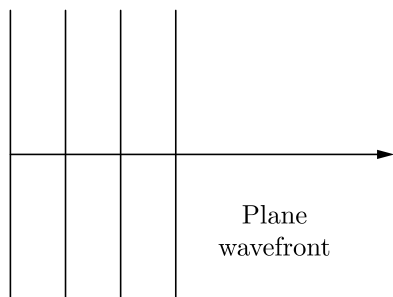




(b)

3. **Plane Wave-front :** When the source is at infinite distance, the radius of curvature of wave-front will be very large.

A small portion of spherical wave-front with source at infinite distance is a plane wave-front. (Fig. (c)).



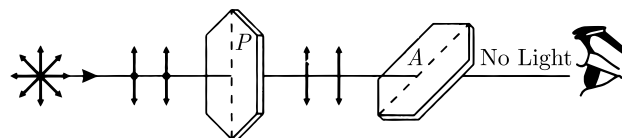
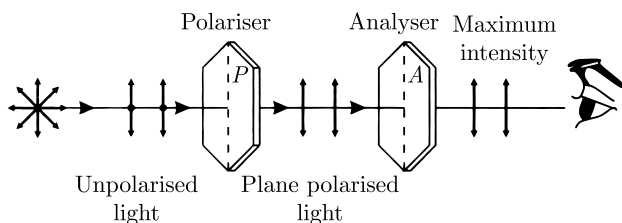
(c)

115. Describe an experiment to show that light waves are transverse in nature.

Ans :

Delhi 2018, Comp 2013

When ordinary (unpolarised) light is passed through a tourmaline crystal P , called polariser, its intensity is cut down to half. We get plane polarised light. This is possible only if light is a transverse wave. To further confirm this, a similar crystal A called analyser, is placed in the path of the beam. When the axes of the two crystals are parallel, maximum intensity is transmitted by A . If crystal A is rotated in its own plane, the intensity of light gradually decreases until it becomes zero when the axes of the two crystals are perpendicular to each other. This confirms the transverse nature of light waves.



116. (i) State the essential conditions for diffraction of light.
 (ii) Explain diffraction of light due to a narrow single slit and the formation of pattern of fringes on the screen.
 (iii) Find the relation for width of central maximum in terms of wavelength λ , width of slit a , and separation between slit and screen D .
 (iv) If the width of the slit is made double the original width, how does it affect the size and intensity of the central band?

Ans :

OD 2016

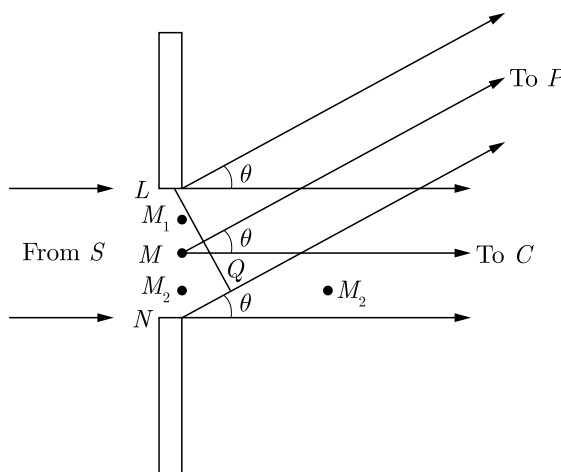
- (i) Essential conditions for diffraction of light
 (a) Sources of light should be monochromatic.
 (b) Wavelength of the light used should be comparable to the size of the obstacle.
 (ii) Single slit diffraction is explained by treating different parts of the wavefront at the slit as sources of secondary wavelets.

At the central point C on the screen, θ is zero, All path differences are zero and give maximum intensity at C .

At any other point P , the path difference between two edges of the slit is $NP - LP = NQ$

$$= a \sin \theta \simeq a\theta$$

Any point P , in direction θ , is a location of minima if $a\theta = n\lambda$



This can be explained by dividing the slit into even number of parts. The path difference between waves from successive parts is 180° out of phase and hence cancel each other leading to a minima.

Any point P , in direction Q , is a location of maxima if $a\theta = (n + \frac{1}{2})\lambda$

This can be explained by dividing the slit into odd number of parts. The contributions from successive parts cancel in pairs because of 180° phase difference. The unpaired part produce intensity at P , leading to a minima.

(iii) If θ is the direction of first minima, then $a\theta = \lambda$

$$\theta = \frac{\lambda}{a}$$

$$\text{Angular width of central maxima} = 2\theta = \frac{2\lambda}{a}$$

Linear width of central maxima,

$$\beta = 2\theta \cdot D = \frac{2\lambda D}{a}$$

(iv) If a is doubled, β becomes half and intensity becomes 4 times.

117. (a) (i) 'Two independent monochromatic sources of light cannot produce a sustained interference pattern'. Give reason.
- (ii) Light waves each of amplitude a and angular frequency ω , emanating from two coherent light sources superpose at a point. If the displacements due to these waves is given by $y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$ where ϕ is the phase difference between the two. Obtain the expression for the resultant intensity at the point.
- (b) In Young's double slit experiment, using monochromatic light of wavelength λ , the intensity of light at a point on the screen where path difference is λ , is K units. Find out the intensity of light at a point where path difference is $\frac{\lambda}{3}$.

Ans :

OD 2014

- (a) (i) The light waves, originating from two independent monochromatic sources, will not have a constant phase difference for wave reaching at a point. Therefore, these sources will not be coherent and therefore would not produce a sustained interference pattern.
- (ii) The resultant displacement will be given by

$$\begin{aligned} y &= y_1 + y_2 \\ &= a \cos \omega t + a \cos(\omega t + \phi) \\ &= a[\cos \omega t + \cos(\omega t + \phi)] \\ &= 2a \cos \frac{\phi}{2} \cos\left(\omega t + \frac{\phi}{2}\right) \end{aligned}$$

The amplitude of the resultant displacement is $2a \cos(\frac{\phi}{2})$. The intensity of light is directly proportional to the square of amplitude of the

wave. The resultant intensity will be given by

$$I = 4a^2 \cos^2 \frac{\phi}{2}$$

- (b) A path difference of λ , corresponds to phase difference of 2π

$$\text{Intensity, } I = 4a^2$$

$$a^2 = \frac{1}{4}$$

A path difference of $\frac{\lambda}{3}$, corresponds to a phase difference of $\frac{2\pi}{3}$

$$\text{Intensity} = 4 \times \frac{1}{4} \cdot \cos^2 \frac{2\pi}{3} = \frac{1}{4}$$

118. (a) In Young's double slit experiment, deduce the conditions for (i) constructive and (ii) destructive interference at a point on the screen. Draw a graph showing variation of the resultant intensity in the interference pattern against position X on the screen.
- (b) Compare and contrast the pattern which is seen with two coherently illuminated narrow slits in Young's experiment with that seen for a coherently illuminated single slit producing diffraction.

Ans :

Comp 2012, OD 2016

- (a) **Conditions of Constructive and Destructive Interference :** When two difference travel in the same direction along a straight line simultaneously, they superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. The phenomenon of redistribution of intensity due to superposition of two waves of same frequency and constant initial phase difference is called the interference. The waves of same frequency and constant initial phase difference are called coherent waves. At points of medium where the waves arrive in the same phase, the resultant intensity is maximum and the interference at these points is said to be constructive. On the other hand, at points of medium where the waves arrive in opposite phase, the resultant intensity is minimum and the interference at these points is said to be destructive. The positions of maximum intensity are called maxima while those of minimum intensity are called minima. The interference takes place in sound and light both.

Mathematical Analysis : Suppose two coherent waves travel in the same direction along a straight line, the frequency of each wave is $\frac{\omega}{2\pi}$ and amplitudes of electric field are a_1 and a_2 respectively. If at any time t , the electric fields of waves at a point are y_1 and y_2 respectively and phase difference is ϕ , then equation of waves may be expressed as

$$y_1 = a_1 \sin \omega t \quad \dots(1)$$

$$y_2 = a_2 \sin(\omega t + \phi) \quad \dots(2)$$

According to Young's principle of superposition, the resultant displacement at that point will be

$$y = y_1 + y_2 \quad \dots(3)$$

Substituting values of y_1 and y_2 from (1) and (2) in (3), we get

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

Using trigonometric relation,

$$\sin(\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi$$

$$y = a_1 \sin \omega t + a_2 (\sin \omega t \cos \phi + \cos \omega t \sin \phi)$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + (a_2 \sin \phi) \cos \omega t \quad \dots(4)$$

$$\text{Let, } a_1 + a_2 \cos \phi = A \cos \theta \quad \dots(5)$$

$$\text{and } a_2 \sin \phi = A \sin \theta \quad \dots(6)$$

Where A and θ are new constants

Then equation (4) gives

$$y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t \\ = A \sin(\omega t + \theta) \quad \dots(7)$$

This is equations of the resultant disturbance. Clearly the amplitude of resultants disturbance is A and phase difference from first wave is θ . The values of A and θ are determined by (5) and (6).

Squaring (5) and (6) and then adding, we get

$$(a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2 = A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

$$a_1^2 + a_2^2 \cos^2 \phi + 2a_1 a_2 \cos \phi + a_2^2 \sin^2 \phi = A^2 (\cos^2 \theta + \sin^2 \theta)$$

As, $\cos^2 \theta + \sin^2 \theta = 1$, we get

$$A^2 = a_1^2 + a_2^2 (\cos^2 \phi + \sin^2 \phi) + 2a_1 a_2 \cos \phi$$

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

$$\text{Amplitude, } A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi} \quad \dots(8)$$

As the intensity of a wave is proportional to its amplitude in arbitrary units $I = A^2$

Intensity of resultant wave,

$$I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi \quad \dots(9)$$

Clearly the intensity of resultant wave at any point depends on the amplitudes of individual waves and the phase difference between the waves at the point.

Constructive Interference : For maximum intensity at any point $\cos \phi = +1$

or phase difference $\phi = 0, 2\pi, 4\pi, 6\pi \dots$

$$= 2n\pi (n = 0, 1, 2 \dots) \quad \dots(10)$$

The maximum intensity,

$$I_{\max} = a_1^2 + a_2^2 + 2a_1 a_2$$

$$= (a_1 + a_2)^2 \quad \dots(11)$$

$$\text{Path difference, } \Delta = \frac{\lambda}{2\pi} \times \text{Phase difference}$$

$$= \frac{\lambda}{2\pi} \times 2n\pi = n\lambda \quad \dots(12)$$

Clearly, the maximum intensity is obtained in the region of superposition at those points where waves meet in the same phase or the phase difference between the waves is even multiple of π or path difference between them is the integral multiple of λ and maximum intensity is $(a_1 + a_2)^2$ which is greater than the sum intensities of individual waves by an amount $2a_1 a_2$.

Destructive Interference : For minimum intensity at any point $\cos \phi = -1$

or phase difference, $\phi = \pi, 3\pi, 5\pi, 7\pi, \dots$

$$= (2n - 1)\pi, n = 1, 2, 3, \dots \dots(13)$$

In this case the minimum intensity,

$$I_{\max} = a_1^2 + a_2^2 - 2a_1 a_2$$

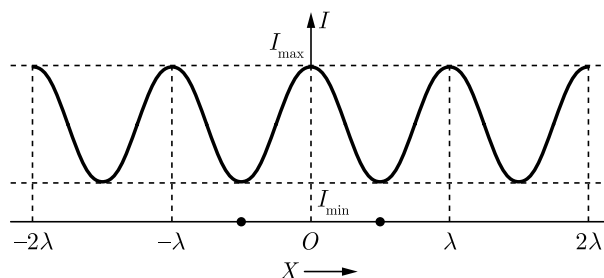
$$= (a_1 - a_2)^2 \dots(14)$$

$$\text{Path difference, } \Delta = \frac{\lambda}{2\pi} \times \text{Phase difference}$$

$$= \frac{\lambda}{2\pi} \times (2n - 1)\pi$$

$$= (2n - 1) \frac{\lambda}{2}$$

Clearly, the minimum intensity is obtained in the region of superposition at those points where waves meet in opposite phase or the phase difference between the waves is odd multiple of π or path difference between the waves is odd multiply of $\frac{\lambda}{2}$ and minimum intensity $= (a_1 - a_2)^2$ which is less than the sum of intensities of the individual waves by an amount $2a_1 a_2$.



From equations (12) and (14), it is clear that the intensity $2a_1 a_2$ is transferred from positions of minima to maxima. This implies that the interference is based on conservation of energy i.e., there is no wastage of energy.

Variation of intensity of light with position x is shown in figure.

$$\begin{aligned}\theta_{n+1} - \theta_n &= \frac{\lambda}{d} \\ \Delta \theta_n &= \frac{\lambda}{d} \\ \frac{y}{D} &= \frac{\lambda}{d} \quad \dots(2)\end{aligned}$$

Thus we find that equation (1) is twice the equation (2) showing there by that the central maxima is twice any secondary maxima.

If the width of slit is increased then as y decreases which shows that pattern will become narrower as the width of the slit is increased.

113. What are unpolarised and polarised waves? Explain polarisation, taking an example of mechanical waves.

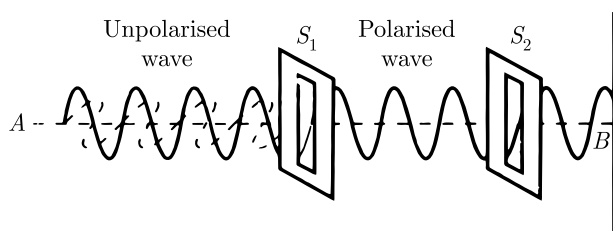
Ans :

OD 2015, Comp 2007

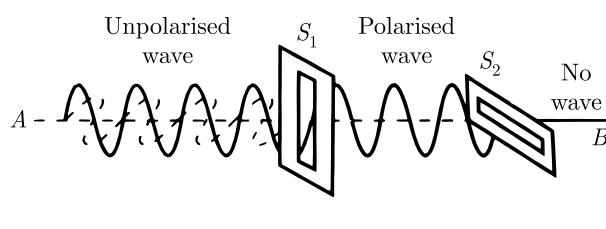
A transverse wave in which vibrations are present in all possible directions, in a plane perpendicular to the direction of propagation is said to be **unpolarised**. If the vibrations of a wave are present in just one direction in a plane perpendicular to the direction of propagation, the wave is said to be **polarised** or **plane polarised**. The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called **polarisation of waves**.

Experimental Demonstration with Mechanical Waves

Consider a long string AB passing through two rectangular slits S_1 and S_2 , as shown in figure. The end B of the string is tied to a hook in a wall and the free end A is jerked in all possible directions perpendicular to the length of the string so as to generate transverse waves in it. The portion AS_1 of the string has vibrations in all directions perpendicular to AB , so that the wave is unpolarised. The first slit S_1 will permit only those vibrations to pass through it which are parallel to the slit S_1 and will cut off all other vibrations. Thus the wave emerging from the slit S_1 is plane polarised. The slit S_1 is called the polariser. If the second slit S_2 , called the analyser, is held parallel to S_1 will pass through S_2 unchanged. If S_2 is held perpendicular to S_1 , no vibrations will emerge from the slit S_2 . This indicates that the slit S_1 has polarised the incoming wave in the vertical plane.



(a)



(b)

114. What do you mean by wave front? Write down its type briefly?

Ans :

OD 2020

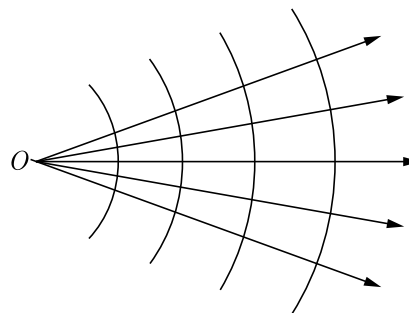
Wave front is defined as the locus of the neighboring particles of the medium which are in the same phase of disturbance.

Wave front are classified in the following types :

1. **Spherical Wave-front** : A wave-front due to point source in a three dimensional isotropic space is called spherical wave-front.

Consider a point source of light placed in an isotropic medium i.e., a medium in which the velocity of light is the same in all directions. The disturbance from the point source travels equally in all directions.

If c = velocity of light, then after time t , light will have travelled a distance equal to ct in all directions. The locus of all points where light reaches after time t will be a sphere. A set of spherical wave-front represents a diverging beam of light (Fig. (a)).



(a)

2. **Cylindrical Wave-front** : When the slit is a source of light, all points equidistant from the source, lie on a cylinder. Therefore, wave-front is cylindrical (Fig. (b)).

1. When $\phi = 0$, then

$$I_R = 5I + 4I \cos 0 = 9I$$

2. When $\phi = \frac{\pi}{2}$, then

$$I_R = 5I + 4I \cos \frac{\pi}{2} = 5I$$

3. When $\phi = \pi$, then

$$I_R = 5I + 4I \cos \pi = 5I - 4I = I$$

- 124.** Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2×10^{-4} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

Ans :

Delhi 2020

For maxima other than central maxima.

$$a \cdot \theta = \left(n + \frac{1}{2}\right) \lambda$$

and

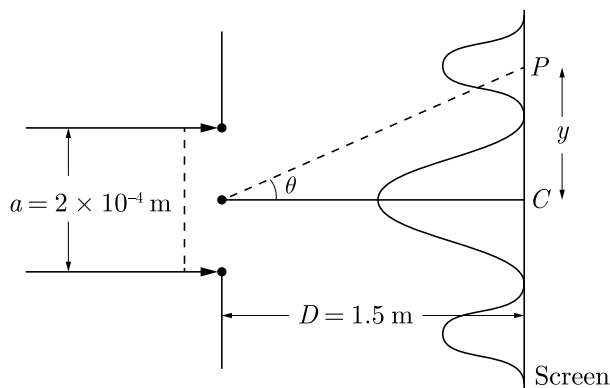
$$\theta = \frac{y}{D}$$

Hence,

$$a \cdot \frac{y}{D} = \left(n + \frac{1}{2}\right) \lambda$$

For light of wavelength $\lambda_1 = 590$ nm

$$\begin{aligned} 2 \times 10^{-4} \times \frac{y_1}{1.5} &= \left(1 + \frac{1}{2}\right) \times 590 \times 10^{-9} \\ y_1 &= \frac{3}{2} \times \frac{590 \times 10^{-9} \times 1.5}{2 \times 10^{-4}} \\ &= 6.64 \text{ mm} \end{aligned}$$



For light of wavelength $\lambda_2 = 596$ nm

$$\begin{aligned} 2 \times 10^{-4} \times \frac{y_2}{1.5} &= \left(1 + \frac{1}{2}\right) \times 596 \times 10^{-9} \\ y_2 &= \frac{3}{2} \times \frac{596 \times 10^{-9} \times 1.5}{2 \times 10^{-4}} \\ &= 6.705 \text{ mm} \end{aligned}$$

Separation between two positions of first maxima

$$\begin{aligned} \Delta y &= y_2 - y_1 \\ &= 6.705 - 6.64 = 0.065 \text{ mm} \end{aligned}$$

- 125.** A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffracting pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Calculate the width of the slit.

Ans :

OD 2013, Comp 2002

The distance of the n th minimum from the centre of the screen is

$$x_n = \frac{nD\lambda}{d} \quad \dots(1)$$

where, D = distance of slit from screen

λ = wavelength of the light

d = width of the slit for first minimum,

$n = 1$

$$x_n = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$$

$$D = 1 \text{ m}$$

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

Putting this values in equations (1), we get

$$\begin{aligned} 2.5 \times 10^{-3} &= \frac{1(1)(500 \times 10^{-9})}{d} \\ d &= 2 \times 10^{-4} \text{ m} = 0.2 \text{ mm} \end{aligned}$$

- 126.** In Young's double slit experiment, the slits are 0.2 mm apart and the screen is 1.5 m away. It is observed that the distance between the central bright fringe and fourth dark fringe is 1.8 cm. Find the wavelength of light used.

Ans :

Foreign 2013

Given,

$$d = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m}$$

$$D = 1.5 \text{ m}$$

$$x'_4 = 1.8 \text{ cm} = 1.8 \times 10^{-2} \text{ m}$$

The distance of n^{th} dark fringe from the central bright fringe is given by,

$$x'_n = (2n - 1) \frac{D\lambda}{2d}$$

Hence,

$$x'_4 = \frac{7}{2} \cdot \frac{D\lambda}{d}$$

or

$$\begin{aligned} \lambda &= \frac{2dx'_4}{7D} \\ &= \frac{2 \times 0.2 \times 10^{-3} \times 1.8 \times 10^{-2}}{7 \times 1.5} \\ &= 6.86 \times 10^{-7} \text{ m} \end{aligned}$$

- 127.** (a) In Young's double slit experiment, two slits are 1 mm apart and the screen is placed 1 m away from the slits. Calculate the fringe width when light of wavelength 500 nm is used.

- (b) **Comparison between two Slit Young's Interference pattern and Single slit diffraction pattern :** Both patterns are the result of wave nature of light; both patterns contain maxima and minima. Interference pattern is the result of superposing two coherent wave while the diffraction pattern is the superposition of large number of waves originating from each point on a single slit.

Differences :

- In Young's two slit experiment; all maxima are of same intensity while in diffraction at a single slit, the intensity of central maximum is maximum and it falls rapidly for first, second order secondary maxima on either side of it.
- In Young's interference the fringes are of equal width while in diffraction at a single slit, the central maximum is twice as wide as other maxima. The intensity falls as we go to successive maxima away from the centre on either side.
- In a single slit diffraction pattern of width a , the first minimum occurs at $\frac{\lambda}{a}$; while in two slit interference pattern of slit separation a , we get maximum at the same angle $\frac{\lambda}{a}$.

NUMERICAL QUESTIONS

- 119.** How is linearly polarised light obtain by the process of scattering of light ? find the Brewster angle for air-glass interface, when the refractive index of glass = 1.5

Ans :

Comp 2017

According to Brewster law

$$\tan i_B = \mu$$

$$i_B = \tan^{-1}(\mu)$$

$$i_B = \tan^{-1}(1.5)$$

$$i_B = 56.30$$

- 120.** In double slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1° . What is the spacing between the two slits ?

Ans :

Delhi 2021, SQP 2005

Wavelength of light,

$$\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$$

$$\text{Angular width of fringe} \quad \theta = 0.1^\circ = \frac{0.1\pi}{180} \text{ rad}$$

$$\text{Using the formulae,} \quad \theta = \frac{\lambda}{d}$$

Spacing between the slits,

$$d = \frac{\lambda}{\theta} = \frac{600 \times 10^{-9} \times 180}{0.1 \times \pi}$$

$$d = 3.44 \times 10^{-4} \text{ m}$$

Thus, the spacing between the two slits is $3.44 \times 10^{-4} \text{ m}$.

- 121.** Two plane monochromatic waves propagating in the same direction with amplitudes A and $2A$ and differing in phase by $\frac{\pi}{3}$ superpose. Calculate the amplitude of the resultant wave.

Ans :

OD 2020

Given,

$$A_1 = A$$

$$A_2 = 2A$$

$$\phi = \frac{\pi}{3}$$

Hence,

$$\begin{aligned} A_R &= \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi} \\ &= \sqrt{A^2 + (2A)^2 + 2A \times 2A \cos \frac{\pi}{3}} \\ &= \sqrt{5A^2 + 4A^2 \times \frac{1}{2}} \\ &= 7A^2 = \sqrt{7} A \end{aligned}$$

- 122.** In Young's double-slit experiment, the angular width of fringe formed on a distance screen is 0.1° . If wavelength of light is $6000 \text{ \AA}$. What is the spacing between the slits?

Ans :

Delhi 2008

Angular width of fringe, $\theta = 0.1^\circ$

$$= 1.745 \times 10^{-3} \text{ rad}$$

Wavelength of light, $\lambda = 6000 \text{ \AA} = 6000 \times 10^{-3} \text{ m}$

Spacing between the slits,

$$\begin{aligned} d &= \frac{\lambda}{\theta} = \frac{6000 \times 10^{-10}}{1.745 \times 10^{-3}} \\ &= 3.4 \times 10^{-4} \text{ m} \end{aligned}$$

- 123.** Two sources of intensity I and $4I$ are used in an interference experiment. Find the intensity at points where the waves from two sources superimpose with a phase difference (1) zero (2) $\frac{\pi}{2}$ and (3) π .

Ans :

Delhi 2014

The resultant intensity at a point where phase difference is ϕ is,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

As,

$$I_1 = I$$

and

$$I_2 = 4I$$

Therefore,

$$\begin{aligned} I_R &= I + 4I + 2\sqrt{I \cdot 4I} \cos \phi \\ &= 5I + 4I \cos \phi \end{aligned}$$

- 131.** In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width X are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the interfering sources in the two arrangements.

Ans :

Delhi 2019, OD 2014

Fringe width X is same in both cases.

$$\text{In first case, } X = \frac{D_1 \lambda_1}{d}$$

$$\text{In second case, } X = \frac{D_2 \lambda_2}{\frac{d}{2}}$$

$$\text{Hence, } \frac{D_1 \lambda_1}{d} = \frac{D_2 \lambda_2}{\frac{d}{2}}$$

$$\text{or } \frac{D_1}{D_2} = 2 \cdot \frac{\lambda_2}{\lambda_1} = \frac{2 \times 600}{400} = \frac{3}{1} = 3:1$$

- 132.** In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. Calculate the fringe width if the entire apparatus is immersed in a liquid medium of refractive index 1.33.

Ans :

SQP 2009

$$\text{Given, } \beta = 2.0 \text{ mm}$$

$$\mu = 1.33$$

Refractive index of liquid,

$$\mu = \frac{\text{Wavelength of light in vacuum}}{\text{Wavelength of light in liquid}}$$

$$\text{or } \lambda' = \frac{\lambda}{\mu}$$

$$\text{Fringe width in air, } \beta = \frac{D\lambda}{d}$$

$$\begin{aligned} \text{Fringe width in liquid, } \beta' &= \frac{D\lambda'}{d} = \frac{D\lambda}{d\mu} = \frac{\beta}{\mu} \\ &= \frac{2.0 \text{ mm}}{1.33} = 1.5 \text{ mm} \end{aligned}$$

- 133.** Find the ratio of intensities of two points P and Q on a screen in a Young's double slit experiment when waves from sources S_1 and S_2 have phase difference of (1) $\frac{\pi}{3}$ and (2) $\frac{\pi}{2}$ respectively.

Ans :

Delhi 2020

As we know that

$$I = 2I_0(1 + \cos \phi)$$

$$I \propto (1 + \cos \phi)$$

$$\text{Hence, } \frac{I_P}{I_Q} = \frac{1 + \cos \frac{\pi}{3}}{1 + \cos \frac{\pi}{2}} = \frac{1 + \frac{1}{2}}{1 + 0} = \frac{3}{2} = 3:2$$

- 134.** In Young's double slit experiment, two slits are separated by 3 mm distance and illuminated by light of wavelength 480 nm. The screen is 2 m from the plane of the slits. Calculate the separation between the 8th bright fringe and the 3rd dark fringe observed with respect to the central bright fringe.

Ans :

OD 2017

As know that,

$$\begin{aligned} \text{Required distance} &= x_8 - x'_3 \\ &= \frac{8D\lambda}{d} - \frac{5D\lambda}{2d} \\ &= \frac{11D\lambda}{2d} \\ &= \frac{11 \times 2 \times 480 \times 10^{-9}}{2 \times 3 \times 10^{-3}} \\ &= 1.76 \times 10^{-3} \text{ m} = 1.76 \text{ mm} \end{aligned}$$

- 135.** Width of two slits in Young's experiment are in ratio of 1 : 4 What is the ratio of intensities of light wave from them?

Ans :

OD 2012

The width of a slit is directly proportional to the amplitude of light (i.e. electric vector).

If W_1 and W_2 are the width of slits and a_1 and a_2 are their respective amplitude of light waves, then

$$W_1 \propto a_1^2$$

$$\text{and } W_2 \propto a_2^2$$

$$\text{Hence, } \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2}$$

$$\frac{1}{4} = \frac{a_1^2}{a_2^2}$$

$$\frac{a_1}{a_2} = \frac{1}{2} \quad \dots(1)$$

If, $I_{\max}$ and $I_{\min}$ are maximum and minimum values of intensities of light, then

$$\begin{aligned} \frac{I_{\max}}{I_{\min}} &= \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{a_1}{a_2} + 1\right)^2 \\ &= \left(\frac{1}{2} + 1\right)^2 = \frac{9}{1} = 9 \end{aligned}$$

- 136.** What is the ratio of slit widths, when amplitude of light wave from them have a ratio $\frac{\sqrt{2}}{1}$?

Ans :

OD 2017

If W_1 and W_2 are the widths of slits and a_1 and a_2 respectively, the amplitude of light waves then, we have

$$\begin{aligned} \frac{W_1}{W_2} &= \frac{a_1^2}{a_2^2} = \left(\frac{a_1}{a_2}\right)^2 \\ &= \left(\frac{\sqrt{2}}{1}\right)^2 = \frac{2}{1} \end{aligned}$$

- (b) What should be the width of each slit in order to obtain 10 maxima of the double slits pattern within the central maximum of the single slit pattern ?
- (c) The intensity at the central maxima in Young's double slit experiment is I_0 . Find out the intensity at a point where the path difference is $\frac{\lambda}{6}$, $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$

Ans :

SQP 2016

- (a) Fringe width is given by

$$\begin{aligned}\beta &= \frac{\lambda D}{d} \\ &= \frac{500 \times 10^{-9} \times 1}{10^{-3}} = 0.5 \text{ mm} \\ &= 0.5 \times 10^{-3} \text{ m} \\ &= 5 \times 10^{-4} \text{ m}\end{aligned}$$

- (b) $\beta_0 = \frac{2\lambda D}{d} = 10\beta$

$$\begin{aligned}d &= \frac{2 \times 500 \times 10^{-9} \times 1}{10 \times 5 \times 10^{-4}} \\ &= 2 \times 10^{-4} \text{ m}\end{aligned}$$

- (c) The general expression, for the intensity, at a point is

$$I = I_0 \cos^2 \frac{\phi}{2}$$

- (i) For path difference $= \frac{\lambda}{6}$, $\phi = 60^\circ$

$$I = \frac{3I_0}{4}$$

[For path difference λ , phase difference $\phi = 2\pi$]

- (ii) For path difference $= \frac{\lambda}{4}$, $\phi = 90^\circ$

$$I = \frac{I_0}{2}$$

- (iii) For path difference $= \frac{\lambda}{3}$, $\phi = 120^\circ$

$$I = \frac{I_0}{4}$$

- 128.** In Young's double experiment, the two parallel slits are made one millimetre apart and a screen is placed one metre away. What is the fringe separation when blue green of wavelength 500 nm is used?

Ans :

OD 2015, Foreign 2001

Given, $d = 1 \text{ mm} = 10^{-3} \text{ m}$
 $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$

Fringe width, $\beta = \frac{D\lambda}{d}$

$$\begin{aligned}&= \frac{1 \times 500 \times 10^{-9}}{10^{-3}} \text{ m} \\ &= 5 \times 10^{-4} \text{ m} = 0.5 \text{ mm}\end{aligned}$$

- 129.** Yellow light of wavelength $6000 \text{ \AA}$ produces fringes of width 0.8 mm in Young's double slit experiment. What will be the fringe width if the light source is replaced by another monochromatic source of wavelength $7500 \text{ \AA}$ and the separation between the slits is doubled?

Ans :

Delhi 2017

Given, $\lambda_1 = 6000 \text{ \AA}$
 $\beta_1 = 0.8 \text{ mm}$
 $\lambda_2 = 7500 \text{ \AA}$

Fringe width in first case,

$$\beta_1 = \frac{D\lambda_1}{d}$$

Fringe width in second case,

$$\beta_2 = \frac{D\lambda_2}{2d}$$

Hence, $\frac{\beta_2}{\beta_1} = \frac{\frac{D\lambda_2}{2d}}{\frac{D\lambda_1}{d}} = \frac{1}{2} \cdot \frac{\lambda_2}{\lambda_1}$

$$\begin{aligned}\beta_2 &= \frac{1}{2} \cdot \frac{\lambda_2}{\lambda_1} \cdot \beta_1 \\ &= \frac{1}{2} \times \frac{7500 \text{ \AA}}{6000 \text{ \AA}} \times 0.8 \text{ mm} \\ &= 0.5 \text{ mm}\end{aligned}$$

- 130.** The fringe width in a Young's double slit interference pattern is $2.4 \times 10^{-4} \text{ m}$, when red light of wavelength $6400 \text{ \AA}$ is used. By how much will it change, if blue light of wavelength $4000 \text{ \AA}$ is used.

Ans :

Comp 2012

Given, $\beta_1 = 2.4 \times 10^{-4} \text{ m}$
 $\lambda_1 = 6400 \text{ \AA}$
 $\lambda_2 = 4000 \text{ \AA}$

As know that

$$\begin{aligned}\beta &= \frac{D\lambda}{d} \\ \frac{\beta_2}{\beta_1} &= \frac{\lambda_2}{\lambda_1} = \frac{4000}{6400} = \frac{5}{8}\end{aligned}$$

or

$$\begin{aligned}\beta_2 &= \frac{5}{8} \times \beta_1 \\ &= \frac{5}{8} \times 2.4 \times 10^{-4} \\ &= 1.5 \times 10^{-4} \text{ m}\end{aligned}$$

Decrease in fringe width

$$\begin{aligned}&= \beta_1 - \beta_2 \\ &= (2.4 - 1.5) \times 10^{-4} \\ &= 0.9 \times 10^{-4} \text{ m}\end{aligned}$$

- 142.** Green light of wavelength $5100 \text{ \AA}$ from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, What is the distance between the slits?

Ans :

Comp 2014

Wave length of green light,

$$\lambda = 5100 \text{ \AA} = 5100 \times 10^{-10} \text{ m}$$

Number of fringe, $n = 10$

Distance between slits and screen,

$$D = 200 \text{ cm} = 2 \text{ m}$$

and separation of fringes,

$$x = 2 \text{ cm} = 2 \times 10^{-3} \text{ m}$$

Separation of fringes,

$$\begin{aligned} x &= \frac{n\lambda D}{d} \\ d &= \frac{n\lambda D}{x} \\ &= \frac{10 \times (5100 \times 10^{-10}) \times 2}{2 \times 10^{-3}} \\ &= 5.1 \times 10^{-4} \text{ m} \end{aligned}$$

- 143.** In double slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1° . What is the spacing between the two slits ?

Ans :

Delhi 2016

Wavelength of light,

$$\begin{aligned} \lambda &= 600 \text{ nm} \\ &= 600 \times 10^{-9} \text{ m} \end{aligned}$$

Angular width of fringe, $\theta = 0.1^\circ = \frac{0.1\pi}{180} \text{ rad}$

Using the formulae, $\theta = \frac{\lambda}{d}$

Spacing between the slits,

$$\begin{aligned} d &= \frac{\lambda}{\theta} = \frac{600 \times 10^{-9} \times 180}{0.1 \times \pi} \\ d &= 3.44 \times 10^{-4} \text{ m} \end{aligned}$$

Thus, the spacing between the two slits is $3.44 \times 10^{-4} \text{ m}$

- 144.** In Young's double slit experiment, distance between slits is 0.28 mm and distance between slits and screen is 1.4 m. Distance between central bright fringe and third bright fringe is 0.9 cm. What is the wavelength of used light?

Ans :

Delhi 2017, SQP 2005

Distance between slits,

$$\begin{aligned} d &= 0.28 \text{ mm} \\ &= 0.28 \times 10^{-3} \text{ m} \end{aligned}$$

Distance between slits and screen,

$$D = 1.4 \text{ m}$$

Distance between central bright fringe and third bright fringe,

$$x_3 = 0.9 \text{ cm} = 0.9 \times 10^{-2} \text{ m}$$

and number of bright fringe,

$$n = 3$$

Distance of third bright fringe from the central fringe,

$$\begin{aligned} x_3 &= \frac{n\lambda D}{d} \\ \lambda &= \frac{x_3 \times d}{nD} \\ &= \frac{(0.9 \times 10^{-2}) \times (0.28 \times 10^{-3})}{3 \times 1.4} \\ &= 6 \times 10^{-7} \text{ m} = 6000 \text{ \AA} \end{aligned}$$

where,

λ = Wavelength of used light

- 145.** When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet, if the central fringe has to shift to the position occupied by 20th bright fringe?

Ans :

OD 2020

Initial thickness of sheet,

$$t_1 = 4.8 \text{ mm}$$

Initial no. of central bright fringes,

$$n_1 = 30$$

and final number of central bright fringe,

$$n_2 = 20$$

Final thickness of the sheet,

$$\begin{aligned} t_2 &= \frac{n_2}{n_1} \times t_1 \\ &= \frac{20}{30} \times 4.8 = 3.2 \text{ mm} \end{aligned}$$

- 146.** In a Young's double slit experiment, the intensity of light at a point on the screen where the path difference is λ is k units. Find the intensity at a point where the path difference is

$$(1) \frac{\lambda}{4}, (2) \frac{\lambda}{3} \text{ and } (3) \frac{\lambda}{2}.$$

Ans :

OD 2018

Intensity at any point on the screen,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Let I_0 be the intensity of either source.

Then, $I_1 = I_2 = I_0$

and $I = 2I_0(1 + \cos \phi) = 4I_0 \cos^2 \frac{\phi}{2}$

When, $p = \lambda$

$$\phi = 2\pi$$

- 137.** The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is $3.0 \times 10^8 \text{ ms}^{-1}$).

Ans :

Delhi 2011

As we know that, $\mu = \frac{c}{v}$

Here, $C = 3.0 \times 10^8 \text{ m/s}$

$$\mu = 1.5$$

$$\text{or } v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} \\ = 2 \times 10^8 \text{ ms}^{-1}$$

- 138.** What is the value of maximum amplitude produced due to interference of two waves is given by $y_1 = 4 \sin \omega t$ and $y_2 = 3 \cos \omega t$?

Ans :

Comp 2018

We have $y_1 = 4 \sin \omega t \quad \dots(1)$

$$y_2 = 3 \cos \omega t$$

$$y_2 = 3 \sin\left(\frac{\pi}{2} + \omega t\right) \quad \dots(2)$$

Compare Eq. (1) and (2) with a standard equation, we get

$$A_1 = 4$$

$$A_2 = 3$$

Initial phase difference, $\phi = \frac{\pi}{2}$

Resultant amplitude,

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\cos\phi = 1$$

For maximum amplitude,

Hence, $\phi = 0^\circ$

$$A = \sqrt{(A_1 + A_2)^2}$$

$$\text{So, } A_{\max} = A_1 + A_2 \\ = 3 + 4 = 7$$

- 139.** Two coherent sources have intensities in the ratio 25:16. Find the ratio of the intensities of maxima to minima, after interference of light occurs.

Ans :

Comp 2019, Delhi 2006

$$\text{Amplitude ratio, } r = \sqrt{\frac{I_1}{I_2}} \\ = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$\text{Hence, } \frac{I_{\max}}{I_{\min}} = \frac{(r+1)^2}{(r-1)^2} \\ = \frac{\left(\frac{5}{4} + 1\right)^2}{\left(\frac{5}{4} - 1\right)^2} = 81:1$$

- 140.** If the ratio of intensities of two waves causing interference be 9:4, What is the ratio of maximum and minimum intensities?

Ans :

Foreign 2015

Intensity of first wave,

$$I_1 = 9I$$

and intensity of second wave,

$$I_2 = 4I$$

Intensity of a wave,

$$I = 2\pi^2 v^2 a^2 \rho \quad v \propto a^2 \\ a \propto \sqrt{I}$$

$$\text{Therefore, } \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{9I}{4I}} = \frac{3}{2}$$

We also know that ratio of maximum to minimum intensities,

$$\left(\frac{I_{\max}}{I_{\min}}\right) = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \\ = \frac{(3 + 2)^2}{(3 - 2)^2} = \frac{25}{1}$$

$$I_{\max} : I_{\min} = 25 : 1$$

- 141.** Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B . What is the difference between the resultant intensities at A and B ?

Ans :

SQP 2010

Intensity of first beam,

$$I_1 = I$$

and intensity of second beam,

$$I_2 = 4I$$

Phase difference between the beams at point,

$$A(\phi_A) = \frac{\pi}{2}$$

and phase difference between the beams at point,

$$B(\phi_B) = \pi$$

The resultant intensity of beams at,

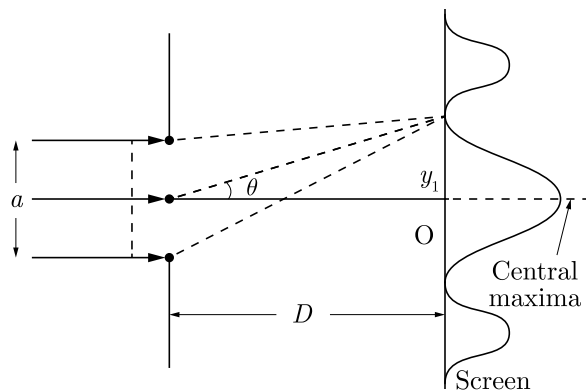
$$A(I_A) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi_A \\ = I + 4I + 2\sqrt{I \times 4I} \cos\left(\frac{\pi}{2}\right) \\ = 5I$$

Similarly, resultant intensity of beam at,

$$B(I_B) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi_B \\ = I + 4I + 2\sqrt{I \times 4I} \cos \pi = I$$

Therefore, difference between the resultant intensities at A and B

$$= I_A - I_B = 5I - I = 4I$$



Ans :

Foreign 2013, OD 2018

From condition of diffraction,

$$\sin \theta = n\lambda \quad (\text{for minima})$$

$$= \left(n + \frac{1}{2}\right)\lambda \quad (\text{for maxima})$$

Given $n = 1, 2, 3, \dots$

and $n = 0$ for central maxima

From condition of minima.

$$a \sin \theta = \lambda (n = 1)$$

Since the value of λ is of nm, so

$$a \cdot \theta = \lambda$$

$$a \cdot \frac{y}{D} = \lambda \quad \left[\text{angle} = \frac{\text{arc}}{\text{radius}} \right]$$

$$a = \frac{\lambda D}{y} = \frac{500 \times 10^{-9} \times 1}{2.5 \times 10^{-3}} \text{ m}$$

$$= 2 \times 10^{-4} \text{ m}$$

CASE BASED QUESTIONS

150. A child is observing a thin film such as a layer of oil on water show beautiful colours when illuminated by white light. He feels happy and surprised to see this. His teacher explains him the reason behind it. The child then gives an example of spreading of kerosene oil on water to prevent malaria and dengue.

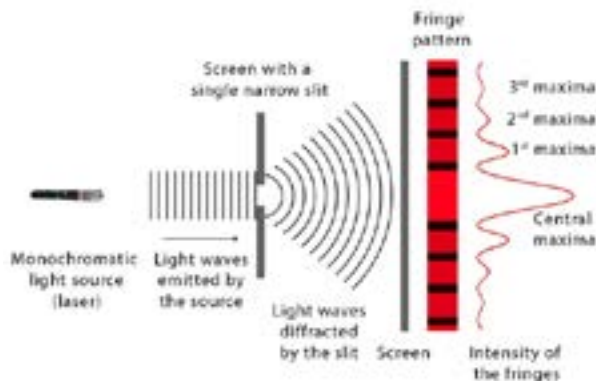


- Name the phenomenon involved ?
- What do you mean by interference?
- Write any four the necessary conditions for interference of light.

Ans :

- The phenomenon involved in a thin film is interference. Different colours of light interfere at different points in space and hence child is able to see different colours.
- Interference is defined as the phenomenon of superimposition of two or more waves of same frequency emitted by coherent sources such that the resultant wave is equal to the sum of the amplitude of the individual waves. Depending on whether the interference is constructive or destructive, the amplitude of the resultant wave can either be larger or smaller than the individual waves.
- The conditions for the interference of light are as under
 - Light source should be monochromatic.
 - Both the waves should be of same frequency.
 - Both the waves should be travel in the same direction.
 - The amplitude of both the waves should be nearly same or equal.
 - Both the sources(slits) should be very thin.

151. Sunil is using yellow light in a single slit diffraction experiment with slit width of 0.6 mm. When his friend replaced yellow light by X-rays then diffraction pattern is not visible. They became frightened and asked the teacher about such thing. The teacher replaces X-rays by yellow light and diffraction pattern appears again. Then teacher explained all the facts about diffraction properly



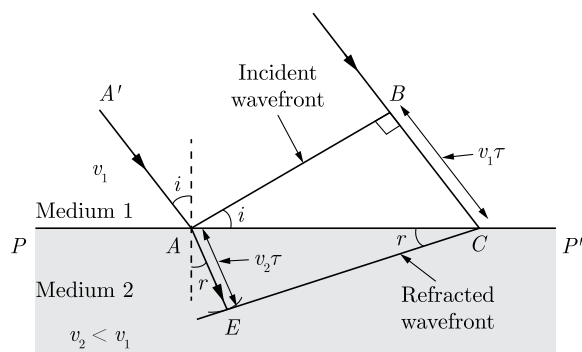
- What is the effect of wavelength of light on the fringe pattern?
- Give the necessary condition for the diffraction ?
- Why is diffraction pattern not visible by using X-rays in such case ?

Ans :

- (i) Increasing the wavelength of the light increases the spacing between different fringes since the spacing between different fringes is wavelength dependent.
- (ii) The size of the aperture/obstacle should be comparable to the wavelength of the light.
- (iii) As the wavelength of X -rays is much smaller than that of yellow light, so the diffraction pattern is not seen when the yellow light is replaced by X -rays in such condition.

- 152.** Waves change direction when they pass from one medium to another. This phenomenon is called refraction. Refraction can occur in different types of mediums like sound, water, etc. Refraction is the bending of light waves. The amount of refraction depends on the change in the speed of light and the angle at which the waves are bending.

Snell's law shows the relationship between angles of incidence and refraction, and the refractive index of each medium. The refractive index refers to the extent to which a medium can increase or decrease the speed of light.



- (i) What is the angle made by the ray of light on the wavefront?
- (ii) Which parameter remains unchanged while a ray of light propagates from one medium to another?
- (iii) Write the correct expression for Snell's law for given diagram?

Ans :

- (i) 90°
- (ii) Frequency.
- (iii) $n_1 \sin i = n_2 \sin r$

- 153.** Sonu and Kabir were going to their friend's house by walk. It was a sunny day in the afternoon. It was very hot. Sonu was finding it very difficult to see around him. He had to strain his eyes to see. Suddenly, Kabir took his cooling glasses from his pocket and asked him to wear them and later, Sonu slowly managed to see, Kabir advised Sonu on the necessity of wearing Sun glasses during summer season.

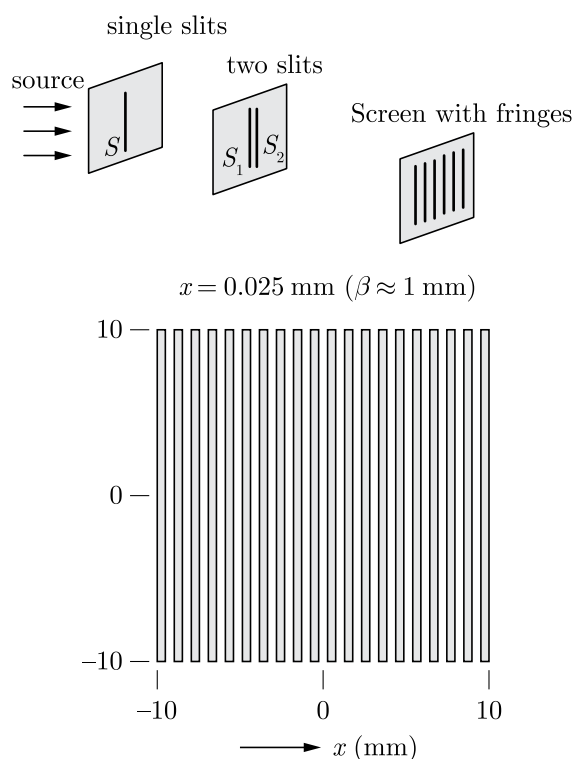


- (i) Why do we people wear sunglasses during hot days?
- (ii) Name the phenomenon based on which cooling glasses reduce the glare.
- (iii) What is the resultant intensity of light if both polariser and analyser are rotated thorough same angle ?

Ans :

- (i) On a bright, sunny day, sunlight can bounce off from anything at any time. Wearing sunglasses with UV protection can protect you from extreme brightness from driving during a sunny day. This allows you to focus on the road better as you get to your destination.
- (ii) Polarisation
- (iii) No change in intensity of light.

- 154.** Young's double-slit experiment uses two coherent sources of light placed at a small distance apart. Usually, only a few orders of magnitude greater than the wavelength of light are used. Young's double-slit experiment helped in understanding the wave theory of light, which is explained with the help of a diagram.



- (i) What is the path difference between the two light waves coming from coherent sources which produces 3rd maxima?
- (ii) What is the correct expression for fringe width (β)?
- (iii) What is the phase difference between two interfering waves producing 1st dark fringe?
- (iv) The ratio of the widths of two slits in Young's double slit experiment is 4:1. Evaluate the ratio of intensities of maxima and minima in the interference pattern?
- (v) In a Young's double slit experiment, the separation between the slits is 0.1 mm, the wavelength of light used is 600 nm and the interference pattern is observed on a screen 1m away. Find the separation between bright fringes?

Ans :

- (i) 3λ
- (ii) $\frac{\lambda D}{d}$
- (iii) π
- (iv) 9:1
- (v) 6.0 mm.

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CHAPTER 11

Dual Nature of Radiation and Matter

SUMMARY

1. ELECTRON EMISSION

The phenomenon is the process which gives emission of electrons from the surface of a metal. The minimum energy needed by an electron to come out from a metal surface is known as work function of the metal. It is denoted by ϕ_0 or W_0 and measured in electron volt (eV).

Work function, $W = h\nu_0 = \frac{hc}{\lambda_0}$

The electron emission can be obtained from the following physical processes.

Thermionic Emission

It is the phenomenon of emission of electrons from the metal surface when heated suitably.

Photoelectric Emission

It is the phenomenon of emission of electrons from the surface of metal when light radiations of suitable frequency fall on it.

Field Emission or Cold Cathode Emission

It is the phenomenon of emission of electrons from the surface of a metal under the application of a strong electric field.

2. PHOTOELECTRIC EFFECT

It is the phenomenon of emission of electrons from the surface of metals, when light radiations of suitable frequency fall on them.

Laws of Photoelectric Emission

The laws of photoelectric effect are as follows-

1. For a given metal and frequency of incident radiation, the number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.
2. For a given metal, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons takes place. This frequency is known as threshold frequency.

3. Above the threshold frequency, the maximum kinetic energy of the emitted photo-electron is independent of the intensity of incident light but depends only upon the frequency (or wavelength) of the incident light.
4. The photoelectric emission is an instantaneous process. The time lag between the incidence of radiation and emission of photoelectrons is very small, less than 10^{-9} second.

Photoelectric Current

Photoelectric current depends on the intensity of incident light and the potential difference applied between the two electrodes.

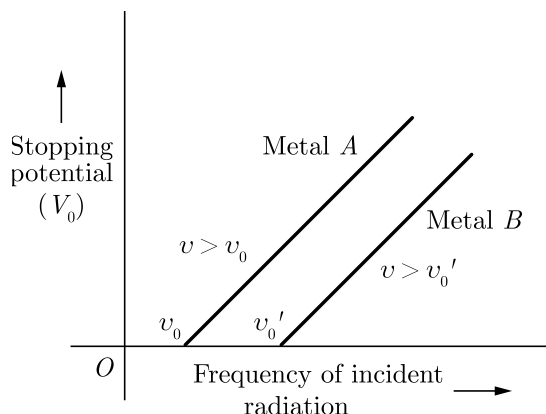
Stopping Potential

The minimum negative potential given to anode plate w.r.t. to cathode plate at which the photoelectric current becomes zero is known as stopping potential or cut off potential. It is denoted by V_0 . If e is the charge on the photo-electron, then

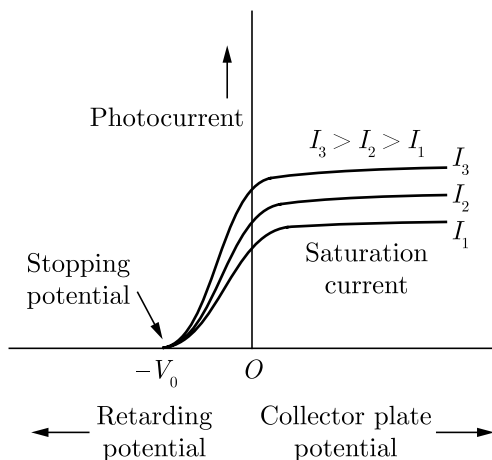
$$K_{\max} = eV_0 = \frac{1}{2}mv_{\max}^2$$

where, m is the mass of photo-electron and $V_{\max}$ is the maximum velocity of emitted photoelectrons.

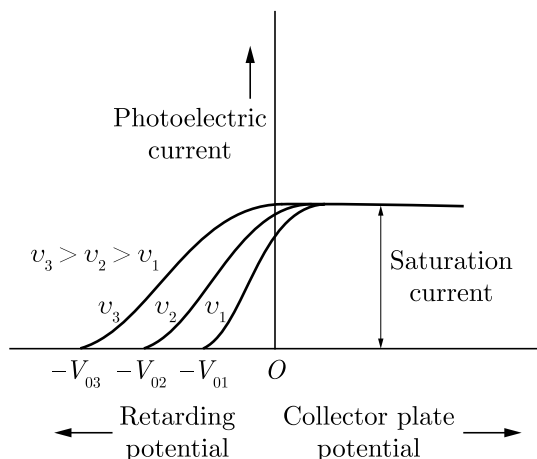
1. Variation of stopping potential V_0 with frequency ν of incident radiation.



2. Variation of photo-current with collector plate potential for different intensity of incident radiation.



3. Variation of photo-current with collector plate potential for different frequencies of incident radiation.



3. EINSTEIN'S PHOTOELECTRIC EQUATION

If a light of frequency ν is incident on a photosensitive material having work function (ϕ_0), then maximum kinetic energy of the emitted electron is given as,

$$K_{\max} = h\nu - \phi_0$$

For, $\nu > \nu_0$

$$\text{or } eV_0 = h\nu - \phi_0 = h\nu - h\nu_0$$

$$\text{or } eV_0 = K_{\max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

where, ν_0 = threshold frequency

λ_0 = threshold wavelength

λ = incident wavelength

Einstein's photoelectric equation is in accordance with the law of conservation of energy.

4. DUAL NATURE OF RADIATION

Wave theory of electromagnetic radiation explains the phenomenon of interference, diffraction and polarisation. On the other hand, photoelectric effect is supported by particle nature of light. Hence, we assume dual nature of light.

5. THE PHOTONS

These are the packets of energy (or energy particles) which are emitted by a source of radiation. The photons emitted from a source, travel through space with same speed c (equal to the speed of light).

1. Energy of a photon,

$$E = h\nu = \frac{hc}{\lambda}$$

where,

ν = frequency, λ = wavelength

h = Plank's constant,

c = speed of the light

2. Momentum of photon is,

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

3. The rest mass of photon is zero.

4. The moving mass m of photon is

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2}$$

5. All photons of light of a particular frequency ν or wavelength λ have the same energy $E(= h\nu = \frac{hc}{\lambda})$ and momentum $p(= \frac{h\nu}{c} = \frac{h}{\lambda})$, whatever be the intensity of radiation.

6. Photon energy is independent of intensity of radiation.

7. Photons are not deflected by electric and magnetic fields.

8. In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved.

9. Number of photons emitted per second of frequency ν from a lamp of power P is,

$$n = \frac{P}{h\nu} = \frac{P\lambda}{hc}$$

6. DE BROGLIE WAVES (MATTER WAVES)

Radiation has dual nature, wave and particle. The nature of experiment determines whether a wave or a particle description is best suited for understanding the experimental result. Reasoning that radiation and matter should be symmetrical in nature, Louis Victor de Broglie attributed a wave like character to matter (material particles). The waves associated with the

moving material particles are known as matter waves or de Broglie waves.

De Broglie Wavelength

The De Broglie wavelength associated with a moving particle is related to its momentum as,

$$\text{De Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{mv}$$

where, m is the mass of the particle, v is the velocity of the particle, p is the momentum of the particle.

- De Broglie wavelength is independent of the charge and nature of the material particle.
- In terms of kinetic energy K , De Broglie wavelength is given by,

$$\lambda = \frac{h}{\sqrt{2mK}}$$

- If a particle of charge q is accelerated through a potential difference V , its De Broglie wavelength is given by,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For an electron,

$$\lambda = \left(\frac{150}{V} \right)^{1/2} \text{ \AA}$$

- For a gas molecule of mass m at temperature is given by,

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

where, k is the Boltzmann constant.

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OBJECTIVE QUESTIONS

- Which one of the following metals does not exhibit emission of electrons from its surface when irradiated by visible light?

- (a) Rubidium (b) Sodium
(c) Cadmium (d) Caesium

Ans :

OD 2023

Certain metals like zinc, cadmium, magnesium etc. respond only to UV light to cause electron emission from its surface.

Thus (c) is correct option.

- A particle of mass m and charged q is accelerated through a potential V . The De-Broglie wavelength of the particle will be:

- (a) $\frac{Vh}{\sqrt{2qm}}$ (b) $\frac{q}{\sqrt{2mV}}$
(c) $\frac{h}{\sqrt{2qmV}}$ (d) $\frac{mh}{\sqrt{2qV}}$

Ans :

SQP 2018, Comp 2004

A particle of mass m and charged q is accelerated through a potential V .

Kinetic energy gained by the particle,

$$K = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

Work done on electron = qV .

$$\text{Hence, } \frac{P^2}{2m} = qV$$

$$P = \sqrt{2mqV}$$

Hence, the de-Broglie wavelength of the particle,

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mqV}}$$

Thus (c) is correct option.

- The energy of a photon of wavelength λ is

- (a) $hc\lambda$ (b) $\frac{hc}{\lambda}$
(c) $\frac{h\lambda}{c}$ (d) $\frac{\lambda}{hc}$

Ans :

OD 2016

The energy of photon is given by,

$$E = h\nu$$

where, ν = frequency of photon

$$\text{Hence, } \nu = \frac{c}{\lambda}$$

$$E = \frac{hc}{\lambda}$$

Thus (b) is correct option.

16. The de-Broglie wavelength λ depends upon mass m and energy E according to the relation represented as
 (a) $mE^{1/2}$ (b) $m^{1/2}E$
 (c) $m^{-1/2}E^{-1/2}$ (d) $m^{-1/2}E^{1/2}$

Ans :

Foreign 2004

$$\text{Mass} = m$$

$$\text{and Energy} = E$$

de-Broglie wavelength,

$$\lambda = \frac{h}{p} \\ = \frac{h}{\sqrt{2mE}} \propto \frac{1}{\sqrt{mE}} \propto m^{-1/2}E^{-1/2}$$

Thus (c) is correct option.

17. Mass of a photon of frequency ν is given by

$$\begin{aligned} \text{(a) } m &= \frac{h}{\lambda} & \text{(b) } m &= \frac{h\nu}{c} \\ \text{(c) } m &= \frac{h\nu}{c} & \text{(d) } m &= \frac{h\nu}{c^2} \end{aligned}$$

Ans :

SQP 2014

Frequency of photon $= \nu$

Energy of a photon,

$$E = h\nu \quad \dots(1)$$

Energy of a photon,

$$E = mc^2 \quad \dots(2)$$

Equating these two equations of energies, we get

$$h\nu = mc^2$$

or

$$m = \frac{h\nu}{c^2}$$

where,

 m = mass of the photon

and

 c = velocity of light

Thus (d) is correct option.

18. If the kinetic energy of an electron doubles, its de-Broglie's wavelength changes by a factor
 (a) 2 (b) $\sqrt{2}$
 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

Ans :

Delhi 2008

Final kinetic energy of electron,

$$E_2 = 2E_1$$

(where, E_1 = Initial kinetic energy of electron)

de-Broglie's wavelength of an electron,

$$\lambda = \frac{h}{mv} \\ = \frac{h}{\sqrt{2mE}} \propto \frac{1}{\sqrt{E}}$$

$$\begin{aligned} \text{Therefore, } \frac{\lambda_1}{\lambda_2} &= \sqrt{\frac{E_2}{E_1}} \\ &= \sqrt{\frac{2E_1}{E_1}} = \sqrt{2} \\ \text{or } \lambda_2 &= \frac{\lambda_1}{\sqrt{2}} \end{aligned}$$

Hence, de-Broglie's wavelength of an electron changes by the factor $\frac{1}{\sqrt{2}}$.

Thus (d) is correct option.

19. In photoelectric effect, the no. of electrons ejected per second is proportional to
 (a) intensity of light
 (b) wavelength of light
 (c) frequency of light
 (d) frequency of the metal

Ans :

Foreign 2006, Delhi 2015

In photoelectric effect for a given photosensitive material and frequency of incident light above the threshold frequency of the material, the number of electrons ejected per second is directly proportional to the intensity of light.

Thus (a) is correct option.

20. If the intensity of light falling on a metal is increased, then
 (a) K.E. of photoelectrons increases
 (b) photoelectric current increases
 (c) K.E. of photoelectrons decreases
 (d) photoelectric current remains constant

Ans :

Delhi 2016

For a given photosensitive material and frequency of incident radiation above the threshold frequency, emission of photoelectrons (i.e., photoelectric current) is directly proportional to the intensity of the incident light. Therefore if the intensity of light falling on a metal is increased, then photoelectric current will also increase.

Thus (b) is correct option.

21. When ultraviolet rays are incident on a metal plate, then the photoelectric effect does not occurs. It occurs by the incidence of
 (a) X-rays (b) radio wave
 (c) infrared rays (d) green house effect

Ans :

OD 2004

Phenomenon of emission of electrons from a metal surface when light of sufficient frequency falls upon it, is known as photoelectric effect. We also know that photoelectrons are ejected by the radiations of short

11. The kinetic energy of an electron, which is accelerated in the potential difference of 100V is

- (a) $1.6 \times 10^{-17} \text{ J}$ (b) $1.6 \times 10^{-14} \text{ J}$
(c) $1.6 \times 10^{-10} \text{ J}$ (d) $1.6 \times 10^{-8} \text{ J}$

Ans :

Foreign 2000

Potential difference, $V = 100 \text{ Volt}$

Kinetic energy of an electron accelerating in the potential difference,

$$K = eV$$

Here, e = Charge on electron equal to $1.6 \times 10^{-19} \text{ C}$

Hence,
$$K = (1.6 \times 10^{-19}) \times 100$$
$$= 1.6 \times 10^{-17} \text{ J}$$

Thus (a) is correct option.

12. There are n_1 photons of frequency ν_1 in a beam of light. In an equally energetic beam, there are n_2 photons of frequency ν_2 . Their correct relation is

- (a) $\frac{n_1}{n_2} = 1$ (b) $\frac{n_1}{n_2} = \frac{\nu_1}{\nu_2}$
(c) $\frac{n_1}{n_2} = \frac{\nu_2}{\nu_1}$ (d) $\frac{n_1}{n_2} = \frac{\nu_1^2}{\nu_2^2}$

Ans :

OD 2009, Delhi 2012

In First Beam

$$\text{No. of photons} = n_1$$

$$\text{Frequency of each photon} = \nu_1$$

In Second Beam, No. of photons = n_2

and Frequency of each photon = ν_2

Energy of n_1 photons in first beam of light,

$$E_1 = n_1 h \nu_1$$

Similarly, energy of n_2 photons in second beam of light,

$$E_2 = n_2 h \nu_2$$

Since both the beams of light are equally energetic,

$$\text{Therefore, } n_1 h \nu_1 = n_2 h \nu_2$$

or
$$\frac{n_1}{n_2} = \frac{\nu_2}{\nu_1}$$

where, h = Planck's constant

Thus (c) is correct option.

13. If a radiation of energy E falls normally on a perfectly reflecting surface, then the momentum transferred to the surface is

- (a) Ec (b) Ec^2
(c) $\frac{E}{c^2}$ (d) $\frac{2E}{c}$

Ans :

Delhi 2015

Energy of radiation = E

Velocity of radiation before reflection,

$$v_1 = c$$

where,

c = Velocity of light

Since, the surface is perfectly reflecting, therefore velocity of radiation after reflection,

$$v_2 = -c$$

(minus sign due to opposite direction)

Momentum of radiation before reflection,

$$p_1 = \frac{E}{c}$$

and momentum of radiation after reflection,

$$p_2 = \frac{E}{(-c)} = -\frac{E}{c}$$

Therefore, momentum transferred to the surface

$$= p_1 - p_2$$
$$= \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}$$

Thus (d) is correct option.

14. If momentum of a particle is doubled, then its de-Broglie's wavelength will

- (a) be half (b) be two times
(c) be four times (d) remain unchanged

Ans :

SQP 2010

Final momentum of particle,

$$p_2 = 2p_1$$

(where, p_1 = Initial momentum of particle)

de-Broglie's wavelength of particle,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \propto \frac{1}{p}$$

where

h = plank constant

Therefore,
$$\frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = \frac{2p_1}{p_1} = 2$$

$$\lambda_2 = \frac{\lambda_1}{2}$$

Thus (a) is correct option.

15. If the velocity of an electron increases, then its de-Broglie wavelength will

- (a) increase (b) decrease
(c) remain the same (d) first a then c

Ans :

OD 2002

de-Broglie wavelength of electronic,

$$\lambda = \frac{h}{mv} \propto \frac{1}{v}$$

(where, v = Velocity of electron)

Therefore, if velocity increases, the de-Broglie's wavelength will decreased and vice-versa.

Thus (b) is correct option.

4. The number of Photons of frequency 10^{14} Hz in radiation of 6.62 J will be

- (a) 10^{10} (b) 10^{15}
(c) 10^{20} (d) 10^{25}

Ans :

OD 2018

Energy, $E = 6.62 \text{ J}$

Frequency, $\nu = 10^{14} \text{ Hz}$

$$\begin{aligned} \text{Number of Photons} &= \frac{\text{Energy}}{\text{Energy of one Photon}} \\ &= \frac{6.62}{h\nu} = \frac{6.62}{6.62 \times 10^{-34} \times 10^{14}} \\ &= 10^{20} \end{aligned}$$

Thus (c) is correct option.

5. The energy of emitted photo electron depends upon

- (a) Intensity of light
(b) Wavelength of light
(c) Work-function of metal
(d) None of these

Ans :

Delhi 2001, SQP 2014

Energy of photon is given by,

$$E = h\nu$$

where,

h = Planck's constant

ν = Frequency of light radiation

Since,

$$\nu = \frac{c}{\lambda}$$

Hence,

$$E = \frac{hc}{\lambda}$$

Here, h and c are constant $E \propto \frac{1}{\lambda}$.

Therefore the energy of emitted photo-electron depends upon the wavelength of light.

Thus (b) is correct option.

6. The value of 1 MeV is

- (a) $1.6 \times 10^{-19} \text{ J}$ (b) $1.6 \times 10^{-16} \text{ J}$
(c) $1.6 \times 10^{-13} \text{ J}$ (d) $1.6 \times 10^{-11} \text{ J}$

Ans :

Foreign 2009

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Therefore,

$$\begin{aligned} \text{Value of 1 MeV} &= 10^6 \text{ eV} \\ &= 10^6 \times (1.6 \times 10^{-19}) \\ &= 1.6 \times 10^{-13} \text{ J} \end{aligned}$$

Thus (c) is correct option.

7. Which one of following is charge less?

- (a) Alpha particle (b) Beta particle
(c) Photon particle (d) Proton

Ans :

SQP 2017

Photon is the charge less (neutral particle) because electromagnetic wave does not have charge in it and it produced by photon particles.

Thus (c) is correct option.

8. If an electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum, then its final velocity will be

- (a) $\frac{eV}{2m}$ (b) $\frac{eV}{m}$
(c) $\sqrt{\frac{2eV}{m}}$ (d) $\sqrt{\frac{eV}{m}}$

Ans :

Delhi 2012

Mass of electron = m

Charge = e

and Potential difference = V

Kinetic energy of an electron

$$= \frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}}$$

(where, v = Final velocity of electron)

Thus (c) is correct option.

9. Cathode rays can be deflected by

- (a) electric field (b) magnetic field
(c) both types of fields (d) none of these fields

Ans :

Cathode rays are the stream of moving electrons. Therefore, they can be deflected by both the electric and magnetic fields.

Thus (c) is correct option.

10. If a photon has velocity c and frequency ν , then which of the following represents its wavelength?

- (a) $\frac{hc}{E}$ (b) $\frac{h\nu}{c}$
(c) $\frac{h\nu}{c^2}$ (d) $h\nu$

Ans :

OD 2002

Velocity of photon = c

and Frequency = ν

Energy of a photon,

$$E = \frac{hc}{\lambda}$$

or

$$\lambda = \frac{hc}{E}$$

(where, λ = Wavelength of photon)

Thus (a) is correct option.

wavelength. Since, wavelength of X-rays is less than of ultraviolet rays. Therefore, photoelectric effect occurs by the incidence of X-rays.

Thus (a) is correct option.

ASSERTION AND REASON

- 22. Assertion :** Photo-sensitivity of a metal is high if its work function is small.

Reason : Work function $= hf_0$ where f_0 is the threshold frequency.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

The photo-sensitivity of a metal is high when its work function is small. Work function of a metal depends not on the threshold frequency but on the nature of the metal.

Thus (b) is correct option.

- 23. Assertion :** The energy (E) and momentum (p) of a photon are related by $p = E/c$.

Reason : The photon behaves like a particle.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

Photon when behaves as a particle carries momentum equal to $\frac{E}{c}$.

Thus (a) is correct option.

- 24. Assertion :** Photoelectric effect demonstrates the wave nature of light.

Reason : The number of photoelectrons is proportional to the frequency of light.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

Photoelectric effect demonstrate the particle nature of light.

The number of photo-electron is proportional to the intensity of light.

Thus (d) is correct option.

- 25. Assertion :** Kinetic energy of photo electrons emitted by a photosensitive surface depends upon the intensity of incident photon.

Reason : The ejection of electrons from metallic surface is possible with frequency of incident photon below the threshold frequency.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

Intensity of incident photon decides the number of electrons ejected and not the kinetic energy.

Ejection of electron from metallic surface is possible only when frequency of incident photon is more than threshold frequency.

Thus (d) is correct option.

- 26. Assertion :** Mass of moving photon varies inversely as the wavelength.

Reason : Energy of the particle $= \text{mass} \times (\text{speed of light})^2$

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) The Assertion is correct but Reason is incorrect.
- (d) Both the Assertion and Reason are incorrect.

Ans :

For photon, $mv = \frac{h}{\lambda}$

mass varies inversely as the wavelength. For particle $E = mc^2$, E is energy of particle if its mass is converted into energy. Both are uncorrelated.

Thus (b) is correct option.

- 27. Assertion :** Photoelectric saturation current increases with the increase in frequency of incident light.

Reason : Energy of incident photons increases with increase in frequency and as a result photoelectric current increases.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.

- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Photoelectric saturation current is independent of frequency. It only depends on intensity of light.

Thus (d) is correct option.

- 28. Assertion :** The photoelectrons produced by a monochromatic light beam incident on a metal surface, have a spread in their kinetic energies.

Reason : The work function of the metal varies as a function of depth from the surface.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Electrons being emitted as photoelectrons have different velocities. Actually all the electrons do not occupy the same level of energy but they occupy continuous band and levels. So, electrons being knocked off from different levels come out with different energies. Work function is the energy required to pull the electron out of metal surface. Naturally electrons on the surface will require less energy to be pulled out hence will have lesser work function as compared with those deep inside the metal. So, assertion and reason are correct and reason correctly explains the assertion.

Thus (a) is correct option.

- 29. Assertion :** The photoelectrons produced by a monochromatic light beam incident on a metal surface have a spread in their kinetic energies.

Reason : The work function of the metal is its characteristics property.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The kinetic energy of emitted photoelectrons varies from zero to a maximum value. Work function depends on metal used.

Thus (b) is correct option.

- 30. Assertion :** In photosensitive cell inert gas is used.

Reason : Inert gas in the photoemissive cell gives greater current.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The photoemissive cell contain two electrodes are enclosed in a glass bulb which may be evacuated or contain an inert gas at low pressure. An inert gas in the cell gives greater current but causes a time lag in the response of the cell to very rapid changes of radiation which may make it unsuitable for some purpose.

Thus (a) is correct option.

- 31. Assertion :** In process of photoelectric emission, all emitted electrons do not have same kinetic energy.

Reason : If radiation falling on photosensitive surface of a metal consists of different wavelength then energy acquired by electrons absorbing photons of different wavelengths shall be different.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Both statement I and II are true; but even if radiation of single wavelength is incident on photosensitive surface, electrons of different KE will be emitted.

Thus (a) is correct option.

- 32. Assertion :** If the speed of charged particle increases both the mass as well as charge increases.

Reason : If m_0 = rest mass and m be mass at velocity v then

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where c = speed of light.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) Both the Assertion and Reason are incorrect.
 (d) The Assertion is incorrect but the Reason is correct.

Ans :

When speed of particle increases, the mass increases as

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ charge does not alter}$$

Thus (d) is correct option.

VERY SHORT ANSWER QUESTIONS

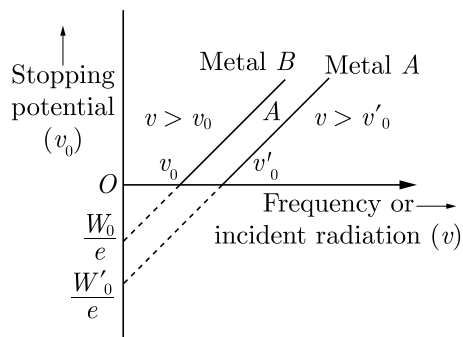
33. Define the term 'threshold frequency' in relation for photoelectric effect.

Ans :

Delhi 2021

Threshold frequency is defined as the minimum frequency of incident radiation which can cause photoelectric emission. It is different for different metal.

34. The graph shows the variation of stopping potential with frequency of incident radiation for two photosensitive metals A and B. Which one of the two has higher value of work function? Justify your answer.



Ans :

Comp 2021

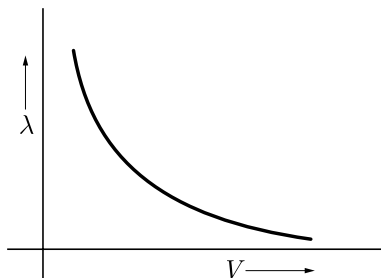
Metal A has higher value of work function because the intercept of the line of the given graph depends on work function.

35. Show graphically the variation of the de-Broglie wavelength (λ) with the potential (V) through which an electron is accelerated from rest.

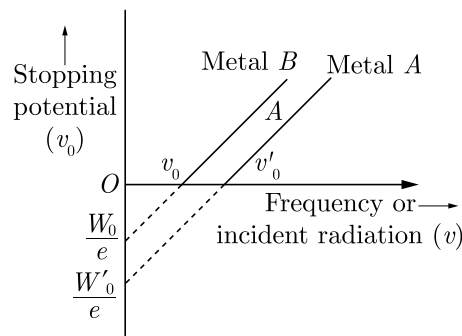
Ans :

OD 2020, Comp 2019

$\lambda \propto \frac{1}{\sqrt{V}}$, so graph is shown in figure below :

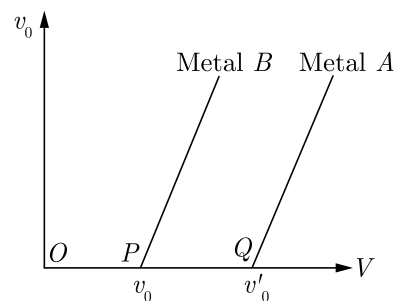


36. The graph shows variation of stopping potential V_0 versus frequency of incident radiation ν for two photosensitive metals A and B which one of the two metals has higher threshold frequency and why?



Ans :

OD 2021



As,

$$OQ > OP$$

Hence,

$$v'_0 > v_0$$

So, the threshold frequency of metal A is greater than metal B.

37. Name the phenomenon which shows the quantum nature of electromagnetic radiation.

Ans :

Delhi 2020

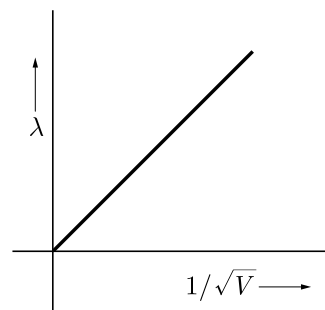
Photoelectric effect.

38. Show on a graph variation of the de-Broglie wavelength (λ) associated with the electron versus $1/\sqrt{V}$, where V is the accelerating potential for the electron.

Ans :

Delhi 2019

$$\lambda = \frac{h}{\sqrt{2meV}}$$



- (iii) On what factors does the number of emitted photoelectrons depend?

Ans :

SQP 2015

In photoelectric effect, an electron absorbs a quantum of energy $h\nu$ of radiation, which exceeds the work function, an electron is emitted with maximum kinetic energy.

$$E_{K \max} = h\nu - W$$

- (i) No, all electrons are bound with different forces in different layers of the metal. So, more tightly bound electron will emerge with less kinetic energy. Hence, all electrons do not have same kinetic energy.
- (ii) No, because an electron cannot emit out if quantum energy $h\nu$ is less than the work function of the metal. The K_E depends on energy of each photon.
- (iii) Number of emitted photoelectrons depends on the intensity of the radiations provided the quantum energy $h\nu$ is greater than the work function of the metal.
81. Plot a graph showing the variation of photoelectric current with intensity of light. The work function for the following metals is given

Na : 2.75 eV and Mo : 4.175 eV

Which of these will not give photoelectron emission from a radiation of wavelength 33000 Å from a laser beam? What happened if the source of laser beam is brought closer?

Ans :

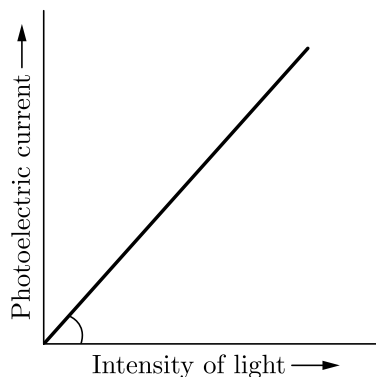
Foreign 2013

As we know that,

$$\begin{aligned} \text{Energy of photon, } E &= \frac{hc}{\lambda} \text{ J} = \frac{hc}{e\lambda} \text{ eV} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 3.3 \times 10^{-7}} \text{ eV} \\ &= 3.75 \text{ eV} \end{aligned}$$

Since W_0 of Mo is greater than E . Therefore will not give photoemission.

There will be no effect of bringing source closer in the case of Mo. In case of Na, photocurrent will increase.

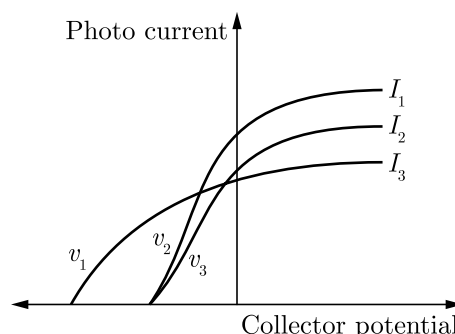


82. (a) Define the term 'intensity of radiation' in photon picture.
- (b) Plot a graph showing the variation of photo current v/s collector potential for three different intensities $I_1 > I_2 > I_3$, two of which (I_1 and I_2) have the same frequency ν and the third has frequency $\nu_1 > \nu$.
- (c) Explain the nature of the curves on the basis of Einstein's equation.

Ans :

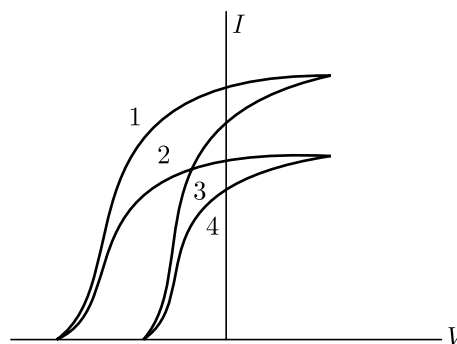
Delhi 2017, Comp 2004

- (a) The amount of light energy or photon energy incident per metre square per second is called intensity of radiation.
- (b)



- (c) As per Einstein's equation,
- (i) The stopping potential is same for I_1 and I_2 as they have the same frequency.
- (ii) The saturation currents are as shown in figure because $I_1 > I_2 > I_3$.

83. The given graph shows the variation of photoelectric current (I) with the applied voltage (V) for two different materials and for two different intensities of the incident radiations. Identify and explain using Einstein's photoelectric equation for the pair of curves that correspond to (i) different materials but same intensity of incident radiation, (ii) different intensities but same materials.



Ans :

OD 2015

- (i) 1 and 2 correspond to same intensity but different material.

50. Write the basic features of photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based.

Ans :

Foreign 2014, OD 2006

Features of the photons are as :

1. Photons are particles of light having energy $E = h\nu$ and momentum $p = \frac{h}{\lambda}$, where, h is Planck's constant.
2. Photons travel with the speed of light in vacuum, independent of the frame of reference.
3. Intensity of light depends on the number of photons crossing unit area in a unit time.

51. Use de-Broglie's hypothesis to write the relation for n th radius of Bohr orbit in terms of Bohr's quantization condition of orbital angular momentum.

Ans :

OD 2010

According to de-Broglie's hypothesis,

$$\lambda = \frac{h}{p} = \frac{h}{mv_n} \quad \dots(1)$$

According to de-Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de-Broglie wavelength.

$$2\pi r_n = n\lambda : n = 1, 2, 3 \quad \dots(2)$$

Substituting value of λ from (1) in (2), we get

$$2\pi r_n = n \frac{h}{mv_n}$$

$$r_n = \frac{nh}{2\pi mv_n}$$

52. A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why?

Ans :

OD 2015

De-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

Where,

$$K = E_K$$

For given KE ,

$$\lambda \propto \frac{1}{\sqrt{m}}$$

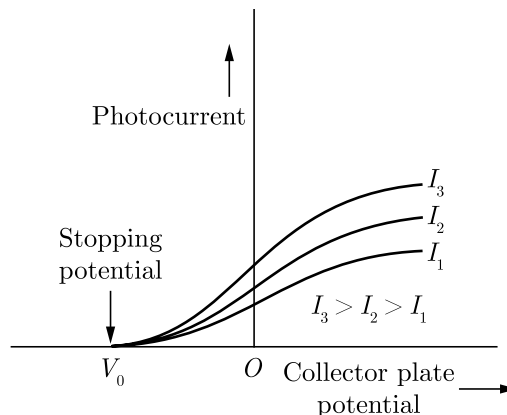
Electron have smaller mass, $\lambda_e > \lambda_p$ [So $m_e < m_p$].

For given kinetic energy, electrons have greater wavelength as these have smaller mass.

53. Show the variation of photo-current with collector plate potential for different intensity but same frequency of incident radiation.

Ans :

OD 2018



SHORT ANSWER QUESTIONS

54. What are photons? State their important properties.

Ans :

Comp 2021

According to Plank's quantum theory of radiation, an electromagnetic wave travels in the form of discrete packets of energy called quanta. One quantum of light radiation is called a photon.

The main features of photons are as follows:

1. A photon travels with the speed of light.
2. The frequency of a photon does not change as it travels from one medium to another.
3. The speed of a photon changes as it travels through different media due to the change in its wavelength.
4. The rest mass of a photon is zero i.e., a photon cannot exist at rest.
5. Energy of a photon,

$$E = h\nu = \frac{hc}{\lambda}$$

6. Momentum of a photon,

$$p = mc = \frac{hv}{c} = \frac{h}{\lambda}$$

7. From Einstein's mass-energy relationship, the equivalent mass m of a photon is given by,

$$E = mc^2 = hv \text{ or } m = \frac{hv}{c^2}$$

55. Why photoelectric effect cannot be explained on the basis of wave nature of light?

Ans :

OD 2021

Failure of Wave Theory to Explain Photoelectric Effect

According to wave theory, light is an electromagnetic wave consisting of electric and magnetic fields with continuous distribution of energy over the region over which the wave extends. This theory failed to explain the following experimental facts:

$$(b) \quad \frac{1}{2} m v_{\max}^2 = h\nu - \phi_0$$

$$V_{\max}^2 = \left(\frac{2h}{m}\right)v - \left(\frac{2\phi_0}{m}\right)$$

Hence, Slope = $\frac{2h}{m} = \frac{l}{n}$

$$h = \frac{ml}{2n}$$

Intercept = $\frac{2\phi_0}{m} = l$

Hence, $\phi_0 = \frac{ml}{2}$

72. Write three characteristic features, in photoelectric effect which cannot be explained on the basis of wave theory of light, but can be explained only using Einstein's equation.

Ans :

Comp 2015

The three characteristic features which cannot be explained by wave theory are as follows :

1. Kinetic energy of emitted electrons is found to be independent of the intensity of incident light.
 2. There is no emission of electrons if frequency of incident light is below a certain frequency (threshold frequency).
 3. Photoelectric effect is an instantaneous process.
73. Write three basic properties of photons which are used to obtain Einstein's photoelectric equation. Use this equation to draw a plot of maximum kinetic energy of the electrons emitted versus frequency of incident radiation.

Ans :

Comp 2014

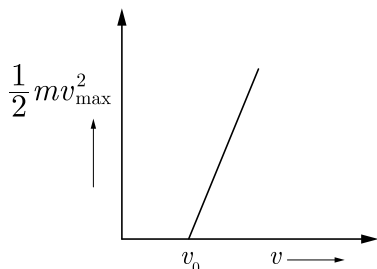
Three basic properties of photons are given as below :

- (i) Photons are quanta or discrete carriers of energy.
- (ii) Energy of a photon is proportional to the frequency of light.
- (iii) The photon gives all its energy to the electron with which it interacts.

Einstein's photoelectric equation

$$\frac{1}{2} m v_{\max}^2 = h\nu - W$$

The plot is shown as below



74. Show mathematically how Bohr's postulate of quantization of orbital angular momentum in hydrogen atom is explained by de-Broglie's hypothesis.

Ans :

Delhi 2016, Foreign 2011

According to de-Broglie's hypothesis,

$$\lambda = \frac{h}{mv} \quad \dots(1)$$

According to de-Broglie's condition of stationary orbits, the stationary orbit are those which contain complete de-Broglie wavelength.

$$2\pi r = n\lambda \quad \dots(2)$$

Substituting value of λ from (1) in (2), we get

$$2\pi r = n \frac{h}{mv}$$

$$mvr = n \frac{h}{2\pi} \quad \dots(3)$$

This is Bohr's postulate of quantisation of energy levels.

75. When an electron in hydrogen atom jumps from the third excited state to the ground state. How would the de-Broglie wavelength associated with the electron change? Justify your answer.

Ans :

OD 2011

We have $\lambda = \frac{h}{p} = \frac{h}{mv}$

or $mv = \frac{h}{\lambda}$

or $mvr = \frac{hr}{\lambda}$

or $\lambda = \frac{2\pi}{nh} \times hr = \frac{2\pi r}{n}$

As, $r \propto n^2$

$$\lambda \propto \frac{1}{n}(n^2) = n$$

Thus, we can say that,

$$\frac{\lambda_3}{\lambda_1} = \frac{3}{1} \text{ or } \lambda_1 = \frac{\lambda_3}{3}$$

Thus, wavelength decreases 3 times as an electron jumps from third excited state to the ground state.

76. A proton and an electron have same velocity. Which one has greater de-Broglie wavelength and why?

Ans :

OD 2019

De-Broglie wavelength (λ) is given as $\lambda = \frac{h}{mv}$

Given, $v_p = v_e$

where, v_p = velocity of proton

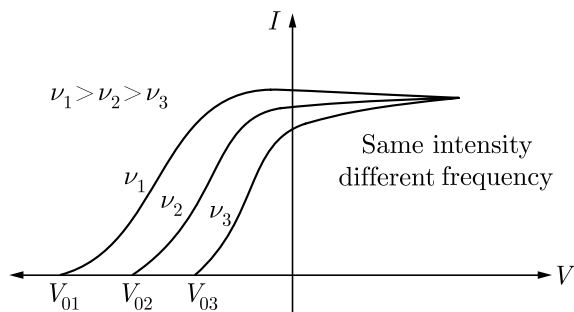
and v_e = velocity of electron

Since, $m_p > m_e$

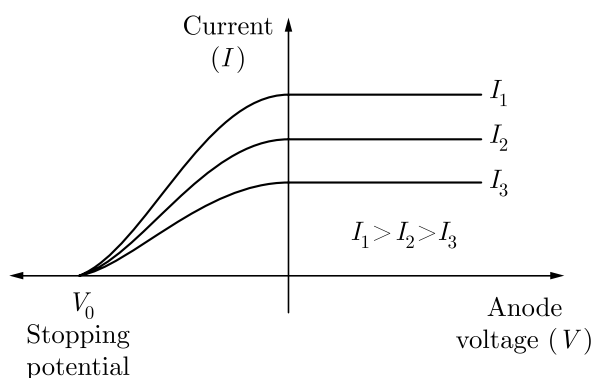
From the given relation

Ans :

OD 2017



(1)



(2)

60. Write the relationship of de-Broglie wavelength λ associated with a particle of mass m in terms of its kinetic energy E .

Ans :

Comp 2010, SQP 2013

Kinetic energy, $E_K = \frac{p^2}{2m}$

where,

 p = momentum m = mass and E_K = kinetic energy

$$p = \sqrt{2mE_K}$$

de-Broglie wavelength,

$$\lambda = \frac{h}{p}$$

where,

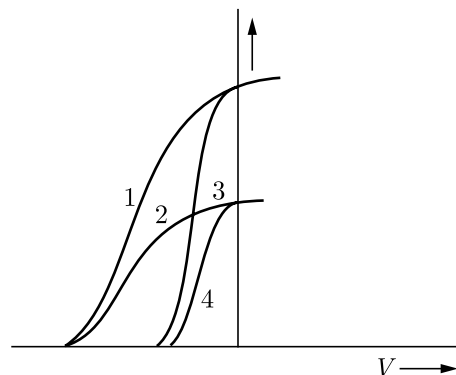
$$p = \sqrt{2mE_K}$$

Hence,

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

61. The given graph shows the variation of photoelectric current I versus applied voltage V for two different photosensitive materials and for two different intensities of the incident radiations. Identify the

pairs of curves that corresponds to different materials but same intensity of incident radiation.



Ans :

Delhi 2016

Curves 1 and 2 correspond to similar materials while curves 3 and 4 represent different materials, since the value of stopping potential for the pair of curves (1 and 2) and (3 and 4) are the same. For given frequency of the incident radiation, the stopping potential is independent of its intensity.

So, the pairs of curves (1 and 3) and (2 and 4) correspond to different materials but same intensity of incident radiation.

62. A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has.
1. Greater value of de-Broglie wavelength associated with it.
 2. Less momentum?

Give reasons to justify your answer.

Ans :

OD 2013

1. de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Here, V is same for proton and deuteron.

As mass of proton $<$ mass of deuteron and $q_p = q_d$.

Therefore, $\lambda_p > \lambda_d$ for same accelerating potential.

2. Momentum $= \frac{h}{\lambda}$.

Therefore, $\lambda_p > \lambda_d$

So, momentum of proton will be less than that of deuteron.

63. Show on a plot the nature of variation of photoelectric current with the intensity of radiation incident on a photosensitive surface.

Ans :

Comp 2019

Graph of variation of photoelectric current with the intensity of radiation incident on a photosensitive surface is given as below.

39. State de-Broglie hypothesis.

Ans :

Comp 2016

According to hypothesis of de-Broglie “The atomic particles of matter moving with a given velocity, can display the wave like properties.”

i.e., $\lambda = \frac{h}{mv}$ (mathematically)

40. What is photoelectric effect?

Ans :

OD 2018

The phenomenon of emission of electrons from a metal surface, when electromagnetic radiations of sufficiently high frequency are incident on it, is called photoelectric effect. The photo (light)-generated electrons are called photoelectrons.

41. In photoelectric effect, why should the photoelectric current increase as the intensity of monochromatic radiation incident on a photosensitive surface is increased? Explain.

Ans :

Foreign 2014

The photoelectric current increases proportionally with the increase in intensity of incident radiation. Larger the intensity radiation, larger is the number of incident photons and hence larger is the number of electrons ejected from the photosensitive surface.

42. Define intensity of radiation on the basis of photon picture of light. Write its SI unit.

Ans :

Delhi 2017

The amount of light energy or photon energy incident per metre square per second is called intensity of radiation. Its SI unit is $\frac{W}{m^2}$ or $J/s\ m^2$.

43. What do you mean by stopping potential.

Ans :

Comp 2011

The particular value of negative potential V_0 at which photoelectric current becomes zero that is known as stopping potential.

44. What is de-Broglie wave? Write an expression for its wavelength.

Ans :

OD 2013

The wave which is associated with movable matter particle is known as the de-Broglie wave or matter wave. The wavelength of de-Broglie wave is given by following expression:

Wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv}$ (where, p is momentum)

45. Why does thermionic emission take place from a metal surface only?

Ans :

SQP 2010

Thermionic emission take place from a metal surface

because the metal has free electron. When a metal is heated, its free electrons get sufficient thermal energy and they can overcome surface barrier.

46. Define threshold frequency, work function and stopping potential with reference to photoelectric effect.

Ans :

OD 2018, Foreign 2004

1. **Threshold Frequency :** The minimum value of the frequency of incident radiation below which the photo electric emission stops.
2. **Work Function :** It is the minimum amount of energy required to remove an electron from the surface of given metal.
3. **Stopping Potential :** The value of the retarding potential at which the photoelectric current becomes zero is called cut off or stopping potential.

47. Write down the some important uses of photo-cells.

Ans :

Comp 2020

Some important uses of photo-cells in daily life are as follows :

1. In cinematography, photo-cells are used for the reproduction of sound.
2. As light meters in photographic cameras.
3. In burglar's alarms.
4. In counting devices.
5. In automatic control of street light system.
6. In industries for detecting minor flaws or holes in metal sheets.

48. What is meant by work function of a metal? How does the value of work function influence the kinetic energy of electrons liberated during photoelectron emission?

Ans :

Delhi 2016

The minimum energy required to free an electron from metallic surface is called the work function.

Smaller the work function, larger the kinetic energy of emitted electron.

49. Write the expression for the de-Broglie wavelength associated with a charged particle having charge q and mass m , when it is accelerated by a potential.

Ans :

Comp 2017

A charged particle having charge q and mass m , then kinetic energy of the particle is equal of the particle is equal to the work done on it by the electric field.

i.e., $K = qV$

$$\frac{1}{2}mv^2 = qV$$

$$\frac{p^2}{2m} = qV$$

$$p = \sqrt{2mqV}$$

$$\begin{aligned}\text{Hence, } \frac{\lambda_p}{\lambda_a} &= \sqrt{\frac{m_a q_a}{m_p q_p}} \\ &= \sqrt{\frac{4m_p}{m_p} \cdot \frac{2e}{e}} = \sqrt{8} = 2\sqrt{2}\end{aligned}$$

Clearly, $\lambda_p > \lambda_a$

Hence, proton has a greater de-Broglie wavelength.

(ii) Kinetic energy,

$$E_K = qV$$

For same V ,

$$E_K \propto q$$

$$\frac{E_{K_p}}{E_{K_a}} = \frac{q_p}{q_a}$$

$$= \frac{e}{2e} = \frac{1}{2}$$

Clearly, $E_{K_p} < E_{K_a}$

69. Write two characteristic features observed in photoelectric effect which support the photon picture of electromagnetic radiation.

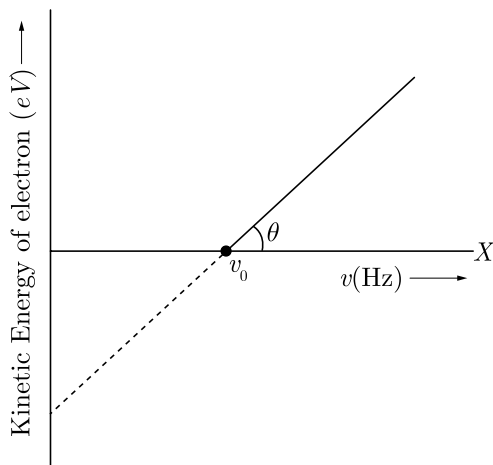
Draw a graph between the frequency of incident radiation (ν) and the maximum kinetic energy of the electrons emitted from the surface of a photosensitive material. State clearly how this graph can be used to determine (i) Planck's constant and (ii) work function of the material.

Ans :

Delhi 2012

- All photons of light of a particular frequency ' ν ' have same energy and momentum whatever the intensity of radiation may be.
- Photons are electrically neutral and are not affected by presence of electric and magnetic fields.
- From this graph, the Planck constant can be calculated by the slope of the current.

$$h = \frac{\Delta(KE)}{\Delta\nu}$$



- Work function is the minimum energy required to eject the photoelectron from the metal surface.

$$\phi = h\nu_0$$

where ν_0 = Threshold frequency.

70. (a) Define the term intensity of radiation in terms of photon picture of light.
- Two monochromatic beams, one red and the other blue, have the same intensity in which case
 - the number of photons per unit area per second is larger,
 - the maximum kinetic energy of the photoelectrons is more? Justify your answer.

Ans :

SQP 2009, OD 2015

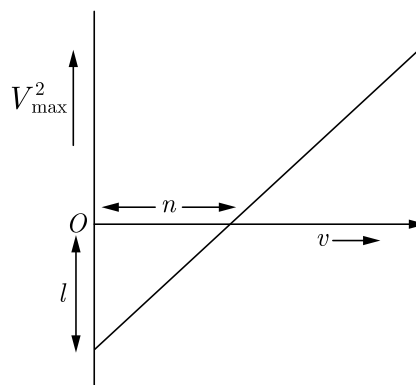
- The number of photons incident normally per unit area per unit time is determined the intensity of radiations.
- Red light, because the energy of red light is less than that of blue light

$$(h\nu)_R < (h\nu)_B$$
 - Blue light, because the energy of blue light is greater than that of red light.

$$(h\nu)_B > (h\nu)_R$$

71. (a) Give a brief description of the basic elementary process involved in the photoelectric emission in Einstein's picture.
- When a photosensitive material is irradiated with the light of frequency ν , the maximum speed of electrons is given by $V_{\max}$. A plot of $V_{\max}^2$ is found to vary with frequency ν as shown in the figure.

Use Einstein's photoelectric equation to find the expressions for (i) Planck's constant and (ii) work function of the given photosensitive material, in terms of the parameters l , n and mass m of the electron.



Ans :

OD 2016

- According to Einstein, packets of energy called photons, which are absorbed completely by electrons. This absorbed energy is used to reject the electron and also provide kinetic energy to the emitted electron.

1. A higher intensity of incident radiation should liberate photoelectrons of higher kinetic energy. But $K_{\max}$ is found to be independent of intensity.
 2. No matter what the frequency of incident radiation is, a light wave of sufficiently high intensity should be able to eject the electrons from the metal surface. Thus the wave theory fails to explain the existence of threshold frequency.
 3. An electron intercepts a very small amount of the energy of a light wave and so it should require a finite time to escape from metal surface. But actually, the emission is almost instantaneous.
56. 1. Show that the de Broglie wavelength of electron of energy E is given by the relation,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

2. Why is the belt of a Van de Graaff generator made of insulating material?

Ans :

Delhi 2013

1. Let an electron of charge e having mass m be accelerated from rest through a potential difference v , than.

Gain in kinetic energy of an electron,

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}} \quad \dots(1)$$

The wavelength associated with moving charge is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

From equation (1),

$$\lambda = \frac{h}{\sqrt{2mE}}$$

2. Because this charges taken by belt do not spread out but it resides on same place. Charges are send by the belt to the pointed conductor. If the belt is conductor material then charges spread out on the whole belt and the system does not run systematically.
57. Write Einstein's photoelectric equation and point out any two characteristic properties of photons on which this equation is based.

Ans :

Delhi 2013

If radiation of frequency ν greater than threshold frequency (ν_0) irradiate the metal surface, electrons emitted out from the metal. So Einstein's photoelectric equation can be given as,

$$E_{\max} = \frac{1}{2}m v_{\max}^2 = h\nu - h\nu_0$$

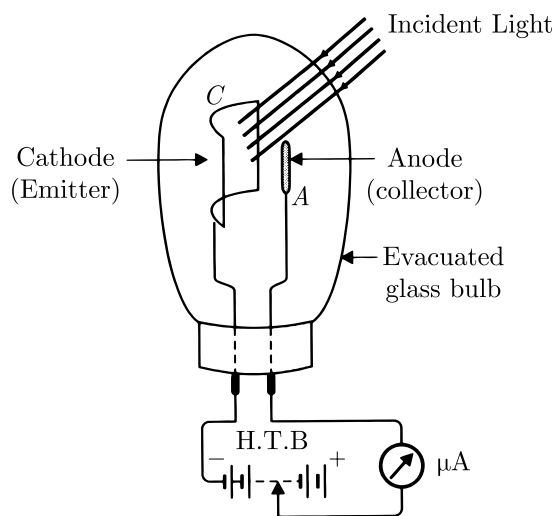
Characteristic properties of photons-

1. Energy of photon is directly proportional to the frequency (or inversely proportional to the wavelength).
 2. In photon-electron collision, total energy and momentum of the system of two constituents remains constant.
 3. In the interaction of photons with the free electrons, the entire energy of photon is absorbed.
58. Describe the construction and work of a photocell (or photo-emissive cell).

Ans :

Delhi 2020, OD 2010

Photo-emissive cell works on the principle of photoelectric emission.



Construction

It consists of an evacuated glass tube which encloses two electrodes, as shown in figure. The cathode or emitter is a parabolic metal plate coated with a layer of some photosensitive material like oxide of Cs. The anode or collector is a thin rod of Pt or Ni. The two electrodes are connected externally to a high tension battery and a micro-ammeter (μA).

Working

When light of frequency greater than the threshold frequency falls on the cathode, photoelectrons are emitted which are attracted by the collector. The circuit gets completed and a current starts flowing in the circuit. As the number of photoelectrons emitted is proportional to the intensity of incident light, the photoelectric current indicated by the micrometer gives a measure of the intensity of light.

59. Draw the graphs showing the variations of Photoelectric current with the change in anode potential of a photocell for the following conditions.
1. the same intensity but different frequencies.
 2. the same frequency but different intensities.

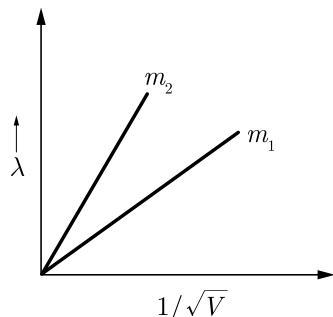
$$\lambda \propto \frac{1}{m}, \quad \text{Hence } \lambda_p < \lambda_e$$

Thus, electron has greater de-Broglie wavelength, if accelerated with same speed.

77. Plot a graph showing variation of de-Broglie wavelength λ versus $\frac{1}{\sqrt{V}}$ where V is accelerating potential for two particles A and B carrying same charge but of masses m_1, m_2 ($m_1 > m_2$). Which one of the two represents a particle of smaller mass and why?

Ans :

Comp 2017



As,
$$\lambda = \frac{h}{\sqrt{2mqV}}$$

or
$$\lambda = \left(\frac{h}{\sqrt{2q}} \cdot \frac{1}{\sqrt{m}} \right) \frac{1}{\sqrt{V}}$$

or
$$\frac{\lambda}{\frac{1}{\sqrt{V}}} = \frac{h}{\sqrt{2q}} \cdot \frac{1}{\sqrt{m}}$$

As the charge of two particles is same, we get

$$\text{Slope} \propto \frac{1}{\sqrt{m}}$$

Hence, particle with lower mass (m_2) will have greater slope.

78. (i) Describe briefly three experimentally observed features in the phenomenon of photoelectric effect.
(ii) Discuss briefly how wave theory of light cannot explain these features.

Ans :

Comp 2010

- (i) Three experimentally observed features in the phenomenon of photoelectric effect.
- (a) **Intensity-** When intensity of incident light increases as one photon ejects one electron, the increase in intensity will increase the number of ejected electrons. Frequency has no effect on photoelectrons.
- (b) **Frequency-** When the frequency of incident photon increase, the kinetic energy of the emitted electrons increases, intensity has no effect on kinetic energy of photoelectrons.
- (c) **No time lag-** When energy incident photon is greater than the work function, the photoelectrons are immediately ejected. Thus, there is no time lag between the incidence of

light and emission of photoelectrons.

- (ii) These features cannot be explained in the wave theory of light because wave nature of radiation cannot explain the following.
- (a) The instantaneous ejection of the photoelectrons.
- (b) The existence of threshold frequency for a metal surface.
- (c) The fact that kinetic energy of the emitted electrons is independent of the intensity of light and depend upon its frequency.

79. Define the terms (i) 'cut-off voltage' and (ii) 'threshold frequency' in relation to the phenomenon of photoelectric effect.

Using Einstein's photoelectric equation show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph.

Ans :

Delhi 2012, OD 2019

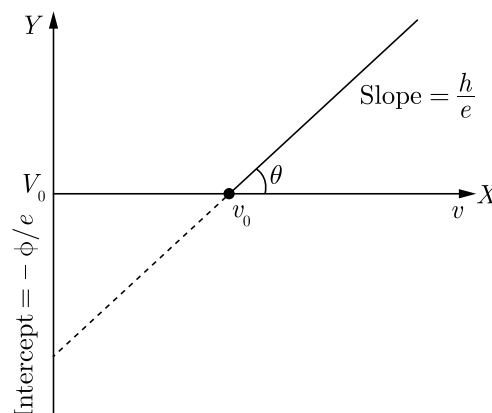
- (i) Cut-off or stopping potential is that minimum value of negative potential at anode which just stops the photoelectric current.
- (ii) For a given material, there is a minimum frequency of light below which no photoelectric emission will take place, this frequency is called as threshold frequency.

$$E_{K_{\max}} = \frac{hc}{\lambda} - \phi = hv - hv_0$$

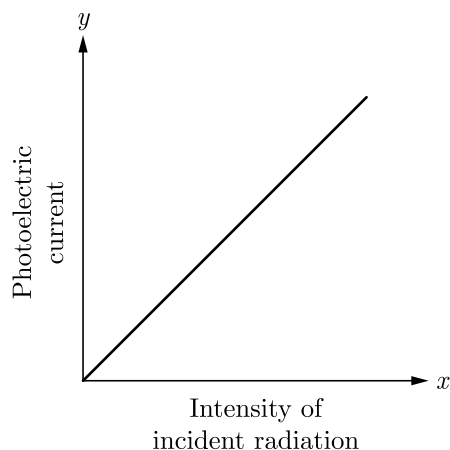
$$eV_0 = hv - hv_0$$

$$V_0 = \frac{h}{e}v - \frac{h}{e}v_0$$

Clearly, $V_0 - v$ graph is a straight line.



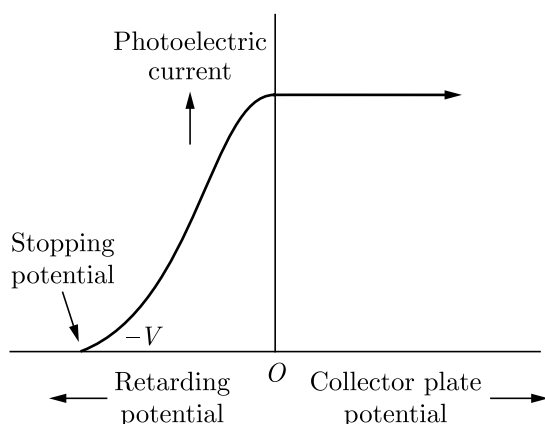
80. A beam of monochromatic radiation is incident on a photosensitive surface. Answer the following questions.
- (i) Do the emitted photoelectrons have the same kinetic energy?
- (ii) Does the kinetic energy of the emitted electrons depend on the intensity of incident radiation?



64. Plot a graph showing variation of photoelectric current with collector plate potential at a given frequency and intensity of incident radiation. What does the intercept of the graph with potential axis signify?

Ans :

SQP 2015



Intercept of the graph with potential axis gives the stopping potential.

65. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de-Broglie wavelength associated with it would be affected.

Ans :

Delhi 2018

de-Broglie wavelength,

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ \lambda &\propto \frac{1}{v}\end{aligned}$$

Also,

$$v \propto \frac{1}{n}$$

Hence,

$$\lambda \propto n$$

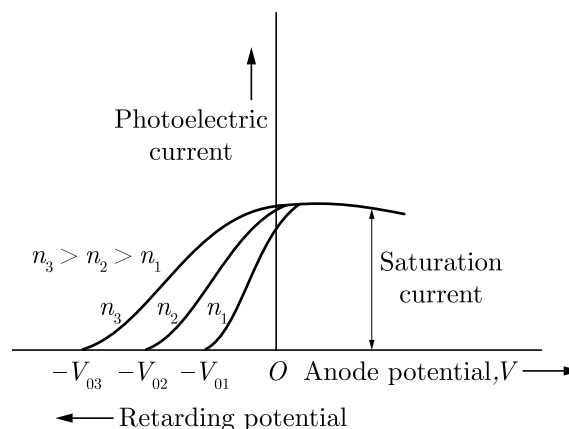
Hence, de-Broglie wavelength will increase.

66. Show the variation of photo-current with collector plate potential for different frequencies but same intensity of incident radiation.

Ans :

SQP 2012

The variation of photo-current with collector plate potential for different plate frequencies is shown as below.



67. Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory.

Ans :

OD 2018, Comp 2012

According to photon picture of light the photoelectric effect, electron absorbs a quantum of energy ($h\nu$) of radiation. If this quantum of energy needed for the electron to escape from the metal surface, the electron is emitted with some kinetic energy, the maximum value of which can be given by,

$$K_{\max} = h\nu - \phi_0$$

This is known as Einstein's photoelectric equation. ϕ_0 is the work function, which is the minimum energy needed by a surface electron to come out.

The two features of photoelectric effect which cannot be explained by wave theory, are

1. Instantaneous emission
2. Existence of threshold frequency.

68. A proton and an alpha particle are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it and (ii) less kinetic energy? Give reason to justify your answer.

Ans :

Foreign 2011

- (i) de-Broglie wavelength,

$$\text{Since, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For same V ,

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

- (ii) 3 and 4 correspond to same intensity but different material.

This is because the saturation current are same and stopping potentials are different.

or

- (i) 1 and 3 correspond to different intensity but same material.
 (ii) 2 and 4 correspond to different intensity but same material.

This is because the stopping potentials are same but saturation currents are different.

84. Sketch the graphs showing variation of stopping potential with frequency of incident radiations for two photosensitive materials A and B having threshold frequencies $\nu_A > \nu_B$.

- (i) In which case is the stopping potential more and why?
 (ii) Does the slope of the graph depend on the nature of the material used? Explain.

Ans :

Delhi 2018

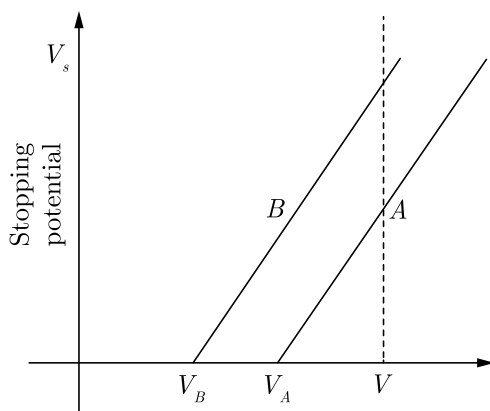
- (i) From the graph for the same value of ' ν ' stopping potential is more for material ' B '.

From Einstein's photoelectric equation

$$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0 = \frac{h}{e}(\nu - \nu_0)$$

Hence, V_0 is higher for lower value of ν_0



- (ii) No, as slope is given by $\frac{h}{e}$ which is a universal constant.

85. State two important properties of photon which are used to write Einstein's photoelectric equation. Define (i) stopping potential and (ii) threshold frequency using Einstein's equation and drawing necessary plot between relevant quantities.

Ans :

OD 2016, SQP 2001

Characteristic properties of photons are as follows :

- (a) Energy of photon is directly proportional to the frequency (or inversely proportional to the wavelength)
 (b) In photon electron collision, total energy and momentum of the system of two constituents remains constant.
 (c) In the interaction of photons with the free electrons, the entire energy of photon is absorbed.

If radiation of frequency (ν) greater than threshold frequency (ν_0) irradiate the metal surface, electrons emitted out from the metal. So, Einstein's photoelectric equation can be given as

$$E_{K\max} = \frac{1}{2}mv_{\max}^2 = h\nu - h\nu_0$$

- (i) Cut off or stopping potential is that minimum value of negative potential at anode which just stops the photoelectric current.
 (ii) For a given material, there is a minimum frequency of light below which no photoelectric emission will take place, this frequency is called as threshold frequency.

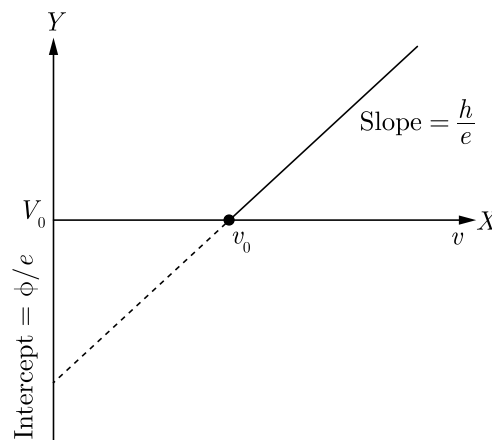
By Einstein's photoelectric equation

$$E_{K\max} = \frac{hc}{\lambda} - \phi = h\nu - h\nu_0$$

$$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0$$

Clearly, $V_0 - \nu$ graph is a straight line.



86. Define the term "cut off frequency" in photoelectric emission. The threshold frequency of a metal is f . When the light of frequency $2f$ is incident on the metal plate, the maximum velocity of photoelectrons is v_1 . When the frequency of the incident radiation is increased to $5f$, the maximum velocity of photoelectrons is v_2 . Find the ratio of v_1 and v_2 .

- 103.** If the wavelength of a photon is $4000 \text{ \AA}$. What is the energy of photon? ($c = 3 \times 10^8 \text{ m-s}^{-1}$)

Ans :

Delhi 2019, OD 2012

$$\begin{aligned}\text{Wavelength of photon, } \lambda &= 4000 \text{ \AA} \\ &= 4000 \times 10^{-10} \text{ m}\end{aligned}$$

$$\text{Velocity of light, } c = 3 \times 10^8 \text{ m-s}^{-1}$$

$$\text{Energy of a photon, } E = \frac{hc}{\lambda}$$

where, h = Planck's constant equal to $6.6 \times 10^{-34} \text{ J-s}$.

$$\begin{aligned}E &= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{(4000 \times 10^{-10})} \\ &= 4.95 \times 10^{-19} \text{ J}\end{aligned}$$

- 104.** What is the velocity of an electron having de-Broglie's wavelength of 10^{-10} m ? (Take $h = 6.6 \times 10^{-34} \text{ J-s}$)

Ans :

SQP 2011

De-Broglie's wavelength of electron,

$$\lambda = 10^{-10} \text{ m}$$

Planck's constant $h = 6.6 \times 10^{-34} \text{ J-s}$

$$\text{Velocity of electron, } v = \frac{h}{m\lambda}$$

$$= \frac{6.6 \times 10^{-34}}{(9.1 \times 10^{-31})(10^{-10})}$$

(where, m = Mass of electron to $9.1 \times 10^{-31} \text{ kg}$)

$$= 7.25 \times 10^6 \text{ m-s}^{-1}$$

- 105.** If the work function of aluminium is 4.125 eV , what is the cut off wavelength for photoelectric effect for aluminium is (Take $h = 6.6 \times 10^{-34} \text{ J-s}$)

Ans :

Foreign 2014

Work function of the aluminium,

$$\begin{aligned}\phi_0 &= 4.125 \text{ eV} \\ &= 4.125 \times (1.6 \times 10^{-19}) \\ &= 6.6 \times 10^{-19} \text{ J}\end{aligned}$$

and Planck's constant,

$$h = 6.6 \times 10^{-34} \text{ J-s}$$

Cut off wavelength of aluminium,

$$\begin{aligned}\lambda_0 &= \frac{hc}{\phi_0} \\ &= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{6.6 \times 10^{-19}} \\ &= 3 \times 10^{-7} \text{ m} \\ &= 300 \times 10^{-9} \text{ m} \\ &= 300 \text{ nm}\end{aligned}$$

(where, c = Velocity of light equal to $3 \times 10^8 \text{ m-s}^{-1}$)

- 106.** The kinetic energy of the fastest moving photoelectron from a metal of work function 2.8 eV is 2 eV . If the frequency of light is doubled, What is the kinetic energy of photoelectron?

Ans :

Delhi 2013

Work function of metal,

$$\phi_0 = 2.8 \text{ eV}$$

Initial maximum kinetic energy of photoelectrons,

$$E_{\text{max}_1} = 2 \text{ eV}$$

Initial frequency of light,

$$\nu_1 = \nu$$

and Final frequency of light,

$$\nu_2 = 2\nu$$

Initial kinetic energy of photoelectrons,

$$E_{\text{max}_1} = h\nu_1 - \phi_0$$

or

$$\begin{aligned}h\nu_1 &= K_{\text{max}_1} + \phi_0 \\ &= 2 + 2.8 = 4.8 \text{ eV}\end{aligned}$$

Similarly, final kinetic energy of photoelectrons,

$$\begin{aligned}E_{\text{max}_2} &= h\nu_2 - \phi_0 \\ &= h(2\nu_1) - \phi_0 \\ &= 2h\nu_1 - \phi_0 \\ &= (2 \times 4.8) - 2.8 \\ &= 9.6 - 2.8 \\ &= 6.8 \text{ eV}\end{aligned}$$

- 107.** If threshold wavelength for photoelectric effect on sodium is $5000 \text{ \AA}$, What is the work function of sodium? (Take $h = 6.6 \times 10^{-34} \text{ J-s}$)

Ans :

Comp 2017

Threshold wavelength of sodium,

$$\lambda_0 = 5000 \text{ \AA} = 5000 \times 10^{-10} \text{ m}$$

and Planck's constant,

$$h = 6.6 \times 10^{-34} \text{ J-s}$$

Work function of sodium,

$$\begin{aligned}\phi_0 &= \frac{hc}{\lambda_0} \\ &= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{500 \times 10^{-10}}\end{aligned}$$

(where, c = Velocity of light equal to $3 \times 10^8 \text{ m-s}^{-1}$)

$$= 3.96 \times 10^{-19} \text{ J}$$

$$= \frac{3.96 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.5 \text{ eV}$$

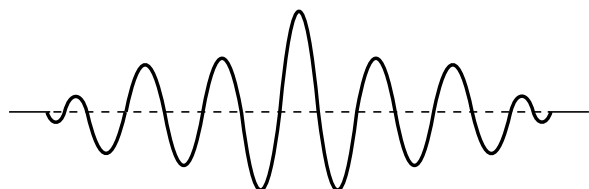
- 108.** Light of two different frequencies, whose photons have energies of 1 eV and 2.5 eV successively illuminate

$$= \frac{12.27}{\sqrt{V}} \times 10^{-10} \text{ m}$$

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

This is the required expression for de-Broglie wavelength associated with electron accelerated to potential of V volt.

The diagram of wave packet describing the motion of a moving electron is shown.



LONG ANSWER QUESTIONS

89. Write Einstein's photoelectric equation. State clearly the three salient features observed in photoelectric effect which can explain on the basis of this equation. The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2 . Derive the expressions to the threshold wavelength λ_0 and work function for the metal surface.

Ans :

Comp 2014

Einstein's photoelectric equation is $E_K = h\nu - W$ for a single photon ejecting a single electron.

- Explanation of frequency law-** When frequency of incident photon (ν), increases, the kinetic energy of emitted electron increases. Intensity has no effect on kinetic energy of photoelectrons.
- Explanation of intensity law-** When intensity of incident light increases, the number of incident photons increases, as one photon ejects one electron; the increase in intensity will increase the number of ejected electrons. In other words, photocurrent will increase with increase of intensity. Frequency has no effect on photocurrent.
- Explanation of no time lag law-** When the energy of incident photon is greater than work function, the photoelectron is immediately ejected. Thus, there is no time lag between incidence of light and emission of photoelectrons.

For wavelength λ_1 ,

$$\frac{hc}{\lambda_1} = \phi_0 + E_K = \phi_0 + eV_0 \quad \dots(1)$$

where,

$$E_K = eV_0$$

For wavelength λ_2 ,

$$\frac{hc}{\lambda_2} = \phi_0 + 2eV_0 \quad \dots(2)$$

because E_K is doubled

From equations (1) and (2), we get

$$\begin{aligned} \frac{hc}{\lambda_2} &= \lambda_2 + 2\left(\frac{hc}{\lambda_1} - \phi_0\right) \\ &= \phi_0 + \frac{2hc}{\lambda_1} - 2\phi_0 \\ \phi_0 &= \frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} \end{aligned}$$

For threshold wavelength λ_0 , kinetic energy $E_K = 0$ and work function $\phi_0 = \frac{hc}{\lambda_0}$

$$\begin{aligned} \frac{hc}{\lambda_0} &= \frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} \\ \frac{1}{\lambda_0} &= \frac{2}{\lambda_1} - \frac{1}{\lambda_2} \\ \lambda_0 &= \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1} \end{aligned}$$

$$\text{Work function, } \phi_0 = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}$$

90. (i) Why photoelectric effect cannot be explained on the basis of wave nature of light? Give reasons.
(ii) Write the basic features of photon picture of electro magnetic radiation on which Einstein's photoelectric equation is based.

Ans :

Delhi 2016, OD 2011

- The photoelectrons effect cannot be explained on the basis of wave nature of light because wave nature of radiation cannot explain the following.
 - The instantaneous ejection of photoelectrons.
 - The existence of threshold frequency for a metal surface.
 - The fact that kinetic energy of the emitted electrons is independent of the intensity of light and depends upon its frequency.
- Photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based on particle nature of light. Its basic features are given as below.
 - In interaction of radiation with matter, radiation behaves as if it is made up of particle called photons.
 - Each photon has energy $E(=h\nu = \frac{hc}{\lambda})$ and momentum $p(=h\nu/c = h/\lambda)$, where c is the speed of light, h is Planck's constant, ν and λ are frequency and wavelength of radiation.
 - All photons of light of a particular frequency ν or wavelength λ have the same energy $E(=h\nu = \frac{hc}{\lambda})$ and momentum $p(= \frac{h\nu}{c} = \frac{h}{\lambda})$ whatever the intensity of radiation may be.

Ans :

SQP 2008, 12

Cut off frequency is that maximum frequency of incident radiation below which no photoemission takes place from a photoelectric material.

According to Einstein's photoelectric equation

$$E_{K_{\max}} = \frac{hc}{\lambda} - \phi = hf - \phi$$

Given that threshold frequency of the metal is f . If light of frequency, $2f$ is incident on metal plane, maximum velocity of photoelectrons is v_1 then,

$$\begin{aligned}\frac{1}{2}mv_1^2 &= h(2f - f) \\ \frac{1}{2}mv_1^2 &= hf \quad \dots(1)\end{aligned}$$

If light of frequency, $5f$ is incident and maximum velocity of photoelectron is v_2 .

$$\begin{aligned}\frac{1}{2}mv_2^2 &= h(5f - f) \\ \frac{1}{2}mv_2^2 &= 4hf \quad \dots(2)\end{aligned}$$

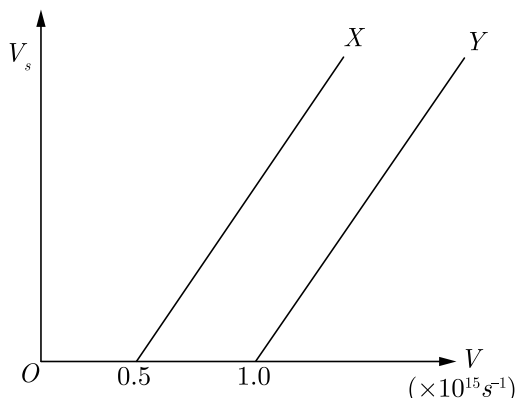
Dividing (1) by (2), we get

$$\begin{aligned}\left(\frac{v_1}{v_2}\right)^2 &= \frac{1}{4} \\ \frac{v_1}{v_2} &= \frac{1}{2}\end{aligned}$$

Hence, $v_1 : v_2 = 1 : 2$

87. The following graph shows the variation of stopping potential V_s with the frequency (ν) of the incident radiation for two photosensitive metals X and Y.

- Which of the metals has larger threshold wavelength? Give reason.
- Explain giving reason which metal gives out electrons having larger kinetic energy, for the same wavelength of the incident radiation.
- If the distance between the light source and metal X is halved, what will be the kinetic energy of electrons emitted due to this change? Give reason.



Ans :

Foreign 2011, Delhi 2005

- Threshold frequency of $X = 0.5 \times 10^{15} \text{ s}^{-1}$
Threshold frequency of $Y = 1.0 \times 10^{15} \text{ s}^{-1}$
As threshold wavelength is inversely proportional to threshold frequency, so metal X has larger threshold wavelength.
- Kinetic energy $E_K = \frac{hc}{\lambda} - h\nu_0$
As threshold frequency ν_0 is smaller for X, so for same wavelength of incident radiation, kinetic energy is larger for metal X.
- If the distance between light source and metal X is halved, the intensity of incident radiation becomes one fourth ($I \propto \frac{1}{r^2}$) but kinetic energy of photoelectrons is independent of intensity, so kinetic energy of photoelectrons remains unchanged.

88. Deduce de-Broglie wavelength of electrons accelerated by a potential of V volt. Draw a schematic diagram of a localized wave describing the wave nature of moving electron.

or

Expansion for the de-Broglie wavelength associated with accelerated electrons.

Ans :

Delhi 2017

The de-Broglie wavelength associated with electrons of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots(1)$$

Where m is mass and v is velocity of electron. If E_K is the kinetic energy of electron, then

$$\begin{aligned}E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} \\ &\quad \text{(since } p = mv = v = \frac{h}{\lambda}) \\ p &= \sqrt{2mE_K}\end{aligned}$$

Equation (1) gives

$$\lambda = \frac{h}{\sqrt{2mE_K}} \quad \dots(2)$$

If V volt is accelerating potential of electron, then kinetic energy

$$E_K = eV$$

Equation (2) gives

$$\lambda = \frac{h}{\sqrt{2meV}} \quad \dots(3)$$

Substituting the given values, we get

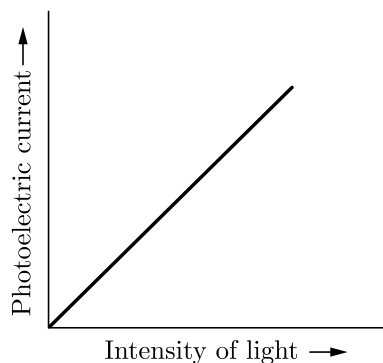
$$m = 9.1 \times 10^{-31} \text{ kg},$$

$$e = 1.6 \times 10^{-19} \text{ C},$$

$$h = 6.63 \times 10^{-34} \text{ Js, we get}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

1. **Intensity of Light on Photoelectric Current :** If we allow radiations of a fixed frequency to fall on plate P and the accelerating potential difference between the two electrodes is kept fixed, then the photoelectric current is found to increase linearly with the intensity of incident radiation, as shown in Figure. Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, this implies that the number of photoelectrons emitted per second is proportional to the intensity of incident radiation.



2. **Effect of Potential :** As shown in Figure, if we keep the intensity I_1 and the frequency of incident radiation fixed, and increase the positive potential (called accelerating potential) on plate A gradually, it is found that the photoelectric current increases with the increase in accelerating potential till a stage is reached when the photoelectric current becomes maximum and does not increase further with the increase in the accelerating potential. This maximum value of the photoelectric current is called the saturation current. At this stage, all the electrons emitted by the plate C are collected by the plate A .

Now, if we apply a negative potential on plate A with respect to plate C and increase its magnitude gradually, it is seen that the photoelectric current decreases rapidly until it becomes zero for a certain value of negative potential on plate A . The value of the retarding potential at which the photoelectric current becomes zero is called cut off or stopping potential for the given frequency of the incident radiation. At the stopping potential V_0 , when no photoelectrons are emitted, the work done by stopping potential on the fastest electron must be equal to its kinetic energy. Hence,

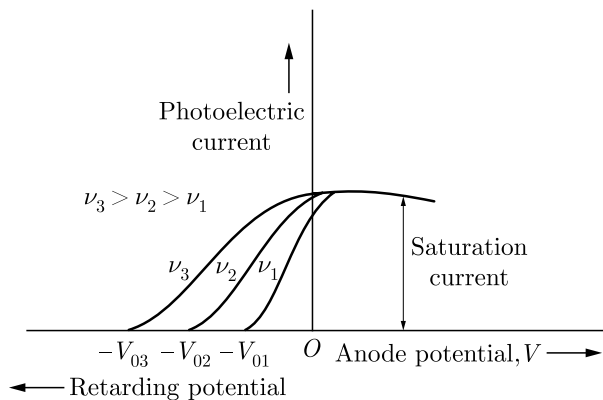
$$K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_0$$

If we repeat the experiment with incident radiation of the same frequency but of higher intensity I_2 and I_3 ($I_3 > I_2 > I_1$), we find that the values of

saturation currents have increased in proportion to the intensity of incident radiation, while the stopping potential is still the same. Thus, for a given frequency of incident radiation, the stopping potential is independent of its intensity.

3. **Effect of Frequency of Incident Radiation on Stopping Potential :** To study the effect of frequency on photoelectric effect, the intensity of incident radiation at each frequency is adjusted in such a way that the saturation current is same each time when the plate A is at a positive potential. The potential on the plate A is gradually reduced to zero and then increased in the negative direction till stopping potential is reached. The experiment is repeated with radiations of different frequencies.

As shown in Figure, the value of stopping potential increases with the frequency of incident radiation. For frequencies $\nu_3 > \nu_2 > \nu_1$, the corresponding stopping potentials vary in the order $V_{03} > V_{02} > V_{01}$.



If we plot a graph between the frequency of incident radiation and the corresponding stopping potential for different metals, we get straight line graphs, as shown in Figure. These graphs reveal the following facts :

- (a) The stopping potential or the maximum kinetic energy of the photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity.
- (b) There exists a certain minimum cut-off frequency for which the stopping potential is zero. The minimum value of the frequency of incident radiation below which the photoelectric emission stops altogether is called threshold frequency.
- (c) For two different metals A and B , these graphs have same slope. But the threshold frequencies are different.

- (d) By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.
- (e) Photon are electrically neutral and are not deflected by electric and magnetic fields.
- (f) In a photon particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, number of photons may not be conserved.
- (g) The velocity of photon in different media is different which is due to the change in its wavelength.

91. Draw a graph showing the variation of stopping potential with the frequency of incident radiation in relation to photoelectric effect.

1. What does the slope of this graph represent?
2. How can the value of Planck's constant be determined from this graph?
3. How can the value of work function of the material be determined from this graph.

Ans :

Comp 2020

Determination of Planck's Constant and Work Function

According to Einstein's photoelectron equation, the maximum K.E. of a photoelectron is given by,

$$K_{\max} = h\nu - W_0$$

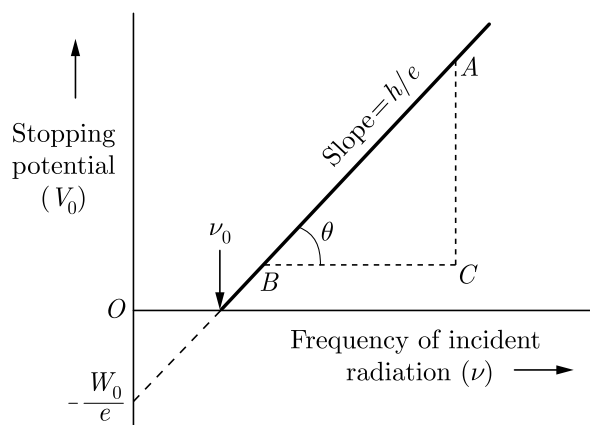
If V_0 is the stopping potential, then

$$K_{\max} = eV_0$$

Hence, $eV_0 = h\nu - W_0$

$$\text{or } V_0 = \left(\frac{h}{e}\right)\nu - \frac{W_0}{e} \quad \dots(1)$$

It follows from the above equation that V_0 versus ν graph is a straight line, as shown in figure below.



V_0 Versus ν Graph for a Photosensitive Material

1. Clearly, slope of V_0 - ν graph $= \frac{h}{e}$.
2. To determine the slope, take two points A and B on the straight line graph. Then,

$$m = \tan \theta = \frac{AC}{BC} = \frac{h}{e}$$

$$h = e \times \frac{AC}{BC} = e \times \text{slope of } V_0\text{-}\nu \text{ graph}$$

Thus, the Planck's constant h can be determined.

3. Moreover, the intercept on vertical axis $= -\frac{W_0}{e}$. Hence, $W_0 = e \times \text{Magnitude of the intercept on vertical axis}$.

In this way, the work function W_0 can be determined.

92. Describe an experimental arrangement to study photoelectric effect. Explain the effect of:

1. Intensity of light on photoelectric current
2. Potential on photoelectric current
3. Frequency of incident radiation on stopping potential.

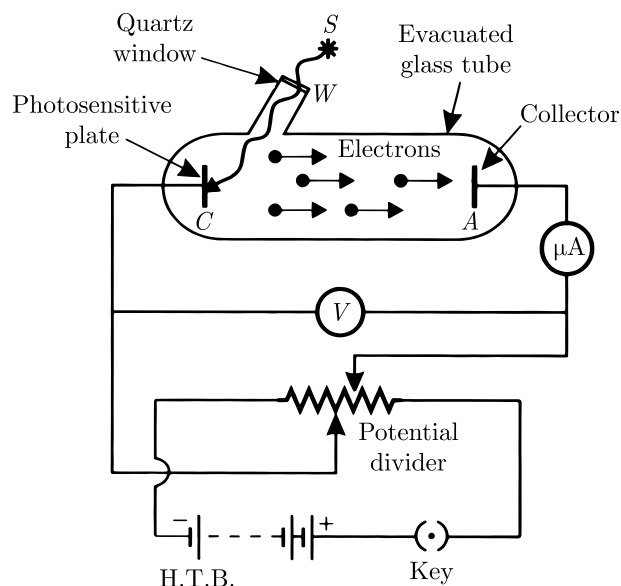
or

Describe suitable experiments to study the laws of photoelectric emission.

Ans :

Delhi 2021, Comp 2012

Figure shows the experimental arrangement used for the study of photoelectric effect. It consists of an evacuated glass/quartz tube which encloses a photosensitive plate C and another metal plate A . The two electrodes are connected to a high tension battery, a potential divider arrangement and a microammeter μA . When monochromatic radiations of a sufficiently high frequency fall on the plate C , electrons are emitted which are collected by the plate A . So a current, called photoelectric current, flows in the outer circuit which is measured by the microammeter μA .



the metal is a single event which involves transfer of energy in one lump instead of the continuous absorption of energy as in the wave theory of light. Hence there is no time lag between the incidence of photon and the emission of a photoelectrons.

95. Briefly describe the observations of Hertz, Hallwach's and Lenard's in regard of photoelectric effect.

Ans :

OD 2021, Comp 2005

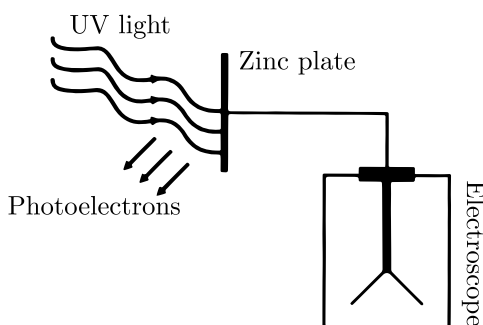
Hertz's Observations

The phenomenon of photoelectric effect was discovered by Heinrich Hertz in 1887. While demonstrating the existence of electromagnetic waves, Hertz found that high voltage sparks passed across the metal electrodes of the detector loop more easily when the cathode was illuminated by ultraviolet light from an arc lamp. The ultraviolet light falling on the metal surface caused the emission of negatively charged particles, which are now known to be electrons, into the surrounding space and hence enhanced the high voltage sparks.

Hallwach's and Lenard's Observations

During the years 1886-1902, Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of photoelectric emission in detail.

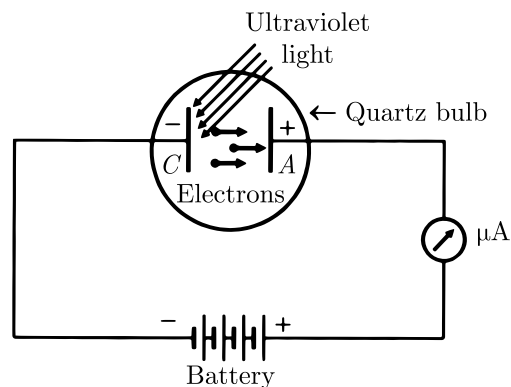
As shown in Figure (a), Hallwachs connected a zinc plate to an electroscope. He allowed ultraviolet light to fall on a zinc plate. He observed that the zinc plate became (1) uncharged if initially negatively charged, (2) positively charged if initially uncharged and (3) more positively charged if initially positively charged. From these observations, he concluded that some negatively charged particles were emitted by the zinc plate when exposed to ultraviolet light.



(a) Demonstration of Photoelectric Effect

A few years later, Lenard observed that when ultraviolet radiations are allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (cathode C and anode A), a current flows in the circuit, as shown in Figure (b). As soon as ultraviolet radiations are stopped, the current also

stops. In 1900, Lenard argued that when ultraviolet light is incident on the emitter plate, it causes the emission of electrons from its surface. These electrons are attracted by the positive collector plate so that the circuit is completed and a current flows. This current was called photoelectric current.



(b) Production of Photoelectric Production

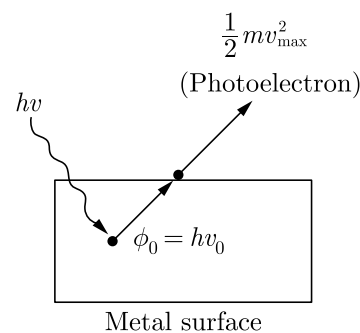
Hallwachs and Lenard also observed that when the frequency of the incident light was less than a certain minimum value, called the threshold frequency, no photoelectrons were emitted at all.

96. Write down Einstein's photoelectric equation and explain the photoelectric effect on its basis.

Ans :

OD 2018

When a quantum of light radiation of energy $h\nu$ falls on a metal surface, then this energy is absorbed by the electron and is used in following two ways:



1. A part of energy is used to overcome the surface barrier and come out of the metal surface. This part of energy is called work function. It is expressed as $\phi_0 = h\nu_0$.
2. The remaining part of the energy is used in giving a velocity v to the emitted photoelectrons. This is equal to the maximum kinetic energy of the photoelectrons. $(\frac{1}{2}mv^2_{\max})$, where m is the mass of the photoelectrons.

93. Explain briefly the reasons why wave theory of light is not able to explain the observed features of photoelectric effect.

Ans :

OD 2012

The observed characteristics of photoelectric effect could not be explained on the basis of wave theory of light due to the following reasons :

- (i) According to wave theory, the light propagates in the form of wavefronts and the energy is distributed uniformly over the wavefronts. With increase of intensity of light the amplitude of waves and the energy stored by waves will increase. These waves will then, provide more energy to electrons of metal; consequently, the energy of electrons will increase.
Thus, according to wave theory, the kinetic energy of photoelectrons must depend on the intensity of incident light, but according to experimental observations, the kinetic energy of photoelectrons does not depend on the intensity of incident light.
- (ii) According to wave theory, the light of any frequency can emit electrons from metallic surface provided the intensity of light be sufficient to provide necessary energy for emission of electrons, but according to experimental observations, the light of frequency less than threshold frequency cannot emit electrons, whatever the intensity of incident light may be.
- (iii) According to wave theory, the energy transferred by light waves will not go to a particular electron, but it will be distributed uniformly to all electrons present in the illuminated surface. Therefore, electrons will take some time to collect the necessary energy for their emission. The time for emission will be more for light of less intensity and vice-versa. But experimental observations show that the emission of electrons take place instantaneously after the light is incident on the metal; whatever the intensity of light may be.

94. What is photoelectric emission effect? What are the laws of photoelectric emission effect? How did Einstein explain these laws of photoelectric effect?

or

Explain the phenomenon of photoelectric emission. What are the laws of photoelectric effect? How are they explained by Einstein?

Ans :

Delhi 2016, OD 2014

Photoelectric Emission

It is the process in which electrons are emitted from a metal surface when electromagnetic radiations of sufficiently high frequency are incident on it. The emitted electrons are called photoelectrons but their rate of emission is very low.

Laws of Photoelectric Emission

On the basis of the experimental results on photoelectric effect, Lenard and Millikan gave the following laws of photoelectric emission.

1. For a given photosensitive material and frequency of incident radiation, (above the threshold frequency), the photoelectric current is directly proportional to the intensity of light. The saturation current is directly proportional to the intensity of incident radiation.
2. For a given photosensitive material, there exists a certain minimum cut-off frequency below which no photoelectrons are emitted, howsoever high is the intensity of incident radiation. This frequency is called threshold frequency.
3. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the photoelectrons is directly proportional to the frequency of incident radiation, but is independent of its intensity.
4. The photoelectric emission is an instantaneous process. The time lag between the incidence of light radiation and the emission of photoelectrons is very small, even less than 10^{-9} s.

Explanation of Law of Photoelectric Emission

1. **Explanation of Effect of Intensity :** The increase of intensity mean the increase in the number of photons striking the metal surface per unit time. As each photon ejects only one electron, so the number of ejected photoelectrons increases with the increases in intensity of incident radiation.
2. **Explanation of Threshold Frequency :** If $\nu < \nu_0$ i.e., the frequency of incident radiation is less than the threshold frequency, the kinetic energy of photo electrons becomes negative. This has no physical meaning. So photoelectric emission does not occur below the threshold frequency.
3. **Explanation of Kinetic Energy :** If $\nu > \nu_0$, then

$$K_{\max} = \frac{1}{2} m v_{\max}^2 \propto \nu$$

i.e., above the threshold frequency, the maximum kinetic energy of the electrons increases linearly with the frequency ν of the incident radiation. Moreover, the increase in intensity increases only the number of incident photons and not their energy. Hence the maximum kinetic energy of the photoelectrons is independent of the intensity radiation.

4. **Explanation of Time Lag :** Photoelectric emission is the result of an elastic collision between a photon and an electron. Thus the absorption of energy from a photon by a free electron inside

99. Calculate the de-Broglie wavelength of the electron orbiting in the $n = 2$ state of hydrogen atom.

Ans :

OD 2020

Kinetic energy for the second state,

$$\begin{aligned} E_K &= \frac{13.6 \text{ eV}}{4} = \frac{13.6 \text{ eV}}{2^2} \\ &= \frac{13.6 \text{ eV}}{4} \\ &= 3.4 \times 1.6 \times 10^{-19} \text{ J} \end{aligned}$$

De-Broglie wavelength,

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2mE_K}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} \\ &= 0.67 \text{ nm} \end{aligned}$$

100. A proton and an α -particle have the same de-Broglie wavelength. Determine the ratio of (1) their accelerating potentials (2) their speeds.

Ans :

Comp 2018

De-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where,

m = mass of charge particle

q = charge of particle

V = potential difference

$$(1) \quad \lambda^2 = \frac{h^2}{2mqV}$$

$$V = \frac{h^2}{2mq\lambda^2}$$

$$\text{Hence, } \frac{V_p}{V_\alpha} = \frac{2m_\alpha q_\alpha}{2m_p q_p} = \frac{2 \times 4m_2 q}{2mq} = \frac{8}{1}$$

$$V_p : V_\alpha = 8 : 1$$

$$(2) \quad \lambda = \frac{h}{mv}, \lambda_p = \frac{h}{m_p v_p}, \lambda_\alpha = \frac{h}{m_\alpha v_\alpha}$$

$$\lambda_p = \lambda_\alpha$$

$$\frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha}$$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4}{1} = 4 : 1$$

101. An electron and a proton, each have de-Broglie wavelength of 1.00 nm.

- Find the ratio of their momenta.
- Compare the kinetic energy of the proton with that of the electron.

Ans :

OD 2015, Comp 2009

$$1. \quad \lambda_e = \frac{h}{p_e} \text{ and } \lambda_p = \frac{h}{P_p}, \lambda_e = \lambda_p = 1.00 \text{ nm}$$

$$\text{So, } \frac{\lambda_e}{\lambda_p} = \frac{P_p}{P_e} = \frac{1}{1}$$

$$\frac{P_p}{P_e} = \frac{1}{1} = 1 : 1$$

2. From relation,

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$E_{Ke} = \frac{P_e^2}{2m_e}$$

and

$$K_p = \frac{P_p^2}{2m_p}$$

$$\frac{E_{kp}}{E_{ke}} = \frac{P_p^2}{2m_p} \times \frac{2m_e}{P_e^2} = \frac{m_e}{m_p}$$

$$\text{Since, } m_e \ll m_p.$$

$$\text{So, } E_{Kp} \ll E_{Ke}$$

$$\begin{aligned} \frac{E_{Kp}}{E_{Ke}} &= \frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} \\ &= 5.4 \times 10^{-4} \end{aligned}$$

102. 1. The mass of a particle moving with velocity 5×10^6 m/s has de-Broglie wavelength associated with it to be 0.135 nm. Calculate its mass.
2. In which region of the electromagnetic spectrum does this wavelength lie?

Ans :

Comp 2012

1. de-Broglie matter wave equation is given by,

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{h}{\sqrt{2mE_K}} \quad (\because p = \sqrt{2mE_K}) \end{aligned}$$

where,

m = mass of proton

E_K = kinetic energy of proton

According to the question, kinetic energy of proton,

$$E_K = m_e c^2 \text{ (Einstein's mass-energy relation)}$$

$$\lambda = \frac{h}{\sqrt{2m(m_e c^2)}}$$

$$= \frac{h}{\sqrt{2} c \sqrt{mm_e}}$$

$$= \frac{h}{\sqrt{2} c \times (m_e) \sqrt{1836}}$$

$$(\because m = 1836m_e)$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.414 \times (3 \times 10^8) \times 9.1 \times 10^{-31} \times 42.8}$$

2. This region of electromagnetic spectrum is X-ray.

According to the law of conservation of energy,

$$\begin{aligned} hv &= \phi_0 + \frac{1}{2}mv_{\max}^2 \\ &= hv_0 + \frac{1}{2}mv_{\max}^2 \end{aligned}$$

$$\begin{aligned} \text{Hence, } \frac{1}{2}mv_{\max}^2 &= K_{\max} \\ &= h(v - v_0) \\ &= hv - \phi_0 \end{aligned}$$

$$\text{Hence, } K_{\max} = hv - \phi_0$$

This equation is called Einstein's photoelectric equation.

Verification of Laws of Photoelectric Emission Based on Einstein's Photoelectric Equation

Einstein's photoelectric equation is,

$$\begin{aligned} K_{\max} &= \frac{1}{2}mv_{\max}^2 \\ &= h(v - v_0) \end{aligned}$$

This equation successfully explain the laws of photoelectric emission. These are as follows:

1. If $v < v_0$, then $\frac{1}{2}mv_{\max}^2$ is negative which is not possible therefore, for photoelectric emission to take place $v > v_0$.
2. Since, one photon emits one electron, so the number of photoelectrons emitted per second is directly proportional to the intensity of incident light.
3. It is clear that $\frac{1}{2}mv_{\max}^2 \propto v$, as h and v_0 are constants. This shows that kinetic energy of the photoelectrons is directly proportional to the frequency of the incident light.
4. Photoelectric emission is due to elastic collisions between a photon and an electron. As such there cannot be any significant time lag between the incidence of photon and emission of photoelectrons.

NUMERICAL QUESTIONS

97. A light of wavelength $6000 \text{ \AA}$ is incident on a metal of work function 2 eV . Will electrons be emitted? If yes, find maximum energy of emitted electron.

Ans :

Comp 2021, OD 2014

$$\text{Wavelength of light, } \lambda = 6000 \text{ \AA} = 6000 \times 10^{-10} \text{ m}$$

$$\text{Work function, } \phi = 2 \text{ eV}$$

$$\begin{aligned} \text{Energy of photon, } E &= h\nu = h\frac{c}{\lambda} \quad \left[\text{Since, } \nu = \frac{c}{\lambda} \right] \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10} \text{ m}} \end{aligned}$$

$$= 3.31 \times 10^{-19} \text{ J}$$

$$= \frac{3.31 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.07 \text{ eV}$$

As $E > \phi$, so electron will be emitted.

According to Einstein's photoelectric emission, we have

$$\begin{aligned} KE_{\max} &= E - \phi \\ &= 2.07 - 2.0 = 0.07 \text{ eV} \end{aligned}$$

De-broglie wavelength of the emitted electron,

$$\begin{aligned} \lambda_{\text{emin}} &= \frac{12.27}{\sqrt{KE_{\max}(\text{eV})}} \\ &= \frac{12.27}{\sqrt{0.07}} = 46.37 \text{ \AA} \\ \lambda &> \lambda_{\text{emin}} \end{aligned}$$

98. If work function of a metal surface is 2.14 eV , then evaluate

1. The threshold frequency of metal
2. The wavelength of incident light, if the electron emitted with maximum kinetic energy is stopped with reverse voltage of 0.60 V .

Ans :

Delhi 2020

Work function of metal surface,

$$\begin{aligned} \phi_0 &= 2.14 \text{ eV} \\ &= 2.14 \times 1.6 \times 10^{-19} \text{ J} \end{aligned}$$

Now

$$\begin{aligned} \phi &= hv_0 \\ v_0 &= \frac{\phi}{h} \\ &= \frac{2.14 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} \\ &= 5.17 \times 10^{14} \text{ Hz} \end{aligned}$$

Thus, for frequencies less than this threshold frequency no photoelectrons are rejected.

According to the Einstein's photo-electric equation,

$$\begin{aligned} eV_0 &= hv - \phi_0 \\ eV_0 &= \frac{hc}{\lambda} - \phi_0 \quad \left[\text{Since, } \nu = \frac{c}{\lambda} \right] \\ \frac{hc}{\lambda} &= eV_0 + \phi_0 \\ \lambda &= \frac{hc}{eV_0 + \phi_0} \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{(0.60 \text{ eV} + 2.14 \text{ eV})} \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2.74 \times 1.6 \times 10^{-19}} \\ &= 4.53 \times 10^{-7} \text{ m} \\ &= 453 \text{ nm} \end{aligned}$$

a metal whose work function is 0.5 eV. What is the ratio of maximum velocities of the emitted electrons?

Ans :

OD 2016, SQP 2003

Energy of first photon,

$$E_1 = 1 \text{ eV}$$

Energy of second photon,

$$E_2 = 2.5 \text{ eV}$$

and work function of the metal,

$$\phi_0 = 0.5 \text{ eV}$$

Maximum kinetic energy of an electron,

$$K_{\max} = \frac{1}{2} m v_{\max}^2$$

Maximum kinetic energy of the emitted electron,

$$\frac{1}{2} m v_{\max}^2 = E - \phi_0$$

$$\text{or} \quad v_{\max} = \sqrt{\frac{2(E - \phi_0)}{m}} \propto \sqrt{(E - \phi_0)}$$

$$\begin{aligned} \text{Therefore,} \quad \frac{v_{\max_1}}{v_{\max_2}} &= \sqrt{\frac{E_1 - \phi_0}{E_2 - \phi_0}} \\ &= \sqrt{\frac{1 - 0.5}{2.5 - 0.5}} \\ &= \sqrt{\frac{0.5}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \end{aligned}$$

$$\text{or} \quad v_{\max_1} : v_{\max_2} = 1 : 2$$

109. Find the energy that should be added to an electron, to reduce its de-Broglie's wavelength from 10^{-10} m to $0.5 \times 10^{-10} \text{ m}$.

Ans :

OD 2017

Initial de-Broglie's wavelength,

$$\lambda_1 = 10^{-10} \text{ m}$$

and final de-Broglie's wavelength,

$$\lambda_2 = 0.5 \times 10^{-10} \text{ m}$$

Velocity of electron,

$$v = \frac{h}{m\lambda}$$

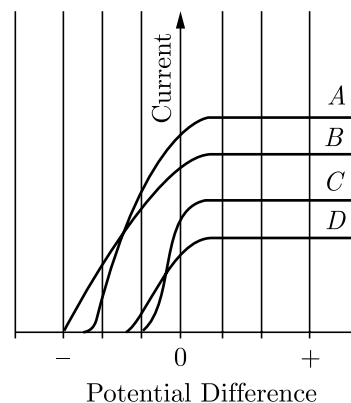
and kinetic energy of electron,

$$\begin{aligned} E &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} m \left(\frac{h}{m\lambda} \right)^2 \\ &= \frac{h^2}{2m\lambda^2} \propto \frac{1}{\lambda^2} \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \quad \frac{E_2}{E_1} &= \left(\frac{\lambda_1}{\lambda_2} \right)^2 \\ &= \left(\frac{10^{-10}}{0.5 \times 10^{-10}} \right)^2 = 4 \\ E_2 &= 4E_1 \end{aligned}$$

CASE BASED QUESTIONS

110. Figure shows the variation of photoelectric current measured in a photo cell circuit as a function of the potential difference between the plates of the photo cell when light beams A, B, C and D of different wavelengths are incident on the photo cell. Examine the given figure and answer the following questions :



- Which light beam has the highest frequency and why ?
- Which light beam has the longest wavelength and why ?
- Which light beam eject photoelectrons with maximum momentum and why ?
- What is the effect on threshold frequency and stopping potential on increasing the frequency of incident beam of light? Justify your answer.

Ans :

- Here B has the highest frequency because higher the frequency of incident light higher the value of stopping potential and *vice-versa*.
- Since, frequency and wavelength are inversely proportional to each other. The wave with the greatest frequency has the shortest wavelength. So, beam C with lowest frequency have longest wavelength.
- Photon energy and momentum are related by

$$\rho = EC$$

$$\rho \propto E$$

$$\rho \propto \frac{1}{\lambda}$$

beam B has the lowest wavelength, then it has the maximum momentum related to it.

- The frequency of the light for which its energy is equal to the work function is called threshold frequency ν_0 on increasing the frequency of the incident light, there is no effect on the threshold

frequency (it is a property of material). The stopping potential varies with frequency of incident light and increase in frequency of the incident light will increase the kinetic energy of the emitted electrons therefore greater retarding potential is needed to stop them.

111. “Know your face beauty through complexion meter” was one of the stall on a science exhibition. A student interested to know his/her face beauty was made to stand on a platform and light from a lamp was made to fall on his/her face. The reading of complexion meter indicated the face beauty of the student which might be very fair, fair, semi fair, semi dark and dark etc.



- What is the basic concept used in the working of complexion meter?
- How is the face beauty recorded by face complexion meter?

Ans :

- The basic concept used in the working of complexion meter is photoelectric effect.
 - The reading of complexion meter depends on the current generated from the light reflected from the face. Fair the colour more is the light reflected and vice versa.
112. Kamal had knowledge that energy and momentum of an electron are related to frequency and wavelength of the associated matter (de-Broglie) wave by the relations, $E = hv$, $p = \frac{h}{\lambda}$. But when he heard from his friend that value of λ has no physical significances. Kamal requested Ravi to explain such thing clearly. Ravi said to him that according to de-Broglie a particle behaves as a wave, but it is now established that a particle cannot be equivalent to a single wave, but it is equivalent to a group of waves or a wave packet.
- As $p = \frac{h}{\lambda}$, so in the discussion of matter waves, a wave packet is significant and hence only wavelength, λ is significant. As a single wave is insignificant, so phase velocity (velocity of a single wave, $V = v\lambda$) is insignificant and thus, frequency v is also insignificant. Calculate the
- momentum and
 - de-Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Ans :

Mass of electron, $m = 9.1 \times 10^{-31}$ kg.

- (i) Momentum,

$$\begin{aligned} p &= \sqrt{2mE_K} \\ &= \sqrt{2meV} \\ &= \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 56} \\ &= 4.04 \times 10^{-24} \text{ kg ms}^{-1} \end{aligned}$$

- (ii) de-Broglie wavelength,

$$\begin{aligned} &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34}}{4.04 \times 10^{-24}} \\ &= 1.64 \times 10^{-10} \text{ m} \\ &= 0.164 \text{ nm.} \end{aligned}$$

113. A function was organised in the school auditorium. There was 500 sitting arrangement in the auditorium. When entry started students entered in groups and so counting became a great problem. Then principle of the school ordered science students to take responsibility at the gate. The science students managed the situation and now all the students used to enter the hall one by one. This helped them to maintain discipline and counting became easy with the help of a scientific device used by these students.



- Why do we use photocell?
- Name the scientific device which is based on application of photoelectric effect.
- What is the principle of such scientific device?

Ans :

- Photocells and motion sensors are electronic devices you can use to manage indoor or outdoor lighting. These sensors improve the security and safety of your home, automatically turning on lights when it gets dark or they detect motion. They also save energy by turning themselves off when extra light is unnecessary.
- Photocell.

- (iii) A person approaching a doorway may interrupt a light beam which is incident on a photocell. The abrupt change in photo-current records every interruption of the light beam caused by person passing across the beam. In this way help count the person entering the auditorium provided they enter the hall one by one.

114. Heena wanted to give a gift to her cousin a burglar alarm (A burglar alarm system consists of a series of electrical components that are connected to a property. via sensors and contacts, they detect movement or the opening of doors and windows, upon which a loud alarm is produced to alert those nearby of the unauthorised entry.). But she did not know about its working principle and its significance. She discussed it with her friends Surbhi. Surbhi explained her that in burglar alarm, ultraviolet light is continuously made to fall on a photocell installed at doorway. A person entering the door interrupts the beam falling on the photocell. The abrupt change in photo-current is used to start an electric bell ringing.



An electron and a photon each have a wavelength of 1.00 nm. Find

- their momenta
- the energy on the photon, and
- the kinetic energy of electron.

Ans :

Given, $\lambda = 1.0\text{nm} = 1.00 \times 10^{-9}\text{m}$

- (i) Momentum of electron = Momentum of photon

$$\begin{aligned}\frac{hc}{\lambda} &= \frac{6.63 \times 10^{-34}}{1.00 \times 10^{-9}} \\ &= 6.63 \times 10^{-25} \text{ kgm/s}\end{aligned}$$

- (ii) Energy of photon,

$$\begin{aligned}E &= \frac{hc}{\lambda} = pc \\ &= 6.63 \times 10^{-25} \times 3 \times 10^8 \\ &= 1.989 \times 10^{-16} \text{ J} \\ &= \frac{1.989 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 1.24 \times 10^3 \text{ eV} \\ &= 1.24 \text{ KeV}\end{aligned}$$

- (iii) Kinetic energy of electron,

$$\begin{aligned}E_K &= \frac{p^2}{2m} \\ &= \frac{(6.63 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \\ &= 2.413 \times 10^{-19} \text{ J} \\ &= \frac{2.413 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.51 \text{ eV}\end{aligned}$$

115. Sandeep and his younger brother Vimal went to a shopping mall whose doors were automatic with self close and open. Vimal asked Sandeep that, “Why these doors are automatically opening and closing without even a single touch?” Sandeep was a graduate but, he did not know the exact reason behind this. Sandeep asked the same from his friend Rahul who was junior to him. Rahul explained the whole phenomenon agilerly to Sandeep and told that this whole phenomenon is based on photoelectric effect given by the great scientist Einstein.



- Write photoelectric equation given by Einstein
- Define photoelectric current.

Ans :

- (i) Photoelectric equation, by Einstein's

$$\frac{1}{2}mv^2 = E_{K\text{max}} = E - \phi$$

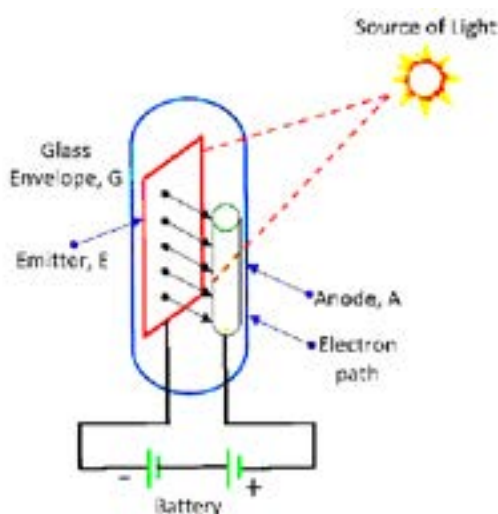
Where, $E_{K\text{max}}$ = maximum kinetic energy of photoelectron,

E = energy of photon incident on the metal surface.

ϕ = work function for the metal

- (ii) **Photocurrent :** It is current developed at the time of flow of photoelectrons in the conductor wire.

116. The photoelectric emission is possible only if the incident light is in the form of packets of energy, each having a definite value, more than the work function of the metal. This shows that light is not of wave nature but of particle nature. It is due to this reason that photoelectric emission was accounted by quantum theory of light.



- What is the name of packet of energy ?
- How much energy associated with each photon?
- Which of the waves can produce photo electric effect?

Ans :

- Quanta.
 - $h\nu$.
 - UV radiation.
117. Rekha's brother was riding the bike on highway and she was sitting behind him. While sitting, at a place traffic signal turned red from green and her brother continued riding without noticing the signal change. Rekha observed the whole situation and asked her brother to stop. Her brother felt happy on his sister's intelligence.



- What are the principles that are used in maintaining traffic signals?
- What is the leading physical quantity in the process? Write an equation for the speed of the photoelectrons.

Ans :

- Principle of photoelectric effect is used to maintain the traffic signal.
- The leading physical quantity of photoelectric current raised due to motion of photoelectrons.

The required equation is

$$v = \sqrt{\frac{2E}{m} - \frac{2\phi}{m}}$$

Here, v = speed of the electron, E = energy of the photon, m = mass of the electron and ϕ = work function for the metal.

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CHAPTER 12

Atoms

SUMMARY

1. IMPACT PARAMETER

The perpendicular distance of the velocity vector of α -particle from the central line of the nucleus of the atom is called impact parameter b .

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{E_K} \cot \frac{\theta}{2}$$

2. DISTANCE OF CLOSEST APPROACH

The smallest distance of approach of α -particle near heavy nucleus is a measure of the size of nucleus. Distance of nearest approach $\approx$ size of nucleus $= \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{E_K}$.

where, E_K = kinetic energy of incident α -particle

Z = atomic number

e = electronic charge

3. BOHR'S THEORY OF HYDROGEN ATOM

Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates. These are

1. **Bohr's First Postulate :** It states that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory.
2. **Bohr's Second Postulate :** This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$. Where h is the Planck's constant ($= 6.6 \times 10^{-34}$ J-s). Thus, the angular momentum (L) of the orbiting electron is quantised,

$$\text{i.e., } L = \frac{nh}{2\pi}$$

As, angular momentum of electron,

$$L = mvr$$

Hence, for any permitted (stationary) orbit,

$$mvr = \frac{nh}{2\pi}$$

where, n = any positive integer i.e., 1, 2, 3,....

It is also called principle quantum number.

3. **Bohr's Third Postulate :** It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

The frequency of the emitted photon is then given by

$$h\nu = E_i - E_f$$

where, E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

3.1 Important Terms Related to Bohr's Model

1. Radius of n^{th} orbit is given by

$$r_n = \frac{\epsilon_0 h^2 n^2}{\pi m Z e^2}$$

For hydrogen atom, $Z = 1$

$$(r_n)_H = \frac{\epsilon_0 h^2 n^2}{\pi m e^2}$$

The radius of first orbit of hydrogen atom is called Bohr's radius. It is denoted by,

$$a_0 = \frac{\epsilon_0 h^2}{\pi m e^2} = 0.529 \times 10^{-10} \text{ m} \\ = 0.529 \text{ \AA}$$

2. Energy of Orbiting Electron

$$\text{Kinetic energy, } E_K = \frac{1}{2} m v^2 = \frac{1}{4\pi\epsilon_0} \frac{Z e^2}{2r}$$

$$\text{Potential energy, } U = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(-e)}{r} \\ = -\frac{1}{4\pi\epsilon_0} \frac{Z e^2}{r}$$

$$\text{Total energy, } E = -\frac{1}{4\pi\epsilon_0} \frac{Z e^2}{2r}$$

For n^{th} orbit, writing E_n for E , we have

$$E_n = -\frac{1}{4\pi\epsilon_0} \frac{Z e^2}{2r_n}$$

After substituting the value of r_n , we get

$$E_n = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{2\left(\frac{\epsilon_0 h^2 n^2}{\pi m Z e^2}\right)}$$

$$= -\frac{mZ^2 e^4}{8\epsilon_0^2 h^2 n^2}$$

For convenience introducing Rydberg constant,

$$R = \frac{me^4}{8\epsilon_0^2 ch^3}$$

We have, $E_n = \frac{Z^2 R h c}{n^2}$

For hydrogen atom, $Z = 1$

Energy of orbiting electron in H -atom,

$$E_n = -\frac{R h c}{n^2}$$

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

The lowest energy of the atom is called the ground state, this state has lowest energy, The energy of this state is -13.6 eV . Therefore the minimum energy required to free the electron from the ground state of the hydrogen atom is -13.6 eV .

4. HYDROGEN SPECTRUM

Hydrogen emission spectrum consists of 5 series.

- (a) **Lyman Series** : This lies in ultraviolet region.
- (b) **Balmer Series** : This lies in the visible region.
- (c) **Paschen Series** : This lies in near infrared region.
- (d) **Brackett Series** : This lies in mid infrared region.
- (e) **Pfund Series** : This lies in mid infrared region.

The atomic hydrogen emits a line spectrum consisting of various series. The frequency of any line in a series can be expressed as

- (a) Lyman series,

$$v = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right); n = 2, 3, 4...$$

[Shortest wavelength = $911.6 \text{ \AA}$, Longest wavelength $1215 \text{ \AA}$]

- (b) Balmer series,

$$v = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right); n = 3, 4, 5...$$

[Shortest wavelength = $3646 \text{ \AA}$ Longest wavelength = $6563 \text{ \AA}$]

- (c) Paschen series,

$$v = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right); n = 4, 5, 6, ...$$

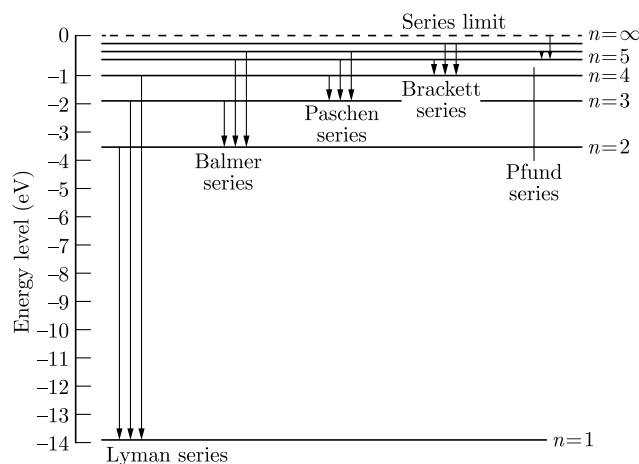
[Shortest wavelength $8204 \text{ \AA}$ Longest wavelength = $18752 \text{ \AA}$].

- (d) Brackett series,

$$v = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right); n = 5, 6, 7...$$

- (e) Pfund series,

$$v = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right); n = 6, 7, 8$$



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OBJECTIVE QUESTIONS

1. A hydrogen atom makes a transition $n = 5$ to $n = 1$ orbit. The wavelength of photon from emitted is λ . The wavelength of photon emitted when it makes a transition from $n = 5$ to $n = 2$ orbit is

- (a) $\frac{8}{7}\lambda$ (b) $\frac{16}{7}\lambda$
(c) $\frac{24}{7}\lambda$ (d) $\frac{32}{7}\lambda$

Ans :

OD 2023

Required formula for calculating wavelength,

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

where, R is the Rydberg constant.

Here, $n_2 = 5, n_1 = 1$

$$\frac{1}{\lambda} = R \left[1 - \frac{1}{25} \right] = R \left(\frac{24}{25} \right)$$

or $\lambda = \frac{25}{24R} = \frac{25}{24\lambda} \dots(i)$

Now, when $n_1 = 2, n_2 = 5$

$$\frac{1}{\lambda'} = R \left[\frac{1}{4} - \frac{1}{25} \right]$$

$$\frac{1}{\lambda'} = R \left(\frac{25-4}{100} \right) = R \left(\frac{21}{100} \right)$$

$$\lambda' = \frac{100}{21 \times R} = \frac{100 \times 24}{21 \times 25} \lambda = \frac{32}{7} \lambda$$

Thus (d) is correct option.

2. The minimum angular momentum of electron in Hydrogen atom will be

- (a) $\frac{h}{\pi}$ Js (b) $\frac{h}{2\pi}$ Js
(c) $h\pi$ Js (d) $2\pi h$ Js

Ans :

OD 2018, Comp 2005

According to Bohr's atomic model, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, h being plank's constant.

$$L = \frac{nh}{2\pi}$$

When, $n = 1$

$$L = \frac{h}{2\pi}$$

Hence, the minimum angular momentum of electron in hydrogen atom will be $\frac{h}{2\pi}$.

Thus (b) is correct option.

3. Which series comes in visible region of hydrogen spectrum?

- (a) Lyman series (b) Balmer series
(c) Paschen series (d) Bracket series

Ans :

OD 2013

When an electron jumps from n^{th} orbit to second orbit in an hydrogen atom then the series emitted is balmer series is in visible region.

Thus (b) is correct option.

4. Rutherford's α -particle experiment showed that the atoms have

- (a) proton (b) nucleus
(c) neutron (d) electrons

Ans :

Foreign 2007

From Rutherford's α particle experiment we know that the atoms have nucleus and the nucleus contains atoms all the mass of an atom.

Thus (b) is correct option.

5. According to the Rutherford's atomic model, the electrons inside the atom

- (a) stationary (b) not stationary
(c) centralized (d) none of these

Ans :

Delhi 2002

According to Rutherford's atomic model that the electron inside the atom are not stationary.

If these electrons would have been stationary, they would have been pulled into the nucleus due to the electrostatic force of attraction between the nucleus and electrons.

Thus (b) is correct option.

6. The hydrogen atom can give spectral lines in the Lyman, Balmer and Paschen series. Which of the following statement is correct?

- (a) Balmer series is in the visible region
(b) Paschen series is in the visible region
(c) Lyman series is in the infra-red region
(d) Balmer series is in the ultraviolet region

Ans :

SQP 2009

Balmer series, all the spectral lines correspond to transition of electrons from higher excited state to the orbit having $n = 2$ i.e., $n_1 = 2, n_2 = 3, 4, 5, \dots$ and maximum wavelength of Balmer series ($n_2 = 3$), $\lambda_{\text{max}} = 6564 \text{ \AA}$ and minimum wavelength of Balmer series ($n_2 = \infty$), $\lambda_{\text{min}} = 3646 \text{ \AA}$. These values of maximum and minimum wavelengths indicate that the series lies in the visible region.

Thus (a) is correct option.

ASSERTION AND REASON

- 18. Assertion :** The specific charge of positive rays is not constant.

Reason : The mass of ions varies with speed.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The specific charge $\frac{e}{m}$ of positive ray is not constant because for different velocities the rest mass of particles change i.e. mass of ions varies with speed. So $\frac{e}{m}$ will differ.

Thus (a) is correct option.

- 19. Assertion :** Balmer series lies in the visible region of the electrons radiate.

Reason : $\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$, where $n = 3, 4, 5$

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Wavelength is Balmer series is,

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

$$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$\text{or } \lambda_{\max} = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} \\ = 6563 \text{ \AA}$$

$$1/\lambda_{\min} = R \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right]$$

$$\text{or } \lambda_{\min} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3646 \text{ \AA}$$

So it lies in the visible region.

Thus (b) is correct option.

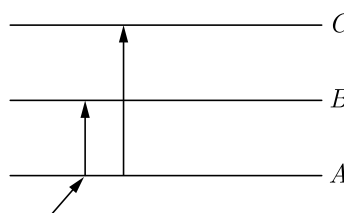
- 20. Assertion :** Between any two given energy levels, the number of absorption transitions is always less than the number of emission transitions.

Reason : Absorption transitions start from the lowest energy level only and may end at any higher energy level. But emission transitions may start from any higher energy level and end at any energy level below it.

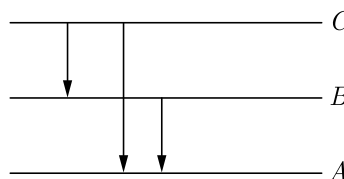
- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Absorption transition



Two possibilities in absorption transition.



Three possibilities in emission transition.

Therefore, absorption transition < emission.

Thus (a) is correct option.

- 21. Assertion :** Bohr had to postulate that the electrons in stationary orbits around the nucleus do not radiate.

Reason : According to classical physics all moving electrons radiate.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Bohr postulated that electrons in stationary orbits around the nucleus do not radiate. This is the one of Bohr's postulate. According to this the moving electrons radiates only when they go from one orbit to the next lower orbit.

Thus (b) is correct option.

Ans :

OD 2001

Radius of ionised beryllium = Radius of hydrogen atom.

Energy level of ground state of hydrogen $n_1 = 1$.

Atomic no. of hydrogen atom $Z_1 = 1$.

and atomic no. of beryllium atom $Z_2 = 4$.

Orbital radius of n^{th} orbit,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2 Z}$$

or

$$n \propto \sqrt{Z}$$

Therefore, $\frac{n_1}{n_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

or

$$n_2 = 2n_1 = 2 \times 1 = 2$$

where, n_2 = Energy level of triply ionised beryllium

Thus (b) is correct option.

14. In which of the following system will the radius of first orbit be minimum ?

- (a) hydrogen atom
(b) deuterium atom
(c) singly ionized helium
(d) doubly ionized lithium

Ans :

Foreign 2014

Energy level of first orbit, $n = 1$

Radius of first orbit,

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2 Z} \propto \frac{1}{Z}$$

Atomic number of hydrogen atom (${}_1\text{H}^1$), deuterium atom (${}_1\text{H}^2$), singly ionized helium (He^+) and doubly ionized lithium (Li^{++}) are 1, 1, 2 and 3 respectively. Since the atomic no. of doubly ionized lithium is maximum, therefore radius of first orbit of doubly ionized lithium will be minimum.

Thus (d) is correct option.

15. If the energy of hydrogen atom in ground state is -13.6 eV , then its energy in the first excited state will be

- (a) -3.4 eV (b) -6.8 eV
(c) -27.2 eV (d) -52.4 eV

Ans :

OD 2007

Energy of hydrogen atom in ground state,

$$E_1 = -13.6 \text{ eV}$$

Energy level of a hydrogen atom in its first excited state,

$$n = 2$$

Therefore energy of hydrogen atom in first excited state,

$$\begin{aligned} E_2 &= \frac{E_1}{n^2} \\ &= -\frac{13.6}{(2)^2} \\ &= -3.4 \text{ eV} \end{aligned}$$

Thus (a) is correct option.

16. The ground state energy of hydrogen atom is -13.6 eV . What is the potential energy of the electron in this state?

- (a) 0 eV (b) 1 eV
(c) 2 eV (d) -27.2 eV

Ans :

Delhi 2004, OD 2007

Total energy of hydrogen atom in ground state,

$$E = -13.6 \text{ eV}$$

Potential energy of the electron in ground state,

$$\begin{aligned} P &= 2 \times E \\ &= 2 \times (-13.6) = -27.2 \text{ eV} \end{aligned}$$

Thus (d) is correct option.

17. As the electron in Bohr's orbit of hydrogen atom passes from state $n = 2$ to $n = 1$, the kinetic energy K and the potential energy u change as

- (a) K four-fold and u two-fold
(b) K two-fold and u four-fold
(c) K two-fold and u also two-fold
(d) K four-fold and u also four-fold

Ans :

OD 2005

Initial energy level, $n_1 = 2$

Final energy level, $n_2 = 1$

Kinetic energy of an electron in the n^{th} orbit,

$$K_n = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} \propto \frac{1}{n^2}$$

and potential energy of an electron in the n^{th} orbit,

$$u_n = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2} \propto -\frac{1}{n^2}$$

Therefore, $\frac{K_1}{K_2} = \left(\frac{n_2}{n_1}\right)^2$
 $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

or

$$K_2 = 4K_1$$

Similarly, $\frac{u_1}{u_2} = \left(\frac{n_2}{n_1}\right)^2$
 $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

or

$$u_2 = 4u_1$$

Thus (d) is correct option.

7. According to classical theory, the path of an electron in Rutherford's atom is

(a) spiral (b) circular
(c) parabolic (d) straight line

Ans :

Delhi 2014

According to classical theory that the path of an electron in Rutherford's atom is spiral towards the nucleus due to the continuous loss of energy of electron.

Thus (a) is correct option.

8. For an electron in the second orbit of Bohr's hydrogen atom, the moment of linear momentum is

(a) πh (b) $2\pi h$
(c) $\frac{h}{\pi}$ (d) $\frac{2h}{\pi}$

Ans :

OD 2007

Energy level of orbit, $n = 2$

Moment of linear momentum of second orbit,

$$L = \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi}$$

Thus (c) is correct option.

9. Nuclear atom model is required to explain the results of

(a) Rydberg's experiment
(b) Rutherford's experiment
(c) Thomson's atom model
(d) Bohr's frequency condition

Ans :

Delhi 2008, Foreign 2012

In Rutherford's only a very small number of α -particles retrace their path.

Therefore a massive and positive part of an atom should be concentrated in a small portion of the atom. This small positive part having almost entire mass of the atom is called nucleus.

Thus (b) is correct option.

10. In the Bohr's atomic model, if the atomic radius of the first orbit is r_0 , then radius of the fourth orbit is

(a) r_0 (b) $4r_0$
(c) $8r_0$ (d) $16r_0$

Ans :

SQP 2017

Atomic radius of first orbit, $r_1 = r_0$

Initial energy level, $n_1 = 1$

Final energy level, $n_2 = 4$

Radius of n^{th} orbit,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \propto n^2$$

Therefore,

$$\begin{aligned} \frac{r_1}{r_2} &= \left(\frac{n_1}{n_2} \right)^2 \\ &= \left(\frac{1}{4} \right)^2 = \frac{1}{16} \end{aligned}$$

Radius of fourth orbit, $r_2 = 16r_1 = 16r_0$

Thus (d) is correct option.

11. In Bohr's atomic model of hydrogen atom, the product of binding energy of electron in n^{th} orbit and its radius in n^{th} orbit is

(a) independent of n
(b) proportional to n
(c) proportional to n^2
(d) inversely proportional to n

Ans :

Foreign 2006

Binding energy of electron in n^{th} orbit of hydrogen atom,

$$E_n = \frac{13.6}{n^2} \quad \dots(1)$$

and radius of electron in n^{th} orbit of hydrogen atom,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \quad \dots(2)$$

Multiplying equations (1) and (2),

$$\begin{aligned} E_n \times r_n &= \left(\frac{13.6}{n^2} \right) \times \left(\frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \right) \\ &= \frac{13.6 \varepsilon_0 h^2}{\pi m e^2} \end{aligned}$$

Thus, it is independent of n .

Thus (a) is correct option.

12. The total energy of an electron in the first excited state of hydrogen atom is about -3.4 eV. Its kinetic energy in this state is

(a) -3.4 eV (b) 3.4 eV
(c) -6.8 eV (d) 6.8 eV

Ans :

SQP 2010

Total energy of electron in first excited state,

$$E = -3.4 \text{ eV}$$

Kinetic energy of electron in first excited state,

$$E_K = -E = -(-3.4) = 3.4 \text{ eV}$$

Thus (b) is correct option.

13. Which state of triply ionised beryllium (Be^{+++}) has the same orbital radius as that of the ground state of hydrogen?

(a) $n = 1$ (b) $n = 2$
(c) $n = 3$ (d) $n = 4$

VERY SHORT ANSWER QUESTIONS

22. What is the ratio of radii of the orbits corresponding to first excited state and ground state, in a hydrogen atom?

Ans :

OD 2021

For first excited states $n = 2$

Ground state occurs for $n = 1$

Hence, $r_n = r_0 n^2$

Since, $r \propto n^2$ [$r_0 = 0.53$]

$$\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$$

$$= \left(\frac{2}{1}\right)^2$$

$$r_1 : r_2 = 4 : 1$$

Where, r_1 and r_2 are radii corresponding to first excited state and ground state of the atom.

23. Define ionisation energy. What is its value for a hydrogen atoms?

Ans :

Delhi 2020

The minimum amount of energy required to remove an electron from the ground state of the atom is known as ionisation energy.

$$\text{Ionisation energy for hydrogen atom} = E_\infty - E_1$$

$$= -(-13.6 \text{ eV}) = 13.6 \text{ eV}$$

24. Where is H_α -line of the balmer series in the emission spectrum of hydrogen atom obtained?

Ans :

Comp 2018, SQP 2015

H_α -line of the balmer series in the emission spectrum of hydrogen atom is obtained in visible region.

25. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model?

Ans :

Comp 2014

According to Bohr model electrons having different energies belong to different levels having different values of n . So, their angular momenta will be different, as

$$L = \frac{nh}{2\pi} \text{ or } L \propto n$$

26. State Bohr's quantisation condition for defining stationary orbits.

Ans :

SQP 2011

According to Bohr's quantisation condition, electrons are permitted to revolve in only those orbits in which the angular momentum of electron is an integral

multiple of $\frac{h}{2\pi}$ i.e.,

$$mvr = \frac{nh}{2\pi} \quad \text{where, } n = 1, 2, 3, \dots$$

m , v , r are mass, speed and radius of electron respectively and h being Planck's constant.

27. In the Rutherford scattering experiment, the distance of closest approach for an α -particles is d_0 . If α -particles is replaced by a proton, then how much kinetic energy in comparison to α -particle will be required to have the same distance of closest approach d_0 ?

Ans :

Delhi 2017

Distance of closest approach,

$$d_0 = \frac{2kZe^2}{K_\alpha} \quad \left(\text{where, } k = \frac{1}{4\pi\epsilon_0}\right)$$

Since, For given distance of closest approach,

Kinetic energy $\propto Z$ (atomic number)

$$\frac{K_{\text{proton}}}{K_\alpha} = \frac{Z_{\text{proton}}}{Z_\alpha} = \frac{1}{2}$$

$$K_{\text{proton}} : K_\alpha = 1 : 2$$

28. Why is the classical Rutherford model for an atom of electron orbiting around the nucleus not able to explain the atomic structure?

Ans :

OD 2013

The classical method could not explain the atomic structure as the electron revolving around the nucleus are accelerated and emits energy as the result, the radius of the circular paths goes on decreasing. Ultimately electrons fall into the nucleus, which is not in practical.

29. In the Rutherford scattering experiment, the distance of closest approach for an α -particle is d_0 . If α -particle is replaced by a proton, then how much kinetic energy in comparison to α -particle will be required to have the same distance of closest approach d_0 ?

Ans :

Comp 2021

Distance of closest approach

$$d_0 = \frac{2kZe^2}{K_X} \quad \text{Where, } k = \frac{1}{4\pi\epsilon_0}$$

Since, for given distance of closest approach, kinetic energy $\propto Z$ (atomic number)

$$\frac{K_{\text{proton}}}{K_\alpha} = \frac{Z_{\text{proton}}}{Z_\alpha} = \frac{1}{2}$$

$$K_{\text{proton}} : K_\alpha = 1 : 2$$

30. Define ionization energy. How would the ionization energy change when electron in hydrogen atom is replaced by a particle of mass 200 times that of the electron but having the same charge?

Ans :

Delhi 2019

The minimum energy, required to free the electron from the ground state of the hydrogen atom, is known as ionization energy.

$$E_0 = \frac{me^4}{8\varepsilon_0^2 h^2}, \text{ i.e., } E_0 \propto m$$

Therefore, ionization energy will become 200 times.

$$= 13.6 \text{ eV}$$

Hence, Ratio of energies of photon

$$= \frac{10.2}{13.6} = \frac{3}{4} = 3 : 4$$

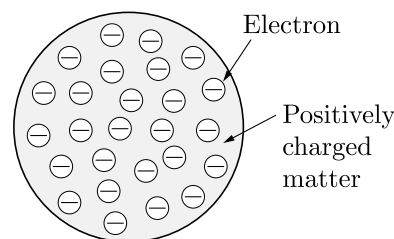
33. Briefly describe Thomson's model of an atom, Why was this model discarded later on?

Ans :

Delhi 2020

Thomson's Model of an Atom

An atom consists of a sphere of positively charged matter in which the negatively charged electrons are uniformly embedded like the plums in a pudding, as shown in figure. The mutual repulsions between electrons are balanced by their attractions with the positively charged matter. Thus the atom as a whole is stable and neutral.



Failure of Thomson's Model

Thomson model was discarded due to the following drawbacks:

1. It could not explain the origin of several spectral series in the case of hydrogen and other atoms.
 2. It failed to explain the large angle scattering of α -particles in Rutherford's experiment.
34. Define the terms excitation and ionisation energies; and excitation and ionisation potentials.

Ans :

OD 2014, Delhi 2019

Excitation Energy

The excitation energy of an atom is defined as the energy required by its electron to jump from the ground state to any one of the excited states.

First excitation energy of hydrogen

$$= E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

Ionisation Energy

It is defined as the energy required to knock an electron completely out of the atom, i.e., the energy required to take an electron from its ground state to the outermost orbit ($n = \infty$). After the removal of the electron, the atom is left with positive charge and it is said to be ionised.

Ionisation energy of hydrogen

$$= E_\infty - E_1 = 0 - (-13.6) = 13.6 \text{ eV}$$

SHORT ANSWER QUESTIONS

31. (a) What is meant by ionisation energy? Write its value for hydrogen atom.
(b) Define the term, mass defect. How is it related to stability of the nucleus?

Ans :

OD 2023

- (a) The ionization energy is the quantity of energy that an isolated, gaseous atom in the ground electronic state must absorb to discharge an electron, resulting in a cation. The energy is usually expressed in kJ/mol. It is also called an ionization potential, which is the amount of energy required to remove an electron from an isolated atom or molecule. The value of ionisation energy for hydrogen atom is 13.6 eV.
(b) Mass defect is the difference between the actual atomic mass and the predicted mass. The actual mass of the atomic nucleus is always less than the mass of protons and neutrons present in the nucleus. When a nucleus is formed, energy is released. This energy is removed in the form of a reduction in total mass. This missing mass is known as the mass defect.
The mass defect (Δm) can be calculated by subtracting the original atomic mass (M_n) from the sum of the mass of protons and neutrons present in the nucleus.

32. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its
(i) second permitted energy level to the first permitted level and
(ii) the highest permitted energy level to the first permitted level.

Ans :

Delhi 2021

- (i) Since, the second permitted energy level to the first level $= E_2 - E_1 =$ energy of photon released.
 $(-3.4 \text{ eV}) - (-13.6 \text{ eV}) = 10.2 \text{ eV}$
(ii) The highest permitted energy level to the first permitted level

$$E_\infty - E_1 = 0 - (-13.6)$$

Excitation Potential

It is that accelerating potential which gives to a bombarding electron, sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit.

First excitation potential of hydrogen

$$= -3.4 - (-13.6) = 10.2 \text{ V}$$

Second excitation potential of hydrogen

$$= -1.51 - (-13.6) = 12.09 \text{ V}$$

Ionisation Potential

It is that accelerating potential which gives to a bombarding electron, sufficient energy the target atom by knocking one of its electrons completely out of the atom.

Ionisation potential of hydrogen

$$= 0 - (-13.6) = 13.6 \text{ V}$$

35. Describe Rutherford's model of the atom. Give its limitations.

Ans :

OD 2020

Rutherford's Model of an Atom

On the basis of the α -particle scattering experiment, Rutherford proposed the following model of an atom:

1. An atom consists of a small and massive central core in which the entire positive charge and almost the whole mass of the atom are concentrated. This core is called the nucleus.
2. The size of the nucleus ($\approx 10^{-15} \text{ m}$) is very small as compared to the size of atom ($\approx 10^{-10} \text{ m}$).
3. The nucleus is surrounded by a suitable number of electrons so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.
4. The electrons revolve around the nucleus in various orbits just as planets revolve around the sun. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.

Limitations of Rutherford's Atomic Model

Rutherford's model has two main difficulties in explaining the structure of atom:

1. It predicts that atoms are unstable because the accelerated electrons revolving around the nucleus must radiate energy and move spirally into the nucleus. This contradicts the stability of matter.
 2. It cannot explain the characteristic line spectra of atoms of different elements.
36. State the drawbacks of Bohr's atomic theory.

Ans :

Comp 2015

Limitations of Bohr's Atomic Model

1. This model is applicable only to hydrogenic atoms. It cannot be extended to even two electron atoms such as He.
2. This model is unable to explain the relative intensities of the frequencies emitted by hydrogenic atoms.
3. It does not explain the fine structure of spectral lines even in hydrogen atom.
4. It does not explain why only circular orbits should be chosen when elliptical orbits are also possible.
5. It does not take into account the wave properties of electrons.

37. Write the expression for Bohr's radius in hydrogen atom.

Ans :

Foreign 2015, OD 2003

Expression for Bohr's radius in hydrogen atom,

$$r = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$$

$$r = \frac{n^2 h^2}{4\pi^2 m k e^2}$$

where, n = principal quantum number

m = mass of electron

$$k = \frac{1}{4\pi\epsilon_0}$$

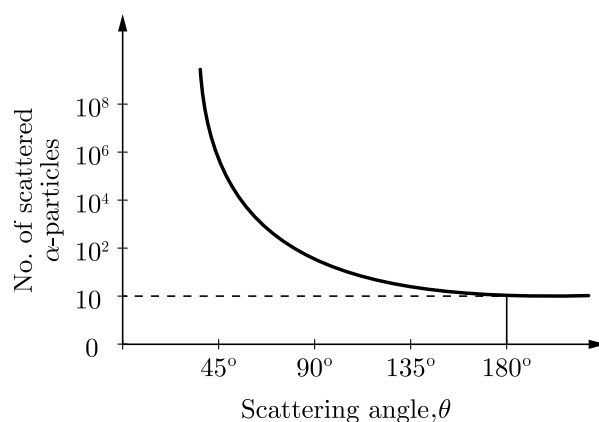
$$= 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

Z = atomic number of atom = 1

h = Planck's constant

38. In an experiment of α -particle scattering by a thin foil of gold, draw a plot showing the number of particle scattered versus the scattering angle θ .

Why is it that a very small fraction of the particles are scattered at $\theta > 90^\circ$?



Ans :

SQP 2013

A small fraction of the alpha particles scattered at angle $\theta > 90^\circ$ is due to the reason.

$$\text{Total energy, } E = E_K + U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{2r} \quad \dots(3)$$

Comparing equation (1), (2), (3), we have

$$E_K = -E \text{ and } U = 2E$$

$$\text{Given, } E = -13.6 \text{ eV}$$

[For ground state, $n = 1$]

$$\text{Hence, Kinetic energy, } E_K = 13.6 \text{ eV}$$

$$\text{Potential energy, } U = 2 \times (-13.6 \text{ eV}) \\ = -27.2 \text{ eV}$$

For second excited state, $n = 3$

$$\text{Hence, } E_K = -E = \frac{+13.6}{9} \text{ eV} = 1.51 \text{ eV}$$

$$\text{and } U = 2E \\ = \frac{2 \times (-13.6 \text{ eV})}{9} = -3.02 \text{ eV}$$

69. The electron in a given Bohr orbit has a total energy of -1.5 eV . Calculate its

1. Kinetic energy
2. Potential energy
3. Wavelength of radiation emitted, when this electron makes a transition to the ground state.

(Given, energy in the ground state $= -13.6 \text{ eV}$ and Rydberg's constant $= 1.09 \times 10^7 \text{ m}^{-1}$)

Ans :

OD 2017

1. The kinetic energy E_K of the electron in an orbit is equal to negative of its total energy E .

$$E_K = -E \\ = -(-1.5) = 1.5 \text{ eV}$$

2. The potential energy U of the electron in an orbit is equal to twice its total energy E .

$$\text{i.e., } U = 2E \\ = -1.5 \times 2 = -3 \text{ eV}$$

3. As, a result of transition of electron from excited state of ground state.

$$\text{Energy of radiation} = -1.5 - (-13.6)$$

$$\text{Hence, Ground state energy of H-atom} \\ = -13.6 \text{ eV}$$

$$E = hv = h \frac{v}{\lambda}$$

$$\frac{hc}{\lambda} = 12.1 \text{ eV} = \text{energy of radiation}$$

$$\text{Hence, } \frac{1}{\lambda} = \frac{12.1 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$$\lambda = 1.025 \times 10^{-7} \text{ m} \\ = 1025 \text{ \AA}$$

70. The ground state energy of hydrogen atom is -13.6 eV . If an electron makes a transition from an energy level -0.85 eV to 3.4 eV , Calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

Ans :

Delhi 2013

$$\text{We have } E_n = \frac{-13.6}{n^2} \text{ eV} \quad \dots(1)$$

$$\text{For, } n = 1, E_1 = -13.6 \text{ eV}$$

When electron undergoes transition from,

$$E_A = 0.85 \text{ eV to } E_B = -3.4 \text{ eV}$$

Then, from equation (1),

$$-0.85 = \frac{-13.6}{n_A^2}$$

$$n_A = 4$$

$$\text{Similarly, } -3.4 = \frac{-13.6}{n_B^2}$$

$$n_B = 2$$

Hence, electron transits from $n = 4$ to $n = 2$. It corresponds to Balmer series.

$$\frac{1}{\lambda} = R \left(\frac{1}{n_B^2} - \frac{1}{n_A^2} \right)$$

$$\text{Here, } n_A = 4, n_B = 2, R = 1.097 \times 10^7 \text{ m}^{-1}$$

$$\text{Then, } \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \\ \lambda = 4862 \text{ \AA}$$

71. The value of ground state energy of hydrogen atom is -13.6 eV .

1. Find the energy required to move an electron from the ground state to the first excited state of the atom.
2. Determine (a) the kinetic energy and (b) orbital radius in the first excited state of the atom.

[Given, the value of bohr's radius $= 0.53 \text{ \AA}$]

Ans :

OD 2015, Comp 2011

1. Energy of electron in n th orbit of hydrogen atom.

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

$$\text{For } n = 1,$$

$$E_1 = 13.6 \text{ eV}$$

$$\text{For } n = 2,$$

$$E_2 = \frac{-13.6}{4} = -3.4 \text{ eV}$$

Energy required to move an electron

$$= E_2 - E_1 = -3.4 - (-13.6) \\ = -3.4 + 13.6 = 10.2 \text{ eV}$$

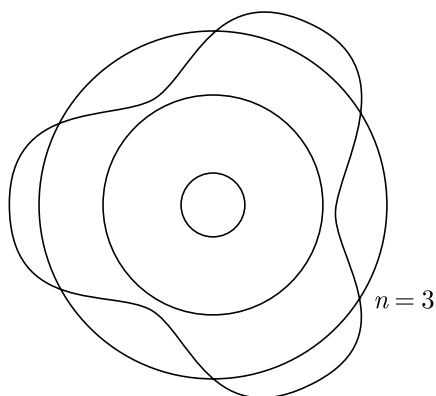
$$2\pi r = n\lambda \quad \dots(2)$$

Substituting value of λ from eq. (2) in eq.(1), we get

$$2\pi r = n \frac{h}{mv}$$

$$mvr = n \frac{h}{2\pi} \quad \dots(3)$$

This is Bohr's postulate of quantisation of energy levels.



43. A hydrogen atom initially in its ground absorbs a photon and is in the excited state with energy 12.5 eV. Calculate the longest wavelength of the radiation emitted and identify the series to which it belongs.

[Take Rydberg constant $R = 1.1 \times 10^7 \text{ m}^{-1}$]

Ans :

Comp 2010

Let n_i and n_f are the quantum numbers of initial and final states, then we have

$$\frac{1}{\lambda_{\max}} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

The energy of the incident photon = 12.5 eV

Energy of ground state = -13.6 eV

Hence, Energy after absorption of photon can be -1.1 eV

This means that electron can go to the excited state $n_i = 3$.

It emits photon of maximum wavelength on going to $n_f = 2$, therefore,

$$\frac{1}{\lambda_{\max}} = \left(\frac{1}{2^2} - \frac{1}{3^2} \right) R$$

$$\lambda_{\max} = \frac{36}{5R} = \frac{36}{5 \times 1.1 \times 10^7}$$

$$= 6.555 \times 10^{-7} \text{ m} = 6555 \text{ \AA}$$

It belongs to balmer series.

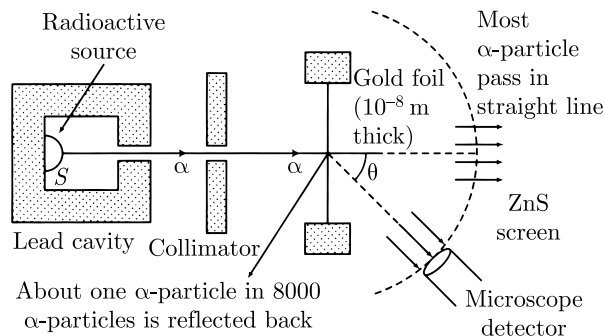
44. In the study of Geiger-marsdon experiment on scattering of α -particles by a thin foil of gold, draw the trajectory of α -particles in the coulomb field of target nucleus. Explain briefly how can gets the

information on the size of the nucleus from this study.

From the relation $R = R_0 A^{\frac{1}{3}}$, where, R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A .

Ans :

Foreign 2013



From this experiment, the following is observed.

- Most of the α -particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms.
- About one α -particle in every 8000 α -particles deflects by more than 90° . As most of the α -particles go undeflected and only a few get deflected, this shows that most of the space in an atom is empty and at the centre of the atom, there exists a nucleus. By the number of α -particles get deflected, the information regarding size of the nucleus can be known.

If m is the average mass of the nucleus and R is the nuclear radius, then mass of nucleus = mA , where A is the mass number of the element.

Volume of the nucleus, $V = \frac{4}{3}\pi R^3$

$$V = \frac{4}{3}\pi (R_0 A^{\frac{1}{3}})^3$$

$$V = \frac{4}{3}\pi R_0^3 A$$

Density of nuclear matter, $\rho = \frac{mA}{V}$

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$\rho = \frac{3m}{4\pi R_0^3}$$

This shows that the nuclear density is independent of A .

45. Using de-Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantization of energy levels in a hydrogen atom.

Ans :

Comp 2018, OD 2013

Bohr's Quantization Condition of Angular Momentum

Consider the motion of an electron in a circular orbit of radius r around the nucleus of atom. According to

That if impact parameter ' b ' reduces to zero, coulomb force increases, hence, alpha particles are scattered at angle $\theta > 90^\circ$, and only one alpha particle is scattered at angle 180° .

39. (i) State bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition.
 (ii) An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond?

Ans :

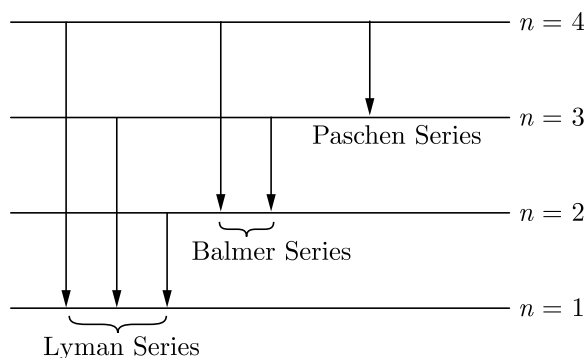
OD 2014

- (i) Bohr's third postulate it states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by

$$h\nu = E_i - E_f$$

Where E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

- (ii) Electron jumps from fourth to first orbit in an atom



Maximum number of spectral lines can be

$$4C_2 = \frac{4!}{2!2!} = \frac{4 \times 3}{2} = 6$$

The line responds to Lyman series (e^- jumps to 1st orbit), Balmer series (e^- jumps to 2nd orbit), paschen series (e^- jumps to 3rd orbit).

40. Using Bohr's second postulate of quantisation of orbital angular momentum show that the circumference of the electron in the n th orbital state in hydrogen atom is n -times the de-Broglie wavelength associated with it.

Ans :

OD 2016, Comp 2008

Bohr's second postulate states that the electron revolves around the nucleus in certain privileged orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$

where, h is Plank's constant,

$$\text{i.e., } L = mvr = \frac{nh}{2\pi}$$

where, m = mass of electron

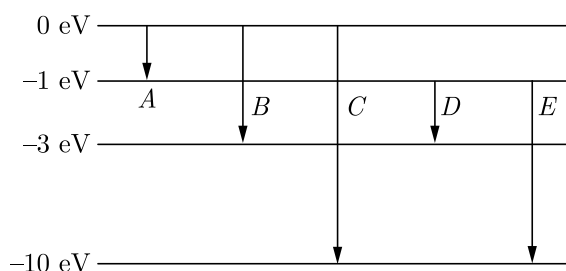
v = speed of electron

and r = radius of orbit of electron

$$2\pi r = n\left(\frac{h}{mv}\right)$$

Circumference of electron in n^{th} orbit = $n \times$ de-Broglie wavelength associate with electron.

41. The energy levels of an atom are given below in the diagram.



Which of the transitions belong to lyman and balmer series? Calculate the ratio of the shorter wavelengths of the lyman and the balmer series of the spectra.

Ans :

Delhi 2015

Transition C and E belong to lyman series.

Reason : In lyman series, the electron jumps to lowest energy level from any higher energy levels.

Transition B and D belong to balmer series.

Reason : The electron jumps from any higher energy level to the level just above the ground energy level. The wavelength associated with the transition is given by,

$$\lambda = \frac{hc}{\Delta E}$$

Ratio of the shortest wavelength

$$\lambda_L : \lambda_B = \frac{hc}{\Delta E_L} : \frac{hc}{\Delta E_B}$$

$$\frac{1}{0 - (-10)} : \frac{1}{0 - (-3)} = 3 : 10$$

42. Using de-Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantisation of energy levels in a hydrogen atom.

Ans :

OD 2014

According, to de-Broglie's hypothesis.

$$\lambda = \frac{h}{mv} \quad \dots(1)$$

According to de Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de-Broglie wavelength.

The wavelength of line in these series can be expressed by the following formulae.

For Lyman Series :

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

where, $n = 2, 3, 4, \dots$

For Balmer Series :

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where, $n = 3, 4, 5, \dots$

For Paschen Series :

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right),$$

where, $n = 4, 5, 6, \dots$

For Brackett Series :

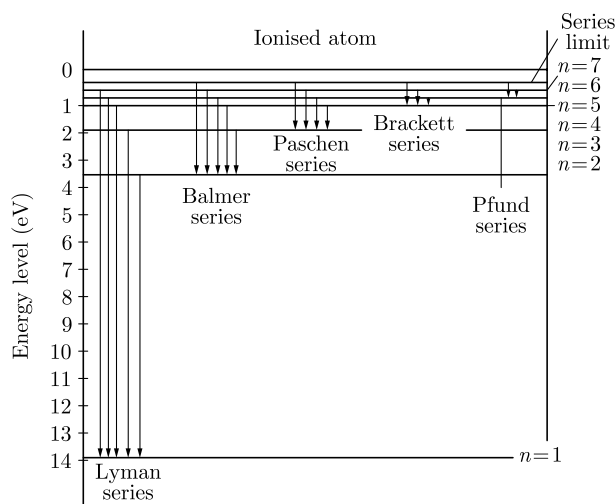
$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right),$$

where, $n = 5, 6, 7, \dots$

For Pfund Series :

$$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$$

where, $n = 6, 7, 8, \dots$

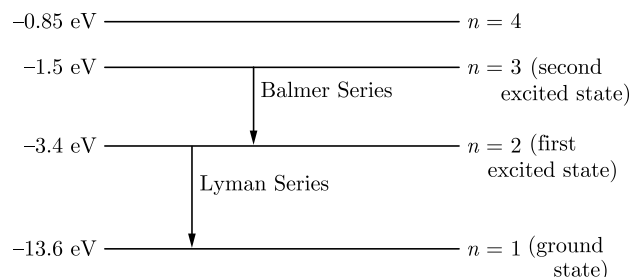


47. (i) In hydrogen atom, an electron undergoes transition from second excited state to the first excited state and then to the ground state. Identify the spectral series to which these transitions belong.
- (ii) Find out the ratio of the wavelength of the emitted radiations in the two cases.

Ans :

Foreign 2012, Comp 2009

- (i) An electron undergoes transition from second excited state to the first excited state is Balmer series and then to the ground state is Lyman series.
- (ii) The wavelength of the emitted radiations in the two cases.



For $n_2 \xrightarrow{\lambda_L} n_1$

$$\Delta E = (-3.40 + 13.6) = 10.20 \text{ eV}$$

$$\lambda_L = \frac{12.43 \times 10^{-7}}{10.2}$$

$$= 1.218 \times 10^{-7} \text{ m}$$

$$\lambda_L = 1218 \text{ \AA}$$

For $n_3 \xrightarrow{\lambda_B} n_2$

$$\Delta E = (-1.5 + 3.4) = 1.9 \text{ eV}$$

$$\lambda_B = \frac{12.43 \times 10^{-7}}{1.9}$$

$$= 6.54 \times 10^{-7} \text{ m} = 6540 \text{ \AA}$$

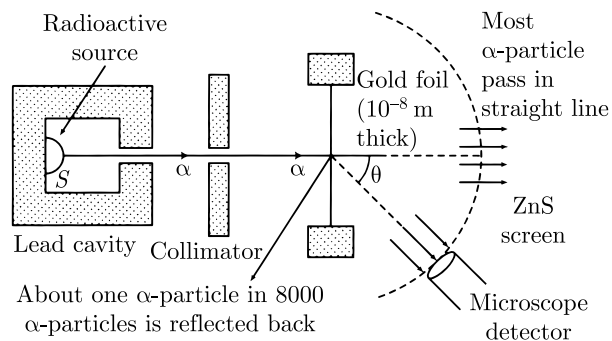
$$\frac{\lambda_B}{\lambda_L} = \frac{6540 \text{ \AA}}{1218 \text{ \AA}} = 5.36$$

48. Describe Rutherford's model of atom developed on the basis of his alpha particle scattering experiment.

Ans :

Delhi 2020

This experiment was suggested by Rutherford in 1911 as given in the figure below.



In this experiment, H Geiger and E Marsden took $^{214}_{83}\text{Bi}$ as a source for α -particles. A collimated beam of α -particles of energy 5.5 MeV was allowed to fall on $2.1 \times 10^{-7} \text{ m}$ thick gold foil. The α -particles were observed through a rotatable detector consisting of a zinc sulphide screen and microscope and it was found that α -particles got scattered. These scattered α -particles produced scintillations on the zinc sulphide screen. Now, these scintillation were counted at different angles from the direction of incident beam.

de-Broglie hypothesis, this electron is also associated with wave character. Hence a circular orbit can be taken to be a stationary energy state only if it contains an integral number of de-Broglie wavelengths, i.e., we must have

$$2\pi r = n\lambda$$

But de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

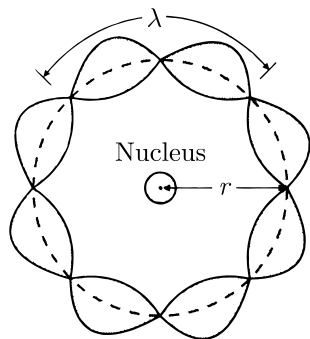
Hence,
$$2\pi r = \frac{nh}{mv}$$

The angular momentum L of the electron must be,

$$L = mvr = \frac{nh}{2\pi}$$

$$n = 1, 2, 3, \dots$$

This is the famous **Bohr's quantisation condition for angular momentum**. Thus only those circular orbits can be the allowed stationary states of an electron in which its angular momentum is an integral multiple of $\frac{h}{2\pi}$.



LONG ANSWER QUESTIONS

46. Give postulates of Bohr's theory. Explain hydrogen spectrum on the basis of Bohr's theory.

Ans :

SQP 2015

Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates.

These three postulates are as follows:

1. **Bohr's First Postulate** was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist and each possible state has definite total energy. These are called the stationary states of the atom.
2. **Bohr's Second Postulate** states that the electron revolves around the nucleus only in those orbits

for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant ($= 6.63 \times 10^{-34} \text{ J-s}$)

Thus, the angular momentum (L) of the orbiting electron is quantised,

i.e.,
$$L = \frac{nh}{2\pi}$$

As, angular momentum of electron $= mvr$

Hence, for any permitted (stationary) orbit,

$$mvr = \frac{nh}{2\pi}$$

where, n = any positive integer 1, 2, 3, ...

It is also called principal quantum number.

3. **Bohr's Third Postulates** states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

The frequency of the emitted photon is given by

$$h\nu = E_i - E_f$$

where, E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

Hydrogen Spectrum or Line Spectra of Hydrogen Atom

Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as **hydrogen emission spectrum**. There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as absorption spectrum.

Blamer found an empirical formula by the observation of a small part of this spectrum and it is represented by.

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where, $n = 3, 4, 5, \dots$

and, R = is a constant called Rydberg constant

and its value is $1.097 \times 10^7 \text{ m}^{-1}$

So,
$$\frac{1}{\lambda} = 1.522 \times 10^6 \text{ m}^{-1}$$

$$= 656.3 \text{ nm for } n = 3$$

Other series of spectra for hydrogen were subsequently discovered and known by the name of their discoverers. The lines of Balmer series are found in the visible part of the spectrum. Other series were found in the invisible parts of the spectrum.

e.g. Lyman series in the ultraviolet region and Paschen, Brackett and Pfund in the infrared region.

Ans :

OD 2020, Comp 2004

Energy Level Diagram

It is a diagram in which the energies of the different stationary states of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale.

Energy Levels of Hydrogen

From Bohr's theory, the total energy of an electron in n^{th} orbit is given by,

$$E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2 h^2}$$

For hydrogen $Z = 1$, therefore,

$$E_n = -\frac{2\pi^2 mk^2 e^4}{h^2} \cdot \frac{1}{n^2}$$

Energy of the electron in the first orbit ($n = 1$) is given by,

$$\begin{aligned} E_1 &= -\frac{2 \times (3.14)^2 \times 9.11 \times 10^{-31} \times (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4}{(6.63 \times 10^{-34})^2} \cdot \frac{1}{1^2} \\ &= -21.76 \times 10^{-19} \text{ J} \\ &= -\frac{21.76 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \end{aligned}$$

or $E_1 = -13.6 \text{ eV}$ [since $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$]

Hence we can write,

$$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

Setting $n = 2, 3, 4$, etc., we get the energies for the higher orbits as follows:

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}$$

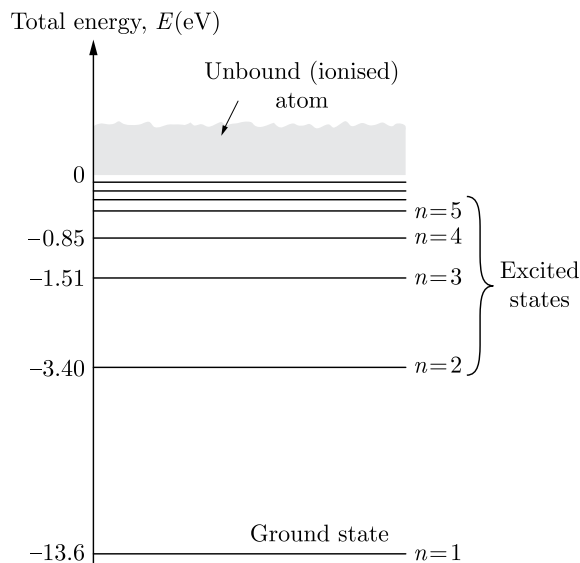
$$E_4 = \frac{-13.6}{4^2} = -0.85 \text{ eV}$$

$$E_5 = \frac{-13.6}{5^2} = -0.54 \text{ eV}$$

$$E_6 = \frac{-13.6}{6^2} = -0.37 \text{ eV}$$

Clearly, an electron can have only certain definite values of energy while revolving in different orbits. This is called energy quantisation.

The energy state corresponding to $n = 1$, has the lowest energy equal to -13.6 eV . This state is called the ground state of the atom. Normally the electron in the hydrogen atom remains in the ground state. When the hydrogen atom receives energy from outside by processes such as electron collisions, the electron may make transition to some higher energy state like E_2 , E_3 , E_4 , etc. which are called the excited states. The energy level diagram for the hydrogen atom is shown in figure.

**NUMERICAL QUESTIONS**

51. Calculate the shortest wavelength of the spectral lines emitted in balmer series.

[Given Rydberg constant, $R = 10^7 \text{ m}^{-1}$]

Ans :

Delhi 2021

For hydrogen spectra,

$$\frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Where, n = orbit on which electron jump; m = orbit from which electron jump.

For balmer series.

$$n = 2$$

But for shortest wavelength, $n = \infty$

$$\text{Hence, } \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

$$\frac{1}{\lambda} = \frac{R}{4}$$

$$\lambda = \frac{4}{R} = 4 \times 10^{-7} \text{ m}$$

52. Use Bohr model of hydrogen atom to calculate the speed of the electron in the first excited state.

Ans :

Delhi 2016, OD 2010

Bohr's quantum condition is

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr}$$

and

$$r = \frac{1}{k} \frac{n^2 h^2}{4\pi^2 m e^2}$$

So,

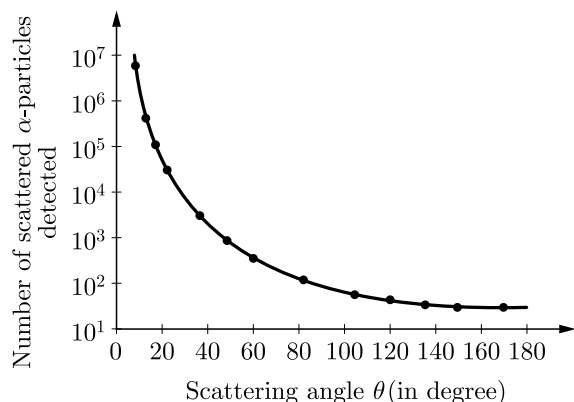
$$v = k \frac{2\pi e^2}{nh}$$

Observations

Rutherford made the following observations from his experiment that are given below

1. Most of the α -particles passed through the gold foil without any appreciable deflection.
2. Only about 0.14% of the incident α -particles scattered by more than 1° .
3. About one α -particle in every 8000 α -particles deflected by more than 90° .

The total number of α -particles (N) scattered through an angle (θ) is as shown in the below figure:



4. The number of α -particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\frac{\sin^4 \theta}{2}$.

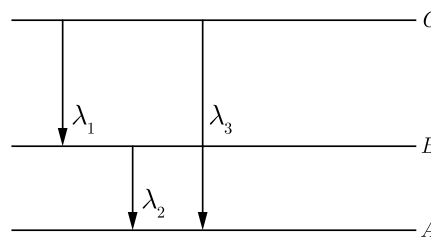
$$N(\theta) \propto \frac{1}{\frac{\sin^4 \theta}{2}}$$

5. The force between α -particles and nucleus is given by,

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(Ze)}{r^2}$$

Where, r is the distance between the α -particles and the nucleus. This force is directed along the line joining the α -particle and the nucleus. The magnitude and direction of this force on α -particle continuously changes as it approaches the nucleus and recedes away from it.

49. 1. State Bohr's quantization condition for defining stationary orbits. How does the Broglie hypothesis, explain the stationary orbits?
2. Find the relation between the three wavelengths λ_1 , λ_2 and λ_3 from the energy level diagram shown below.



Ans :

Delhi 2019

1. Only those orbits are stable for which the angular momentum of revolving electron is an integral multiple of $\left(\frac{h}{2\pi}\right)$, where h is the planck's constant. Thus, the angular momentum (L) of the orbiting electron is quantised. That is,

$$L = \frac{nh}{2\pi}$$

According to de-Broglie hypothesis

Linear momentum = $\frac{h}{\lambda}$ and for circular orbit,

$$L = r_n P$$

where, r_n is the radius of quantised orbits $\alpha = r_n \frac{h}{\lambda}$.

$$\text{Also, } L = \frac{nh}{2\pi}$$

$$\text{Hence, } \frac{r_n h}{\lambda} = \frac{nh}{2\pi}$$

$$\text{or } 2\pi r_n = n\lambda$$

Hence, circumference of permitted orbits are integral multiples of the wavelength.

$$2. \quad E_C - E_B = \frac{hc}{\lambda_1} \quad \dots(1)$$

$$E_B - E_A = \frac{hc}{\lambda_2} \quad \dots(2)$$

$$E_C - E_A = \frac{hc}{\lambda_3} \quad \dots(3)$$

Adding (1) and (2), we have

$$E_C - E_A = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad \dots(4)$$

From (3) and (4), we have

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

50. What is the energy level diagram for an atom? Calculate the energies of the various energy levels of a hydrogen atom and draw and energy level diagram for it.

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Hence,

$$r_n = r_0 n^2$$

Hence,

$$r \propto n^2$$

$$\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2$$

Hence,

$$n_1 : n_2 = 4 : 1$$

Where, r_1 and r_2 are radii corresponding to first excited state and ground state of the atom.

57. The electron in a given Bohr orbit has a total energy of -1.5 eV. Calculate its

- kinetic energy
- potential energy
- wavelength of radiation emitted, when this electron makes a transition to the ground state.

[Given, energy in the ground state $= -13.6$ eV and Rydberg's constant $= 1.09 \times 10^7 \text{ m}^{-1}$]

Ans :

Comp 2020

- The kinetic energy (K_K) of the electron in an orbit is equal to negative of its total energy (E).

$$E_K = -E = -(-1.5) = 1.5 \text{ eV}$$

- The potential energy (U) of the electron in an orbit is equal to twice its total energy (E)

i.e., $U = 2E = -1.5 \times 2 = -3 \text{ eV}$

- As, a result of transition of electron from excited state of ground state.

$$\text{Energy of radiation} = -1.5 - (-13.6)$$

(Since, Ground state energy of H-atom $= -13.6$ eV)

$$E = hv = h \frac{c}{\lambda}$$

$$\frac{hc}{\lambda} = 12.1 \text{ eV} = \text{energy of radiation}$$

$$\text{Hence, } \frac{1}{\lambda} = \frac{12.1 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$$\lambda = 1.025 \times 10^{-7} \text{ m} = 1025 \text{ \AA}$$

58. The ground state energy of hydrogen atom is -13.6 eV. If an electron makes a transition from an energy level -0.85 eV to -3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

Ans :

Delhi 2020, SQP 2013

As we know that,

$$E_n = \frac{-13.6}{n^2} \text{ eV} \quad \dots(1)$$

For $n = 1$, $E_1 = -13.6$ eV

When electron undergoes transitions from

$$E_A = -0.85 \text{ eV to } E_B = -3.4 \text{ eV}$$

Then, from equation (1)

$$-0.85 = \frac{-13.6}{n_A^2}$$

$$n_A = 4$$

$$\text{Similarly, } -3.4 = \frac{-13.6}{n_B^2}$$

$$n_B = 2$$

Hence, electron transits from $n = 4$ to $n = 2$. It corresponds to Balmer series.

$$\text{Now } \frac{1}{\lambda} = R \left(\frac{1}{n_B^2} - \frac{1}{n_A^2} \right)$$

$$\text{Here, } n_A = 4, n_B = 2, R = 1.097 \times 10^7 \text{ m}^{-1}$$

$$\text{Then, } \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$$

$$\lambda = 4862 \text{ \AA}$$

59. The value of ground state energy of hydrogen atom is -13.6 eV.

- Find the energy required to move an electron from the ground state to the first excited state of the atom.
- Determine (a) the kinetic energy and (b) orbital radius in the first excited state of the atom.

[Given, the value of Bohr's radius $= 0.53 \text{ \AA}$]

Ans :

SQP 2014

- Energy of electron in n th orbit of hydrogen atom.

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

For $n = 1$

$$E_1 = -13.6 \text{ eV}$$

For $n = 2$

$$E_2 = \frac{-13.6}{4} = -3.4 \text{ eV}$$

Energy required to move an electron

$$= E_2 - E_1 = -3.4 - (-13.6)$$

$$= -3.4 + 13.6 = 10.2 \text{ eV}$$

- (a) Kinetic energy $= -$ (Total energy of the electron in first excited state).

$$= -(-3.41) = 3.4 \text{ eV}$$

- (b) Orbital radius in the state $\propto n^2$

$$[\text{For first excited state, } n = 2]$$

$$= n \times 0.53 = 2.12 \text{ \AA}$$

60. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of $Z = 80$, stops and reverse its direction.

Ans :

Delhi 2015

Let, r be the distance between the alpha particle and the nucleus ($Z = 80$). When the alpha particle is at the stopping point, then

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{2\pi e^2}{nh} = \frac{e^2}{2\epsilon_0 h} \cdot \frac{1}{n} \\
 v &= \frac{e^2}{2\epsilon_0 h} \cdot \frac{1}{n} \\
 &= \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34}} \left(\frac{1}{n}\right) \\
 &= \frac{2.18 \times 10^6}{n} \text{ m/s}
 \end{aligned}$$

For $n = 2$,

$$v_2 = \frac{2.18 \times 10^6}{2} = 1.09 \times 10^6 \text{ m/s}$$

For $n = 3$, $v_3 = \frac{2.18 \times 10^6}{3} = 7.27 \times 10^5 \text{ m/s}$

Obviously the speed of electron goes on decreasing with increasing n .

- 53.** In a Geiger-marsdon experiment, calculate the difference of closest approach to the nucleus of $Z = 80$ when an α -particles of 8 MeV energy impinges on it before it comes to momentarily rest and reverses its direction.

How will the distance of closest approach be affected when the kinetic energy of the α -particle is doubled?

Ans :

OD 2021

Given, $Z = 80$, $E_K = K = 8 \text{ MeV}$
 $= 8 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$

Energy conservation law,

$$K = \frac{(Ze)(2e)}{4\pi\epsilon_0 r_0}$$

where, r_0 = distance of closest approach.

$$\begin{aligned}
 r_0 &= \frac{2Ze^2}{4\pi\epsilon_0(K)} \\
 r_0 &= \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{8 \times 10^6 \times 1.6 \times 10^{-19}} \\
 r_0 &= 2.88 \times 10^{-14} \text{ m} \\
 r_0 &= \frac{1}{K}
 \end{aligned}$$

If E_K gets doubled, distance of closest approach reduces to half.

- 54.** The energy E of a hydrogen atom with principal quantum no. n is given by $E = -\frac{13.6}{n^2} \text{ eV}$. Find the energy ejected when the electron jumps from $n = 3$ state to $n = 2$ state of hydrogen.

Ans :

Relation for energy, $E = -\frac{13.6}{n^2} \text{ eV}$
 (where, n is quantum no.)

Initial energy level, $n_1 = 3$

Final energy level, $n_2 = 2$

Energy ejected when electron jumps from one energy state to another energy state,

$$\begin{aligned}
 E &= -13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \\
 &= -13.6 \left[\frac{1}{3^2} - \frac{1}{2^2} \right] \\
 &= -13.6 \left[\frac{1}{9} - \frac{1}{4} \right] \\
 &= -13.6 \times \frac{-5}{36} \\
 &= 1.9 \text{ eV}
 \end{aligned}$$

- 55.** The ground state energy of hydrogen is -13.6 eV . What is the kinetic and potential energies of the electron in this state?

or

Given the value of the ground state energy of hydrogen atom as -13.6 eV , find out its kinetic and potential energy in the ground and second excited states ?

Ans :

Delhi 2021, Foreign 2010

Kinetic energy, $E_K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r} \quad \dots(1)$

Potential energy, $U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r} \quad \dots(2)$

Total energy, $E = E_K + U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{2r} \quad \dots(3)$

Comparing equations (1), (2), (3), we have

$$E_K = -E \text{ and } U = 2E$$

Given, $E = -13.6 \text{ eV}$

For ground state $n = 1$

Hence, kinetic energy, $E_K = 13.6 \text{ eV}$

Potential energy, $U = 2 \times (-13.6 \text{ eV})$
 $= -27.2 \text{ eV}$

For second excited state, $n = 3$

$$\begin{aligned}
 E_K &= -E \\
 &= \frac{+13.6}{9} \text{ eV} = 1.51 \text{ eV}
 \end{aligned}$$

and

$$\begin{aligned}
 U &= 2E \\
 &= \frac{2 \times (-13.6 \text{ eV})}{9} \\
 &= -3.02 \text{ eV}
 \end{aligned}$$

- 56.** What is the ratio of radii of the orbits corresponding to first excited state and ground state, in a hydrogen atom?

Ans :

Comp 2019

For first excited states, $n = 2$

Ground state occurs for, $n = 1$

series is given by,

$$\begin{aligned} n_1 &= 2, n_2 = \infty \\ \frac{1}{\lambda} &= R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) \\ \frac{1}{\lambda} &= R \times \frac{1}{4} \\ \frac{1}{\lambda} &= \frac{R}{4} \\ \lambda &= \frac{4}{R} = \frac{4}{1.097 \times 10^7} \\ &[\because R = 1.097 \times 10^7 \text{ m}^{-1}] \\ &= 3.64 \times 10^{-7} \text{ m} \end{aligned}$$

The lines of Balmer series are found in the visible part of the spectrum.

65. What is H_α line in the emission spectrum of hydrogen atom obtained? Calculate the frequency of the photon emitted during this transition.

Ans :

Delhi 2012

The line with the longer wavelength of the Balmer series is called H_α .

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where, λ = wavelength

$$R = 1.097 \times 10^7 \text{ m}^{-1} \quad (\text{Rydberg constant})$$

When the electron jumps from the orbit with $n = 3$ to $n = 2$.

We have,

$$\begin{aligned} \frac{1}{\lambda} &= R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \\ \frac{1}{\lambda} &= \frac{5}{36} R \end{aligned}$$

The frequency of photon emitted is given by,

$$\begin{aligned} v &= \frac{c}{\lambda} = c \times \frac{5}{36} R \\ &= 3 \times 10^8 \times \frac{5}{36} \times 1.097 \times 10^7 \text{ Hz} \\ &= 4.57 \times 10^{14} \text{ Hz} \end{aligned}$$

66. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelength and the corresponding series of the lines emitted.

Ans :

Foreign 2009

$$\text{Given, } E = 12.5 \text{ eV} = 12.5 \times 1.6 \times 10^{-19} \text{ J}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$h = 6.6 \times 10^{-34} \text{ J-s}$$

$$\text{Hence, } P = \sqrt{2mE}$$

$$P = \sqrt{2 \times 9.1 \times 10^{-31} \times 12.5 \times 1.6 \times 10^{-19}}$$

$$\frac{h}{\lambda} = \sqrt{2 \times 9.1 \times 10^{-31} \times 12.5 \times 1.6 \times 10^{-19}} \quad \left(\because P = \frac{h}{\lambda} \right)$$

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{364 \times 10^{-50}}} \\ &= \frac{6.6 \times 10^{-34}}{19.07 \times 10^{-25}} = 0.34 \times 10^{-19} \text{ m} \\ &= 3.4 \times 10^{-10} \text{ m (Ultraviolet)} \end{aligned}$$

67. 1. The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11} \text{ m}$. Calculate its radius in $n = 3$ orbit.
2. The total energy of an electron in the first excited state of the hydrogen atom, is -3.4 eV . Find out its (a) kinetic energy and (b) potential energy in this state.

Ans :

SQP 2011, Comp 2011

$$\text{Bohr's radius, } r_0 = 5.3 \times 10^{-11} \text{ m}$$

$$n = 3$$

Total energy of an electron in first excited state

$$= -3.4 \text{ eV}$$

1. Radius of orbit is given by,

$$r_n = n^2 r_0$$

For

$$n = 3$$

$$r_3 = (3)^2 \times 5.3 \times 10^{-11}$$

$$r_3 = 47.7 \times 10^{-11}$$

$$r_3 = 4.77 \times 10^{-10} \text{ m}$$

2. (a) Kinetic energy

$$= - \text{Total energy}$$

$$= -(-3.4 \text{ eV}) = 3.4 \text{ eV}$$

- (b) Potential energy

$$= -2 \times \text{Kinetic energy}$$

$$= -2 \times 3.4 = -6.8 \text{ eV}$$

68. The ground state energy of hydrogen is -13.6 eV . What is the kinetic and potential energies of the electron in this state?

or

Given the value of the ground state energy of hydrogen atom as -13.6 eV , find out its kinetic and potential energy in the ground and second excited states?

Ans :

Delhi 2019

$$\text{Kinetic energy, } E_K = \frac{1}{2} mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{2r} \quad \dots(1)$$

$$\text{Potential energy, } U = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r} \quad \dots(2)$$

$$E_K = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(2e)}{r}$$

or

$$\begin{aligned} r &= \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{E_K} \\ &= \frac{9 \times 10^9 \times 2 \times 80e^2}{4.5 \text{ MeV}} \\ &= \frac{9 \times 10^9 \times 2 \times 80 \times 1.6 \times 10^{-19}}{4.5 \times 10^6} \\ &= \frac{9 \times 160 \times 16}{4.5} \times 10^{-16} \\ &= 512 \times 10^{-16} \text{ m} \\ &= 5.12 \times 10^{-14} \text{ m} \end{aligned}$$

61. In the ground state of hydrogen atom, its Bohr radius is given as $5.3 \times 10^{-11} \text{ m}$. The atom is excited such that the radius becomes $21.2 \times 10^{-11} \text{ m}$. Find:

1. The value of the principal quantum number.
2. Total energy of the atom in this excited state.

Ans :

OD 2017

1. Given,

$$r_1 = 5.3 \times 10^{-11} \text{ m}$$

$$r_2 = 21.2 \times 10^{-11} \text{ m}$$

$$n_1 = 1$$

$$r \propto n^2$$

$$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2}$$

$$n_1 = 1$$

$$\frac{1}{n_2^2} = \frac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}}$$

$$n_2^2 = 4$$

$$n_2 = 2$$

- 2.

$$\begin{aligned} E &= \frac{-13.6}{n^2} \\ &= \frac{-13.6}{4} = -3.4 \text{ eV} \end{aligned}$$

62. Calculate the wavelength of radiation emitted when electron in a hydrogen atom jumps from $n = \infty$ to $n = 1$.

Ans :

Comp 2013

$$\text{As we know that, } \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Where,

$$n_1 = 1$$

$$n_2 = \infty$$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right]$$

$$\frac{1}{\lambda} = R$$

$$\lambda = \frac{1}{R} = \frac{1}{1.097 \times 10^7} \text{ m}$$

$$= 9.116 \times 10^{-8} \text{ m} \simeq 912 \text{ \AA}$$

63. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Up to which energy level the hydrogen atoms would be excited? Calculate the wavelengths of the first members of Lyman and first member of Balmer series.

Ans :

OD 2016

The energy of gaseous hydrogen at room temperature are,

$$E_1 = -13.6 \text{ eV}$$

$$E_2 = -3.4 \text{ eV}$$

$$E_3 = -1.51 \text{ eV}$$

$$E_4 = -0.85 \text{ eV}$$

$$E_3 - E_1 = -1.51 - (-13.6) = 12.09 \text{ eV}$$

and

$$E_4 - E_1 = -0.85 - (-13.6) = 12.75 \text{ eV}$$

As both the values does not match the given value, but it is nearest to $E_4 - E_1$.

Hence, up to E_4 energy level the hydrogen atoms would be excited.

Lyman Series

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{n^2} \right]$$

For first member, $n = 2$

$$\begin{aligned} \text{Hence, } \frac{1}{\lambda_1} &= R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \\ &= 1.097 \times 10^7 \left[\frac{4-1}{4} \right] \end{aligned}$$

$$\lambda_1 = 1.215 \times 10^{-7} \text{ m}$$

Balmer Series

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

For first member, $n = 3$

$$\begin{aligned} \text{Hence, } \frac{1}{\lambda_1} &= R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] \\ &= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9} \right] \end{aligned}$$

$$\lambda_1 = 6.56 \times 10^{-7} \text{ m}$$

64. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared visible, ultraviolet) of hydrogen spectrum does this wavelength lie?

Ans :

Comp 2018, OD 2011

Since, we know that for Balmer series,

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), n = 3, 4, 5, \dots$$

For shortest wavelength in Balmer series, the spectral

2. (a) Kinetic energy
(Total energy of the electron in first excited state)
- $$= -(-3.41) = 3.4 \text{ eV}$$

(b) Orbital radius in the state $\propto n^2$
[For first excited state, $n = 2$]

$$= 4 \times 0.53 = 2.12 \text{ \AA}$$

72. A 12.3 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited? Calculate the wavelengths of the second member of Lyman series and second member of Balmer series.

Ans :

SQP 2010

The energy of electron in the n th orbit of hydrogen atom is,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

When the incident beam of energy 12.3 eV is absorbed by hydrogen atom. Let the electron jump from $n = 1$ to $n = n$ level.

$$\begin{aligned} E &= E_n - E_1 \\ 12.3 &= -\frac{13.6}{n^2} - \left(-\frac{13.6}{1^2}\right) \\ 12.3 &= 13.6 \left[1 - \frac{1}{n^2}\right] \\ \frac{12.3}{13.6} &= 1 - \frac{1}{n^2} \\ n^2 &= 10 \\ n &= 3 \end{aligned}$$

That is the hydrogen atom would be excited upto second excited state.

For Lyman Series

$$\begin{aligned} \frac{1}{\lambda} &= R \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \\ \frac{1}{\lambda} &= 1.097 \times 10^7 \left[\frac{1}{1} - \frac{1}{9} \right] \\ \frac{1}{\lambda} &= 1.097 \times 10^7 \times \frac{8}{9} \\ \lambda &= \frac{9}{8 \times 1.097 \times 10^7} \\ &= 1.025 \times 10^{-7} \\ &= 102.5 \text{ nm} \end{aligned}$$

For Balmer Series

$$\begin{aligned} \frac{1}{\lambda} &= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{16} \right] \\ \frac{1}{\lambda} &= 1.097 \times 10^7 \times \frac{3}{16} \\ \lambda &= 4.86 \times 10^{-7} \text{ m} = 486 \text{ nm} \end{aligned}$$

73. The energy of electron in first Bohr orbit of hydrogen atoms is -13.6 eV . What is energy of electron in its 2nd Bohr-orbit?

Ans :

Comp 2016

From Bohr's theory, the total energy of an electron in n^{th} orbit is given by,

$$\begin{aligned} E_n &= \frac{2\pi^2 mk^2 e^4}{n^2 h^2} \\ &= -13.6 n^2 \text{ eV} = \frac{E_1}{n^2} \end{aligned}$$

Here, $E_1 = -13.6$

For $n = 2$, $E_n = \frac{E_1}{n^2} = -\frac{13.6}{(2)^2} = -3.4 \text{ eV}$

74. An electron in a hydrogen atom, moves from n_1 to n_2 state. If time-period of the electron in the initial state is eight times that of final state, What is the possible values of n_1 and n_2 ?

Ans :

Comp 2014 OD 2004

Initial energy level $= n_1$

Final energy level $= n_2$

Initial time-period, $T_1 = 8T_2$
(where, T_2 is final time-period)

Radius of n^{th} orbit,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \propto n^2$$

Therefore, $\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 \quad \dots(1)$

and velocity of electron in n^{th} orbit,

$$v_n = \frac{e^2}{2\varepsilon_0 h n} \propto \frac{1}{n}$$

Therefore, $\frac{v_1}{v_2} = \frac{n_2}{n_1} \quad \dots(2)$

Time-period of electron in n^{th} orbit,

$$T_n = \frac{2\pi r_n}{v_n} \propto \frac{r_n}{v_n}$$

Therefore, $\frac{T_1}{T_2} = \frac{r_1}{r_2} \times \frac{v_2}{v_1} = \left(\frac{n_1}{n_2}\right)^2 \times \frac{n_1}{n_2} = \left(\frac{n_1}{n_2}\right)^3$

or $\frac{8T_2}{T_2} = \left(\frac{n_1}{n_2}\right)^3$

or $\frac{n_1}{n_2} = 2$

Thus possible value of n_1 and n_2 are $n_1 = 4$ and $n_2 = 2$.

75. An α -particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. What is the order of distance of the closest approach?

Ans :

OD 2012, 15

Given,

Energy of α -particle,

$$\begin{aligned}
 E &= 5 \text{ MeV} \\
 &= (5 \times 10^6) \times (1.6 \times 10^{-19}) \\
 &= 8 \times 10^{-13} \text{ J}
 \end{aligned}$$

Atomic no. of uranium $Z = 90$.

Distance of the closet approach,

$$r = k \times \frac{2Ze^2}{E}$$

where, k = Electrostatic force constant equal to 9×10^9

$$\begin{aligned}
 &= (9 \times 10^9) \times \frac{2 \times 92 \times (1.6 \times 10)^2}{8 \times 10^{-13}} \\
 &= 5.3 \times 10^{-14} \text{ m} \\
 &= 5.3 \times 10^{-12} \text{ cm}
 \end{aligned}$$

Therefore, distance to the closest approach is of the order of 10^{-12} cm.

- 76.** In a hydrogen atom, when an electron jumps from second orbit to first orbit, What is the wavelength of spectral line emitted by hydrogen atom?

Ans :

Comp 2018

Initial energy level, $n_1 = 2$ Final energy level, $n_2 = 1$

$$\begin{aligned}
 \text{Change in energy, } \Delta E &= -13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \\
 &= -13.6 \left[\frac{1}{(2)^2} - \frac{1}{(1)^2} \right] \\
 &= -13.6 \left[\frac{-3}{4} \right] \\
 &= 10.2 \text{ eV}
 \end{aligned}$$

Wavelength of emitted spectral line,

$$\begin{aligned}
 \lambda &= \frac{12375}{\Delta E} \\
 &= \frac{12375}{10.2} = 1213 \text{ \AA}
 \end{aligned}$$

- 77.** The first member of Balmer's series of hydrogen has a wavelength of $6563 \text{ \AA}$. What is the wavelength of its second member?

Ans :

SQP 2010

In Balmer series, first wavelength corresponds to $n_1 = 3$ and second wavelength corresponds to $n_2 = 4$. Relation for the wavelength of first member in Balmer series λ_1 is,

$$\frac{1}{\lambda_1} = R_H \left[\frac{1}{(2)^2} - \frac{1}{(n_1)^2} \right]$$

$$\begin{aligned}
 \frac{1}{6563} &= R_H \left[\frac{1}{(2)^2} - \frac{1}{(3)^2} \right] \\
 &= R_H \left[\frac{1}{4} - \frac{1}{9} \right] \\
 &= \frac{5R_H}{36} \quad \dots(1)
 \end{aligned}$$

Similarly, relation for the wavelength of second member in Balmer series λ_2 is,

$$\begin{aligned}
 \frac{1}{\lambda_2} &= R_H \left[\frac{1}{(2)^2} - \frac{1}{(n_2)^2} \right] \\
 &= \frac{1}{\lambda_2} \times R_H \left[\frac{1}{(2)^2} - \frac{1}{(4)^2} \right] \\
 &= R_H \left[\frac{1}{4} - \frac{1}{16} \right] \\
 &= \frac{3R_H}{16} \quad \dots(2)
 \end{aligned}$$

Dividing equation (1) and (2),

$$\begin{aligned}
 \frac{\frac{1}{6563}}{\frac{1}{\lambda_2}} &= \frac{\frac{5R_H}{36}}{\frac{3R_H}{16}} \\
 \frac{\lambda_2}{6563} &= \frac{5}{36} \times \frac{16}{3} = \frac{80}{108} \\
 \lambda_2 &= \frac{6563 \times 80}{108} = 4861 \text{ \AA}
 \end{aligned}$$

- 78.** If the atom ${}_{100}\text{Fm}^{257}$ follows the Bohr's atomic model and radius of ${}_{100}\text{Fm}^{257}$ is m times the Bohr radius, what is the value of m ?

Ans :

OD 2019, Foreign 2015

$$\text{Atom} = {}_{100}\text{Fm}^{257}$$

Radius of atoms, $r_n = m r_0$ (where, r_0 is Bohr radius)

Principal quantum no. of ${}_{100}\text{Fm}^{257}$ atom,

$$n = 5$$

Radius of ${}_{100}\text{Fe}^{257}$ atom,

$$r_n = \frac{n^2 r_0}{Z}$$

or

$$m r_0 = \frac{n^2 r_0}{Z}$$

or

$$\begin{aligned}
 m &= \frac{n^2}{Z} = \frac{(5)^2}{100} \\
 &= \frac{25}{100} = \frac{1}{4}
 \end{aligned}$$

- 79.** Atomic radius of first orbit of hydrogen atom is $0.53 \text{ \AA}$. What is the radius of its fifth orbit?

Ans :

Delhi 2009

Radius of first orbit, $r_1 = 0.53 \text{ \AA}$;

Initial energy level, $n_1 = 1$ and

Final energy level, $n_2 = 5$

Bohr's atomic model that radius of n^{th} orbit,

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \propto n^2$$

Therefore,

$$\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$$

$$= \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

or

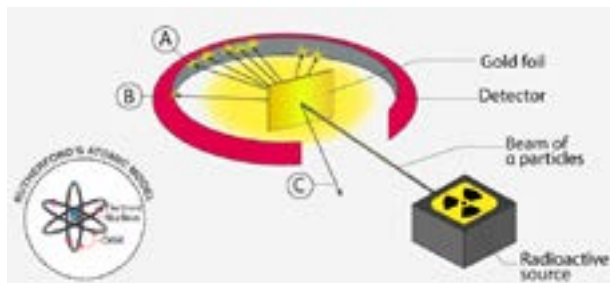
$$r_2 = 25r_1 = 25 \times 0.53$$

Radius of fifth orbit,

$$r_5 = 12.25 \text{ Å}$$

CASE BASED QUESTIONS

80. Rutherford's atomic model was a reasonably good model of structure of atom based on the famous α -particle scattering experiment. However, it was subsequently modified by Neils Bohr and later on, by others. The resulting improvements in the understanding of atomic structure have greatly contributed to further scientific advancement. There are many other similar examples in scientific field when original contributors happily accept modifications in their ideas.



- Write two major drawbacks of Rutherford's atomic model ?
- How were these drawbacks overcome in Neils Bohr Model ?

Ans :

OD 2013

- Drawbacks
 - An accelerated electron must emit radiation and should fall into the nucleus.
 - An atom should emit continuous spectrum which is contrary to experimental results.
- Neils Bohr's Improvement
 - The electron can revolve only in specific orbits called stationary orbits. In these orbits, the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ i.e., $mvr = \frac{nh}{2\pi}$ where n is an integer.
 - While revolving in such orbits, the electrons, do not emit radiations.

81. Rutherford was the pioneer who postulated and established the existence of the atomic nucleus. At Rutherford's suggestion, Geiger and Marsden performed their classic experiment: on the scattering of α -particles from thin gold foils. Their experiments revealed that the distance of closest approach to a gold nucleus of an α -particle of kinetic energy 5.5 MeV is about $4.0 \times 10^{-14} \text{ m}$. The scattering of α -particle by the gold sheet could be understood by Rutherford by assuming that the coulomb repulsive force was solely responsible for scattering. Since the positive charge is confined to the nucleus, the actual size of the nucleus has to be less than $4.0 \times 10^{-14} \text{ m}$. If we use α -particles of higher energies than 5.5 MeV, the distance of closest approach to the gold nucleus will be smaller and at some point the scattering will begin to be affected by the short range nuclear forces, and differ from Rutherford's calculations. Rutherford's calculations are based on pure coulomb repulsion between the positive charges of the α particle and the gold nucleus. From the distance at which deviations set in, nuclear sizes can be inferred. By performing scattering experiments in which fast electrons, instead of α particles, are projectiles that bombard targets made up of various elements, the sizes of nuclei of various elements have been accurately measured. It has been found that a nucleus of mass number A has a radius $R = R_0 A^{\frac{1}{3}}$, where $R_0 = 1.2 \times 10^{-15} \text{ m}$. This means the volume of the nucleus, which is proportional to R^3 is proportional to A . Thus the density of nucleus is a constant, independent of A , for all nuclei. Different nuclei are like drop of liquid of constant density. The density of nuclear matter is approximately $2.3 \times 10^{17} \text{ kg m}^{-3}$. This density is very large compared to ordinary matter, say water, which is 10^3 kg m^{-3} . This is understandable, as we have already seen that most of the atom is empty. Ordinary matter consisting of atoms has a large amount of empty space.

- What is relative density of nucleus with respect to water?
- What is the kinetic energy of α -particles bombarded towards the gold nucleus in Geiger and Marsden classic experiment?
- What is the range of volume of hydrogen nucleus?

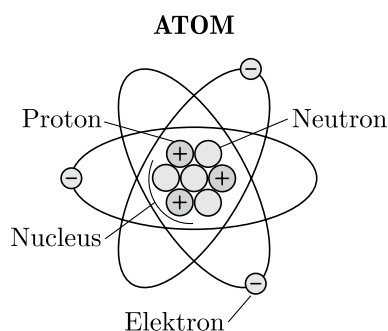
Ans :

Comp 2006, OD 2014

- $2.3 \times 10^{14} \text{ kg m}^{-3}$
- $8.8 \times 10^{-13} \text{ Joule}$
- 10^{-45} m

82. Everything around us which has mass and occupies space is matter. It cannot be broken down further

using any chemical means because it is the basic building block of an element. Every state of matter solid, liquid, gas, and plasma is composed of either atom either it is neutral (unionized), or ionized atoms. An atom is made up of three particles known as protons, neutrons, and electrons. And these particles are also made up from sub-particles. Among these three particles, protons have a positive charge while electrons carry a negative charge and the third particle neutrons have no electrical charge. And the charge of atoms depends on the number of protons and electrons, i.e. an atom is electrically neutral if the number of protons and electrons are equal. If an atom has more or fewer electrons than protons, then it has an overall negative or positive charge, respectively. These atoms are extremely small or you can say their typical sizes are around 100 picometers. So the dense region consisting of protons and neutrons at the center of an atom is known as the atomic nucleus of an atom. Every atom is composed of such nucleus and some electrons will be surrounding it. Studying these atoms and Nuclei will help us to have a thorough understanding of matter. Studying about the nucleus and its reactions will help us to understand more about nuclear energy, which is a very useful renewable energy. That's why it is very important to study about Atoms and Nuclei.



- (i) What is the basic unit of matter?
- (ii) Which particle is responsible for the ionization of the atom?
- (iii) Which is the most dense part of an atom?

Ans :

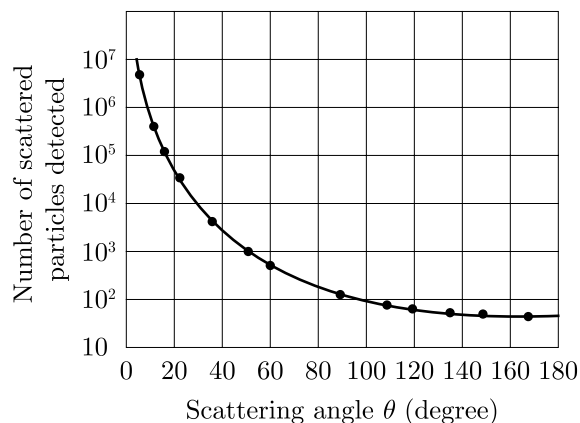
Comp 2009

- (i) Atom.
- (ii) Electron.
- (iii) The region at the center of atom containing neutrons and protons.

83. A typical graph of the total number of α -particles scattered at different angles, in a given interval of time, is shown in fig. The dots in this figure represent the data points and the solid curve is the theoretical

prediction based on the assumption that the target atom has a small, dense, positively charged nucleus. Many of the α -particles pass through the foil. It means that they do not suffer any collisions. Only about 0.14% of the incident α -particles scatter by more than 1° ; and about 1 in 8000 deflect by more than 90° . Rutherford argued that, to deflect the α -particle backwards, it must experience a large repulsive force. This force could be provided if the greater part of the mass of the atom and its positive charge were concentrated tightly at its centre. Then the incoming α -particle could get very close to the positive charge without penetrating it, and such a close encounter would result in a large deflection. This agreement supported the hypothesis of the nuclear atom. This is why Rutherford is credited with the discovery of the nucleus.

In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away. The electrons would be moving in orbits about the nucleus just as the planets do around the sun. Rutherford's experiments suggested the size of the nucleus to be about 10^{-15}m to 10^{-14}m . From kinetic theory, the size of an atom was known to be 10^{-10}m , about 10,000 to 100,000 times larger than the size of the nucleus. Thus, the electrons would seem to be at a distance from the nucleus of about 10,000 to 100,000 times the size of the nucleus itself. Thus, most of an atom is empty space. With the atom being largely empty space, it is easy to see why most α -particles go right through a thin metal foil. However, when α -particle happens to come near a nucleus, the intense electric field there scatters it through a large angle. The atomic electrons, being so light, do not appreciably affect the α -particles. The scattering data shown in fig. can be analysed by employing Rutherford's nuclear model of the atom. As the gold foil is very thin, it can be assumed that α -particles will suffer not more than one scattering during their passage through it. Therefore, computation of the trajectory of an alpha-particle scattered by a single nucleus is enough. Alpha particles are nuclei of helium atom. The charge of the gold nucleus is Ze , where Z is the atomic number of the atom; for gold $Z = 79$. Since the nucleus of gold is about 50 times heavier than α -particle, it is reasonable to assume that it remains stationary throughout the scattering process. Under these assumptions, the trajectory of an alpha-particle can be computed employing Newton's second law of motion and the coulomb's law for electrostatic force of repulsion between the alpha-particle and the positively charged nucleus.



- What percentage of α particle scattered at an angle more than 90° ?
- Why the nucleus of gold is about remains stationary throughout the scattering process?
- Why electrons around the gold nucleus were unable to deflect α particles?
- What is the ratio of charge on α particle and gold nucleus?

Ans :

SQP 2013

- .0125%
- Because its mass is 200 times the mass of proton.
- α particles is much heavier than electron.
- .025

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CHAPTER 13

Nuclei

SUMMARY

1. ATOMIC MASSES

The masses of atoms, nuclei, etc., are expressed in terms of atomic mass unit represented by amu or u . For this mass of C-12 is taken as standard.

$$\begin{aligned}\text{i.e., } 1u &= \frac{\text{mass of one } ^{12}\text{C}_6 \text{ atom}}{12} \\ &= 1.660565 \times 10^{-27} \text{ Kg}\end{aligned}$$

Mass of proton, $m_p = 1.00727 \text{ u}$

Mass of neutron, $m_n = 1.00866 \text{ u}$

Mass of electron, $m_e = 0.000549 \text{ u}$

2. COMPOSITION OF NUCLEUS

The composition of a nucleus can be described by using the following terms and symbols.

1. **Atomic Number (Z)** : Atomic mass number of an element is the number of protons present inside nucleus of an atom of the element

$$\begin{aligned}\text{Atomic number} &= \text{Number of protons} \\ &= \text{Number of electrons}\end{aligned}$$

2. **Mass Number (A)** : (Atomic mass) Mass number of an element is the total number of protons and neutrons inside the atomic nucleus of the element.

Mass number

$$\begin{aligned}&= \text{Number of protons} + \text{Number of neutrons} \\ &= \text{Number of electrons} + \text{Number of neutrons}\end{aligned}$$

$$\text{i.e., } A = Z + N$$

3. SIZE OF NUCLEUS

According to experimental observations, the radius of the nucleus of an atom of mass number A is

$$R = R_0 A^{1/3}$$

$$\text{where, } R_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$$

4. MASS ENERGY EQUIVALENCE RELATION

According to Einstein, the mass and energy are equivalent i.e., mass can be converted into energy and vice-versa. The mass energy equivalence relation is $E = mc^2$

Accordingly, 1 kg mass, is equivalent to energy

$$\begin{aligned}&= 1 \times (3 \times 10^8)^2 \\ &= 9 \times 10^{16} \text{ joules}\end{aligned}$$

and 1 amu ($= \frac{1}{6.02 \times 10^{26}} \text{ kg}$) mass is equivalent to energy 931 MeV.

5. BINDING ENERGY

The binding energy of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest and infinite distance apart. Using Einstein's mass-energy relation,

$$\begin{aligned}\Delta E &= (\Delta m) c^2 \\ \Delta E &= [Zm_p + (A - Z)m_n - M] c^2 \\ 1 \text{ amu} &\cong 931.5 \text{ MeV}\end{aligned}$$

The difference in mass of nucleus and its constituents is called mass defect.

6. BINDING ENERGY PER NUCLEON

This mass defect is in the form of binding energy of nucleus, which is responsible for binding the nucleons into a small nucleus.

$$\text{Hence, Binding energy of nucleus} = (\Delta m) c^2$$

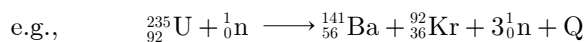
$$\text{and Binding energy per nucleon} = \frac{(\Delta m) c^2}{A}$$

7. NUCLEAR ENERGY

Transmutation of less stable into more tightly bound nuclei provides excellent possibility of releasing nuclear energy.

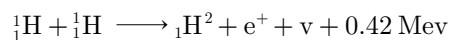
Two distinct ways of obtaining energy from nucleus are given as below

1. **Nuclear Fission** : The phenomenon of splitting of heavy nuclei (mass number > 120) into smaller nuclei of nearly equal masses is known as nuclear fission,



(where, Q = energy release in the nuclear process)

2. **Nuclear Fusion** : The phenomenon of conversion of two lighter nuclei into a single heavy nucleus is called nuclear fusion e.g.,



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OBJECTIVE QUESTIONS

1. The curve of binding energy per nucleon as a function of atomic mass number has a sharp peak for helium nucleus. This implies that helium nucleus is
- radioactive
 - unstable
 - easily fissionable
 - more stable nucleus than its neighbours

Ans :

OD 2023

The sharp peak implies that nuclear is very stable. Binding energy per nucleon for helium nucleus is 7.0 MeV.

Thus (d) is correct option.

2. The atomic number and mass number for a specimen are Z and A respectively. The number of neutrons in the atoms will be

- A
- A
- $A + Z$
- $A - Z$

Ans :

OD 2018

Atomic Number, Z = Number of Protons

Atomic Mass, A = Number of Protons

+ Number of Neutrons

Hence, $A = Z + \text{Number of Neutrons}$

Number of Neutrons = $A - Z$

Thus (d) is correct option.

3. The quantities, which remain conserved in a nuclear reaction
- Total charge
 - Angular momentum
 - Linear momentum
 - All the above

Ans :

Delhi 2018, Comp 2007

Nuclear reactions conserve of course energy, momentum, angular momentum, charge and the total baryon number which means the total number of proton and neutrons.

Thus (d) is correct option.

4. The number of neutrons in an atom of ${}_{90}\text{Th}^{230}$ is
- 90
 - 140
 - 230
 - 320

Ans :

Foreign 2012

Number of neutron, N = Mass number A

– Atomic number Z

Here, Mass number, $A = 230$

$$\begin{aligned}\text{Atomic number, } Z &= 90 \\ \text{Hence, } N &= 230 - 90 \\ &= 140\end{aligned}$$

Thus (b) is correct option.

5. As the mass number A increase which of the following quantities related to a nucleus does not change?

- (a) Mass (b) Volume
(c) Density (d) Binding energy

Ans :

OD 2009

Density of nuclear matter is the ratio of mass of nucleus and its volume. It is given by,

$$\rho = \frac{3m}{4\pi R_0^3}$$

Hence density is not depends on the mass number therefore when mass number A increase than density remains constant.

Thus (c) is correct option.

6. The isotopes have
(a) same atomic number and mass number
(b) same mass number but different atomic number
(c) different atomic number and mass number
(d) same atomic number but different mass number

Ans :

SQP 2017

Isotopes of an element have same atomic number, but different masses number.

Thus (d) is correct option.

7. The elements with same atomic mass, but different atomic number, are called
(a) isobars (b) isotones
(c) isotopes (d) isochores

Ans :

Foreign 2016, OD 2005

The elements with same atomic mass, but different atomic nos., are called isobars.

Thus (a) is correct option.

8. Which of the following is an essential requirement for initiating a fusion reaction?
(a) critical mass (b) thermal neutrons
(c) high temperature (d) critical temperature

Ans :

Delhi 2008

In a fusion reaction, two or more light nuclei get fused together to form a heavy nucleus, if they possess sufficiently high kinetic energy to overcome the force of repulsion and they can possess high kinetic energy only at very high temperature (of the order of 10^8 C).

Thus (c) is correct option.

9. Elements having different atomic number as well as different mass number, but same number of neutrons, are known as

- (a) isobars (b) isotones
(c) isotopes (d) isodiaphers

Ans :

OD 2007

The elements having different atomic number as well as different mass number, but same number of neutrons, are known as isotones.

Thus (b) is correct option.

10. If elements with principal quantum $n > 4$ were not allowed in nature, the number of possible elements would have been

- (a) 4 (b) 32
(c) 60 (d) 64

Ans :

Delhi 2014

Principal quantum number allowed in nature, $n = 4$. Number of possible elements for principal quantum number,

$$\begin{aligned}N &= \sum_{n=1}^4 2n^2 \\ &= 2[(1)^2 + (2)^2 + (3)^2 + (4)^2] \\ &= 2[30] = 60\end{aligned}$$

Thus (c) is correct option.

11. For uranium nucleus, how does its mass vary with volume ?

- (a) $m \propto V$ (b) $m \propto V^2$
(c) $m \propto \sqrt{V}$ (d) $m \propto \frac{1}{V}$

Ans :

SQP 2012

Mass of nucleus, $m = \text{Density} \times \text{Volume}$

$$= \rho V$$

Since density of nucleus is constant, therefore its mass varies with volume i.e., $m \propto V$.

Thus (a) is correct option.

12. Numerically, 1 Curie is equal to

- (a) 3.7×10^7 dps (b) 3.7×10^6 dps
(c) 3×10^{10} dps (d) 3.7×10^{10} dps

Ans :

OD 2016

$$1 \text{ Curie} = 3.7 \times 10^{10} \text{ dps}$$

It is the unit of rate of decay of a radioactive substance.

Thus (d) is correct option.

13. If A is the mass number of an element, then volume of the nucleus of an atom of this element is proportional to

Ans :

Radioactive nuclei emit β -particles. This β -particle comes from the splitting of neutron into β -particle and proton.

Thus (c) is correct option.

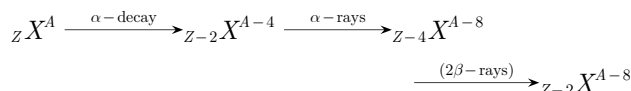
25. **Assertion :** ${}_Z X^A$ undergoes 2α -decays, 2β -decays and 2γ -decays and the daughter produce is ${}_{Z-2} X^{A-8}$.

Reason : In α -decays the mass number decreases by 4 and atomic number decreases by 2. In 2β -decays the mass number remains unchanged, but atomic number increases by 1 only.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The reason given is true. If we test the authenticity of assertion.



Thus (a) is correct option.

26. **Assertion :** Neutrons penetrate mater more readily as compared to protons.

Reason : Neutrons are slightly more massive than protons.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Neutron is penetrate more readily as compared to protons because neutrons do not carry any charge so there is no repulsion between nucleus and neutrons. So assertion is true reason is also true as mass of neutron is more than proton but reason does not explain assertion.

Thus (b) is correct option.

27. **Assertion :** The ionising power of β -particle is less compared to α -particles but their penetrating power is more.

Reason : The mass of β -particle is less than the mass of α -particle.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.

- (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

β -particles, being emitted with very high speed compared to α -particles, pass for very little time near the atoms of the medium. So the probability of the medium. So the probability of the atoms being ionised is comparatively less. But due to this reason, their loss of energy is very slow and they can penetrate the medium through a sufficient depth.

Thus (b) is correct option.

28. **Assertion :** The binding energy per nucleon, for nuclei with a atomic mass number $A > 100$, decreases with A .

Reason : The nuclear forces are weak for heavier nuclei.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Binding energy per nucleon decreases with A for nuclei with atomic mass number $A > 100$ due to weak nuclear forces. It can be explain as :

At short distances, the nuclear force is stronger than the Coulomb force; it can overcome the Coulomb repulsion of protons inside the nucleus. At typical nucleon separation (1.3 fm) it is a very strong attractive force (104 newtons). Beyond about 1.3 fm separation, the force exponentially dies off to zero. However, the Coulomb force between protons has a much larger range and become the only significant force between protons when their separation exceeds about (2.5 fm, $A > 100$).

Thus (a) is correct option.

29. **Assertion :** Cobalt-60 is useful in cancer therapy.

Reason : Cobalt-60 is a source of γ -radiations capable of killing cancerous cells.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

19. An element has binding energy 8 eV per nucleon. If it has total binding energy of 128 eV, then the number of nucleons are

- (a) 8 (b) 16
(c) 32 (d) 64

Ans :

Delhi 2011

Binding energy per nucleon = 8 eV
and Total binding energy = 128 eV
Number of nucleons,

$$N = \frac{\text{Total binding energy}}{\text{Binding energy per nucleon}} \\ = \frac{128}{8} = 16$$

Thus (b) is correct option.

ASSERTION AND REASON

20. **Assertion :** Isobars are the elements having same mass number but different atomic number.

Reason : Neutrons and protons are present inside nucleus.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

By definition, isobars are elements having same mass number but different atomic number. Presence of neutron and proton inside nucleus has nothing to do with this definition.

Thus (b) is correct option.

21. **Assertion :** If the half life of a radioactive substance is 40 days then 25% substance decay in 20 days.

Reason : $N = N_0 \left(\frac{1}{2}\right)^n$

where, $n = \frac{\text{time elapsed}}{\text{half life period}}$

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) Both the Assertion and Reason are incorrect.
(d) The Assertion is incorrect but the Reason is correct.

Ans :

Half life of radioactive substance is 40 days. It means 50% substance decays in 40 days. During this period rate of decay is on decrease, So, 25% decay must have taken place is less than 20 days.

$$N = N_0 \left(\frac{1}{2}\right)^n$$

where, $n = \frac{\text{time elapsed}}{\text{half life period}}$

Thus (d) is correct option.

22. **Assertion :** Separation of isotope is possible because of the difference in electron numbers of isotope.

Reason : Isotope of an element can be separated by using a mass spectrometer.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) Both the Assertion and Reason are incorrect.
(d) The Assertion is incorrect but the Reason is correct.

Ans :

Isotope of an element can be separated by using a mass spectrometer because isotopes have different atomic mass. Alternative (e) is correct.

Thus (d) is correct option.

23. **Assertion :** It is not possible to use ^{35}Cl as the fuel for fusion energy.

Reason : The binding energy of ^{35}Cl is too small.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

Since, ^{35}Cl is stable so binding energy is high. So it is not capable of disintegration.

Thus (c) is correct option.

24. **Assertion :** Radioactive nuclei emit β^- particles.

Reason : Electrons exist inside the nucleus.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

- (a) A (b) A^2
 (c) A^3 (d) $A^{1/3}$

Ans :

Foreign 2015

Mass number of element = A
 Radius of nucleus, $R = R_0 A^{1/3}$. Therefore,

$$\begin{aligned}\text{Volume of nucleus} &= \frac{4}{3}\pi R^3 \\ &= \frac{4}{3}\pi (R_0 A^{1/3})^3 \\ &= \frac{4}{3}\pi R_0^3 A \propto A\end{aligned}$$

Thus (a) is correct option.

14. M_p denotes the mass of a proton and M_n that of a neutron. A given nucleus, of binding energy B , contains Z protons and N neutrons. The mass $M(N, Z)$ of the nucleus is given by (c = Velocity of light)

- (a) $M(N, Z) = NM_n + ZM_p - B^2$
 (b) $M(N, Z) = NM_n + ZM_p + Bc^2$
 (c) $M(N, Z) = NM_n + ZM_p + \frac{B}{c^2}$
 (d) $M(N, Z) = NM_n + ZM_p - \frac{B}{c^2}$

Ans :

OD 2003

$$\text{Mass of proton} = M_p$$

$$\text{Mass of neutron} = M_n$$

$$\text{Binding energy of the nucleus} = B$$

$$\text{Number of protons in the nucleus} = Z$$

$$\text{Number of neutrons in the nucleus} = N$$

$$\text{Mass of the nucleus} = M(N, Z)$$

$$\text{and Velocity of light} = c$$

Mass defect of nucleus,

$$\Delta m = \frac{B}{c^2}$$

and this mass defect appears in the form of binding energy, which binds the nucleons together in the nucleus.

Therefore mass of the nucleus,

$$\begin{aligned}[M(N, Z)] &= NM_n + ZM_p - \Delta m \\ &= NM_n + ZM_p - \frac{B}{c^2}\end{aligned}$$

Thus (d) is correct option.

15. If M is the atomic mass and A is the mass number, then the packing fraction is given by

- (a) $\frac{M}{M-A}$ (b) $\frac{M-A}{A}$
 (c) $\frac{A}{M-A}$ (d) $\frac{A-M}{A}$

Ans :

SQP 2010

$$\text{Atomic mass} = M$$

$$\text{and Mass number} = A$$

Packing fraction is the ratio of atomic mass M minus mass number A to the mass number A i.e., $\frac{M-A}{A}$.

Thus (b) is correct option.

16. If M is atomic mass, A is mass number of a nucleus, then $\frac{M-A}{A}$ is called

- (a) mass defect (b) fermi energy
 (c) binding energy (d) packing fraction

Ans :

Foreign 2013

$$\text{Atomic mass} = M$$

$$\text{and Mass number} = A$$

Packing fraction of a nucleus is the ratio of atomic mass M minus mass number A to the mass number A .

$$\text{Therefore, Packing fraction} = \frac{M-A}{A}$$

Thus (d) is correct option.

17. The function of heavy water in a nuclear reactor is to

- (a) stop the neutrons
 (b) slow down the neutrons
 (c) increase the speed of neutrons
 (d) none of these

Ans :

SQP 2008, Comp 2015

Heavy water is rich in protons and acts as a moderator in a nuclear fission reaction in nuclear reactors. When fast moving neutrons collide head on with the protons of heavy water, they lose their energy and get slow down.

Thus (b) is correct option.

18. The energy produced in the sun is due to

- (a) fission reaction
 (b) fusion reaction
 (c) chemical reaction
 (d) motion of electrons and ions

Ans :

OD 2002

Source of the huge solar energy is the fusion of lighter nuclei. Fusion of hydrogen nuclei into helium nuclei is continuously taking place in the plasma, with a continuous liberation of energy.

Therefore energy produced in the sun is due to fusion reaction.

Thus (b) is correct option.

Ans :

Cobalt 60 is radioactive isotope of cobalt. γ -radiation emitted by it is used in radiation therapy is cancer as it destroys cancerous cells.

Thus (a) is correct option.

- 30. Assertion :** Heavy water is a better moderator than normal water.

Reason : Heavy water absorbs neutrons more efficiently than normal water.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Heavy water has better ability to slow down neutrons by elastic collision between their protons and neutrons hence they are better moderators. Heavy water does not absorb neutrons.

Thus (c) is correct option.

- 31. Assertion :** ^{90}Sr from the radioactive fall out from a nuclear bomb ends up in the bones of human beings through the milk consumed by them. It cause impairment of the production of red blood cells.

Reason : The energetic β -particles emitted in the decay of ^{90}Sr damage the bone marrow.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

RBC of blood are produced in the bone marrow. The radiation from the radioactive substances destroys of bone marrow which result in hampered production of RBC.

Thus (a) is correct option.

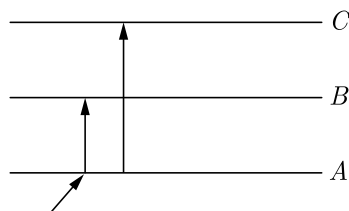
- 32. Assertion :** Between any two given energy levels, the number of absorption transitions is always less than the number of emission transitions.

Reason : Absorption transitions start from the lowest energy level only and may end at any higher energy level. But emission transitions may start from any higher energy level and end at any energy level below it.

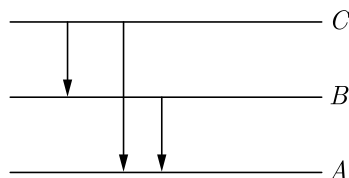
- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Absorption transition



Two possibilities in absorption transition.



Three possibilities in emission transition. Therefore, absorption transition < emission.

Thus (a) is correct option.

- 33. Assertion :** Energy is released in nuclear fission.

Reason : Total binding energy of the fission fragments is larger than the total binding energy of the parent nucleus.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

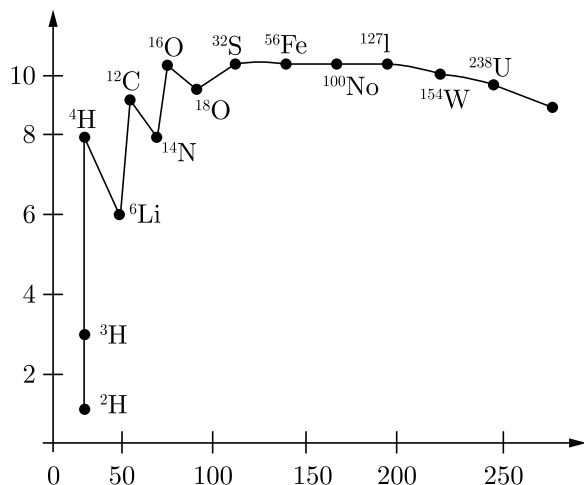
Total binding energy of fragment nucleus is more than total binding energy of parent nucleus. Since, binding energy results in decrease of total energy. Hence there is great decrease in energy fragment nucleus because energy is released in nuclear fission.



Energy of B is decrease but the binding energy of B is increased due to release of energy from it. So, reason supports the assertion.

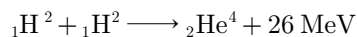
Thus (a) is correct option.

(ii) Binding Energy Curve



Nuclear Fission : When a heavy nucleus ($A > 235$ say) breaks into two lighter nuclei (nuclear fission), the binding energy per nucleon increases i.e. nucleons get more tightly bound. This implies that energy would be released in nuclear fission.

Nuclear Fusion : When two very light nuclei join to form a heavy nucleus, the binding energy per nucleon of fused. Heavier nucleus more than the binding energy per nucleon of lighter nuclei, so again energy would be released in nuclear fusion.



or

- (b) (i) Nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of α -particles by atoms. He found that the scattering result could be explained, if atoms consists of a small central massive and positive core surrounded by orbiting electron. The experiment results indicated that the size of the nucleus is of the order of 10^{-14} meters and is thus, 10,000 times smaller than the size of the atom.

The size of nucleus is obtained by the point of closest approach of an alpha-particle. By shooting alpha particles of kinetic energy 5.5 MeV. The point of closest approach was estimated to be about 4×10^{-14} m. Since, the repulsive force acting here is coulomb repulsion, there is no contact. This means that the size of the nucleus is smaller than 4×10^{-14} m.

Relation between radius and mass number is

$$R = R_0 A^{1/3}$$

where, $R_0 = 1.2 \times 10^{-15} \text{ m}$

and A is the mass number

- (ii) The radius R of nucleus is related to its mass number (A) as :

$$R = R_0 A^{1/3}$$

where, $R_0 = 1.1 \times 10^{-15} \text{ m}$

If m is average mass of nucleon, then mass of nucleus = mA ,

where A is mass number.

$$\begin{aligned} \text{Volume of nucleus} &= \frac{4}{3} \pi R^3 \\ &= \frac{4}{3} \pi (R_0 A^{1/3})^3 \\ &= \frac{4}{3} \pi R_0^3 A \end{aligned}$$

Density of nucleus,

$$\begin{aligned} \rho_N &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{mA}{\frac{4}{3} \pi R_0^3 A} \\ &= \frac{m}{\frac{4}{3} \pi R_0^3} \end{aligned}$$

Clearly, nuclear density, ρ_N is independent of mass number A .

69. Give reason for :

- Lighter elements are better moderators for a nuclear reactor than heavier elements.
- In a natural uranium reactor, heavy water is preferred moderator as compared to ordinary water.
- Cadmium rods are provided in a reactor.
- Very high temperature as those obtained in the interior of the sun are required for fusion reaction.

Ans :

Comp 2016

- A moderator slows down fast neutrons released in a nuclear reactor. The basic principle of mechanics is that the energy transfer in a collision is the maximum when the colliding particles have equal masses. As lighter elements have mass close to that of neutrons, lighter elements are better moderators than heavier elements.
- Ordinary water has hydrogen nuclei (${}_1^1\text{H}$) which have greater absorption capture for neutrons; so ordinary water will absorb neutrons rather than slowing them; on the other hand the heavy hydrogen nuclei (${}_1^2\text{H}$) have negligible absorption capture for neutrons, so they share energy with neutrons and neutrons are slowed down.

can overcome their mutual coulombic repulsions and come closer than the range of nuclear force. That is why a fusion reaction is also called a thermonuclear reaction.

- High density or pressure increases the frequency of collision of light nuclei and hence increases the rate of fusion.

43. A chain reaction dies out sometimes. Why?

Ans :

OD 2003

A chain reaction may die out due to any of the following causes:

- Excessive neutron leakage if the size of the fissionable material is smaller than the critical size.
- Fast neutrons may escape the fissionable material without causing further fissions.
- Some neutrons may suffer non-fission capture by ${}_{92}^{238}\text{U}$ nuclei.

SHORT ANSWER QUESTIONS

44. How is the size of a nucleus estimated? Write the relation between the radius of a nucleus and its mass number.

Ans :

OD 2005

A nucleus can be considered to be spherical in shape. By performing scattering experiments using high energy electrons, we can determine its radius accurately.

Experimental observations show that the volume of a nucleus is directly proportional to its mass number. If R is the radius of a nucleus having mass number A , then

$$\frac{4}{3}\pi R^3 \propto A$$

or $R \propto A^{1/3}$

or $R = R_0 A^{1/3}$

Thus, the radius R of a nucleus is proportional to the cube root of its mass number. The value of constant R_0 depends on the nature of probe particles.

For electrons, $R_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$

45. Briefly describe proton-neutron hypothesis of nuclear composition.

Ans :

OD 2018

Composition of a Nucleus

An atom has a positively charged nucleus. The radius of the nucleus is smaller than the radius of an atom by a factor of 10^4 . More than 99% mass of the atom

is concentrated in the nucleus. According to proton-neutron hypothesis proposed by Heisenberg in 1932, protons and neutrons are the main building blocks of the nuclei of all atoms. Thus a nucleus of mass number A and atomic number Z contains Z protons and $(A-Z)$ neutrons. The protons give positive charge to the nucleus, while protons and neutrons together give it mass. To neutralise the positive charge of the nucleus, i.e., to make the atom electrically neutral, the number of extra-nuclear electrons is Z .

Proton

It is a fundamental particle having a positive charge $1.6 \times 10^{-19} \text{ C}$ and a rest mass of $1.6726 \times 10^{-27} \text{ kg}$. It has an intrinsic (spin) angular momentum equal to $1/2$ and a magnetic moment much smaller than that of an electron.

Neutron

It is a chargeless fundamental particle having rest mass of $1.6749 \times 10^{-27} \text{ kg}$. Its intrinsic angular momentum is equal to $1/2$. It also possesses a small magnetic moment.

46. What is mass defect of a nucleus? Express it mathematically. What light does it throw on the binding energy of nucleus?

Ans :

OD 2012, SQP 2007

The difference between the rest mass of a nucleus and the sum of the rest masses of its constituent nucleons is called its mass defect.

Consider the nucleus ${}_Z^AX$. It has Z protons and $(A-Z)$ neutrons. Therefore, its mass defect will be,

$$\Delta m = Zm_p + (A-Z)m_n - m$$

where, m_p , m_n and m are the rest masses of a proton, neutron and the nucleus ${}_Z^AX$ respectively.

When a nucleus is formed from its nucleons, some of their mass is converted into energy which binds the nucleons together inside the nucleus. This energy, called binding energy, is equivalent of mass defect.

47. What are nuclear forces? Give their important properties.

Ans :

OD 2010

The strong attractive forces acting between the protons and neutrons of a nucleus which keep them bound together inside the tiny nucleus are called nuclear forces.

The important properties of nuclear forces are as follows:

- Nuclear forces are the strongest attractive forces known in nature,

$$F_G : F_E : F_N = 1 : 10^{36} : 10^{38}$$

- 34. Assertion :** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion and

Reason : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z .

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion. Therefore Assertion is correct.

The Reason is incorrect because for heavy nuclei the binding energy per nucleon decreases with increasing Z and for light nuclei, B.E. per nucleon increases with increasing Z .

Thus (c) is correct option.

VERY SHORT ANSWER QUESTIONS

- 35.** Write the relationship between the size of a nucleus and its mass number (A).

Ans :

OD 2012

The relationship between the size of a nucleus and its mass number is given by

$$R = R_0 A^{1/3}$$

where,

R = radius of nucleus

A = mass number

- 36.** How is the radius of a nucleus related to its mass number?

Ans :

OD 2011

Radius of nucleus, $R = R_0 A^{1/3}$

where,

R = radius of nucleus

A = mass number

$$R_0 = 1.2 \text{ fm}$$

$$= 1.2 \times 10^{-15} \text{ m}$$

- 37.** Write any two characteristic properties of nuclear force.

Ans :

OD 2008

Two characteristic of nuclear force are given as below:

- These are short range forces.
- These are strong force of attractive nature.

- 38.** 1. Write the β -decay or tritium in symbolic form.
 2. Why is it experimentally found difficult to detect neutrinos in this process?

Ans :

OD 2012

$$1. \quad {}_1^3\text{H} \xrightarrow{\beta^-} {}_2^3\text{He} + {}_0^{-1}e + \bar{\nu} + Q.$$

2. It is due to very weak interaction with matter.

- 39.** What is Einstein's mass-energy equivalence? What is its importance?

Ans :

OD 2010

In his special theory of relativity, Einstein showed that,

$$E = mc^2$$

This equation expresses equivalence between mass and energy. Thus the energy content of an object is its mass times the square of the speed of light. This principle is central to our understanding of nuclear energy and harnessing it as a source of power. Using this principle, the Q -value of nuclear process can be expressed in terms of initial and final masses.

- 40.** He_2^3 and He_1^3 nuclei have the same mass number. Do they have the same binding energy?

Ans :

OD 2010

Nuclei He_2^3 and He_1^3 have the same mass number. He_2^3 has two protons and one neutron. He_1^3 has one proton and two neutrons. The repulsive force between protons is missing in ${}_1\text{He}^3$, so the binding energy of ${}_1\text{He}^3$ is greater than that of ${}_2\text{He}^3$.

- 41.** What are isotones and isomers? Give suitable examples.

Ans :

OD 2015, Comp 2009

Isotones

The nuclides having the same number of neutrons are called isotones. For example, ${}_{17}^{37}\text{Cl}$ and ${}_{19}^{39}\text{K}$ are isotones, as both contain the same number of neutrons i.e., for both $N = A - Z = 20$.

Isomers

These are the nuclei with same atomic number and same mass number but existing in different energy states. For example, a nucleus in its ground state and the identical nucleus in metastable excited state are isomers.

- 42.** State the necessary conditions for nuclear fusion to occur.

Ans :

OD 2016

The fusion reactions take place under the conditions of extreme temperature and density due to the following reasons:

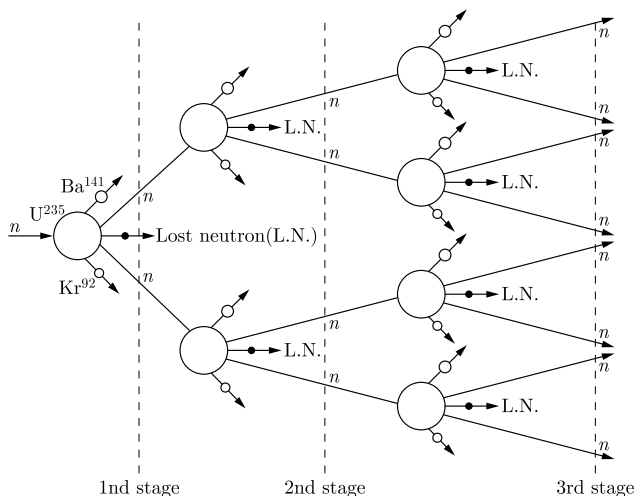
- The high temperature is necessary for the light nuclei to have sufficient kinetic energy so that they

52. What is nuclear fission? Explain how a chain reaction can occur in a fissionable material?

Ans :

OD 2012

Nuclear fission is a peculiar type of reaction which, besides the other fission products, produces the same kind of particles that initiate it, viz., neutrons.



A Chain Reaction in $^{235}_{92}\text{U}$

When a single $^{235}_{92}\text{U}$ nucleus captures a neutron, its fission produces 2.5 neutrons. These freshly produced neutrons can further cause the fission of more uranium nuclei, producing still more neutrons, which can further cause the fission of a larger number of nuclei and so on. The number of fissions taking place at each successive stage goes on increasing at a rapid rate (rather in a geometric progression). Thus a chain reaction is set up, as illustrated in figure. Enrico Fermi first suggested possibility of such a reaction in 1939.

53. What are uncontrolled and controlled chain reactions?

Ans :

OD 2006

Uncontrolled Chain Reaction

If a chain reaction is started in a fissionable material having mass greater than certain critical mass, then the reaction will accelerate at such a rapid rate that the whole material will explode within a microsecond, liberating a huge amount of energy. Such a chain reaction is called uncontrolled chain reaction. It forms the underlying principle of the atomic bombs.

Controlled Chain Reaction

The chain reaction can be controlled and maintained steady by absorbing a suitable number of neutrons at each stage of the reaction, so that on an average one neutron remains available for exciting further fission. Such a reaction is called controlled chain principle of a controlled chain reaction.

54. Define multiplication factor of a fissionable mass. Give its physical significance.

Ans :

OD 2008

The multiplication factor of a fissionable mass is defined as the ratio of the number of neutrons present at the beginning of a particular generation to the number of neutrons present at the beginning of the previous generation.

Thus,

$$k = \frac{\text{Number of neutrons present at the beginning of one generation}}{\text{Number of neutrons present at the beginning of previous generation}}$$

The multiplication factor k gives a measure of the growth rate of the neutrons in a fissionable mass.

If $k > 1$, the chain reaction grows.

If $k = 1$, the chain reaction remains steady.

If $k < 1$, the chain reaction gradually dies out.

55. How the size of a nucleus is experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of nucleus is independent of its mass number.

Ans :

OD 2004, Delhi 2017

The size of the nucleus is experimentally determined using Rutherford's α -scattering experiment and the distance of closest approach and impact parameter.

The relation between radius and mass number of nucleus is

$$R = R_0 A^{1/3}$$

where, $R_0 = 1.2 \text{ Fm}$

A = mass number

R = radius of nucleus

Nuclear density, $\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$

$$= \frac{mA}{\frac{4}{3}\pi(R_0 A^{1/3})^3}$$

where, m = mass of each nucleon

$$\text{Hence, } \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3}$$

From the above formula, it is clear that ρ does not depend on the mass number.

56. What are the utilities of moderator, coolant and controlling rods with reference to nuclear reactor?

Ans :

OD 2014

1. **Moderator** : It is a medium to slow down the fast moving secondary neutrons produce during the fission. Heavy water, graphite, deuterium, paraffins, etc., acts as moderator.

- They are short range forces effective upto 2-3 fermi from a nucleon.
 - They have charge independent nature.
 - They show saturation effect i.e., a nucleon can interact only with a neighbouring nucleon.
 - They are non-central forces i.e., they do not act along the line joining the centres of the two nucleons.
48. Define the terms : nucleons, atomic number, mass number, nuclear mass and nuclide in relation to a nucleus.

Ans :

OD 2010

- Nucleons** : Protons and neutrons which are present in the nuclei of atoms are collectively known as nucleons.
- Atomic Number** : The number of protons present in the nucleus is called the atomic number of the element. It is denoted by Z .
- Mass Number** : The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A .

Number of protons in an atom = Z Number of electrons in an atom = Z Number of nucleons in an atom = A Number of neutrons in an atom = $N = A - Z$

- Nuclear Mass** : The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.
- Nuclide** : When an atom is talked of with particular reference to its nuclear composition, it is called a nuclide. Thus a nuclide is a specific nucleus of an atom characterised by its atomic number Z and mass number A . It is symbolically represented as ${}^A_ZX$.

where, X = chemical symbol of the element Z = atomic number A = mass number

49. What are isotopes and isobars? Give suitable examples.

Ans :

OD 2012

Isotopes

The atoms of an element which have the same atomic number but different mass number are called isotopes. Lithium has two isotopes : ${}^6_3\text{Li}$ and ${}^7_3\text{Li}$. Gold has 32 isotopes, ranging from $A = 173$ to 204.

Isobars

The atoms having the same mass number but different atomic number are called isobars. Such atoms contain different number of protons and electrons.

Examples of isobars are:

- ${}^{37}_{17}\text{Cl}$ and ${}^{37}_{16}\text{S}$, as both have same $A = 37$.
- ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$, as both have same $A = 40$.

50. State the important properties of neutrons.

Ans :

OD 2010

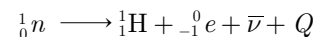
Some important properties of neutrons are as follows :

- Neutron is an elementary particle present in the nuclei of all elements except hydrogen.
- Neutron has no charge and its mass is slightly more than that of a proton,

$$m_n = 1.00866 \text{ amu}$$

$$= 1.6749 \times 10^{-27} \text{ kg}$$

- A free neutron is unstable. It spontaneously decays into a proton, electron and an anti-neutrino with a half life of about 1000 s.



- Neutrons being neutral, have low ionising power.
- Being neutral, neutrons are not repelled or attracted by the nucleus and the electrons of an atom. They can easily penetrate heavy nuclei and induce nuclear reactions.
- They induce radioactivity in many elements.

51. Explain how is fission an enormous source of energy per nuclear fission is about 200 MeV.

Ans :

OD 2017, Delhi 2003

In a nuclear fission, the sum of the masses of the final products is less than the sum of the masses of the reactant components. The difference in masses, called mass defect, is converted into energy according to Einstein's mass-energy relationship ($E = mc^2$). Thus an enormous amount of energy is released in a nuclear fission.

Suppose a nucleus with mass number $A = 240$ breaks into two fragments each of $A = 120$.

Then, Binding energy of per nucleon for $A = 240$ nucleus is about 7.6 MeV.

Binding energy of per nucleon for the two $A = 120$ fragments is about 8.5 MeV.

Hence, gain in B.E. per nucleon $\simeq 0.9$ MeV.

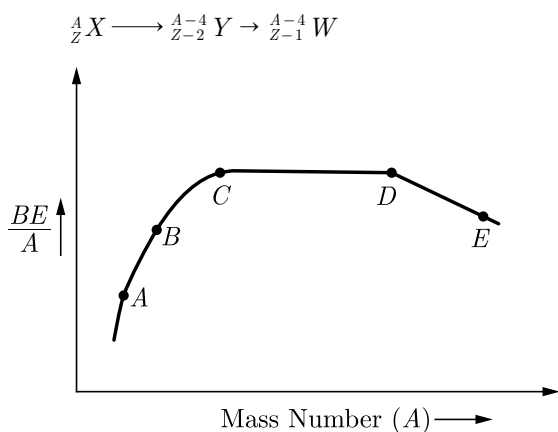
Hence, the total gain in binding energy (by 240 nucleons)

$$\simeq 240 \times 0.9 = 216 \text{ MeV}$$

The disintegration energy of ${}^{235}_{92}\text{U}$ nucleus is about $(235 \times 0.9 \text{ MeV})$ 200 MeV. This energy appears in the form of kinetic energy of the fission products and as γ -rays. One kilogram of uranium will generate on fission 10^{14} J of energy. This is a very huge amount of energy.

reasons, the two process (in terms of A, B, C, D and E), one of which can occur due to nuclear fission and the other due to nuclear fusion.

2. Identify the nature of the radioactive radiations emitted in each step of the decay process given below,



Ans :

OD 2004

1. Nuclear fission of E to D and C ; as there is an increase in binding energy per nucleon.
Nucleon fusion of A and B into C ; as there is an increase in binding energy per nucleon.
 2. First step α particle, as the atomic number decreases by 2 and mass number decreases by 4.
Second step β particle, as the atomic number increases by 1, while mass number remains unchanged.
62. (i) In a typical nuclear reaction. e.g.,
 ${}^2_1\text{H} + {}^2_1\text{H} \longrightarrow {}^3_2\text{He} + n + 3.27$
 Although number of nucleons is conserved yet energy is released. How? Explain.
- (ii) Show that nuclear density in a given nucleus is independent of mass number A .

Ans :

OD 2012

- (i) In a nuclear reaction, the sum of the masses of the larger nucleus (${}^2_1\text{H}$) and the bombarding particle (${}^2_1\text{H}$) may be greater or less than the (${}^3_2\text{He}$) and the outgoing particle (${}_0^1n$). So, from the law of conservation of mass-energy, some energy (3.27 MeV) is evolved or involved in a nuclear reaction. This energy is called Q -value of the nuclear reaction,
- (ii) Density of nuclear matter is the ratio of mass of the nucleus and its volume.

Density of the nuclear matter

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \quad \dots(1)$$

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus $= mA$, where

A is the mass number of the element.

$$\begin{aligned} \text{Volume of the nucleus} &= \frac{4}{3}\pi R^3 \\ &= \frac{4}{3}(\pi R_0 A^{1/3})^3 \\ &= \frac{4}{3}\pi R_0^3 A \end{aligned}$$

Put the value in eq. (1)

$$\text{Thus, density of nucleus} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

As, m and R_0 are constants, therefore density of the nuclear matter is the same for all elements.

Now, using $m = 1.66 \times 10^{-27} \text{ kg}$.

$$= \frac{3 \times 1.66 \times 10^{-27}}{4\pi R_0^3}$$

Using, $R_0 = 1.1 \times 10^{-15} \text{ m}$ and

$$\text{density} = 2.97 \times 10^{17} \text{ kg m}^{-3}$$

Which shows that the density is independent of mass number A .

63. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released. Calculate the energy release in MeV in the deuterium fusion reaction.



Using the data

$$m({}^2_1\text{H}) = 2.014102 \text{ u}$$

$$m({}^3_1\text{H}) = 3.016049 \text{ u}$$

$$m({}^4_2\text{He}) = 4.002603 \text{ u}$$

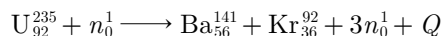
$$m_n = 1.008665 \text{ u}$$

$$1 \text{ u} = 931.5 \frac{\text{MeV}}{c^2}$$

Ans :

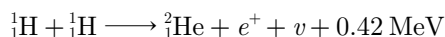
OD 2015, Comp 2002

Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually $A > 230$) into two or more lighter nuclei.



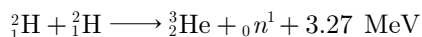
In this case, the energy released per fission of U_{92}^{235} is 200.4 MeV.

Nuclear fusion is the phenomenon of fusion of two or more lighter nuclei to form a single heavy nucleus. The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusing together. This difference in the masses results in the release of tremendous amount of energy. e.g.,



2. **Control Rods** : It is used in nuclear reactors to control the rate of fission of uranium and plutonium. These are made of chemical element capable of absorbing many neutrons without fissioning themselves such as silver, indium, boron and cadmium.
3. **Coolant** : It is a liquid used to remove heat from nuclear reactor core and transfer it to electrical generator and environment. Ordinary water under high pressure is used as coolant.

57. In a typical nuclear reaction, e.g.,

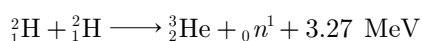


although number of nucleons is conserved, yet energy is released. How? Explain.

Ans :

OD 2013

In nuclear reaction

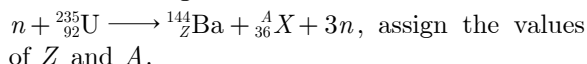


Cause of the energy released

- (i) Binding energy per nucleon of ${}^3_2\text{He}$ becomes more than the (BE/A) of ${}^2_1\text{H}$
- (ii) Mass defect between the reactant and product nuclei.

$$\begin{aligned}\Delta E &= \Delta mc^2 \\ &= [2m({}^2_1\text{H}) - m({}^3_2\text{He}) + m({}^1_0n)]c^2\end{aligned}$$

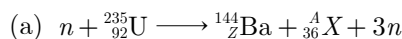
58. (a) In the following nuclear reaction



- (b) If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is the mass converted into energy? Explain.

Ans :

OD 2015



From law of conservation of atomic number,

$$0 + 92 = Z + 36$$

$$Z = 92 - 36 = 56$$

From law of conservation of mass number,

$$1 + 235 = 144 + A + 3 \times 1$$

$$A = 236 - 147 = 89$$

- (b) (i) BE of ${}^{235}_{92}\text{U} < \text{BE of } ({}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{X})$ and due to difference in BE of the nuclides. A large amount of the energy will be released in the fission of ${}^{235}_{92}\text{U}$.
- (ii) Mass number of the reactant and product nuclides are same but there is an actual mass defect. This difference in the total mass of the nuclei on both sides gets converted into energy, i.e., $\Delta E = \Delta mc^2$

59. (i) What characteristic property of nuclear force explains the constancy of binding energy per nucleon (BE/A) in the range of mass number. 'A' lying $30 < A < 170$?
- (ii) Show that the density of nucleus over a wide range of nuclei is constant independent of mass number A.

Ans :

OD 2012, SQP 2015

- (i) Saturation or short range nature of nuclear forces.
- (ii) The radius (size) R of nucleus is related to its mass number (A) as

$$R = R_0 A^{1/3}$$

where, $R_0 = 1.1 \times 10^{-15} \text{ m}$

If m is the average mass of a nucleon, then mass of nucleus = mA , Where A is mass number.

$$\begin{aligned}\text{Volume of nucleus} &= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 \\ &= \frac{4}{3}\pi R_0^3 A\end{aligned}$$

Hence, density of nucleus,

$$\begin{aligned}\rho_N &= \frac{\text{Mass}}{\text{Volume}} \\ &= \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3} = \frac{3m}{4\pi R_0^3}\end{aligned}$$

Clearly, nuclear density ρ_N is independent of mass number A .

60. Answer the following :

- Why is the binding energy per nucleon found to be constant for nuclei in the range of mass number A lying between 30 and 170?
- When a heavy nucleus with mass number $A = 240$ breaks into two nuclei, $A = 120$, energy is released in the process.

Ans :

OD 2011

- The binding energy per nucleon for nucleus of range, $30 < A < 170$ is close to its maximum value. So, the nucleus belongs to this region is highly stable and does not show radioactivity.
- Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e., heavier nuclei are less stable. When a heavier nucleus such as nucleus of mass number 240 splits into lighter nuclei (mass number 120), the BE/nucleon changes from 7.6 MeV to 8.4 MeV. Greater BE of the product nuclei result in the liberation of energy.

61. 1. The figure shows the plot of binding energy (BE) per nucleon as a function of mass number A . The letters A, B, C, D and E represent the positions of typical nuclei on the curve. Point out, giving

	Nuclear Fission	Nuclear Fission
1.	Here a heavy nucleus when excited gets split up into two smaller nuclei of nearly comparable masses.	Here two lighter nuclei fuse together to form a heavier nucleus.
2.	The conditions of high temperature and pressure are not necessary for its occurrence. It can be carried on the earth.	The conditions of extremely high pressure and temperature are necessary for its occurrence. So it cannot be easily carried in a laboratory.
3.	Neutrons are the link particles of this process.	Protons are the link particles of this process.
4.	It is a quick process.	It occurs in several steps. There is sufficient time gap between initial and final steps.
5.	Here the energy available per nucleon is small, about 0.85 MeV.	Here the energy available per nucleon is large, about 6.75 MeV.
6.	The energy obtained from a unit mass of a fissionable material is smaller than that obtained in case of fusion.	The energy obtained from a unit mass of a fusible material is large.
7.	It produces very harmful radioactive wastes.	The products of fusion are harmless.
8.	The stock of fissionable fusion is limited.	The fuel required for fusion is available in plenty.

- Nuclear fission produces energy in a nuclear reactor.
- Nuclear fusion produces energy in the sun.

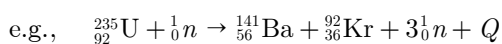
67. What do you mean by Nuclear Fission?

Ans :

OD 2007

The phenomenon of splitting of heavy nuclei (mass number > 120) into smaller nuclei of nearly equal masses is known as nuclear fission.

In nuclear fission, the sum of the masses of the product is less than the sum of masses of the reactants. This difference of mass gets converted into energy $E = mc^2$ and hence some amount of energy is released in a nuclear fission.



$$\begin{aligned}\text{Masses of reactant} &= 235.0439 \text{ amu} + 1.0087 \text{ amu} \\ &= 236.0526 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Masses of product} &= 140.9139 + 91.8973 + 3.0261 \\ &= 235.8373 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Mass defect} &= 236.0526 - 235.8373 \\ &= 0.2153 \text{ amu}\end{aligned}$$

$$\text{Hence, } 1 \text{ amu} = 931 \text{ MeV}$$

$$\begin{aligned}\text{Energy released} &= 0.2153 \times 931 \\ &= 200 \text{ MeV nearly}\end{aligned}$$

Thus, energy is liberated in nuclear fission, if ${}_{92}^{235}\text{U}$.

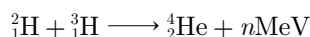
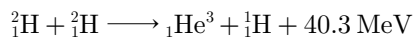
LONG ANSWER QUESTIONS

68. (a) (i) Distinguish between nuclear fission and fusion giving an example of each.
- (ii) Explain the release of energy in nucleus and fission and fusion on the basis of binding energy per nucleon curve.
- (b) (i) How is the size of a nucleus found experimentally? Write the relation between the radius and mass number of a nucleus.
- (ii) Prove that the density of a nucleus is independent of its mass number.

Ans :

OD 2023, Foreign 2012

	Nuclear Fission	Nuclear Fission
1.	When the nucleus of an atom splits into lighter nuclei through a nucleus reaction the process is termed nuclear fission.	Nuclear fusion is a reaction through which two or more light nuclei collide with each other to form a heavier nucleus.
2.	When each atom splits a tremendous amount of energy is released.	The energy released during nuclear fusion is several times greater than the energy released during nuclear fission.
3.	Fission reactions do not occur in nature naturally.	Fusion reactions occur in stars and the sun.
4.	Comparatively, less energy is needed to split an atom in a fission reaction.	High energy is needed to fuse two or more atoms together in a fusion reaction.
5.	Atomic bomb works on this principle.	Hydrogen bomb works on this principle.



$$\begin{aligned}\Delta m &= (2.014102 + 3.016049) \\ &\quad - (4.002603 + 1.008665) \\ &= 0.018883 \text{ u}\end{aligned}$$

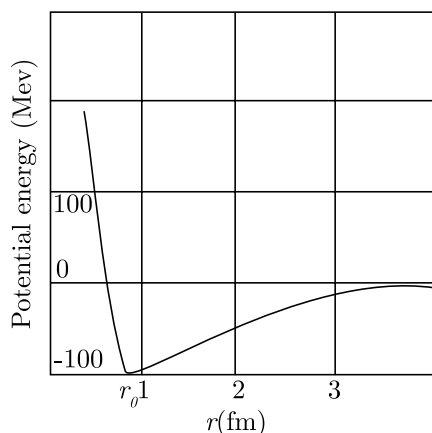
$$\begin{aligned}\text{Energy released, } Q &= 0.018883 \times 931.5 \frac{\text{MeV}}{c^2} \\ &= 17.589 \text{ MeV}\end{aligned}$$

64. (i) Write three characteristic properties of nuclear force.
 (ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions that can be drawn from the graph.

Ans :

OD 2012

- (i) Characteristic properties of nuclear force are as follows :
- Nuclear force act between a pair of neutrons, a pair of protons and also between a neutron-proton pair, with the same strength. This shows that nuclear forces are independent of charge.
 - The nuclear forces are dependent on spin or angular momentum of nuclei.
 - Nuclear forces are non-central forces. This shows that the distribution of nucleons in a nucleus is not spherically symmetric.
- (ii)



From the plot, it is concluded that

- (a) The potential energy is minimum at a distance $r_0 (= 0.8 \text{ fm})$ which means that the force is attractive for distance larger than 0.8 fm and repulsive for the distance less than 0.8 fm between the nucleons.

- (b) Nuclear forces are negligible when the distances between the nucleons is more than 10 fm .

65. Explain giving necessary reaction, how energy is released during:

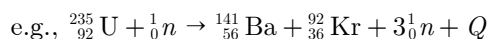
- fission
- fusion

Ans :

OD 2011, SQP 2005

- (i) **Nuclear Fission** : The phenomenon of splitting of heavy nuclei (mass number > 120) into smaller nuclei of nearly equal masses is known as nuclear fission.

In nuclear fission, the sum of the masses of the product is less than the sum of masses of the reactants. This difference of mass gets converted into energy $E = mc^2$ and hence sample amount of energy is released in a nuclear fission.



$$\begin{aligned}\text{Masses of reactant} &= 235.0439 \text{ amu} + 1.0087 \text{ amu} \\ &= 236.0526 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Masses of product} &= 140.9139 + 91.8973 + 3.0261 \\ &= 235.8373 \text{ amu}\end{aligned}$$

$$\begin{aligned}\text{Mass defect} &= 236.0526 - 235.8373 \\ &= 0.2153 \text{ amu}\end{aligned}$$

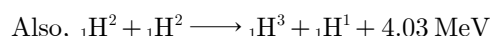
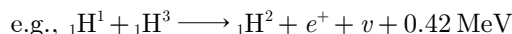
$$\text{Hence, } 1 \text{ amu} = 931 \text{ MeV}$$

$$\begin{aligned}\text{Energy released} &= 0.2153 \times 931 \\ &= 200 \text{ MeV nearly}\end{aligned}$$

Thus, energy is liberated in nuclear fission, if ${}^{235}_{92}\text{U}$.

- (ii) **Nuclear Fusion** : The phenomenon of conversion of two lighter nuclei into a single heavy nucleus is called nuclear fusion.

Since, the mass of the heavier product nucleus is less than the sum of masses of reactant nuclei and therefore certain mass defect occurs which converts into energy as per Einstein's mass-energy relation. Thus, energy is released during nuclear fusion.



66. Differentiate between nuclear fission and nuclear fusion. Which one of these processes produces energy (1) in nuclear reactor and (2) in the sun?

Ans :

OD 2005

The differentiate between nuclear fission and nuclear fusion and nuclear are as follows :

- (c) Cadmium has high absorption capture for neutrons; so cadmium rods are used to absorb extra neutrons; so nuclear fission in a nuclear reactor is controlled; therefore cadmium rods are called control rods.
- (d) In nuclear fusion, two positively charge nuclei fuse together, when two positively charged nuclei come near each other to fuse together, strong electrostatic repulsive force acts between them. To overcome this repulsive force, extremely high temperature of the order of 10^8 K are required. This may be calculated as follows :

For fusion of H-nuclei,

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = kT$$

$$9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{10^{-14}} = 1.38 \times 10^{-23} T$$

$$T = 1.67 \times 10^8 \text{ K}$$

The temperature in the interior of sun is about 2×10^7 K.

Therefore, very high temperature of the order 10^7 K are required for fusion reaction to take place.

70. What do you mean by binding energy of nucleus? Obtain an expression for binding energy. How binding energy per nucleon explains the stability of nucleus?

or

Define the term binding energy. The binding energy of a nucleus ${}^A_ZX$ is given by the formula:

$$\text{B.E.} = [Zm_H + (A - Z)m_n - m({}^A_ZX)] c^2$$

where, $m({}^A_ZX)$ is the atomic mass of X . Derive this equation, state clearly the approximation involved and say it is very safe approximation.

Ans :

OD 2012

The binding energy of a nucleus may be defined as the energy required to break up a nucleus into its constituent protons and neutrons and to separate them to such a large distance that they may not interact with each other.

The binding energy may also be defined as the surplus energy which the nucleons give up by virtue of their attractions when they become bound together to form a nucleus.

If Δm is the mass defect of a nucleus, then its binding energy will be,

$$\Delta E_b = \Delta m \times c^2$$

Expression for Binding Energy

The nucleus ${}^A_ZX$ contains Z protons and $(A - Z)$ neutrons. Its mass defect is,

$$\Delta m = Zm_p + (A - Z)m_n - m_N \quad \dots(1)$$

where m_N is the nuclear mass of ${}^A_ZX$.

The binding energy of the nucleus is,

$$\Delta E_b = \Delta m \times c^2$$

$$= [Zm_p + (A - Z)m_n - m_N] c^2 \quad \dots(2)$$

Now, in an atom the electrons are bound to the nucleus by electrostatic forces. So they have a binding energy of their own, which from the mass-energy equivalence is given by,

$$(\Delta E_b)_e = [(m_N + Zm_e) - m({}^A_ZX)] c^2 \quad \dots(3)$$

where $m({}^A_ZX)$ is the atomic mass. The binding energy of electrons ($\approx$ eV to keV) is negligible compared to the binding energy of nucleons ($\approx 10^3$ MeV). It will be a safe approximation to take,

$$(\Delta E_b)_e = 0$$

$$m_N + Zm_e - m({}^A_ZX) = 0$$

$$m_N = m({}^A_ZX) - Zm_e$$

Thus, in terms of atomic mass the Eq. (2) becomes,

$$\Delta E_b = [Zm_p + (A - Z)m_n - m({}^A_ZX) + Zm_e] c^2$$

$$= [Z(m_p + m_e) + (A - Z)m_n - m({}^A_ZX)] c^2 \quad \dots(4)$$

But, $m_p + m_e = m_H$ = mass of a hydrogen atom

Hence, the Eq. (4) can be written in terms of m_H as,

$$\Delta E_b = [Zm_H + (A - Z)m_n - m({}^A_ZX)] c^2$$

Binding Energy Per Nucleon

The binding energy per nucleon is the average energy required to extract one nucleon from the nucleus. It is obtained by dividing the binding energy of a nucleus by its mass number. The expression for binding energy per nucleon can be written as,

$$\Delta E_{bn} = \frac{\Delta E_b}{A}$$

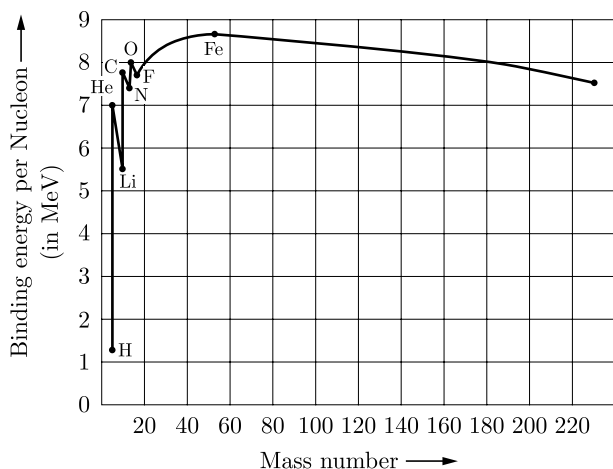
$$= \frac{[Zm_H + (A - Z)m_n - m({}^A_ZX)] c^2}{A}$$

The value of binding energy per nucleon of a nucleus gives a measure of the stability of that nucleus. Greater is the binding energy per nucleon of a nucleus, more stable is the nucleus.

71. Draw the graph showing the variation of binding energy per nucleon with the mass number for a large number of nuclei $2 < A < 240$. What are the main inferences from the graph? How do you explain the constancy of binding energy in the range $30 < A < 170$ using the property that the nuclear force is short-ranged? Explain with the help of this plot the release of energy in the processes of nuclear fission and fusion.

Ans :

OD 2008, Delhi 2015



The variation of binding energy per nucleon versus mass number is shown in figure.

Inferences from Graph

- The nuclei having mass number below 20 and above 180 have relatively small binding energy and hence they are unstable.
 - The nuclei having mass number 56 and about 56 have maximum binding energy – 5.8 MeV and so they are most stable.
 - Some nuclei have peaks, e.g., ${}^2\text{He}^4$, ${}^6\text{C}^{12}$, ${}^8\text{O}^{16}$; this indicates that these nuclei are relatively more stable than their neighbours.
- (a) **Explanation of Constancy of Binding Energy :** Nuclear force is short ranged, so every nucleus interacts with its neighbours only, therefore binding energy per nucleon remains constant.
- (b) **Explanation of Nuclear Fission :** When a heavy nucleus ($A \geq 235$ say) breaks into two lighter nuclei (nuclear fission), the binding energy per nucleon increases i.e., nucleons get more tightly bound. This implies that energy would be released in nuclear fission.
- (c) **Explanation of Nuclear Fusion :** When two very light nuclei ($A \leq 10$) join to form a heavy nucleus, the binding energy per nucleon of fused heavier nucleus is more than the binding energy per nucleon of lighter nuclei, so again energy would be released in nuclear fusion.

NUMERICAL QUESTIONS

72. Calculate the energy equivalent of 1 g of substance.

Ans :

OD 2010

Here, $m = 1 \text{ g} = 10^{-3} \text{ kg}$
 $c = 3 \times 10^8 \text{ ms}^{-1}$

Hence, equivalent energy,

$$\begin{aligned} E &= mc^2 \\ &= 10^{-3} \times (3 \times 10^8)^2 \\ &= 9 \times 10^{13} \text{ J} \end{aligned}$$

73. Express 16 mg mass into equivalent energy in eV.

Ans :

OD 2007

Given, $m = 16 \text{ mg} = 16 \times 10^{-6} \text{ kg}$
 $c = 3 \times 10^8 \text{ ms}^{-1}$

Hence Equivalent energy,

$$\begin{aligned} E &= mc^2 \\ &= 16 \times 10^{-6} \times (3 \times 10^8)^2 \text{ J} \\ &= \frac{16 \times 10^{-6} \times (3 \times 10^8)^2}{16 \times 10^{-19}} \text{ eV} \\ &= 9 \times 10^{30} \text{ eV} \end{aligned}$$

74. A nucleus with mass number, $A = 240$ and $BE/A = 7.6 \text{ MeV}$ breaks into two fragments each of $A = 120$ with $BE/A = 8.5 \text{ MeV}$. Calculate the released energy.

Ans :

OD 2010

Binding energy of nucleus with mass number 240,

$$BE_1 = 240 \times 7.6 = 1824 \text{ MeV}$$

Binding energy of two fragments,

$$\begin{aligned} BE_2 &= 2 \times 120 \times 8.5 \\ &= 2040 \text{ MeV} \end{aligned}$$

Hence, Energy released $= BE_2 - BE_1$
 $= (2040 - 1824) \text{ MeV}$
 $= 216 \text{ MeV}$

75. How many electron volts make up one joule?

Ans :

OD 2015, SQP 2001

By definition, one electron volt is the energy gained by an electron, when accelerated through a potential difference of 1 volt, therefore

$$\begin{aligned} 1 \text{ eV} &= 1.602 \times 10^{-19} \text{ C} \times 1 \text{ V} \\ &= 1.602 \times 10^{-19} \text{ J} \end{aligned}$$

Hence, $1 \text{ J} = \frac{1}{1.602 \times 10^{-19}} \text{ eV}$
 $= 6.242 \times 10^{18} \text{ eV}$

76. What is the value of radius of Ge^{70} : (Given $R_0 = 1.1 \text{ fm}$).

Ans :

OD 2004

Given, $A = 70$
 $R_0 = 1.1 \text{ fm}$

As we know that,

$$\begin{aligned}
 R &= R_0 A^{1/3} = 1.1 \times (70)^{1/3} \\
 &= 1.1 \times 4.12 \\
 &= 4.53 \text{ fm}
 \end{aligned}$$

77. The nuclear mass of ${}^{56}_{26}\text{Fe}$ is 55.85 amu. Calculate its nuclear density.

Ans :

OD 2010

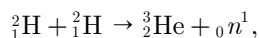
$$\begin{aligned}
 \text{Given, } m_{\text{Fe}} &= 55.85 \text{ amu} \\
 &= 55.85 \times 1.66 \times 10^{-27} \text{ kg} \\
 &= 9.27 \times 10^{-26} \text{ kg}
 \end{aligned}$$

As we know that,

$$\begin{aligned}
 \text{Nuclear radius, } R &= R_0 A^{1/3} \\
 &= 1.1 \times 10^{-15} \times (56)^{1/3} \\
 &[\because A = 56]
 \end{aligned}$$

$$\begin{aligned}
 \rho_{\text{nu}} &= \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{\text{Fe}}}{\frac{4}{3}\pi R^3} \\
 &= \frac{9.27 \times 10^{-26}}{\frac{4}{3}\pi \times (1.1 \times 10^{-15})^3 \times 56} \\
 &= 2.9 \times 10^{17} \text{ kg m}^{-3}
 \end{aligned}$$

78. Calculate the energy in fusion reaction:



where, BE of ${}^2_1\text{H} = 2.23 \text{ MeV}$ and of ${}^3_2\text{He} = 7.73 \text{ MeV}$

Ans :

OD 2011

$$\begin{aligned}
 \text{Initial binding energy, } BE_1 &= 2.23 + 2.23 \\
 &= 4.46 \text{ MeV}
 \end{aligned}$$

$$\text{Final binding energy, } BE_2 = 7.73 \text{ MeV}$$

$$\begin{aligned}
 \text{Hence, Energy released} &= 7.73 - 4.46 \\
 &= 3.27 \text{ MeV}
 \end{aligned}$$

79. When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos)

Given:

1. Mass of ${}^1_1\text{H} = 1.007825 \text{ u}$
2. Mass of helium nucleus = 4.002603 u , $1 \text{ u} = 931 \text{ MeV}/c^2$

Ans :

OD 2003

$$\text{Energy released} = \Delta m \times 931 \text{ MeV}$$

$$\Delta m = 4m({}^1_1\text{H}) - m({}^4_2\text{He})$$

Energy released,

$$\begin{aligned}
 Q &= [4m({}^1_1\text{H}) - m({}^4_2\text{He})] \times 931 \text{ MeV} \\
 &= [4 \times 1.007825 - 4.002603] \times 931 \text{ MeV} \\
 &= 26.72 \text{ MeV}
 \end{aligned}$$

80. Express one atomic mass unit in energy units, first in Joules and then in MeV. Using this, express the mass defect of ${}^{16}_8\text{O}$ in MeV.

Ans :

OD 2018

$$\begin{aligned}
 \text{Given, } m &= 1 \text{ amu} \\
 &= 1.660565 \times 10^{-27} \text{ kg} \\
 c &= 2.9979 \times 10^8 \text{ ms}^{-1} \\
 E &= mc^2 \\
 &= 1.660565 \times 10^{-27} \times (2.9979 \times 10^8)^2 \text{ J} \\
 &= 1.4924 \times 10^{-10} \text{ J} \\
 &= \frac{1.4924 \times 10^{-10}}{1.602 \times 10^{-13}} \text{ MeV} \\
 &[\because 1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}] \\
 &= 931.5 \text{ MeV}
 \end{aligned}$$

The ${}^{16}_8\text{O}$ nucleus contains 8 protons and 8 neutrons.

$$\begin{aligned}
 \text{Mass of 8 protons} &= 8 \times 1.00727 \\
 &= 8.05816 \text{ amu}
 \end{aligned}$$

$$\begin{aligned}
 \text{Mass of 8 neutrons} &= 8 \times 1.00866 \\
 &= 8.06928 \text{ amu}
 \end{aligned}$$

$$\text{Total mass} = 16.12744 \text{ amu}$$

$$\text{Mass of } {}^{16}_8\text{O nucleus} = 15.99053 \text{ amu}$$

$$\text{Mass defect, } \Delta m = 0.13691 \text{ amu}$$

$$\begin{aligned}
 \Delta E_b &= 0.13691 \times 931.5 \text{ MeV} \\
 &= 127.5 \text{ MeV}
 \end{aligned}$$

81. If radius of the ${}^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 fermi, What is the radius of ${}^{125}_{52}\text{Te}$ nucleus?

Ans :

OD 2011, Comp 2009

$$\text{Radius of nucleus } {}^{27}_{13}\text{Al } R_1 = 3.6 \text{ fermi}$$

$$\text{Mass number of } {}^{27}_{13}\text{Al } A_1 = 27 \text{ and}$$

$$\text{Mass number of } {}^{125}_{52}\text{Te } A_2 = 125$$

We also know that radius of nucleus of an atom,

$$R = R_0 A^{1/3} \propto A^{1/3}$$

$$\begin{aligned}
 \text{Therefore, } \frac{R_1}{R_2} &= \left(\frac{A_1}{A_2}\right)^{1/3} \\
 &= \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{or } R_2 &= \frac{5}{3} \times R_1 \\
 &= \frac{5}{3} \times 3.6 = 6 \text{ fermi}
 \end{aligned}$$

(where, R_2 = Radius of nucleus ${}^{125}_{52}\text{Te}$)

82. Atomic weight of boron is 10.81 and it has two isotopes ${}^5_1\text{B}^{10}$ and ${}^5_1\text{B}^{11}$. What is the ratio of atomic

weights of ${}_5\text{B}^{10}$ to ${}_5\text{B}^{11}$?

Ans :

OD 2013

Atomic weight of boron = 10.81

Isotopes of boron = ${}_5\text{B}^{10}$ and ${}_5\text{B}^{11}$

Let, x be the atomic weight of isotope ${}_5\text{B}^{10}$ in boron.

Therefore, atomic weight of isotope ${}_5\text{B}^{11}$ in boron = $(100 - x)$.

Atomic weight of boron

$$\begin{aligned} &= \frac{(x \times 10) + (100 - x) \times 11}{100} \\ &= \frac{10x + 1100 - 11x}{100} \end{aligned}$$

$$\text{or } 10.81 = \frac{1100 - x}{100}$$

$$\text{or } x = 1100 - 1081 = 19$$

Therefore value of isotope,

$$\begin{aligned} {}_5\text{B}^{11} &= 100 - x \\ &= 100 - 19 = 81 \end{aligned}$$

Therefore, ratio of atomic weights of ${}_5\text{B}^{10}$ to ${}_5\text{B}^{11}$

$$= \frac{{}_5\text{B}^{10}}{{}_5\text{B}^{11}} = \frac{19}{81}$$

$$\text{or } {}_5\text{B}^{10} : {}_5\text{B}^{11} = 19 : 81$$

83. If the binding energy of the electron in a hydrogen atom is 13.6 eV. How much energy required to remove the electron from the first excited state of Li^{++} ?

Ans :

OD 2004

Binding energy of electron in hydrogen atom, E_H = 13.6 eV.

Energy level of first excited state, $n = 2$.

Binding energy of hydrogen-like atom (Li^{++}) in n^{th} excited state,

$$\begin{aligned} E_n &= \frac{E_H Z^2}{n^2} \\ &= \frac{13.6 \times (3)^2}{(2)^2} = 30.6 \text{ eV} \end{aligned}$$

(Where, Z = Atomic mass of Li^{++} equal to 3)

84. Calculate the binding energy of an α -particle in MeV. Given,

$$m_p \text{ (mass of proton)} = 1.007825 \text{ amu}$$

$$m_n \text{ (mass of neutron)} = 1.008665 \text{ amu}$$

$$\text{Mass of He nucleus} = 4.002800 \text{ amu}$$

$$1 \text{ amu} = 931 \text{ MeV}$$

Ans :

OD 2003. SQP 2016

An α -particle contains 2 protons and 2 neutrons.

$$\text{Mass of 2 protons} = 2 \times 1.007825$$

$$= 2.015650 \text{ amu}$$

$$\text{Mass of 2 neutrons} = 2 \times 1.008665$$

$$= 2.017330 \text{ amu}$$

Total mass,

$$\text{Mass of He nucleus} = 4.002800 \text{ amu}$$

$$\text{Mass defect, } \Delta m = 0.030180 \text{ amu}$$

Binding energy of α -particle

$$= 0.030180 \times 931$$

$$= 28.097 \text{ MeV}$$

CASE BASED QUESTIONS

85. The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A , so the density of nuclear matter is independent of the size of the nucleus. Thus, the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

Mass of nucleus = mA

$$\begin{aligned} \text{Volume of nucleus} &= \frac{4}{3} \times \pi R^3 = \frac{4}{3} \pi (R_0 A^{1/3})^3 \\ &= \frac{4}{3} \pi R_0^3 A \end{aligned}$$

$$\text{Nuclear density, } \rho_{nu} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$\rho_{nu} = \frac{mA}{\frac{4}{3} \pi R_0^3 A} = \frac{3m}{4 \pi R_0^3}$$

Clearly, nuclear density is independent of mass number A or the size of the nucleus. The nuclear mass density is of the order $10^{17} \text{ kg m}^{-3}$. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$.

- Find the density of nuclear matter of ${}^{16}_8\text{O}$ its radius is $3 \times 10^{-15} \text{ m}$.
- If the nucleus of ${}^{27}_{13}\text{Al}$ has a nuclear radius of about 3.6 fm , then find the radius of ${}^{125}_{52}\text{Te}$.
- Two nuclei have mass number in the ratio 1 : 2. What is the ratio of their nuclei densities ?
- Why do stable nuclei never have more protons than neutrons ?
- Are the nuclear force affected by the charge present inside nucleus.

Ans :

$$\begin{aligned}
 (i) \quad R &= 10^{-15} \text{ m} \\
 m &= 16 \text{ amu} \\
 &= 16 \times 1.66 \times 10^{-27} \text{ kg}
 \end{aligned}$$

$$\begin{aligned}
 \text{Nuclear density } \rho_n v &= \frac{\text{Nuclear mass}}{\text{Nuclear volume}} \\
 &= \frac{16 \times 1.66 \times 10^{-27}}{\frac{4}{3} \pi (3 \times 10^{-15})^3} \\
 &= 2.4 \times 10^{17} \text{ kg/m}^3
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad A_1 &= 27, A_2 = 125 \\
 R_1 &= 3.6 f_m, R_2 = ? \\
 \frac{R_2}{R_1} &= \left(\frac{A_2}{A_1} \right)^{1/3} \\
 &= \left(\frac{125}{27} \right)^{1/3} = \frac{5}{3} \\
 R_2 &= 6 f_m
 \end{aligned}$$

(iii) Nuclear density is independent of mass number, so ratio 1 : 1.

(iv) Protons are positively charged particle and repel each other. This repulsion becomes so great in nuclei with more than 10 protons or so, that an excess of neutrons which produce only attractive (nuclei) forces, is required for stability.

(v) No, nuclear forces are charge independent.

86. The nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of α -particles by atoms. He found that the scattering results could be explained, if atoms consists of a small, central, massive and positive core surrounded by orbiting electrons. The experimental results indicated that the size of the nucleus is of the order of 10^{-14} m and is thus 10000 times smaller than the size of atom.

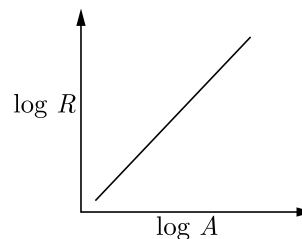
- (i) What is the ratio of mass of nucleus with mass of atom ?
- (ii) if it is radius and A is mass number, then, plot the graph between $\log R$ and $\log A$.
- (iii) What will be the ratio of masses of hydrogen, deuterium and tritium ?
- (iv) Find the ratio of gold isotope $^{197}_{79}\text{Au}$ and silver isotope $^{107}_{47}\text{Au}$.
- (v) What are the isotones ?

Ans :

(i) As 99.9% mass of atom is in nucleus

$$\frac{\text{mass of nucleus}}{\text{mass of atom}} = \frac{99.9}{100} = 0.999 \approx 1.$$

(ii)



$$R = R_0 A^{1/3}$$

Taking log both sides

$$\log R = \log R_0 + \frac{1}{3} \log A$$

Compare it with

$$y = mx + C$$

(iii) As ^1_1H , ^2_1D , ^3_1T , or ^1_1H , ^2_1H , ^3_1H

Then ratio of masses = 1:2:3

$$(iv) \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2} \right)^{1/3} = \left(\frac{197}{107} \right)^{1/3} \approx 1.23$$

(v) Nuclides with same neutrons but different atomic numbers are isotones.

CHAPTER 14

Semiconductor and Electronics Devices

SUMMARY

1. CLASSIFICATION OF SOLIDS ON THE BASIS OF THEIR CONDUCTIVITY

On the basis of the relative values of electrical conductivity (σ) and resistivity ($\rho = 1/\sigma$), the solids are broadly classified as:

1. **Metals** : Those solids which have high conductivity and very low resistivity. The value of conductivity for metals lies in between 10^2 to 10^8Sm^{-1} and of resistivity in between 10^{-2} to 10^8Sm^{-1} and of resistivity in between 10^{-2} to $10^{-8} \Omega\text{m}$.
2. **Insulators** : Those solids which have low conductivity and high resistivity. The value of conductivity for insulators lies between 10^{-11} to 10^{-19}Sm^{-1} and of resistivity between 10^{11} to $10^{19} \Omega\text{m}$.
3. **Semiconductors** : Those solids which have conductivity and resistivity intermediate to metals and insulators. The value of conductivity for semiconductors lies in between 10^5 to 10^{-6}Sm^{-1} and of resistivity between 10^{-5} to $10^6 \Omega\text{m}$.

2. ENERGY BANDS OF SOLIDS OR BAND THEORY OF SOLIDS

2.1 Valence Band

This band contains valence electrons. This band may be partially or completely filled with electrons. This band is never empty. Electrons in this band do not contribute to electric current.

2.2 Conduction Band

In this band, electrons are rarely present. This band is either empty or partially filled. Electrons in the conduction band are known as free electrons. These electrons contribute to the electric current.

2.3 Forbidden Energy Gap or Forbidden Band

The energy gap between the valence band and conduction band is known as forbidden energy gap or

forbidden band. No electrons are present in this gap. It is a measure of energy band gap.

1. The minimum energy required for shifting electrons from valence band to conduction band is known as energy band gap.
2. If λ is the wavelength of radiation used in shifting the electron from valence band to conduction band, then energy band gap is,

$$E_g = h\nu = \frac{hc}{\lambda}$$

Where h is called Planck's constant and c is the speed of light.

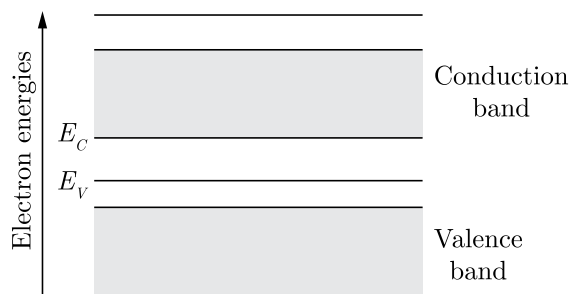
3. The forbidden energy gap E_g in a semiconductor depends upon temperature.
4. Fermi energy is the maximum possible energy possessed by free electrons of a material at absolute zero temperature (i.e. 0 K).

3. DIFFERENCES BETWEEN METALS, INSULATORS AND SEMICONDUCTORS ON THE BASIS OF BAND THEORY

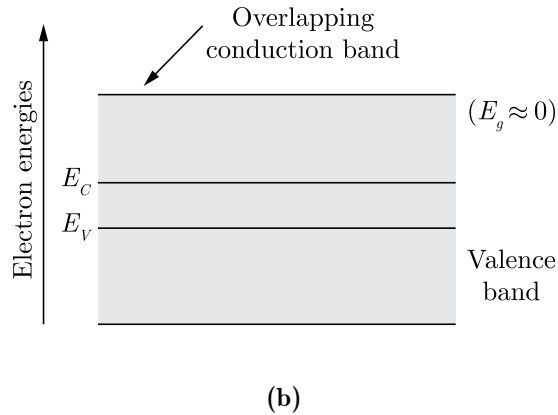
3.1 Metals

In metals either the conduction band is partially filled or conduction band and valence band partially overlap each other.

In metals, there is no forbidden energy gap between the valence and conduction bands.



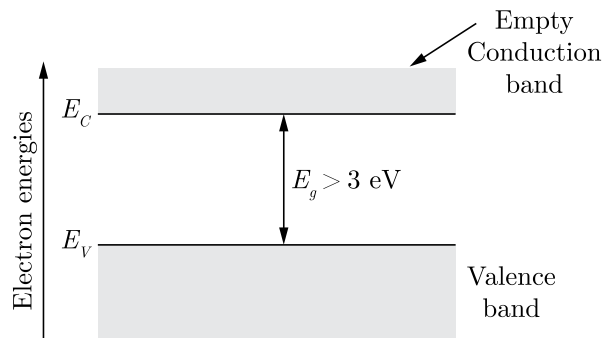
(a)



3.2 Insulators

In insulators, valence band is completely filled and conduction band is completely empty.

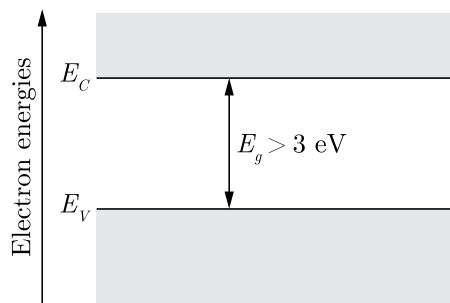
In insulators, there is a very wide forbidden energy gap between the valence and conduction bands. It is of the order of 5 eV or more.



3.3 Semiconductors

In semiconductors, valence band is completely filled and the conduction band is empty.

In semiconductors, there is a small forbidden energy gap between the valence and the conduction bands. It is of the order of 1 eV. For silicon, it is 1.1 eV and for germanium it is 0.72 eV.



At absolute zero, semiconductors behave as a perfect insulator.

Hole : It is a seat of positive charge which is produced when an electron breaks away from a covalent bond in a semiconductor. Hole has a positive charge equal to that of electron. Mobility of hole is smaller than that of electron.

4. INTRINSIC SEMICONDUCTOR

A pure semiconductor which is free from every impurity is known as intrinsic semiconductor. Germanium (Ge) and silicon (Si) are the important examples of intrinsic semiconductors.

1. In intrinsic semiconductor $n_e = n_h = n_i$ where n_e , n_h are number density of electrons in conduction band and number density of holes in valence band, n_i is the intrinsic carrier concentration.
2. When an electric field is applied across an intrinsic semiconductor, electrons and holes move in opposite directions so that total current (I) through the pure semiconductor is given by

$$I = I_e + I_h$$

Where I_e is the free electron current and I_h is the hole current.

3. **Effect of Temperature on Conductivity of Intrinsic Semiconductor :** An intrinsic semiconductor will behave as a perfect insulator at absolute zero. With increasing temperature, the density of hole-electron pairs increases and hence the conductivity of an intrinsic semiconductor increases with increase in temperature. In other words, the resistivity (inverse of conductivity) decreases as the temperature increases.

The semiconductors have negative temperature coefficient of resistance.

5. DOPING

It is a process of deliberate addition of a desirable impurity to a pure semiconductor in order to increase its conductivity. The impurity atoms added are known as dopants.

6. EXTRINSIC SEMICONDUCTOR

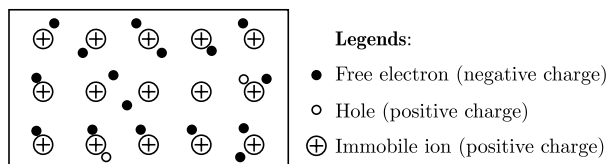
A doped semiconductor is known as extrinsic semiconductor. Extrinsic semiconductors are of two types:

6.1 n-Type Semiconductor

1. When a pure semiconductor of Si or Ge (tetravalent) is doped with a group V pentavalent impurities like arsenic (As), antimony (Sb), phosphorus (P) etc., we obtain a n -type semiconductor. The pentavalent impurity are known as donor atoms.
2. It is called n -type semiconductor because the conduction of electricity in such semiconductor is

due to motion of electrons i.e. negative charges.

- It is called donor type semiconductor, because the doped impurity atom donates one free electron to semiconductor for conduction.
- In n -type semiconductor electrons are majority carriers and holes are minority carriers.
- The representation of n -type semiconductor is as shown in the figure.



- n -type semiconductor is neutral.
- In n -type semiconductor,

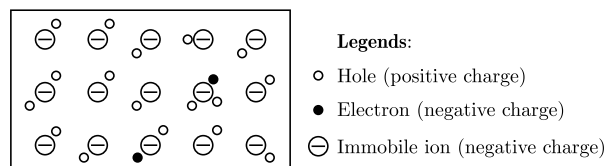
$$n_e \approx N_d \gg n_h$$

where, N_d is the density of donor atoms.

6.2 p -Type Semiconductor

When a pure semiconductor of Si or Ge (tetravalent) is doped with a group III trivalent impurities like aluminium (Al), boron (B), indium (In) etc., we obtain a p -type semiconductor. The trivalent impurity atoms are known as acceptor atoms.

- It is called p -type because the conduction of electricity in such semiconductor is due to motion of holes i.e. positive charges.
- It is called acceptor type semiconductor because doped impurity atom creates a hole in semiconductor which accepts the electron, resulting conduction in p -type semiconductor.
- In p -type semiconductor, holes are majority carriers and electrons are minority carriers.
- The representation of p -type semiconductor is as shown in the figure.



- p -type semiconductor is neutral.
- In p -type semiconductor

$$n_h \approx N_a \gg n_e$$

where, N_a is the density of acceptor atoms.

7. MASS-ACTION LAW

Under thermal equilibrium, the product of the free negative and positive concentrations is a constant

independent of the amount of donor and acceptor impurity doping. This relationship is known as the mass action law and is given by

$$n_e n_h = n_i^2$$

Where, n_e , n_h are number density of electrons and holes respectively and n_i is the intrinsic carriers concentration.

Electrical Conductivity in Semiconductor

The conductivity of the semiconductor is given by $\sigma = e(n_e \mu_e + n_h \mu_h)$ where μ_e and μ_h are the electron and hole densities, e is electronic charge.

- The conductivity of an intrinsic semiconductor is, $\sigma_i = n_i e(\mu_e + \mu_h)$
- The conductivity of n -type semiconductor is, $\sigma_n = eN_d \mu_e$.
- The conductivity of p -type semiconductor is, $\sigma_p = eN_a \mu_h$.

8. p - n JUNCTION

When donor impurities are introduced into one side and acceptors introduced into one side and acceptors into the other side of a single crystal of an intrinsic semiconductor, a p - n junction is formed. It is also known as junction diode. The most important characteristic of a p - n junction is its ability to conduct current in one direction only. In the other (reverse) direction it offers very high resistance. It is symbolically represented by



8.1 Depletion Region

In the vicinity of junction, the region containing the uncompensated acceptor and donor ions is known as depletion region. There is a depletion of mobile charges (holes and free electrons) in this region. Since this region has immobile (fixed) ions which are electrically charged it is also known as the space charge region. The electric field between the acceptor and the donor ions is known as a barrier. The physical distance from one side of the barrier to the other is known as the width of the barrier. The difference of potential from one side of the barrier to the other side is known as the height of the barrier.

- For a silicon p - n junction, the barrier potential is about 0.7 V, whereas for a germanium p - n junction it is approximately 0.3 V.
- The width of the depletion layer and magnitude of potential barrier depend upon the nature of the material of semiconductor and the concentration

OBJECTIVE QUESTIONS

1. In an extrinsic semiconductor, the number density of holes is $4 \times 10^{20} \text{ m}^{-3}$. If the number density of intrinsic carriers is $1.2 \times 10^{25} \text{ m}^{-3}$. The number density of electron in it is

- (a) $1.8 \times 10^9 \text{ m}^{-3}$ (b) $2.4 \times 10^{10} \text{ m}^{-3}$
(c) $3.6 \times 10^9 \text{ m}^{-3}$ (d) $3.2 \times 10^{10} \text{ m}^{-3}$

Ans :

OD 2023

The electron and hole concentration in a semiconductor in thermal equilibrium is given by,

$$\begin{aligned} n_e n_h &= n_i^2 \\ n_e &= \frac{n_i^2}{n_h} = \frac{(1.2 \times 10^{25})^2}{4 \times 10^{20}} \\ &= 0.36 \times 10^{10} \\ &= 3.6 \times 10^9 \text{ m}^{-3} \end{aligned}$$

Thus (c) is correct option.

2. The formation of depletion region in a $p-n$ junction diode is due to

- (a) movement of dopant atoms
(b) diffusion of both electrons and holes
(c) drift of electrons only
(d) drift of holes only

Ans :

OD 2023

The depletion layer in the P-N junction region is caused by diffusion of carriers.

Thus (b) is correct option.

3. A semiconductor is cooled from $T_1 \text{ K}$ to $T_2 \text{ K}$, then its resistance will

- (a) increase
(b) decrease
(c) remain constant
(d) first decrease then increase

Ans :

Foreign 2018, OD 2015

Actually resistivity depends upon the temperature in such way:

$$\rho = \rho_0 [1 + \alpha (T_f - T_i)]$$

where,

α = Coefficient of resistivity

In case of semiconductor ' α ' be negative. Hence if we increases the temperature then resistivity will be decreases, it means resistance will be decreases. Here, semiconductor is cooled, means temperature decreases then, resistivity increases and hence, resistance increases.

Thus (a) is correct option.

4. For n -type Germanium, impurity doped in Germanium is

- (a) Trivalent (b) Tetravalent
(c) Pentavalent (d) None of these

Ans :

Delhi 2017

n -type semiconductor is obtained by doping the tetravalent semiconductor Si (or Ge) with pentavalent impurities such as As, P or Sb group V of the periodic table.

Thus (c) is correct option.

5. Diode is used as

- (a) An amplifier (b) An oscillator
(c) A modulator (d) A rectifier

Ans :

OD 2017

Diode is used as rectifier. Rectifier converts alternating current (AC) into direct current (DC).

Thus (d) is correct option.

6. The majority current-carrier in p -type semiconductor is

- (a) electron (b) hole
(c) photon (d) proton

Ans :

SQP 2016

In the p -type semiconductor hole are the majority charge carriers and electrons are the minority charge carriers.

Thus (b) is correct option.

7. The state of energy of the valence electrons, when the temperature is raised or when electric field is applied, is called

- (a) valence band (b) conduction band
(c) forbidden band (d) none of these

Ans :

Foreign 2010

State of energy of the valence electrons, when the temperature is raised or electric field is applied, is called conduction band.

An increase in the conduction band increases conductivity of the metal.

Thus (b) is correct option.

8. The impurity atom that should be added to the germanium to make it an n -type semiconductor is

- (a) iodine (b) indium
(c) arsenic (d) aluminium

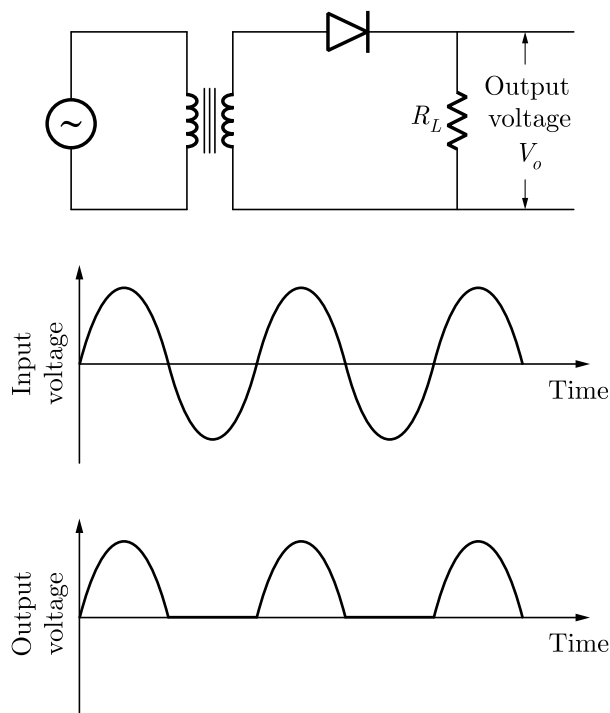
Ans :

Delhi 2015

Arsenic is a pentavalent impurity atom. Therefore when it is added to tetravalent germanium crystal, we obtain n -type semiconductor.

Thus (c) is correct option.

pulsating DC. This action is known as half-wave rectification.



10.3 Ripple Factor

The ripple factor is a measure of purity of the DC output of a rectifier and is defined as

$$r = \frac{\text{rms value of the components of wave}}{\text{average or dc value}}$$

$$= \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1}$$

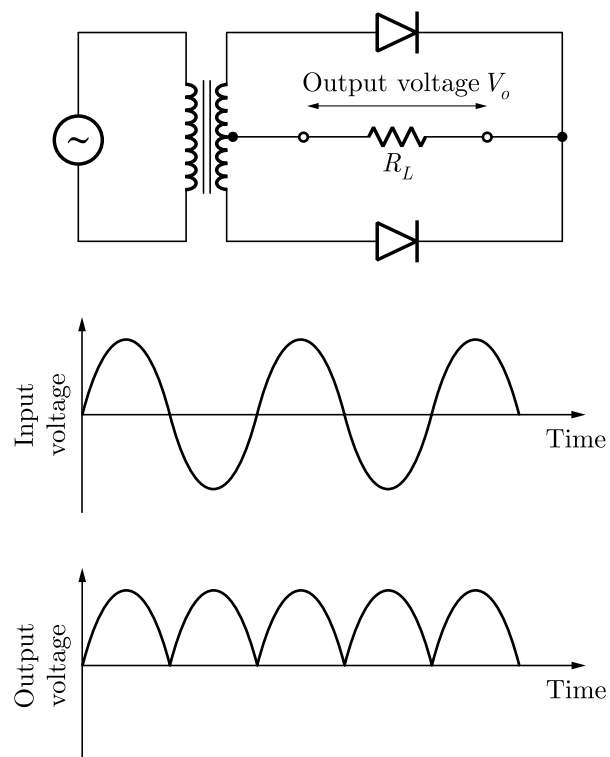
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10.2 Full Wave Rectifier

The circuit diagram, input and output waveforms for a full wave rectifier are as shown in the figure.



of impurity atoms. The thickness of the depletion region is of the order of one tenth of a micrometre.

8.2 Forward Biasing of a p - n Junction

When the positive terminal of external battery is connected to p -side and negative to n -side of p - n junction, then the p - n junction is said to be forward biased.

1. In forward biasing, the width of the depletion region decreases and barrier height reduces.
2. The resistance of the p - n junction becomes low in forward biasing.

8.3 Reverse Biasing of a p - n Junction

When the positive terminal of the external battery is connected to n -side and the negative terminal to p -side of a p - n Junction, then the p - n junction is said to be reverse biased.

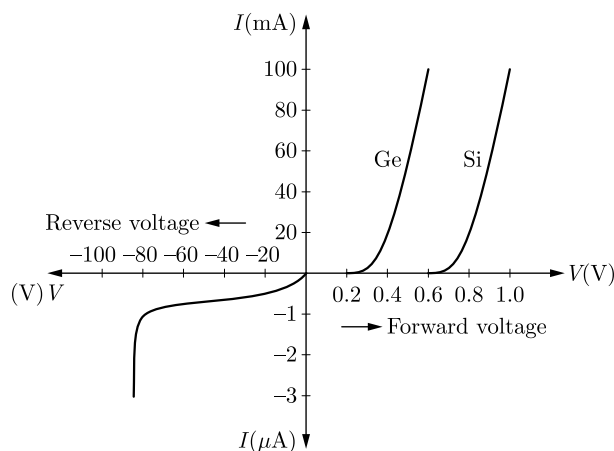
1. In reverse biasing, the width of the depletion region increases and barrier height increases.
2. The resistance of the p - n junction becomes high in reverse biasing.

8.4 Breakdown Voltage

A very small current flows through p - n junction, when it is reverse biased. The flow of the current is due to the movement of minority charge carriers. The reverse current is almost independent of the applied voltage. However, if the reverse bias voltage is continuously increased, for a certain reverse voltage, the current through the p - n junction will increase abruptly. This reverse bias voltage is thus known as breakdown voltage. There can be two different causes for the break down. One is known as zener breakdown and the other is known as avalanche breakdown.

I - V Characteristics of a p - n Junction

The I - V characteristics of a p - n junction do not obey Ohm's law. The I - V Characteristics of a p - n junction are as shown in the figure.

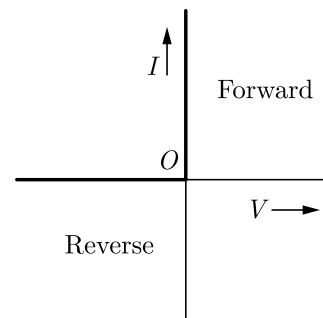


1. **Knee Voltage :** In forward biasing, the voltage at which the current starts to increase rapidly is known as cut-in or knee voltage. For germanium it is 0.3 V while for silicon it is 0.7 V.
2. **Dynamic Resistance :** It is defined as the ratio of a small change in voltage (ΔV) applied across the p - n junction to a small change in current ΔI through the junction.

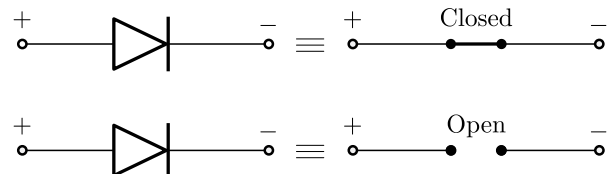
$$r_d = \frac{\Delta V}{\Delta I}$$

9. IDEAL DIODE

A diode permits only unidirectional conduction. It conducts well in the forward direction and poorly in the reverse direction. It would have been ideal if a diode acts as a perfect conductor (with zero voltage across it) when it is forward biased, and as a perfect insulator (with no current-flows through it) when it is reverse biased. The I - V characteristics of an ideal diode as shown in figure.



1. An ideal diode acts like an automatic switch.
2. In forward bias, it acts as a closed switch whereas in reverse bias it acts as an open switch as shown in the figure.



10. RECTIFIER

It is a device which converts AC voltage to DC voltage. Diode is used as a rectifier. Rectifier is based on the fact that, a forward bias p - n junction conducts and a reverse bias p - n junction does not conduct.

10.1 Half Wave Rectifier

Diode conducts corresponding to positive half cycle and does not conduct during negative half cycle, Hence, AC is converted by diode into unidirectional

9. With the increase of temperature, width of the forbidden gap
 (a) decreases (b) increases
 (c) remains same (d) becomes zero

Ans :

OD 2010

With the increase of temperature, the excitation of charge carriers (i.e., electrons) increases. It increases the concentration of charge carriers and conductivity. As a result of this, there is a small decrease in the width of forbidden gap.

Thus (a) is correct option.

10. A pure semiconductor behaves slightly as a conductor at
 (a) low temperature (b) high temperature
 (c) room temperature (d) both (a) and (b)

Ans :

Delhi 2008

At room temperature, a few electrons in valence band acquire energy greater than the forbidden energy gap and jump to the conduction band.

Therefore at room temperature, a pure semiconductor behaves slightly as a conductor.

Thus (c) is correct option.

11. When the conductivity of a semiconductor is only due to breaking of covalent bonds, the semiconductor is called
 (a) extrinsic (b) intrinsic
 (c) *n*-type (d) *p*-type

Ans :

SQP 2014, Comp 2009

In an intrinsic or pure semiconductor, the charge carriers (i.e., electrons and holes) are created due to breaking of covalent bonds.

Therefore conductivity of an intrinsic semiconductor is only due to breaking of covalent bonds.

Thus (b) is correct option.

12. The *n*-type semiconductors are obtained, when germanium is doped with
 (a) arsenic (b) phosphorus
 (c) antimony (d) any one of these

Ans :

Foreign 2012

Arsenic, phosphorous and antimony are pentavalent impurity atoms.

Therefore *n*-type semiconductors are obtained when tetravalent germanium atom is doped with any one of them.

Thus (d) is correct option.

13. In some substances, charge can flow at ordinary temperature, but not at very low temperatures. These substances are called
 (a) conductors (b) insulators
 (c) dielectrics (d) semiconductors

Ans :

OD 2009

In semiconductor, charge can flow at ordinary temperature. And its resistance increases with the decrease in temperature. Therefore at absolute zero temperature, a semiconductor behaves as an insulator. Thus (d) is correct option.

14. At 0 K temperature, a *p*-type semiconductor
 (a) does not have any charge carrier
 (b) has few holes and few free electrons
 (c) has few holes but no free electron
 (d) has equal no. of holes and free electrons

Ans :

Foreign 2005

At 0 K temperature, a *p*-type semiconductor behaves as an insulator, because it has a few holes in its valence band. But there is no free electrons in this state.

Thus (c) is correct option.

15. To obtain a *p*-type semiconductor, germanium atom must be doped with
 (a) arsenic (b) antimony
 (c) indium (d) phosphorus

Ans :

SQP 2008

To obtain a *p*-type semiconductor, germanium atom must be doped with a trivalent impurity atom.

Since indium is a trivalent impurity atom, therefore when tetravalent germanium is doped with trivalent impurity, we obtain a *p*-type semiconductor.

Thus (c) is correct option.

16. When the *p*-end of the *p-n* junction is connected to the negative terminal and the *n*-end to the positive terminal of the battery, then *p-n* junction behaves like a/an
 (a) insulator (b) conductor
 (c) semiconductor (d) superconductor

Ans :

Delhi 2016

When *p*-end of *p-n* junction is connected to the negative terminal and *n*-end to the positive terminal of the battery, then *p-n* junction is reverse biased, which offers high resistance. Therefore *p-n* junction behaves like an insulator.

Thus (a) is correct option.

17. To a germanium sample, traces of gallium are added as an impurity. The resultant sample would behave like a/an

- (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

In a transistor the base is made thin so that base current remains small and we can get output or collector current. We know that

$$I_c = I_e - I_b$$

Reason is incorrect.

Thus (c) is correct option.

- 32. Assertion :** The number of electrons in a *p*-type silicon semiconductor is less than the number of electrons in a pure silicon semiconductor at room temperature.

Reason : It is due to law of mass action.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

$$n_e n_h = h_i^2$$

This formula is based on law of mass action. In *p*-type semiconductor $n_h > n_i$. So, $m_e < n_i$.

Thus (a) is correct option.

- 33. Assertion :** In a common emitter transmitter amplifier the input current is much less than the out put current.

Reason : The common emitter transistor amplifier has very high input impedance.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

In an amplifier output current is always more than input current. Amplifier has low input impedance.

Thus (c) is correct option.

- 34. Assertion :** In common base configuration. The current gain of the transistor is less than unity.

Reason : The collector terminal is reverse biased for amplification.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.

- (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

For common base, Input is I_c and output is I_e .

Current gain = $\frac{I_c}{I_e} = \frac{I_c}{I_c + I_b}$ which is less than unity.

Collector terminal is reversed biased to increase the collector current. Both Assertion and Reason are correct but they are uncorrelated statements.

Thus (b) is correct option.

- 35. Assertion :** The logic gate NOT can be built using diode.

Reason : The output voltage and the input voltage of the diode have 180° phase difference.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

The diode is unidirectional it allows current to pass through it in a particular direction. It does not change the phase of input signal.

Thus (d) is correct option.

- 36. Assertion :** If the temperature of a semiconductor is increased then it's resistance decreases.

Reason : The energy gap between conduction band and valence band is very small.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

In semiconductors the energy gap between conduction band and valence band is small ($\approx 1\text{eV}$). Due to temperature rise, electron in the valence band gain thermal energy and may jump across the small energy gap, (to the conduction band). Thus conductivity increases and hence resistance decreases.

Thus (a) is correct option.

- 37. Assertion :** NAND or NOR gates are called digital building blocks.

Reason : The repeated use of NAND (or NOR) gates can produce all the basis or complicated gates.

Ans :

Delhi 2013

Rectifier is a device which converts an alternating current (or voltage) into direct current (or voltage).

The process of converting alternating current into direct current is called rectification.

Thus (c) is correct option.

26. A p - n junction is used as a/an

- (a) rectifier (b) amplifier
(c) oscillator (d) modulator

Ans :

Foreign 2012

Rectifier is a device which converts an alternating current (or voltage) into direct current (or voltage).

We also know that when an alternating e.m.f. signal is applied across a p - n junction, it permits large current during forward bias, and no current during reverse bias. That is why, a p - n junction is used as rectifier.

Thus (a) is correct option.

27. In comparison to a half wave rectifier, the full wave rectifier gives lower

- (a) efficiency (b) average current
(c) average output voltage (d) none of these

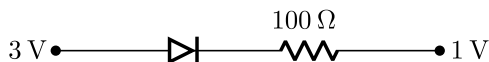
Ans :

SQP 2009

Efficiency, average current as well as average output voltage in case of full wave rectifier are higher as compared to half wave rectifier.

Thus (d) is correct option.

28. Assuming that the p - n junction diode shown in the figure is ideal, the current through the diode is



- (a) 0.02 A (b) 2 A
(c) 0.2 A (d) 20 A

Ans :

OD 2003

Potential at one end, $V_1 = 3$ Volt

Potential at other end, $V_2 = 1$ Volt

External resistance, $R = 100 \Omega$

Change in potentials,

$$\begin{aligned}\Delta V &= V_1 - V_2 \\ &= 3 - 1 = 2 \text{ Volt}\end{aligned}$$

Since the p - n junction diode offers zero resistance when it is ideal, therefore current through the diode,

$$\begin{aligned}I &= \frac{\Delta V}{R} \\ &= \frac{2}{100} = 0.02 \text{ A}\end{aligned}$$

Thus (a) is correct option.

ASSERTION AND REASON

29. **Assertion :** The resistivity of a semi-conductor increases with temperature.

Reason : The atoms of semi-conductor vibrate with larger amplitude as higher temperatures thereby increasing its resistivity.

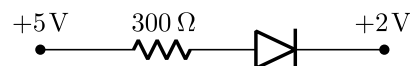
- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

The resistivity of semiconductor decreases with increase in temperature as more electrons jump into conduction band increasing its conductivity.

Thus (d) is correct option.

30. **Assertion :** The value of current through p - n junction in the given figure will be 10 mA.



Reason : In the above figure, p -side is at higher potential than n -side.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) The Assertion is correct but Reason is incorrect.
(d) Both the Assertion and Reason are incorrect.

Ans :

The p -side of the junction diode is at a higher potential than the n -side. So p - n junction is forward biased. Hence a current flows through it and is given by

$$\begin{aligned}I &= \frac{V}{R} = \frac{5 - 2}{300} \\ &= 10^{-2} \text{ A} = 10 \text{ mA}\end{aligned}$$

Thus (b) is correct option.

31. **Assertion :** In a transition the base is made thin.

Reason : A thin base makes the transistor stable.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.

- (a) conductor
- (b) insulator
- (c) *p*-type semiconductor
- (d) *n*-type semiconductor

Ans :**SQP 2010**

Gallium is a trivalent impurity atom. Therefore when the traces of gallium are added to a germanium sample as an impurity, we obtain a *p*-type semiconductor.

Thus (c) is correct option.

- 18.** Which of the following, when added as an impurity atom in the silicon, produces *n*-type semiconductor ?
- (a) phosphorous
 - (b) aluminium
 - (c) magnesium
 - (d) both (b) and (c)

Ans :**Delhi 2014**

Phosphorous is a pentavalent impurity atom. Therefore, when tetravalent silicon atom is doped with a pentavalent impurity atom, it produces an *n*-type semiconductor.

Thus (a) is correct option.

- 19.** When germanium atoms is doped with equal no. of antimony atoms, we get a/an
- (a) superconductor
 - (b) *p*-type semiconductor
 - (c) *n*-type semiconductor
 - (d) intrinsic semiconductor

Ans :**OD 2006**

Antimony is a pentavalent impurity atom. Therefore when it is doped with an equal no. of tetravalent germanium atoms, we get an *n*-type semiconductor.

Thus (c) is correct option.

- 20.** The potential barrier, in the depletion layer, is due to
- (a) ions
 - (b) holes
 - (c) electrons
 - (d) forbidden band

Ans :**Foreign 2001**

The recombination of free and mobile electrons and holes produce the narrow region at the junction called depletion layer.

It is so named because only positive ions are present there which are not free to move.

Thus (a) is correct option.

- 21.** In a *p-n* junction photo cell, the value of the photo-electromotive force, produced by monochromatic light is proportional to
- (a) voltage applied at the *p-n* junction
 - (b) intensity of light falling on the cell

- (c) barrier voltage at the *p-n* junction
- (d) frequency of light falling on the cell

Ans :

In a *p-n* junction photo cell, the value of photo-electromotive force produced by monochromatic light is proportional to the intensity of light falling on the cell.

Thus (b) is correct option.

- 22.** The region with free electrons and holes in a *p-n* junction diode is
- (a) *p*-region
 - (b) *n*-region
 - (c) junction
 - (d) depletion region

Ans :**Delhi 2007**

In a *p-n* junction diode, excess electrons from the *n*-region cross over to the *p*-region across the junction, which are neutralised by the holes and give a chargeless region called depletion region.

Thus (d) is correct option.

- 23.** In a *p-n* junction diode, the holes are due to
- (a) protons
 - (b) neutrons
 - (c) extra electrons
 - (d) missing of electrons

Ans :**Foreign 2017, SQP 2013**

In a *p-n* junction when an electron leaves the *n*-region, a positive charge is left behind it, and a hole is created.

Thus holes are formed due to missing of electrons.

Thus (d) is correct option.

- 24.** The peak voltage in the output of a half wave diode rectifier fed with a sinusoidal signal without filter is 10 V. The DC component of the output voltage is
- (a) $\frac{20}{\pi}$ V
 - (b) $\frac{10}{\sqrt{2}}$ V
 - (c) $\frac{10}{\pi}$ V
 - (d) 10 V

Ans :**SQP 2001**

Peak voltage in the output of a half wave diode rectifier,

$$V_0 = 10 \text{ Volt}$$

DC component of the output voltage,

$$V_{dc} = \frac{I_0}{\pi} = \frac{10}{\pi} \text{ Volt}$$

Thus (c) is correct option.

- 25.** A rectifier converts
- (a) light energy into electrical energy
 - (b) high frequency into low frequency
 - (c) alternating current into direct current
 - (d) mechanical energy into electric energy

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

These gates are called digital building blocks because using these gates only (either NAND or NOR) we can compile all other gates also (like OR, AND, NOT, XOR).

Thus (a) is correct option.

- 38. Assertion :** A transistor amplifier in common emitter configuration has a low input impedance.

Reason : The base to emitter region is forward biased.

- (a) Both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
 (b) Both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
 (c) The Assertion is correct but Reason is incorrect.
 (d) Both the Assertion and Reason are incorrect.

Ans :

Input impedance of common emitter configuration

$$= \left| \frac{\Delta V_{BE}}{\Delta i_B} \right|_{V_{CE} = \text{constant}}$$

where ΔV_{BE} = voltage across base and emitter (base emitter region is forward biased)

Δi_B = base current which is order of few microampere.

Thus (a) is correct option.

VERY SHORT ANSWER QUESTIONS

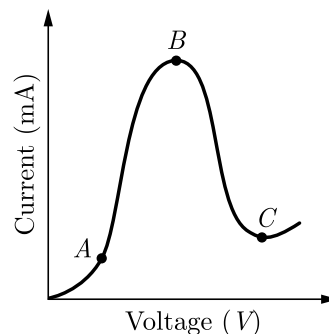
- 39.** What happens to the width of depletion layer of a $p-n$ junction when it is (i) forward biased, (ii) reverse biased?

Ans :

Delhi 2020

- (i) When forward biased, the width of depletion layer decreases.
 (ii) When reverse biased, the width of depletion layer increases.

- 40.** The graph shown in the figure represents a plot of current versus voltage for a given semiconductor. Identify the region at which the semiconductor has a negative resistance.



Ans :

OD 2021

Resistance of a material can be found out by the slope of the curve V versus I . Part BC of the curve shows the negative resistance as with the increase in current and decrease in voltage.

- 41.** What is doping ?

Ans :

Comp 2021, OD 2003

The process of deliberate addition of a desirable impurity to a pure semiconductor so as to increase its conductivity is called doping. The impurity atoms added are called dopants and the semiconductors doped with the impurity atoms are called extrinsic or doped semiconductor.

- 42.** What happens when a forward bias is applied to a $p-n$ junctions

Ans :

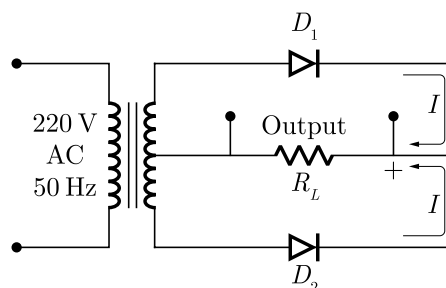
Delhi 2019

When a forward bias is applied to $p-n$ junction, the forward voltage opposes the potential barrier, due to which the potential barrier and the width of the depletion layer decreases. Under the effect of external electric field, hole in the p -region and electrons in the n -region, both move towards the junction and a forward current of milliampere order flows in circuit.

- 43.** Draw the circuit diagram to explain in the working of a full wave $p-n$ junction rectifier.

Ans :

OD 2020

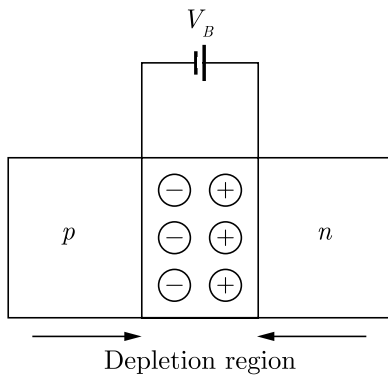


Ans :

Foreign 2016, OD 2011

Potential Barrier : During the formation of a $p-n$ junction, the electrons diffuse from n -region. This forms recombination of charge carriers. In this process, immobile positive ions are collected at a junction toward n -region and negative ions at a junction toward p -region. This causes a potential difference across the unbiased junction. This is called potential barrier.

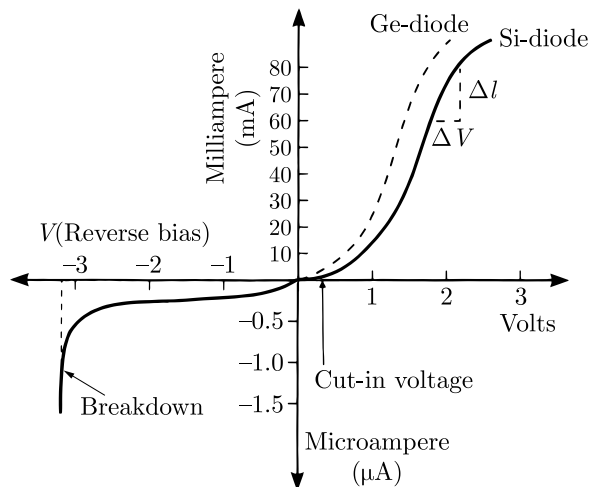
Depletion Region : It is a layer formed near the junction which is devoid of free charge carriers. Its thickness is about $1\mu m$.



52. Draw the voltage-current characteristic curve of a diode and mark its important parameter.

Ans :

OD 2014



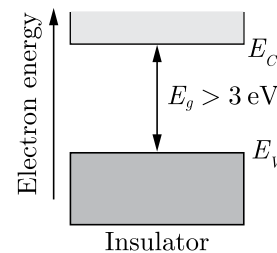
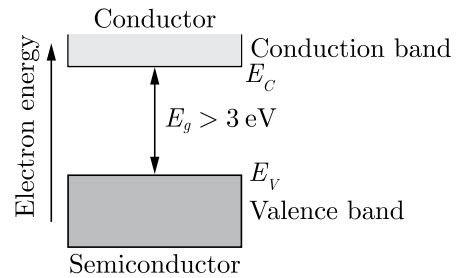
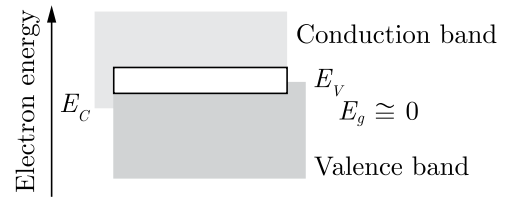
53. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams.

or

Draw the necessary energy band diagrams to distinguish between conductors, semiconductors and insulators. How does the change in temperature affect the behaviour of these materials? Explain briefly.

Ans :

Comp 2014, SQP 2011



Distinguishing Features :

- In conductors :** Valence band and conduction band overlap each other.
In semiconductors Valence band and conduction band are separated by a small energy gap.
In insulators : They are separated by a large energy gap.
- In conductors :** Large number of free electrons are available in conduction band.
In semiconductors A very small number of electrons are available for electrical conduction.
In insulators Conduction band is almost empty i.e., no electron is available for conduction.

Effect of temperature :

- In conductors :** At high temperature, the collision of electrons become more frequent with the atoms/molecules at lattice site in the metals as a result the conductivity decreases (or resistivity increases).
- In semiconductors :** As the temperature of the semiconducting material increases, more electron hole pairs becomes available in the conduction band and valence band and hence the conductivity increases or the resistivity decreases.

	Intrinsic semiconductor	Extrinsic semiconductor
1.	It is a pure semiconductor material with no impurity atoms in it.	It is prepared by doping a small quantity of impurity atoms to the pure semiconductor.
2.	The number of free electrons in the conduction band and the number of holes in valence band is exactly equal.	The number of free electrons and holes is never equal. There is an excess of electrons in n -typed semiconductors and excess of holes in p -type semiconductors.

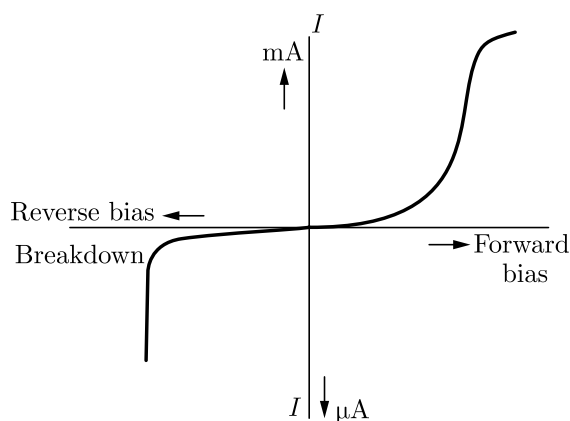
49. Draw $V-I$ characteristics of a $p-n$ junction diode. Answer the following questions, giving reasons:

- Why is the current under reverse bias almost independent of the applied potential upto a critical voltage?
- Why does the reverse current show a sudden increase at the critical voltage?

Name any semiconductor device which operates under the reverse bias in the breakdown region.

Ans :

Delhi 2008



- In the reverse biasing, the current of order of μA is due to movement/drift of minority charge carriers from one region to another through the junction.

A small applied voltage is sufficient to sweep the minority charge carriers through the junction. So, reverse current is almost independent of critical voltage.

- At critical voltage (or breakdown voltage), a large number of covalent bonds break, resulting in the increase of large number of charge carriers. Hence, current increases at critical voltage.

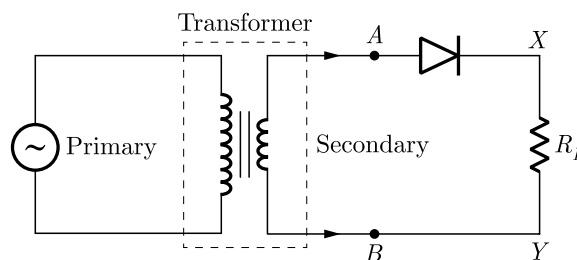
Semiconductor device that is used in reverse biasing, is zener diode.

50. Explain, with the help of a circuit diagram, the working of a $p-n$ junction diode as a half-wave rectifier.

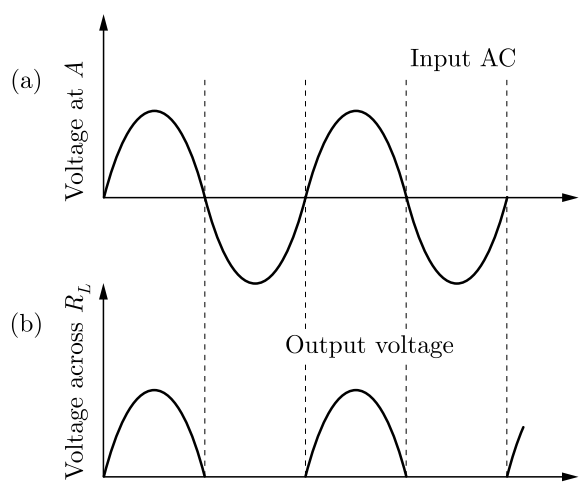
Ans :

Delhi 2017, Comp 2015

- $p-n$ Junction Diode as a Half-Wave Rectifier** : AC voltage to be rectified is connected to the primary coil of a step-down transformer. Secondary coil is connected to the diode through resistors R_L , across which output is obtained.

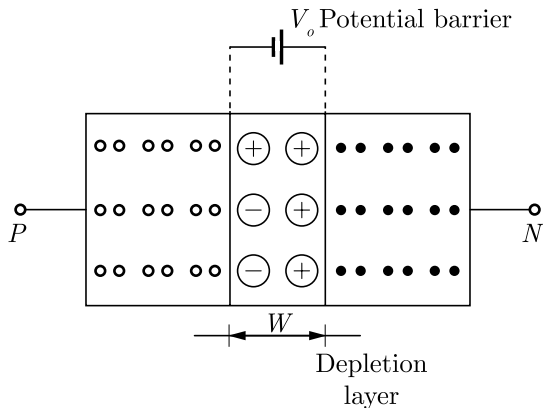
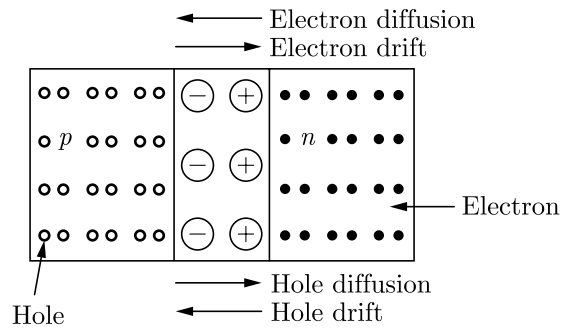


- Working** : During positive half cycle of the input AC, the $p-n$ junction is forward biased. Thus, the resistance in $p-n$ junction becomes low and current flows. Hence, we get output in the load. During negative half cycle of the input AC, the $p-n$ junction is reverse biased. Thus, the resistance of $p-n$ junction is high and current does not flow. Hence, no output in the load. So, for complete cycle of AC, current flows through the load resistance in the same direction.



Input and Output Waveforms

51. Name the important process that occurs during the formation of a $p-n$ junction. Explain briefly, with the help of a suitable diagram, how a $p-n$ junction is formed. Define the term 'barrier potential'.

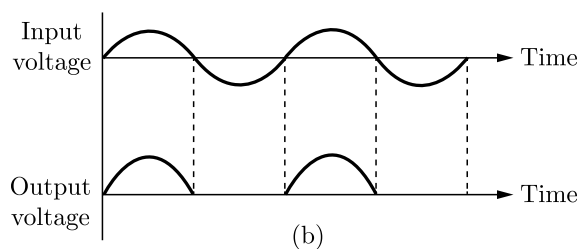
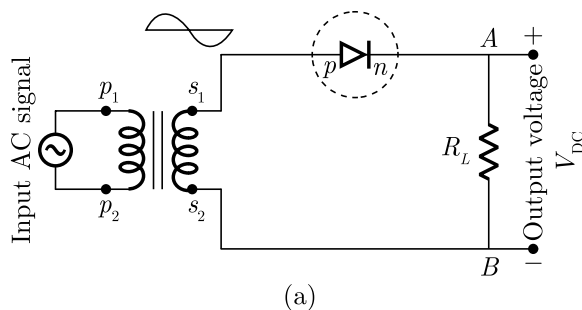


57. Draw the circuit diagram of a half wave rectifier and explain its working.

Ans :

Comp 2016

Half wave rectifier : The circuit diagram for junction diode as half wave rectifier is shown in figure (a).



During first half of the input cycle, the secondary terminal S_1 of transformer be positive relative to S_2 , then the junction diode is forward biased. Therefore, the current flows and its direction of current in load

resistance R_L is from A to B . In next half cycle, the terminal S_1 becomes negative relative to S_2 , then the diode is in reverse bias, therefore no current flows in diode and hence there is no potential difference across load R_L . The cycle repeats. The output current in load flows only when S_1 is positive relative to S_2 . That is during first half cycles of input AC signal, there is a current in circuit and hence a potential difference across load resistance R_L , while no current flows for next half cycle. The direction of current in load is always from A to B which is direct current. Thus, a single $p-n$ junction diode acts as a half wave rectifier.

The input and output waveforms of half wave rectifier are shown in figure (b).

LONG ANSWER QUESTIONS

58. What is rectifier ? How a $p-n$ junction diode can be used in (1) forward biased and (2) reverse biased mode ? Draw a labelled $V-I$ characteristic of $p-n$ junction diode on a graph.

Ans :

OD 2016

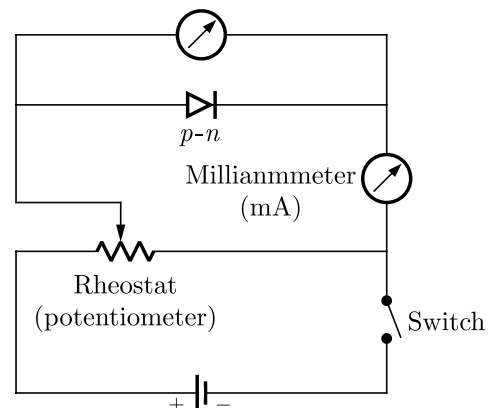
Rectifier

The process of converting alternating current into direct current is called rectification and the device used for this process is called rectifier.

$V-I$ Characteristics of a $p-n$ Junction Diode

A graph showing the variation of current flowing through a $p-n$ junction with the voltage applied across it (both when it is forward and reverse biased) is called the voltage-current or $V-I$ characteristic of $p-n$ junction.

1. **Forward Bias Characteristic :** Figure shows the experimental arrangement for studying the characteristic curve of a $p-n$ junction when it is forward biased.



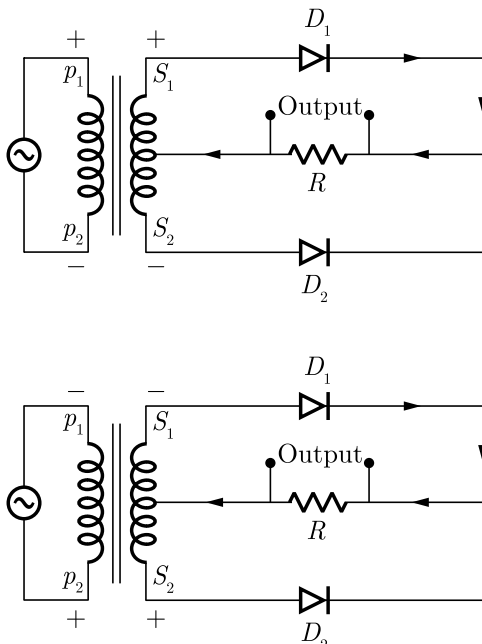
- (c) **In insulators** : The energy band between conduction band and valence band is very large, so it is unsurpassable for small temperature rise. So, there is no change in their behaviour.

54. Draw the circuit diagram of a full-wave rectifier using $p-n$ junction diode. Explain its working and show the output input waveforms.

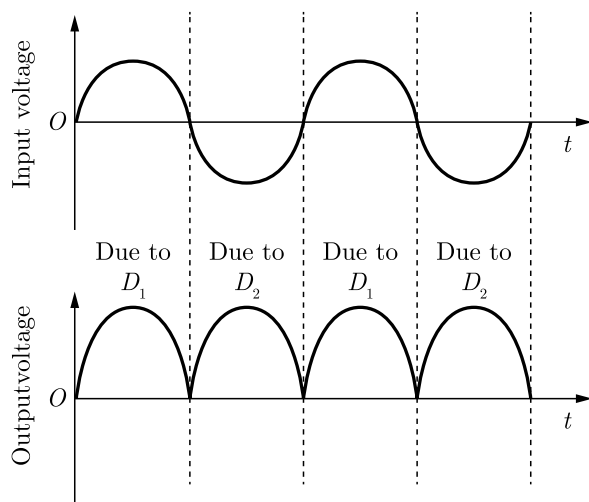
Ans :

OD 2011

The circuit diagram of full-wave rectifier is shown below:

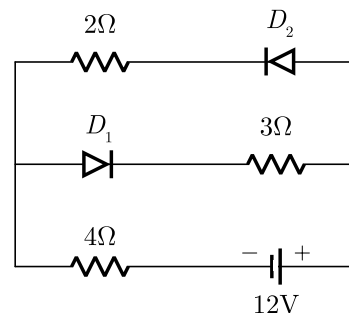


The input and output waveforms have been given below :



Its working based on the principle that junction diode offer very low resistance in forward bias and very high resistance in reverse bias.

55. The circuit shown in the figure has two oppositely connected ideal diodes connected in parallel. Find the current flowing through each diode in the circuit.



Ans :

OD 2013

- (i) Diode D_1 is reverse biased, so it offers an infinite resistance. So, no current flows in the branch of diode D_1 .
- (ii) Diode D_2 is forward biased, and offers no resistance in the circuit. So, current in the branch.

$$I = \frac{V}{R_{eq}} = \frac{12V}{2\Omega + 4\Omega} = 2A$$

56. Describe briefly, with the help of a diagram, the role of the two important process involved in the formation of a $p-n$ junction.

Ans :

OD 2012

Two important processes occurring during the formation of a $p-n$ junction are

- (i) **Diffusion** : In n -type semiconductor, the concentration of electrons is much greater as compared to concentration holes ; while in p -type semiconductor, the concentration of holes is much greater than the concentration of electrons. When a $p-n$ junction is formed, then due to concentration gradient, the holes diffuse from p -side to n -side ($p \rightarrow n$) and electrons diffuse from n -side to p -side ($n \rightarrow p$). This motion of charge carriers gives rise to diffusion current across the junction.
- (ii) **Drift** : The drift of charge carriers occurs due to electric field. Due to built in potential barrier, an electric field directed from n -region to p -region is developed across the junction. This field causes motion of electrons on p -side of the junction to n -side of junction to p -side. Thus, a drift current starts. This current is opposite to the direction of diffusion current.

SHORT ANSWER QUESTIONS

44. Answer the following giving reasons :

- A $p-n$ junction diode is damaged by a strong current.
- Impurities are added in intrinsic semiconductors.

Ans :

OD 2023

- When a strong current passes through the semiconductor it heats up the crystal and covalent bonds are broken. Hence, because of excess number, of free electrons it behaves like a conductor.
- If some impurity is added to intrinsic semiconductor it will increase the concentration or density of charge carriers (either holes or electrons) and which in turn increases its conductivity.

45. Distinguish between n -type and p -type semiconductors.

Ans :

Comp 2015, 13

The difference between n -type and p -type semiconductor are as follows :

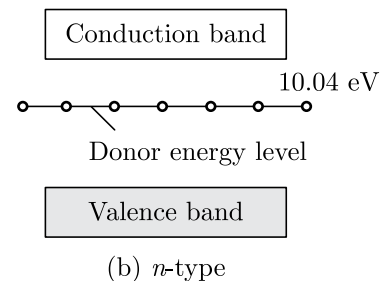
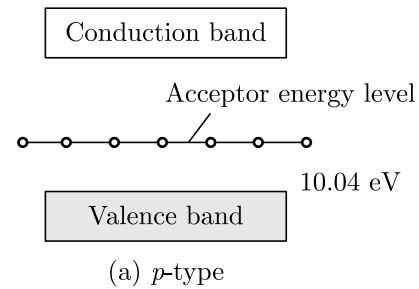
	n -type Semiconductor	p -type Semiconductors
1.	These are extrinsic semiconductors obtained by doping impurity atoms of group V to Ge or Si crystal.	These are extrinsic semiconductors obtained by doping impurity atoms of group III to Ge or Si crystal.
2.	The impurity atoms added provide free electrons and are called donors.	The impurity atoms added create vacancies of electrons (or holes) and are called acceptors
3.	The donor impurity level lies just below the conduction band.	The acceptor impurity level lies just above the valence band.
4.	The electrons are majority charge carriers while holes are minority charge carriers.	The holes are majority charge carriers while electrons are minority charge carriers.
5.	The free electron density is much greater than hole density i.e. $n_e \gg n_h$.	The hole density is much greater than free electron density i.e. $n_h \gg n_e$.

46. Draw energy band diagram of n -typed and p -typed semiconductor at temperature $T > 0$ K. Mark the donor and acceptor energy level with their energies.

Ans :

SQP 2012

The required energy band diagram is shown below.



47. Distinguish between an intrinsic semiconductor and a p -type semiconductor. Give reason why a p -type semiconductor crystal is electrically neutral, although $n_h > n_e$.

Ans :

Comp 2010, Delhi 2001

The difference between intrinsic semiconductor and a p -type semiconductor are as follows :

	Intrinsic semiconductor	p -type semiconductor
(i)	It is a semiconductor in pure form.	It is a semiconductor doped with p -type like Al, in impurity.
(ii)	Intrinsic charge carriers are electrons and holes with equal concentration.	Majority charge carriers are holes and minority charge carriers are electrons.
(iii)	Current due to charge carriers is feeble (of the order of μA).	Current due to charge carriers is significant (of the order of mA).

p -type semiconductor is electrically neutral because every atom, whether it is of pure semiconductor (Ge or Si) or of impurity (Al) is electrically neutral.

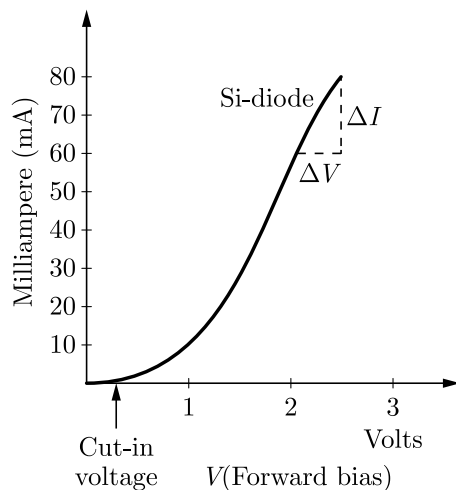
48. Distinguish between 'Intrinsic' and 'Extrinsic' semiconductors?

Ans :

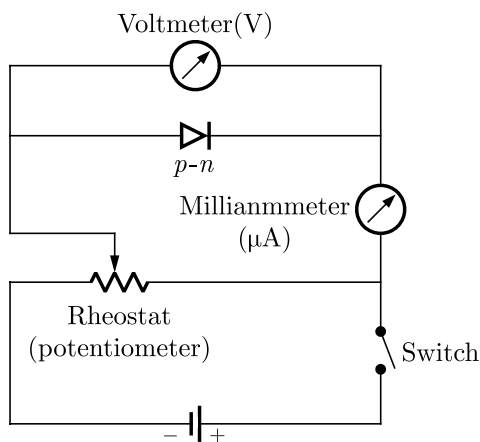
Delhi 2015

The differences between intrinsic and Extrinsic semiconductor are as follows :

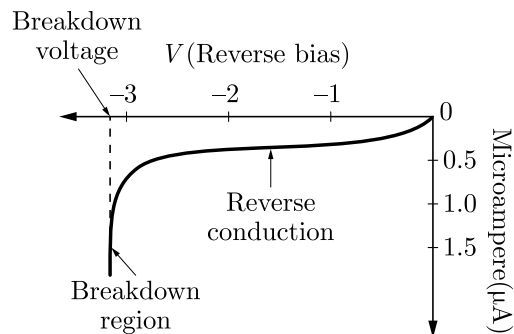
A battery is connected across the p - n junction diode through a potentiometer (or rheostat) so that the voltage applied to the diode can be changed. The milliammeter measures the current through the diode and the voltmeter measures the voltage across the diode. For different values of voltages, the value of current is noted. A graph is plotted between V and I , as shown in Figure. This voltage-current graph is called forward characteristics.



2. **Reverse Bias Characteristic :** Figure shows the experimental arrangement for studying characteristic curve of a p - n junction when it is reverse biased.



Here a micro-ammeter is used to measure the small currents through the reverse biased diode. A V - I graph of the type shown in Figure. It is called reverse characteristic of the junction diode.



59. Explain the band theory of solids. State how on this basis solids are classified into conductors, insulators and semiconductors.

or

What are energy bands in solids ? How are semiconductors, insulators and conductors classified on the basis of band theory?

Ans :

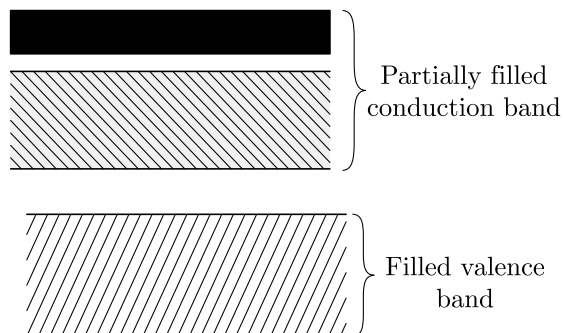
OD 2017, Delhi 2013

Band Theory of Solids

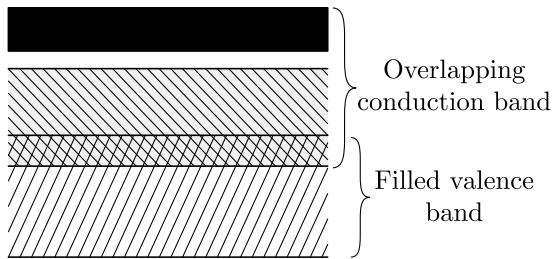
From Bohr's atomic model, we know that the electrons have well defined energy levels in an isolated atom. But due to interatomic interactions in a crystal, the electrons of the outer shells are forced to have energies different from those in isolated atoms. Each energy level splits into a number of energy levels forming a continuous band, called energy band.

Depending on the energy band gap is zero, large or small, the solids may be classified into metals, insulators and semiconductors, as explained below:

- Metals :** Here the last occupied band, called conduction band is partially filled with electrons. Two types of band structures are found in metals:
 - Either there is energy gap between the completely filled valence band and the partially filled conduction band. As shown in figure (i), this band structure is met in alkali metals (Li, Na, K, etc.), noble metals (Cu, Ag, Au) and third group elements like Al, Ga, In and Tl.



(i)

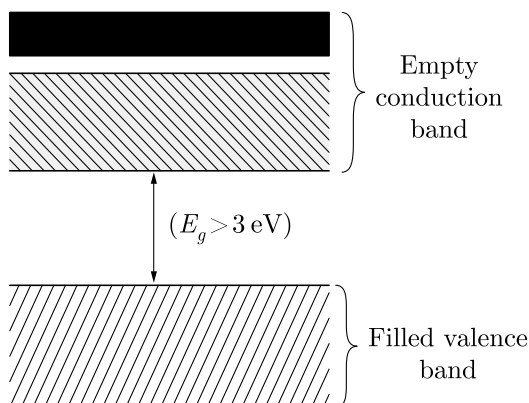


(ii)

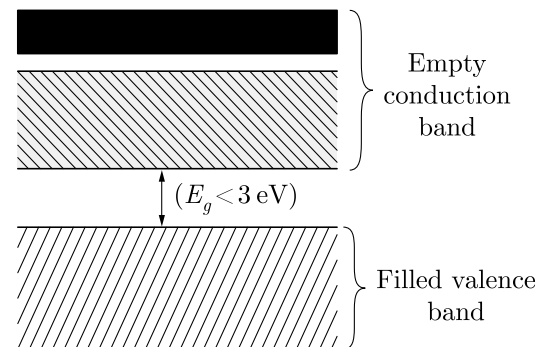
- (b) The conduction and valence band partly overlap. As shown in figure(ii), this band structure is seen in metals like Be ($Z = 4:1s^2 2s^2$), Mg ($Z = 12:1s^2 2s^2 2p^6 3s^2$), Zn etc. Here the valence band is completely filled but the upper unoccupied band ($2p$ with $2s$ in Be, $3p$ with $3s$ in Mg) partly overlaps the valence band.

The highest energy level in the conduction band filled up with electrons at absolute zero is called fermi level and the energy corresponding to the fermi level is called fermi energy. Many electrons after gaining a slight amount of energy from any source get excited to the empty energy levels lying immediately above the fermi level and become free to conduct electricity. This makes available a large number of conduction electrons. So metals have low resistivity or high conductivity. Even a small electric field applied across the metal causes a current flow through it.

2. **Insulators :** In insulators, the valence band is completely filled while the conduction band is empty as shown in Figure (b), there is a larger energy gap ($E_g > 3 \text{ eV}$) between the valence and conduction bands. For example, in case of diamond, $E_g = 6 \text{ eV}$. Even an electric field cannot give this much energy to an electron to make it jump from the valence band into the conduction band. Hence due to the lack of free electrons in the conduction band, the insulators are poor conductors of electricity.



3. **Semiconductors :** At $0K$, the conduction band is empty and the valence band is filled. So the material is essentially insulator at low temperatures. However, the energy gap between conduction and valence bands is small ($E_g < 3 \text{ eV}$). For example, $E_g = 1.17 \text{ eV}$ for Si and $E_g = 0.74 \text{ eV}$ for Ge. At room temperature, some valence electrons acquire enough thermal energy and jump to the conduction band where they are free to conduct electricity (according to Boltzmann law, number of thermally excited electrons $n \propto e^{-E_g/kT}$). Thus the semiconductor acquires a small conductivity at room temperature. The resistance of a semiconductor would not be as high as that of insulator.



60. 1. State briefly the process involved in the formation of $p-n$ junction explaining clearly how the depletion region is formed.
2. Using the necessary circuit diagrams, show how the $V-I$ characteristics of a $p-n$ junction are obtained in:
- Forward biasing
 - Reverse biasing
- How are these characteristics made use of in rectification?

or

Draw the circuit arrangement for studying the $V-I$ characteristics of a $p-n$ junction diode (1) in forward bias and (2) in reverse bias. Draw the typical $V-I$ characteristics of a silicon diode.

Describe briefly the following terms:

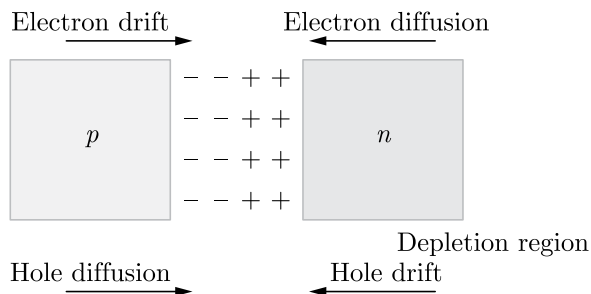
- "minority carrier injection" in forward bias
- "breakdown voltage" in reverse bias.

Ans :

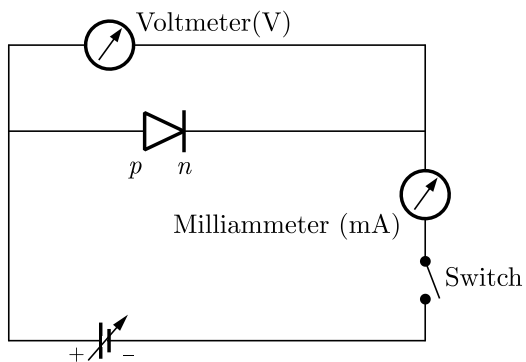
Delhi 2018, OD 2007

- Two processes occur during the formation of a $p-n$ junction are diffusion and drift. Due to the concentration gradient across p and n -sides of the junction, holes diffuse from p -side to n -side ($p \rightarrow n$) and electrons diffuse from n -sides to p -side ($n \rightarrow p$). This movement of charge carriers

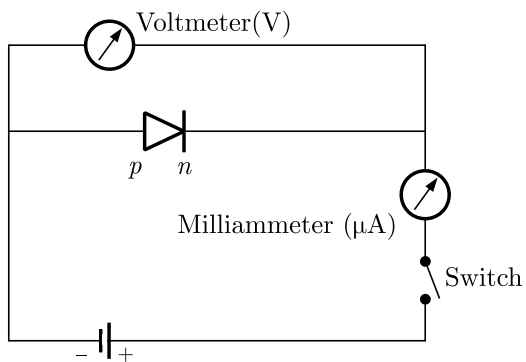
leaves behind ionised acceptors (negative charge ϕ -immobile) on the p -side and donors (positive charge immobile) on the n -side of the junction. This space charge region on either side of the junction together is known as depletion region.



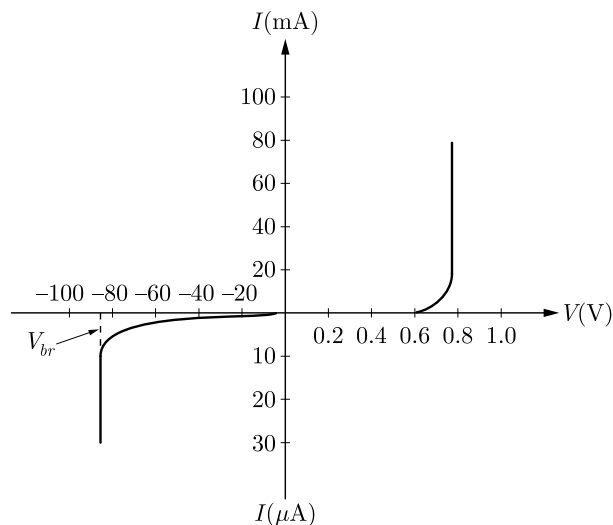
2. The circuit arrangement for studying the $V-I$ characteristics of a diode are shown in figure (a) and (b). For different values of voltages, the value of current is noted. A graph between V and I is obtained as in figure (c). From the $V-I$ characteristic of a junction diode, it is clear that it allows current to pass only when it is forward biased. So, if an alternative voltage is applied across a diode, the current flows only in that part of the cycle when the diode is forward biased. This property is used to rectify alternating voltages.



(a)



(b)



(c)

- Minority Carrier Injection :** Due to the applied voltage, electrons from n -side cross the depletion region and reach p -side (where they are minority carriers). Similarly, holes from p -side cross this junction and reach the n -side (where they are minority carriers). This process under forward bias is known as minority carrier injection.
 - Breakdown Voltage :** It is critical reverse bias voltage at which current is independent of applied voltage.
61. State the principle of working of $p-n$ diode as a rectifier. Explain with the help of a circuit diagram, the use of $p-n$ diode as a full wave rectifier. Draw a sketch of the input and output waveforms.

or

Draw a circuit diagram of a full wave rectifier. Explain the working principle. Draw the input/output waveforms indicating clearly the functions of the two diodes used.

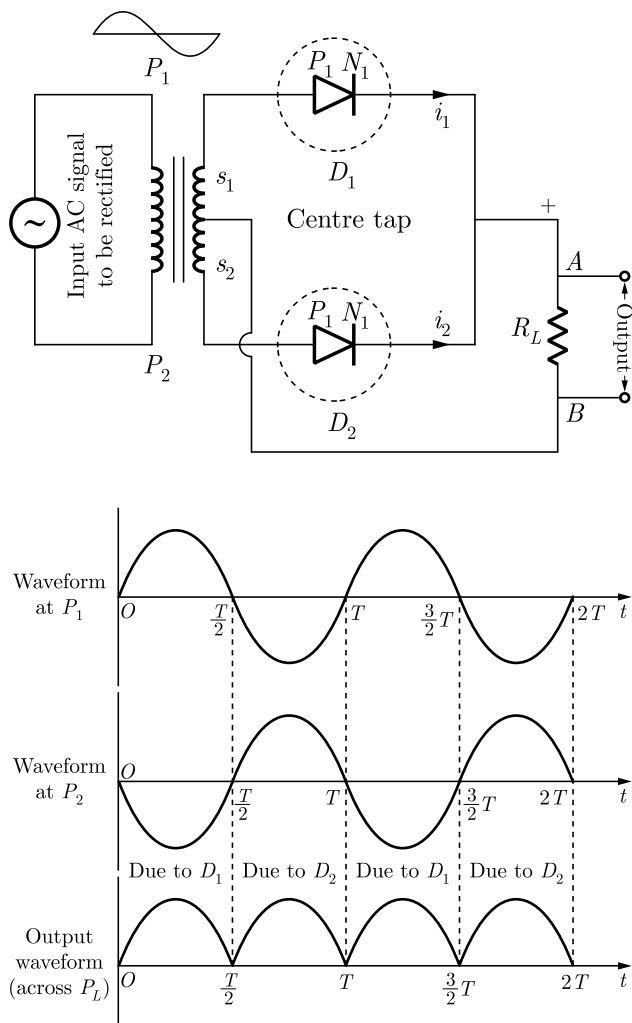
or

With the help of a circuit diagram, explain the working of a junction diode as a full wave rectifier. Draw its input and output waveforms. Which characteristic property makes the junction diode suitable for rectification?

Ans :

OD 2016

Rectification means conversion of AC into DC. A $p-n$ diode acts as a rectifier because an AC changes polarity periodically and a $p-n$ diode allows the current to pass only when it is forward biased. This makes the diode suitable for rectification.



Working

The AC input voltage across secondary s_1 and s_2 changes polarity after each half cycle. Suppose during

the first half cycle of input AC signal, the terminal s_1 is positive relative to centre tap O and s_2 is negative relative to O . Then diode D_1 is forward biased and diode D_2 is reverse biased. Therefore, diode D_1 conducts while diode D_2 does not. The direction of current (i_1) due to diode D_1 in load resistance R_L is direction from A to B . In next half cycle, the terminal s_1 is negative and s_2 is positive relative to centre tap O . The diode D_1 is reverse biased and diode D_2 is forward biased. Therefore, diode D_2 conducts while D_1 does not. The direction of current (i_2) due to diode D_2 in load resistance R_L is still from A to B . Thus, the current in load resistance R_L is in the same direction for both half cycles of input AC voltage. Thus for input AC signal the output current is a continuous series of unidirectional pulses.

In a full wave rectifier, if input frequency is f hertz, then output frequency, will be $2f$ hertz because for each cycle of input, two positive half cycles of output are obtained.

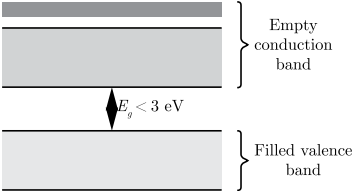
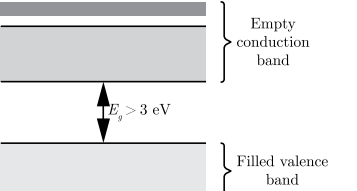
62. What are ‘Energy Bands’? How are these formed? Distinguish between conductors, semiconductors and insulators on the basis of formation of these bands.

Ans : Comp 2021, SQP 2001

Energy Band and its Formation

From Bohr’s atomic model, we know that the electrons have well defined energy levels in an isolated atom. But due to interatomic interactions in a crystal, the electrons of the outer shells are forced to have energies different from those in isolated atoms. Each energy level splits into a number of energy levels forming a continuous band called energy band.

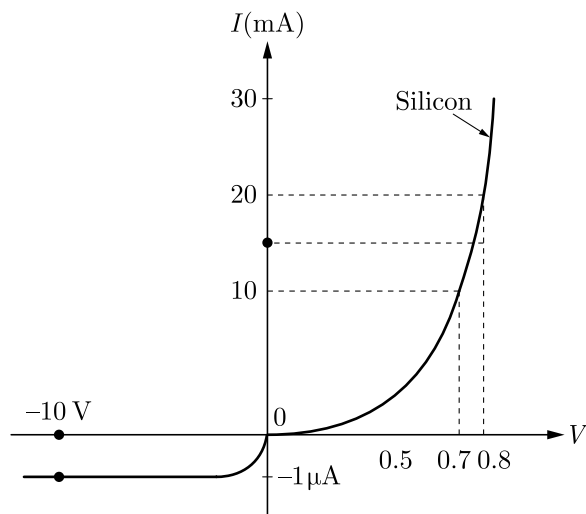
Distinguishing features on the basis of energy band diagrams :

	Conductors	Semiconductors	Insulators
1.	Either the conduction band is partially filled or the valence and conduction bands partly overlap. $E_g = 0$	Valence and conduction bands are separated by a small energy gap. $E_g = 0.2 \text{ eV to } 3 \text{ eV}$	Valence and conduction bands are separated by a large energy gap. $E_g > 3 \text{ eV}$
2.	They have large number of free electrons in the conduction band available for conduction.	They have a very small number of free electrons in the conduction band available for conduction.	They have no free electrons in the conduction band and so do not conduct electricity.
3.			

NUMERICAL QUESTIONS

63. The $V-I$ characteristic of a silicon diode is as shown in the figure. Calculate the resistance of the diode at

- (i) $I = 15 \text{ mA}$
 (ii) $V = -10 \text{ Volt}$.



Ans :

Comp 2020

- (i) $I = 15 \text{ mA}$

By considering straight line of $V-I$ characteristic curve between 10 mA to 20 mA and assuming passing through the origin, draw horizontal and vertical lines from 10 mA and 20 mA , we have

$$I_1 = 10 \text{ mA}$$

$$V_1 = 0.7 \text{ Volt}$$

$$I_2 = 20 \text{ mA}$$

$$V_2 = 0.8 \text{ Volt}$$

Dynamic resistance in forward biasing can be given as,

$$\begin{aligned} R &= \frac{\Delta V}{\Delta I} = \frac{V_2 - V_1}{I_2 - I_1} \\ &= \frac{(0.8 - 0.7) \text{ V}}{(20 - 10) \text{ mA}} \\ &= \frac{0.1 \text{ V}}{10 \text{ mA}} \\ &= \frac{0.1}{10} \times 10^3 \Omega \\ &= 10 \Omega \end{aligned}$$

- (ii) $V = -10 \text{ Volt}$.

At -10 V . The $V-I$ characteristic graph is a straight line parallel to the voltage axis and not showing any variation. So, the static resistance can be given as,

$$V = -10 \text{ Volt}$$

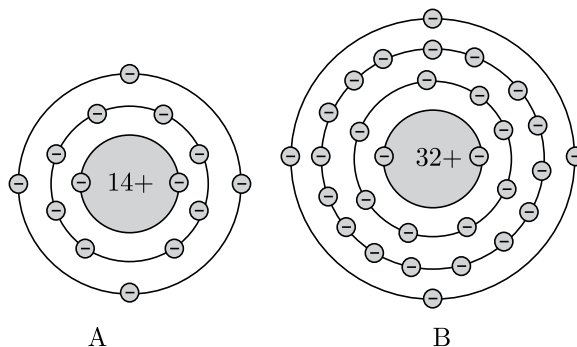
$$\text{and } I = -1 \mu\text{A}$$

Hence, Static resistance,

$$\begin{aligned} R &= \frac{V}{I} = \frac{10 \text{ V}}{1 \mu\text{A}} \\ &= \frac{10}{1 \times 10^{-6}} = 10^7 \Omega \end{aligned}$$

CASE BASED QUESTIONS

64. Anita was thinking that C, Si and Ge have same lattice structure, but C is insulator while Si and Ge intrinsic semiconductors. For its answer, she meet her friend Parul. Parul explained him that the four bonding electrons of C, Si and Ge lie respectively in the second, third and fourth orbit. So, energy required to take out an electron from these atoms known as ionisation energy I_E will be least for Ge, followed by Si and highest for C. Hence number of free electrons for conduction in Ge and Si are significant while negligible small for C.



- (i) Which is better silicon or germanium?
 (ii) If a pure silicon crystal has $5 \times 10^{28} \text{ atoms/m}^3$. It is doped by 1 ppm concentration of pentavalent arsenic.
 If $n_i = 1.5 \times 10^{16} / \text{m}^3$, then calculate the number of electrons and holes.

Ans :

- (i) In present, Silicon is preferred over Germanium for semiconductor. Silicon can be worked at a higher temperature as compared to germanium. The structure of Germanium crystals will be destroyed at higher temperature. Also, Silicon has much smaller leakage current than that of germanium.

- (ii) Given, $n_i = 1.5 \times 10^{16} / \text{m}^3$
 $n = 5 \times 10^{28} \text{ atoms/m}^3$

The number of free electrons n_e in the doped silicon is given by

$$n_e = \frac{1}{10^6} \times n = \frac{5 \times 10^{28}}{10^6}$$

$$= 5 \times 10^{22} / \text{m}^3$$

Now, in a doped semiconductor,

$$n_e n_h = n_i^2$$

$$n_h = \frac{n_i^2}{n_e}$$

$$= \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}}$$

$$= \frac{2.25 \times 10^{32}}{5 \times 10^{22}}$$

$$= 4.5 \times 10^9 / \text{m}^3$$

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